

# The metallicity dependence of the Cepheid *PL*-relation

M. A. T. Groenewegen<sup>1</sup>, M. Romaniello<sup>2</sup>, F. Primas<sup>2</sup>, and M. Mottini<sup>2</sup>

<sup>1</sup> Instituut voor Sterrenkunde, PACS-ICC, Celestijnenlaan 200B, 3001 Leuven, Belgium

<sup>2</sup> ESO, Karl Schwarzschild straÙe 2, 85748 Garching, Germany

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**Abstract.** A sample of 37 Galactic, 10 LMC and 6 SMC cepheids is compiled for which individual metallicity estimates exist and *BVIK* photometry in almost all cases. The Galactic cepheids all have an individual distance estimate available. For the MC objects different sources of photometry are combined to obtain improved periods and mean magnitudes. A multi-parameter Period-Luminosity relation is fitted to the data which also solves for the distance to the LMC and SMC. When all three galaxies are considered, without metallicity effect, a significant quadratic term in  $\log P$  is found, as previously observed and also predicted in some theoretical calculations. For the present sample it is empirically determined that for  $\log P < 1.65$  linear *PL*-relations may be adopted, but this restricts the sample to only 4 LMC and 1 SMC cepheid. Considering the Galactic sample a metallicity effect is found in the zero point in the *VIWK PL*-relation ( $-0.6 \pm 0.4$  or  $-0.8 \pm 0.3$  mag/dex depending on the in- or exclusion of one object), in the sense that metal-rich cepheids are brighter. The small significance is mostly due to the fact that the Galactic sample spans a narrow metallicity range. The error is to a significant part due to the error in the metallicity determinations and not to the error in the fit. Including the 5 MC cepheids broadens the observed metallicity range and a metallicity effect of about  $-0.27 \pm 0.08$  mag/dex in the zero point is found in *VIWK*, in agreement with some previous empirical estimates, but now derived using direct metallicity determinations for the cepheids themselves.

**Key words.** stars: distances – stars: variables: Cepheids

## 1. Introduction

The importance of the cepheid Period-Luminosity relation (*PL*-relation) has long been recognised and is the basis of the determination of the Hubble constant by Mould et al. (2000) and Freedman et al. (2001). The most important uncertainties in this derivation are the zero point of the *PL*-relation based on an adopted distance modulus (DM) to the LMC, and the adopted metallicity correction. Mould et al. (2000) and Freedman et al. (2001) adopt corrections in the Wesenheit-index ( $W = V - 2.55(V - I)$ ) of  $-0.24 \pm 0.16$  and  $-0.2 \pm 0.2$  mag/dex, respectively. This metallicity effect is important as the galaxies surveyed by the HST Key Project span a factor of 30 in oxygen abundance (Ferrarese et al. 2000).

Theoretical pulsation models lead to different results: linear models (e.g. Sandage et al. 1999; Alibert et al. 1999; Baraffe & Alibert 2001) predict a moderate dependence, while non-linear convective models (Bono et al. 1999; Caputo et al. 2000) predict a larger dependence, and in the sense that metal-rich cepheids are fainter (see Table 7 in Groenewegen & Oudmaijer 2000, for values at typical periods). Recently, Fiorentino et al. (2002) suggested that there is also a dependence on the helium abundance.

On the observational side the results seem to indicate consistently that metal-rich cepheids are brighter, and various estimates have been given in the literature,  $-0.88 \pm 0.16$  mag/dex (*BRI* bands, Gould 1994),  $-0.44_{-0.2}^{+0.1}$  mag/dex (*VR* bands, Sasselov et al. 1997),  $-0.24 \pm 0.16$  mag/dex (*VI* bands, Kochanek 1997),  $-0.14 \pm 0.14$  mag/dex (*VI* bands, Kennicutt et al. 1998),  $-0.25 \pm 0.05$  mag/dex (*VI* bands, Kennicutt et al. 2003), and  $-0.21 \pm 0.19$  in *V*,  $-0.29 \pm 0.19$  in *W*,  $-0.23 \pm 0.19$  in *I*,  $-0.21 \pm 0.19$  mag/dex in *K* (Storm et al. 2004). The potential drawback or caveat is that no individual abundance determinations of individual cepheids are being used in these studies but rather abundances of nearby HII regions, or even a mean abundance of the entire galaxy.

The present paper aims at investigating the metallicity dependence from the observational side, but using cepheid individual metallicity determinations. This has become possible because of advances in abundance determinations for the Galaxy (Fry & Carney 1997; Andrievsky et al. 2002a,b,c; Luck et al. 2003) and LMC (Luck et al. 1998), as well as recent advances in individual distance estimates for Galactic cepheids based on surface-brightness relations (Fouqu e et al. 2003), a Bayesian statistical analysis to solve the surface-brightness equations (Barnes et al. 2003), direct measurement of distances based on combining radial velocity data with interferometric observations (Kervella et al. 2003), and distance determinations of cepheids in open clusters (Tammann et al. 2003).

Send offprint requests to: M. Groenewegen,  
e-mail: groen@ster.kuleuven.ac.be

**Table 1.** Comparison of distances (in parsec) for Galactic cepheids with independent determinations.

Name	F03	K03	B03	T03	B02	N02	L02	Adopted (pc)
$\delta$ Cep	261 $\pm$ 5				271 $\pm$ 11	272 $\pm$ 6		271 $\pm$ 5
$\zeta$ Gem		360 $\pm$ 25					362 $\pm$ 38	360 $\pm$ 20
$\eta$ Aql	250 $\pm$ 6	261 $\pm$ 14					320 $\pm$ 32	267 $\pm$ 12
$l$ Car	628 $\pm$ 3	524 $\pm$ 49						620 $\pm$ 30
X Cyg	1214 $\pm$ 9		1101 $\pm$ 28					1130 $\pm$ 25
T Mon	1430 $\pm$ 35		1306 $\pm$ 41	1690 $\pm$ 120				1350 $\pm$ 40
BF Oph	715 $\pm$ 11		713 $\pm$ 63					714 $\pm$ 40
RS Pup	2111 $\pm$ 73		1706 $\pm$ 228	1800 $\pm$ 130				1810 $\pm$ 110
U Sgr	595 $\pm$ 6		672 $\pm$ 49	643 $\pm$ 55				620 $\pm$ 25
WZ Sgr	1808 $\pm$ 40		2513 $\pm$ 196	1790 $\pm$ 125				1960 $\pm$ 90
BB Sgr	801 $\pm$ 10		914 $\pm$ 76	664 $\pm$ 47				760 $\pm$ 30
RY Sco	1268 $\pm$ 20		960 $\pm$ 65					1050 $\pm$ 55
SZ Tau			560 $\pm$ 85	555 $\pm$ 40				556 $\pm$ 35
SV Vul			1846 $\pm$ 69	2320 $\pm$ 170				1910 $\pm$ 70
CV Mon	1576 $\pm$ 25			1755 $\pm$ 300				1600 $\pm$ 115
V Cen	684 $\pm$ 20			682 $\pm$ 49				684 $\pm$ 45
S Nor	959 $\pm$ 14			933 $\pm$ 67				945 $\pm$ 50
V340 Nor	1690 $\pm$ 155			1713 $\pm$ 125				1700 $\pm$ 100
VY Car	2000 $\pm$ 20			2118 $\pm$ 150				2040 $\pm$ 80
RZ Vel	1600 $\pm$ 20			1730 $\pm$ 120				1650 $\pm$ 80
SW Vel	2510 $\pm$ 30			2610 $\pm$ 180				2550 $\pm$ 115
U Car	1560 $\pm$ 25			1960 $\pm$ 140				1740 $\pm$ 90

The paper is organised as follows. In Sect. 2 the datasets on individual distance and metallicity determinations for Galactic cepheids are presented and compared. The Magellanic Cloud sample is also described. In Sect. 3 the model and the results are presented. The conclusions and future prospects are outlined in Sect. 4.

## 2. The data

### 2.1. Galactic cepheids

Distances for Galactic cepheids have been taken from Benedikt et al. (2002, B02), Nordgren et al. (2002, N02), Lane et al. (2002, L02), Fouqué et al. (2003, F03), Barnes et al. (2003, B03), Kervella et al. (2003, K03), and Tammann et al. (2003, T03). Some of them have independent determinations, and these are compared in Table 1. T03 (largely based on Feast 1999) does not quote error bars and here a uniform error bar of 0.15 in DM is assigned as recommended by Feast (1999), that takes into account the error in the Pleiades distance, as well as the uncertainty due to the reddening in the main-sequence fitting method which is the method used for these distances. The agreement between the different determinations is (surprisingly) good in many cases. The finally adopted distances (last column in Table 1) are weighted means, but with the error bar for the F03 determinations multiplied by a factor of 5. The error bars quoted by F03 appear very small when compared to those

in B03 which are also based on the surface-brightness technique, and sometimes appear even smaller than the rms error in the fundamental surface-brightness relations (e.g. Nordgren et al. 2002). The factor of 5 roughly brings the error bars in F03 to the level of B03. When the individual determinations are very different the error bar is slightly increased (WZ Sgr,  $l$  Car).

The absolute magnitudes in  $BVIK$  have been taken from the following sources in order of preference. F03, where appropriate, changed to allow for the finally adopted distance in Table 1, or for the increased error bar in the distance. T03 for the  $BVI$  absolute magnitudes (where appropriate, changed to allow for the finally adopted distance), with observed  $K$  magnitudes from Groenewegen (1999), dereddened using the  $E(B-V)$  listed by T03, and using  $A_K = 0.3 E(B-V)$ . For the stars only listed in B03, the observed  $BVIJK$  magnitudes come from Groenewegen (1999) and from the electronic database of Fernie et al. (1995). Recent  $K$ -band photometry for CF Cas and DL Cas was taken from Hoyle et al. (2003). Dereddening was done using the values in the electronic database using  $A_V = 3.3 E(B-V)$ , and  $A_B = 1.33 A_V$ ,  $A_I = 0.60 A_V$ ,  $A_K = 0.091 A_V$ .

Abundance determinations have been considered from the recent papers by Fry & Carney (1997), Andrievsky et al. (2002a,b,c) and Luck et al. (2003)<sup>1</sup>. A comparison of stars in

<sup>1</sup> For Andrievsky et al. (2002a,c) and Luck et al. (2003) the errors in the abundances are listed in the electronically available tables.

**Table 2.** Comparison of  $[\text{Fe}/\text{H}]$  values for Galactic cepheids with independent determinations.

Name	Fry & Carney (1997)	Andrievsky et al. (2002a)	Luck et al. (2003)	Adopted
V340 Nor	$-0.18 \pm 0.03$	$-0.00 \pm 0.10$		$-0.09 \pm 0.08$
CF Cas	$-0.20 \pm 0.02$	$-0.01 \pm 0.09$		$-0.10 \pm 0.07$
DL Cas	$+0.05 \pm 0.01$	$-0.01 \pm 0.14$		$+0.02 \pm 0.10$
$\delta$ Cep	$-0.01 \pm 0.06$	$+0.06 \pm 0.07$		$+0.03 \pm 0.06$
CV Mon	$-0.05 \pm 0.06$	$-0.03 \pm 0.12$		$-0.04 \pm 0.06$
V Cen	$-0.14 \pm 0.02$	$+0.04 \pm 0.09$		$-0.05 \pm 0.06$
U Sgr	$+0.01 \pm 0.03$	$+0.04 \pm 0.08$		$+0.03 \pm 0.06$
$\eta$ Aql	$+0.07 \pm 0.04$	$+0.05 \pm 0.06$		$+0.06 \pm 0.06$
S Nor	$-0.03 \pm 0.02$	$+0.05 \pm 0.08$		$+0.01 \pm 0.07$
X Cyg	$+0.12 \pm 0.03$	$+0.12 \pm 0.05$		$+0.12 \pm 0.05$
WZ Sgr	$-0.15 \pm 0.03$	$+0.17 \pm 0.08$		$+0.00 \pm 0.15$
SW Vel	$-0.08 \pm 0.03$	$+0.01 \pm 0.08$	$-0.07 \pm 0.09$	$-0.05 \pm 0.06$
T Mon	$+0.09 \pm 0.02$	$+0.13 \pm 0.06$	$+0.13 \pm 0.12$	$+0.11 \pm 0.06$
$\zeta$ Gem	$+0.00 \pm 0.04$	$+0.04 \pm 0.07$		$+0.02 \pm 0.05$
RX Aur	$-0.13 \pm 0.04$	$-0.07 \pm 0.08$		$-0.10 \pm 0.07$
SZ Tau	$-0.01 \pm 0.03$	$+0.08 \pm 0.07$		$+0.04 \pm 0.06$
T Vul	$-0.10 \pm 0.04$	$+0.01 \pm 0.05$		$-0.05 \pm 0.06$
SV Vul	$+0.06 \pm 0.04$	$+0.03 \pm 0.09$		$+0.05 \pm 0.06$

common is given in Table 2, which also lists the adopted value in this study. The disagreement between the studies is at times worrisome, and the impact will be discussed in Sect. 4. It also appears from the work of Andrievsky et al. and Luck et al., as well as from the intercomparison made in Table 2 that the errors listed in Fry & Carney are unrealistically small, and for stars that only appear in that paper a minimum error in  $[\text{Fe}/\text{H}]$  of 0.06 dex has been adopted<sup>2</sup>.

The finally adopted sample of stars with recent distance and metallicity determinations is listed in Table 3, and consists of 37 objects. Listed are the name, observed period, adopted distance modulus with error, corresponding absolute magnitudes in  $BVIK$ , adopted reddening and metallicity with error, and finally a code for fundamental mode ( $\text{FU} = 0$ ) and first overtone ( $\text{FO} = 1$ ) pulsators.

## 2.2. Magellanic Cloud cepheids

The metallicity determination for 10 LMC and 6 SMC cepheids come from Luck et al. (1998) (The values as derived using Kurucz ATLAS9 model atmospheres are being used). An error bar of 0.10 dex had been adopted.

Optical photometry was taken from Martin & Warren (1979), Freedman et al. (1985), Moffett et al. (1998), Barnes et al. (1999), Caldwell et al. (2001), and Sebo et al. (2002), and infrared photometry from Welch et al. (1987) and Laney & Stobie (1986b). Corresponding  $BVIK$  datasets were joined (with offsets sometimes applied), and analysed using the

program “Period98” (Sperl 1998) to find (improved) periods and mean magnitudes (and Fourier coefficients). The results are listed in Table 4. The error quoted for the mean magnitudes is the rms in the fit, but has been used as an error estimate.

Reddening values when available are taken from Caldwell & Laney (1994), Laney & Stobie (1986a, 1994) and Caldwell & Coulson (1985) while for HV 900 and HV 909 an average of 0.06 has been adopted. The effect of increased reddening will be discussed later.

The cepheids are not located all at the same distance because of depth and projection effects. The magnitudes to be added to the observed values to correct for this are listed in Table 4 and have been determined using the position angle and inclination from Van der Marel & Cioni (2001) for the LMC and Caldwell & Laney (1991) for the SMC. The effect is small but should be considered when aiming for the highest accuracies.

## 3. The model and results

The following Period-Luminosity-Colour-Metallicity ( $PLCZ$ )-relation is fitted to the data, where both the zero point and slope are allowed to vary quadratically with metallicity:

$$\begin{aligned}
 M = & \alpha_1 + \alpha_2 \log [\text{Fe}/\text{H}] + \alpha_3 (\log [\text{Fe}/\text{H}])^2 \\
 & + \alpha_4 \left( 1 + \frac{\alpha_5}{\alpha_4} \log [\text{Fe}/\text{H}] + \frac{\alpha_6}{\alpha_4} (\log [\text{Fe}/\text{H}])^2 \right) \log P_0 \\
 & + \alpha_7 \left( 1 + \frac{\alpha_8}{\alpha_7} \log [\text{Fe}/\text{H}] + \frac{\alpha_9}{\alpha_7} (\log [\text{Fe}/\text{H}])^2 \right) (\log P_0)^2 \\
 & + \beta(m_1 - m_2) + \Delta_{\text{LMC}} x_{\text{LMC}} + \Delta_{\text{SMC}} x_{\text{SMC}},
 \end{aligned}$$

<sup>2</sup> This error is also more realistic considering the error in the derived metallicity due to uncertainties in the effective temperature and gravity, as Fry & Carney themselves discuss in their Table 7.

**Table 3.** Basic data of Galactic cepheid sample with individual distances and metallicity determinations.

Name	$\log P$	DM	$M_B$	$M_V$	$M_I$	$M_K$	$E(B - V)$	[Fe/H]	FU/FO
T Vel	0.667	$9.802 \pm 0.285$	-2.050	-2.690	-3.370	-4.260	0.281	$-0.02 \pm 0.08$	0
$\delta$ Cep	0.730	$7.165 \pm 0.193$	-2.950	-3.510	-4.140	-4.890	0.092	$0.03 \pm 0.06$	0
Z Lac	1.037	$11.637 \pm 0.262$	-3.860	-4.560	-5.290	-6.090	0.404	$0.01 \pm 0.08$	0
BN Pup	1.136	$12.950 \pm 0.239$	-3.760	-4.510	-5.270	-6.180	0.438	$0.01 \pm 0.06$	0
RZ Vel	1.310	$11.020 \pm 0.141$	-4.250	-5.040	-5.820	-6.820	0.335	$-0.07 \pm 0.06$	0
VZ Pup	1.365	$13.083 \pm 0.271$	-4.320	-5.010	-5.720	-6.560	0.471	$-0.16 \pm 0.08$	0
RY Vel	1.449	$12.019 \pm 0.155$	-4.690	-5.500	-6.300	-7.280	0.562	$-0.03 \pm 0.09$	0
AQ Pup	1.479	$12.522 \pm 0.216$	-4.650	-5.510	-6.410	-7.400	0.512	$-0.14 \pm 0.06$	0
X Cyg	1.214	$10.265 \pm 0.048$	-3.960	-4.830	-5.610	-6.530	0.288	$0.12 \pm 0.05$	0
SW Vel	1.370	$12.032 \pm 0.096$	-4.240	-5.050	-5.880	-6.920	0.349	$-0.05 \pm 0.09$	0
V340 Nor	1.053	$11.152 \pm 0.124$	-2.990	-3.830	-4.690	-5.680	0.315	$-0.09 \pm 0.08$	0
S Nor	0.989	$9.877 \pm 0.112$	-3.310	-4.070	-4.830	-5.790	0.189	$0.01 \pm 0.07$	0
V Cen	0.740	$9.175 \pm 0.138$	-2.710	-3.300	-3.960	-4.770	0.289	$-0.05 \pm 0.06$	0
CV Mon	0.731	$11.021 \pm 0.151$	-2.490	-3.070	-3.830	-4.680	0.714	$-0.04 \pm 0.06$	0
$\eta$ Aql	0.856	$7.132 \pm 0.095$	-3.080	-3.730	-4.420	-5.310	0.149	$0.06 \pm 0.06$	0
T Mon	1.432	$10.652 \pm 0.063$	-4.240	-5.210	-6.090	-7.220	0.209	$0.11 \pm 0.06$	0
RS Pup	1.617	$11.300 \pm 0.228$	-4.790	-5.760	-6.700	-7.820	0.446	$0.17 \pm 0.10$	0
U Sgr	0.829	$8.961 \pm 0.086$	-2.910	-3.600	-4.340	-5.150	0.403	$0.03 \pm 0.06$	0
WZ Sgr	1.339	$11.461 \pm 0.129$	-4.040	-4.970	-5.890	-7.050	0.467	$0.00 \pm 0.15$	0
EV Sct	0.490	$10.920 \pm 0.150$	-2.150	-2.683	-3.345	-4.077	0.621	$-0.37 \pm 0.06$	1
V1726 Cyg	0.651	$12.690 \pm 0.150$	-2.716	-3.338	-3.986		0.548	$-0.02 \pm 0.14$	0
CF Cas	0.688	$12.690 \pm 0.150$	-2.541	-3.209	-3.901	-4.859	0.531	$-0.10 \pm 0.07$	0
QZ Nor	0.578	$11.170 \pm 0.150$	-2.574	-3.181	-3.813	-4.636	0.286	$0.06 \pm 0.06$	1
V367 Sct	0.799	$11.320 \pm 0.150$	-2.921	-3.543	-4.323	-5.020	1.208	$-0.01 \pm 0.11$	0
DL Cas	0.903	$11.220 \pm 0.150$	-3.071	-3.743	-4.435	-5.444	0.479	$0.02 \pm 0.10$	0
TW Nor	1.033	$11.470 \pm 0.150$	-2.906	-3.697	-4.498	-5.443	1.214	$0.03 \pm 0.06$	0
KQ Sco	1.458	$12.360 \pm 0.150$	-4.241	-5.339	-6.401	-7.666	0.839	$0.16 \pm 0.05$	0
S Vul	1.838	$13.240 \pm 0.150$	-5.588	-6.740	-7.803	-8.862	0.737	$-0.02 \pm 0.05$	0
W Sgr	0.881	$7.876 \pm 0.123$	-2.945	-3.571	-4.230		0.110	$-0.01 \pm 0.07$	0
$\beta$ Dor	0.993	$7.519 \pm 0.132$	-3.157	-3.920	-4.676	-5.572	0.040	$-0.01 \pm 0.12$	0
$\zeta$ Gem	1.007	$7.782 \pm 0.117$	-3.109	-3.897	-4.717		0.010	$0.02 \pm 0.05$	0
Y Oph	1.234	$9.024 \pm 0.164$	-4.331	-5.000	-5.734	-6.537	0.650	$0.05 \pm 0.07$	0
RX Aur	1.065	$11.101 \pm 0.195$	-3.622	-4.337	-5.009		0.270	$-0.10 \pm 0.07$	0
SZ Tau	0.498	$8.725 \pm 0.133$	-2.623	-3.151	-3.761	-4.501	0.290	$0.04 \pm 0.06$	1
EU Tau	0.323	$10.269 \pm 0.159$	-2.258	-2.737	-3.310		0.170	$-0.06 \pm 0.06$	1
T Vul	0.647	$8.920 \pm 0.142$	-2.794	-3.364	-3.985	-4.740	0.060	$-0.05 \pm 0.06$	0
SV Vul	1.653	$11.405 \pm 0.078$	-5.245	-6.066	-6.821	-7.655	0.570	$0.05 \pm 0.06$	0

where  $M$  is the absolute magnitude in a given photometric band,  $P_0$  the fundamental period (in days), [Fe/H] the metallicity on a logarithmic scale relative to Solar,  $(m_1 - m_2)$  represents an arbitrary colour term,  $x_{\text{LMC}}$  and  $x_{\text{SMC}}$  are 1 for the LMC, respectively SMC, cepheids and 0 otherwise. The absolute magnitudes for the MC cepheids are calculated assuming default DM of 18.50 and 18.90, respectively (which represent “rounded” values that are within 0.1 mag of recent determinations, see e.g. Feast 2003; Walker 2003; Storm et al. 2004,

and for the LMC corresponds to the value adopted by Mould et al. and Freedman et al. in the HST key project), and  $\Delta_{\text{LMC}}$  and  $\Delta_{\text{SMC}}$  represent corrections to that.

The observed periods of the first overtone pulsators are “fundamentalised” using  $P_0 = P_1 / (0.716 - 0.027 \log P_1)$  (Feast & Catchpole 1997).

The results of the fitting are listed in Table 5. In the fitting the error in the absolute magnitudes includes the error in the distance modulus for the Galactic cepheids, a 0.01 error

**Table 4.** Basic data of LMC/SMC cepheid sample with individual metallicity determinations.

Name	$P$	[Fe/H]	$E(B - V)$	Depth	$m_V$	$m_B$	$m_I$	$m_K$
HV 879	36.824	$-0.56 \pm 0.10$	0.060	-0.030	$13.401 \pm 0.036$	$13.516 \pm 0.060$	$12.330 \pm 0.022$	$11.103 \pm 0.020$
HV 883	133.432	$-0.45 \pm 0.10$	0.104	-0.005	$12.161 \pm 0.057$	$13.363 \pm 0.121$	$11.014 \pm 0.029$	$9.720 \pm 0.027$
HV 900	47.479	$-0.38 \pm 0.10$	0.06	-0.030	$12.832 \pm 0.012$	$13.853 \pm 0.018$	$11.848 \pm 0.018$	$10.706 \pm 0.054$
HV 909	37.591	$-0.28 \pm 0.10$	0.06	-0.036	$12.805 \pm 0.023$	$13.652 \pm 0.031$	$11.939 \pm 0.021$	$10.880 \pm 0.011$
HV 2257	39.390	$-0.34 \pm 0.10$	0.061	-0.040	$13.105 \pm 0.030$	$14.158 \pm 0.025$	$12.084 \pm 0.022$	$10.950 \pm 0.036$
HV 2338	42.193	$-0.44 \pm 0.10$	0.040	-0.057	$12.823 \pm 0.043$	$13.824 \pm 0.061$	$11.851 \pm 0.023$	$10.730 \pm 0.030$
HV 2447	118.338	$-0.40 \pm 0.10$	0.043	+0.017	$12.021 \pm 0.037$	$13.328 \pm 0.058$	$10.880 \pm 0.031$	$9.614 \pm 0.069$
HV 2827	78.820	$-0.24 \pm 0.10$	0.080	+0.094	$12.324 \pm 0.023$	$13.538 \pm 0.035$	$11.196 \pm 0.019$	$9.850 \pm 0.033$
HV 2883	108.923	$-0.16 \pm 0.10$	0.015	+0.067	$12.511 \pm 0.037$	$13.780 \pm 0.061$	$11.384 \pm 0.030$	$10.111 \pm 0.028$
HV 5497	99.587	$-0.48 \pm 0.10$	0.095	+0.029	$11.940 \pm 0.036$	$13.169 \pm 0.043$	$10.817 \pm 0.039$	$9.475 \pm 0.023$
HV 821	127.253	$-0.84 \pm 0.10$	0.074	-0.073	$11.997 \pm 0.037$	$13.025 \pm 0.054$	$10.936 \pm 0.036$	$9.747 \pm 0.043$
HV 824	65.861	$-0.94 \pm 0.10$	0.030	-0.037	$12.401 \pm 0.026$	$13.288 \pm 0.028$	$11.486 \pm 0.024$	$10.370 \pm 0.020$
HV 829	84.800	$-0.61 \pm 0.10$	0.030	-0.018	$11.957 \pm 0.021$	$12.796 \pm 0.018$	$11.053 \pm 0.021$	$9.952 \pm 0.036$
HV 834	73.566	$-0.59 \pm 0.10$	0.020	+0.006	$12.254 \pm 0.044$	$13.102 \pm 0.068$	$11.365 \pm 0.039$	$10.230 \pm 0.034$
HV 837	42.721	$-0.91 \pm 0.10$	0.042	+0.022	$13.288 \pm 0.021$	$14.202 \pm 0.023$	$12.322 \pm 0.018$	$11.178 \pm 0.023$
HV 11157	69.491	$-0.77 \pm 0.10$	0.080	+0.005	$12.939 \pm 0.010$	$14.021 \pm 0.010$	$11.862 \pm 0.009$	$10.599 \pm 0.025$

in  $E(B - V)$  for all cepheids, the error in the mean magnitudes for the MC cepheids and a 0.01 error in the depth effect for the MC cepheids. The (non-negligible) error in the fit when including a metallicity effect, due to the error therein, will be discussed in detail below.

First, the classical (linear)  $PL$ -relations in  $BVIWK$  are determined for stars with  $\log P > 0.4$  for the Galaxy only, and the fit for the  $V$ -band is shown in Fig. 1 as an example. The slopes in  $BVIWK$ , respectively,  $BVI$  agree at the  $2\sigma$  or better level with those in F03 and T03 (his mean relation). At  $\log P = 1$  the agreement with F03 for the absolute magnitudes is excellent in all filters (typically 0.03 mag or about  $1\sigma$  or better), with T03 the difference is larger (about 0.1 mag fainter, which is also at the  $1\sigma$  level). The reason for the latter effect is probably that in 10 out of 15 cepheids in Table 1 the adopted distance is in the end smaller than the one taken by T03.

Second, the  $PL$ -relations in  $BVIWK$  are determined for all three galaxies combined and allowing for the quadratic term in  $\log P$  but not yet for any metallicity dependence. The result in the  $V$ -band is shown in Fig. 2. This fit and the results in the tables show that there is a significant quadratic term in  $\log P$  when the long-period MC cepheids are included in the fit.

One can also observe that the best fitting distance to, in particular, the SMC seems to have become shorter than the default value of 18.90 (there is some scatter depending on the band, and some dependence on the reddening) and that the error bar in the linear term in  $\log P$  has increased significantly.

Deviations from a linear  $PL$ -relation have long been observed, e.g. Sandage & Tammann (1968) and Martin et al. (1979) at  $\log P \approx 1.7-1.8$ . The presence of a quadratic term in the  $PL$ -relation was predicted by the model calculations of Bono et al. (1999) in the  $V$ -band (see also Fiorentino et al. 2002), and they proposed two linear relations with the break

at  $\log P = 1.4$ . Our results show that the linear relation may be valid to slightly longer periods than  $\log P = 1.4$ .

The upper cut-off in  $\log P$  is determined empirically in such a way that the quadratic term becomes statistically insignificant (in all colours  $BVIWK$ ), and a linear fit in  $\log P$  is therefore justified. This cut-off is  $\log P = 1.65$ , in good agreement with the quoted earlier observational results.

In view of this, two samples will be considered, both with cepheids in the period range  $0.4 < \log P_0 < 1.65$ , viz. a purely Galactic sample, and a second sample that includes all three galaxies but only 1 SMC and 4 LMC stars. For this reason  $\Delta_{\text{LMC}}$  and  $\Delta_{\text{SMC}}$  are no longer fitted in what follows.

Next the effect of metallicity is considered for the Galactic sample, as a possible change of the zero point. As is clear from Fig. 1, any dependence would be heavily influenced by the outlier EV Sct which has  $[\text{Fe}/\text{H}] = -0.37$ . Therefore Table 5 includes both cases, where this star is either kept or is excluded from the sample when fitting the  $PLZ$ -relation in the Galaxy. The coefficient  $a_2$  is strongly negative ( $-0.2$  to  $-0.8$  mag/dex) but only significant at the  $2-3\sigma$  level considering the formal error in the fit. Figure 3 shows the result for the  $V$ -band, excluding EV Sct.

As noted earlier, there are some worrisome discrepancies between independent metallicity determinations for the same object (see Table 2). The fit parameters quoted are based on assuming that there is no error in the independent variables period and metallicity. For the period this is certainly the case but not for the metallicity. Therefore a set of 1000 simulations was run where the metallicity of each star was set to its observed value plus its attributed error multiplied by a number drawn from a Gaussian distribution. The fitting was then repeated and the 23th and 977th ordered values, which correspond to  $\pm 2\sigma$ , of the fit parameters determined. The analysis of the  $VWK$  bands shows that the  $\sigma$  due to the error in the

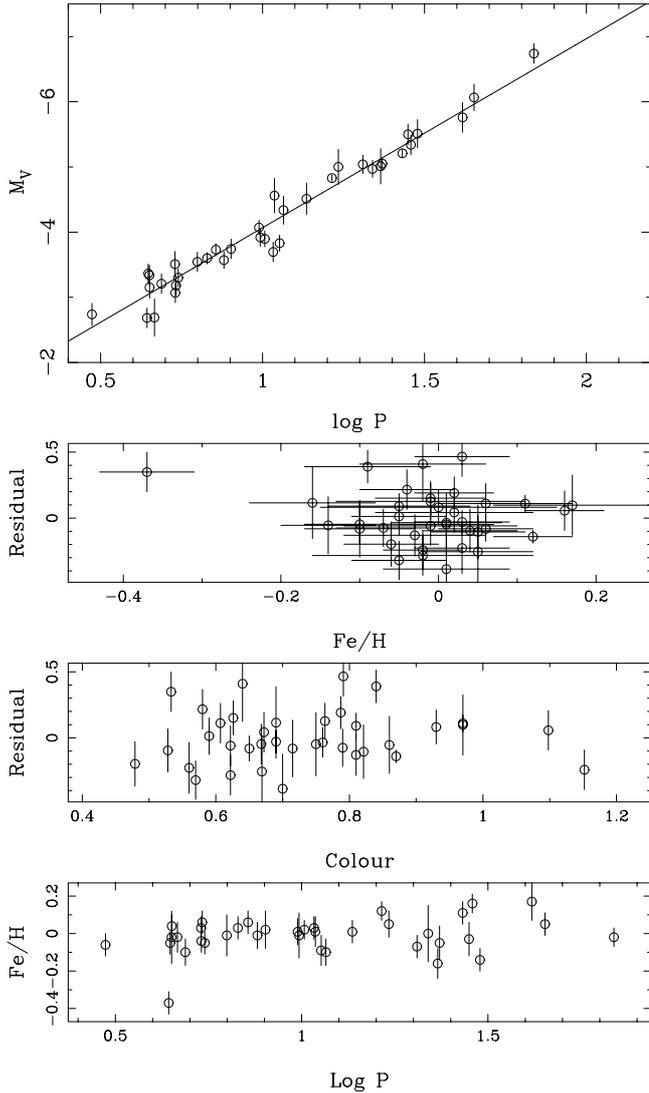
**Table 5.** Results of the fitting. Coefficients refer to Eq. (1). The last column gives the rms in the fit.

	$\alpha_1$	$\alpha_4$	$\alpha_7$	$\alpha_2$	$\Delta_{\text{LMC}}$	$\Delta_{\text{SMC}}$	$\sigma$
<i>PL</i> -relation Galaxy							
<i>B</i>	$-0.881 \pm 0.080$	$-2.444 \pm 0.070$	$0.000 \pm 0.000$	$0.000 \pm 0.000$	$0.000 \pm 0.000$	$0.000 \pm 0.000$	0.234
<i>V</i>	$-1.167 \pm 0.078$	$-2.901 \pm 0.069$	$0.000 \pm 0.000$	$0.000 \pm 0.000$	$0.000 \pm 0.000$	$0.000 \pm 0.000$	0.200
<i>I</i>	$-1.644 \pm 0.076$	$-3.174 \pm 0.067$	$0.000 \pm 0.000$	$0.000 \pm 0.000$	$0.000 \pm 0.000$	$0.000 \pm 0.000$	0.188
<i>W</i>	$-2.346 \pm 0.082$	$-3.637 \pm 0.072$	$0.000 \pm 0.000$	$0.000 \pm 0.000$	$0.000 \pm 0.000$	$0.000 \pm 0.000$	0.216
<i>K</i>	$-2.292 \pm 0.080$	$-3.424 \pm 0.067$	$0.000 \pm 0.000$	$0.000 \pm 0.000$	$0.000 \pm 0.000$	$0.000 \pm 0.000$	0.197
<i>PL</i> -relation Galaxy + LMC + SMC; all periods							
<i>B</i>	$-0.425 \pm 0.142$	$-3.427 \pm 0.210$	$0.481 \pm 0.073$	$0.000 \pm 0.000$	$-0.137 \pm 0.047$	$-0.530 \pm 0.052$	0.289
<i>V</i>	$-0.706 \pm 0.134$	$-3.857 \pm 0.190$	$0.455 \pm 0.061$	$0.000 \pm 0.000$	$0.054 \pm 0.042$	$-0.251 \pm 0.045$	0.235
<i>I</i>	$-1.091 \pm 0.127$	$-4.295 \pm 0.176$	$0.522 \pm 0.053$	$0.000 \pm 0.000$	$0.053 \pm 0.038$	$-0.173 \pm 0.040$	0.216
<i>W</i>	$-2.020 \pm 0.164$	$-4.341 \pm 0.263$	$0.345 \pm 0.103$	$0.000 \pm 0.000$	$0.132 \pm 0.065$	$0.015 \pm 0.074$	0.216
<i>K</i>	$-1.365 \pm 0.138$	$-5.096 \pm 0.184$	$0.700 \pm 0.054$	$0.000 \pm 0.000$	$0.051 \pm 0.034$	$-0.127 \pm 0.037$	0.205
<i>PLZ</i> -relation Galaxy; $\log P < 1.65$ ; EV Sct excluded							
<i>B</i>	$-1.005 \pm 0.091$	$-2.323 \pm 0.086$	$0.000 \pm 0.000$	$-0.212 \pm 0.310$	$0.000 \pm 0.000$	$0.000 \pm 0.000$	0.234
<i>V</i>	$-1.330 \pm 0.090$	$-2.731 \pm 0.085$	$0.000 \pm 0.000$	$-0.634 \pm 0.309$	$0.000 \pm 0.000$	$0.000 \pm 0.000$	0.198
<i>I</i>	$-1.799 \pm 0.089$	$-3.013 \pm 0.084$	$0.000 \pm 0.000$	$-0.638 \pm 0.308$	$0.000 \pm 0.000$	$0.000 \pm 0.000$	0.178
<i>W</i>	$-2.536 \pm 0.094$	$-3.441 \pm 0.088$	$0.000 \pm 0.000$	$-0.653 \pm 0.313$	$0.000 \pm 0.000$	$0.000 \pm 0.000$	0.188
<i>K</i>	$-2.301 \pm 0.097$	$-3.420 \pm 0.089$	$0.000 \pm 0.000$	$-0.574 \pm 0.315$	$0.000 \pm 0.000$	$0.000 \pm 0.000$	0.168
<i>PLZ</i> -relation Galaxy; $\log P < 1.65$ ; EV Sct included							
<i>B</i>	$-1.002 \pm 0.091$	$-2.314 \pm 0.086$	$0.000 \pm 0.000$	$-0.467 \pm 0.254$	$0.000 \pm 0.000$	$0.000 \pm 0.000$	0.236
<i>V</i>	$-1.328 \pm 0.090$	$-2.725 \pm 0.085$	$0.000 \pm 0.000$	$-0.792 \pm 0.253$	$0.000 \pm 0.000$	$0.000 \pm 0.000$	0.199
<i>I</i>	$-1.798 \pm 0.089$	$-3.007 \pm 0.084$	$0.000 \pm 0.000$	$-0.784 \pm 0.252$	$0.000 \pm 0.000$	$0.000 \pm 0.000$	0.178
<i>W</i>	$-2.534 \pm 0.094$	$-3.437 \pm 0.087$	$0.000 \pm 0.000$	$-0.782 \pm 0.255$	$0.000 \pm 0.000$	$0.000 \pm 0.000$	0.186
<i>K</i>	$-2.293 \pm 0.097$	$-3.417 \pm 0.089$	$0.000 \pm 0.000$	$-0.776 \pm 0.255$	$0.000 \pm 0.000$	$0.000 \pm 0.000$	0.167
<i>PLZ</i> -relation Galaxy + LMC + SMC; $\log P < 1.65$ ; no fit to MC distances							
<i>B</i>	$-0.807 \pm 0.073$	$-2.534 \pm 0.061$	$0.000 \pm 0.000$	$+0.053 \pm 0.064$	$0.000 \pm 0.000$	$0.000 \pm 0.000$	0.239
<i>V</i>	$-1.123 \pm 0.071$	$-2.948 \pm 0.058$	$0.000 \pm 0.000$	$-0.274 \pm 0.057$	$0.000 \pm 0.000$	$0.000 \pm 0.000$	0.196
<i>I</i>	$-1.603 \pm 0.066$	$-3.215 \pm 0.052$	$0.000 \pm 0.000$	$-0.274 \pm 0.047$	$0.000 \pm 0.000$	$0.000 \pm 0.000$	0.174
<i>W</i>	$-2.422 \pm 0.079$	$-3.558 \pm 0.067$	$0.000 \pm 0.000$	$-0.237 \pm 0.088$	$0.000 \pm 0.000$	$0.000 \pm 0.000$	0.182
<i>K</i>	$-2.193 \pm 0.070$	$-3.519 \pm 0.051$	$0.000 \pm 0.000$	$-0.299 \pm 0.041$	$0.000 \pm 0.000$	$0.000 \pm 0.000$	0.164
<i>PLZ</i> -relation Galaxy + LMC + SMC; $\log P < 1.65$ ; no fit to MC distances; increased reddening							
<i>B</i>	$-0.552 \pm 0.073$	$-2.812 \pm 0.061$	$0.000 \pm 0.000$	$+0.266 \pm 0.064$	$0.000 \pm 0.000$	$0.000 \pm 0.000$	0.273
<i>V</i>	$-0.885 \pm 0.070$	$-3.209 \pm 0.058$	$0.000 \pm 0.000$	$-0.164 \pm 0.057$	$0.000 \pm 0.000$	$0.000 \pm 0.000$	0.226
<i>I</i>	$-1.406 \pm 0.066$	$-3.432 \pm 0.052$	$0.000 \pm 0.000$	$-0.267 \pm 0.047$	$0.000 \pm 0.000$	$0.000 \pm 0.000$	0.196
<i>W</i>	$-2.424 \pm 0.079$	$-3.556 \pm 0.067$	$0.000 \pm 0.000$	$-0.242 \pm 0.088$	$0.000 \pm 0.000$	$0.000 \pm 0.000$	0.181
<i>K</i>	$-2.155 \pm 0.070$	$-3.560 \pm 0.051$	$0.000 \pm 0.000$	$-0.304 \pm 0.041$	$0.000 \pm 0.000$	$0.000 \pm 0.000$	0.165

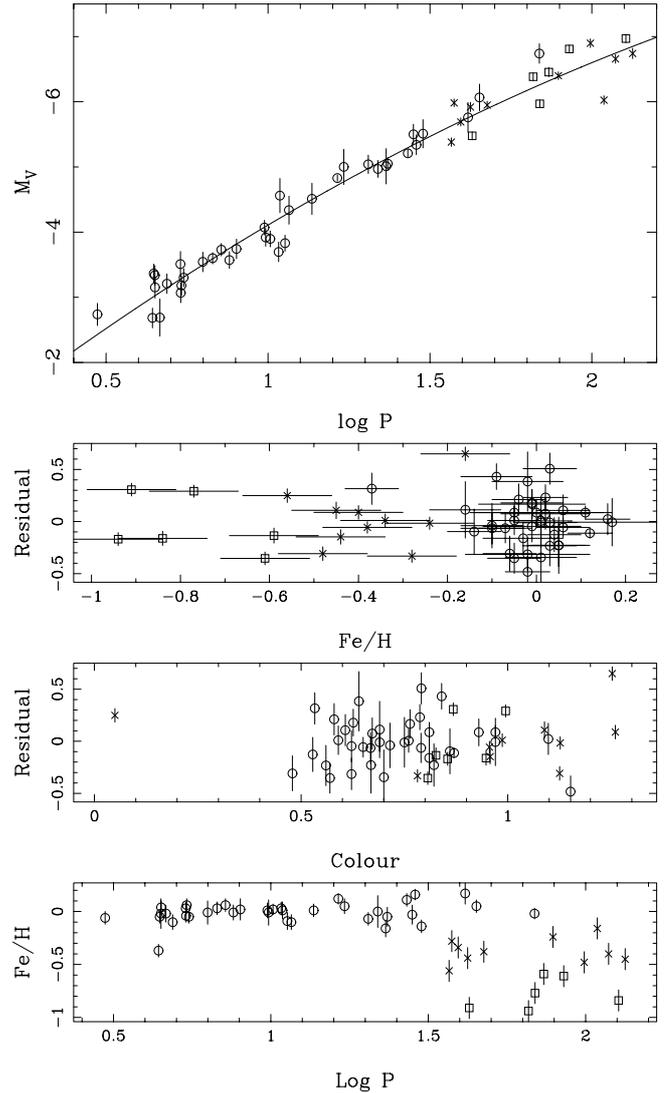
metallicity is  $\approx 0.03$  in both slope and zero point. This error should be quadratically added to the errors in the fit listed in Table 5, but this has only a marginal effect. On the other hand, the  $\sigma$  due to the error in the metallicity  $\alpha_2$  for *VWK* with (without) EV Sct included in the sample is 0.18 (0.24), 0.16 (0.21) and 0.14 (0.17), respectively. The total error in  $\alpha_2$  becomes, respectively, 0.31 (0.39), 0.30 (0.38), 0.29 (0.36) mag/dex. This implies that the metallicity effect in the zero point for the Galactic sample is  $-0.6 \pm 0.4$  mag/dex in *VWK* when EV Sct is excluded and  $-0.8 \pm 0.3$  when EV Sct is included.

These results indicate that to obtain a better estimate of the metallicity effect based on Galactic cepheids alone, one would need more data points at low and high metallicities. Good candidates would be cepheids in the direction of the centre and the anti-centre of the Galaxy. However, these stars would be at considerable distance and probably an enlargement of the Galactic sample would be limited by our ability to accurately determine individual distances.

When all three galaxies are considered, the baseline in metallicity is now much larger (EV Sct is included, but the star does not influence the solution just because the baseline in



**Fig. 1.** *Top:* Period–Luminosity relation in the  $V$ -band for the Galaxy. Shown are the data points and the best fitting linear  $PL$ -relation. The middle panels shown the residuals plotted versus metallicity and  $(B - V)_0$  colour. The bottom panel shows  $[Fe/H]$  versus Period. Stars are plotted at their fundamental period.



**Fig. 2.** *Top:* Period–Luminosity relation in the  $V$ -band showing data points ( $\circ$  = Galactic,  $\square$  = LMC,  $\times$  = SMC) and the fit allowing for a quadratic term in  $\log P$ . Other panels as Fig. 1.

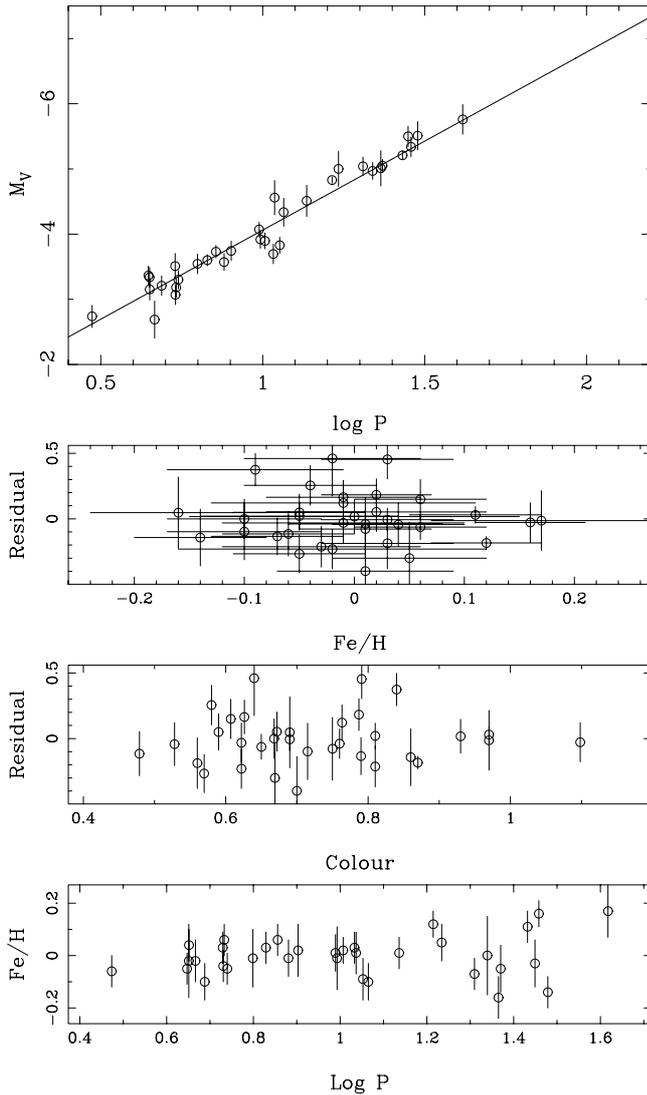
metallicity is much larger now). The coefficient  $\alpha_2$  is now determined at the  $\sim 6\sigma$  level. In the  $B$ -band its value is slightly positive, in  $V$  negative at about  $-0.27$  mag/dex. Figure 4 shows the result for the  $V$ -band. The error bars listed have to be increased to include the uncertainty due to the error in the metallicity itself. As before, Monte Carlo simulations have been performed and  $\sigma$  values of 0.061, 0.035, 0.050, respectively, in  $V$  were determined in  $\alpha_2$ . This implies that the metallicity effect is  $-0.27 \pm 0.08$  mag/dex, and is within the error the same in  $V$ . Error bars of 0.04 should be added quadratically to the listed ones for the slopes and zero points (independent of photometric band) due to the error in the metallicity.

The results in  $B$  and to a lesser extent in  $V$  and  $I$  depend somewhat on the adopted reddenings to the MC cepheids. The last block of results in Table 5 refers to the situation where a uniform reddening of  $E(B - V) = 0.151$  and 0.092 for LMC and SMC, respectively, has been adopted (the mean reddening

of cepheids adopted by the OGLE team). The results in the Wesenheit index and the  $K$ -band are essentially unchanged.

When considering the metallicity effect in Galaxy, LMC and SMC it was so far assumed that the slope in the  $\log P$  dependence does not depend on metallicity. In reality, there is increasing evidence that the slopes in the Galaxy, LMC and SMC are different, see e.g. the recent discussion in Fouqué et al. (2003). Equation (1) allows for an explicit metallicity dependence of the slope in  $\log P$ , and a test calculation allowing  $\alpha_5$  to vary was performed. A non-conclusive ( $1\sigma$ ) result was obtained and this is likely due to the very small numbers of MC cepheids. When an increased sample of MC cepheids with metallicity determinations becomes available the metallicity dependence of the slope in  $\log P$  can be investigated in more detail.

More complicated fitting formulae, for example with a quadratic metallicity dependence, or a colour term, have been



**Fig. 3.** *Top:* Linear Period–Luminosity relation in the  $V$ -band for the Galaxy, with EV Sct excluded and for  $\log P < 1.65$ . Other panels as Fig. 1.

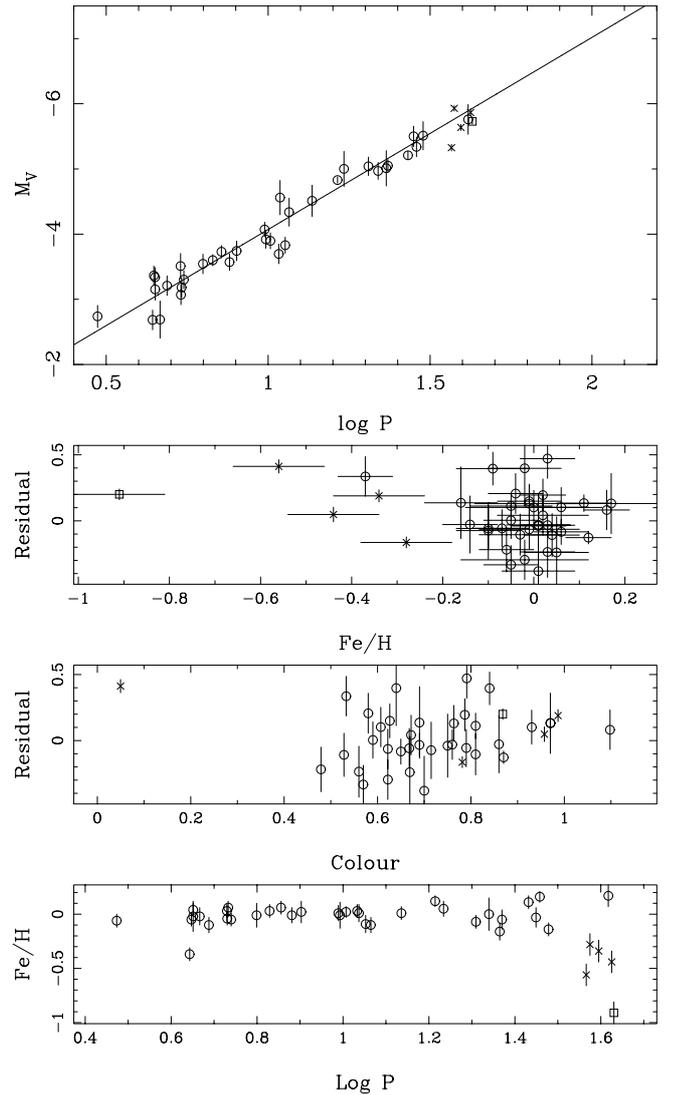
tried but did not result in parameters which are determined in a significant way.

#### 4. Discussion

An attempt has been made, using the currently available data of cepheids with direct metallicity determinations, to quantify the metallicity dependence of the cepheid  $PL$ -relation.

For a purely Galactic sample, the range in metallicity covered is too small to draw any firm conclusions. A metallicity effect of  $-0.6 \pm 0.4$  or  $-0.8 \pm 0.3$  mag/dex in  $VWK$  is derived, depending on whether the only Galactic star with a significantly sub-solar metallicity is excluded or included.

For the combined sample of Galactic, SMC and LMC cepheids the problem is that most of the MC cepheids have such long periods, in the regime where the  $PL$ -relation is no longer linear. Restricting the sample in periods to  $\log P < 1.65$  a metallicity effect of about  $-0.27 \pm 0.08$  mag/dex in the zero point is found (in  $VIWK$ ). However, the sample of



**Fig. 4.** *Top:* Linear Period–Luminosity relation in the  $V$ -band for Galaxy, SMC, LMC for  $\log P < 1.65$ . Other panels as Fig. 1.

MC cepheids is presently too small to additionally solve for a metallicity dependence of the slope in  $\log P$ . Formally, the result for the Galactic sample is in agreement with that comprising all three galaxies.

This result based on the sample including the MC cepheids is in agreement with other recent empirical estimates (most recently by Kennicutt et al. 2003; Storm et al. 2004). However, it is stressed that this study is the first to use *direct* cepheid metallicity determinations to arrive at this result. In agreement with Storm et al. (2004) we find that in the range  $V$  to  $K$  the metallicity effect seems not to depend on wavelength, contrary to theoretical calculations (e.g. Bono et al. 1999), which predicted a decreasing effect towards longer wavelengths.

The method here developed (Eq. (1)) is general and can easily be extended to other functional forms, or to include other galaxies. In principle, the distance to a galaxy and the metallicity dependence can be derived independently. For the present restrictive sample this is not the case however, since the distribution over metallicity is correlated with the parent galaxy.

In other words,  $\alpha_2$ ,  $\Delta_{\text{LMC}}$  and  $\Delta_{\text{SMC}}$  are strongly correlated: a change in the DM to the MCs can be “compensated for” by a change in the metallicity effect. Therefore, it was decided to not solve for the distance moduli. However, in the second block of results in Table 5 (no metallicity effect, but allowing for a quadratic term in  $\log P$ ) there is some indication that the best fitting DM might be slightly longer than 18.50 for the LMC and slightly shorter than 18.90 for the SMC (although there is some scatter depending on the wavelength). Simply adopting values of 18.55 and 18.80, respectively, within the error bars of the current best independent estimates (Feast 2003; Walker 2003), would result in a value for  $\alpha_2$  of  $-0.38 \pm 0.08$  mag/dex in  $V$ , that is, it would make the metallicity effect slightly stronger (and similarly for the other bands).

It is shown that the error in the independent variable metallicity is a significant contributor to the final error in the metallicity effect as it is larger than the error in the fitting in the dependent variable, but of little effect in the final error in slope and zero point. The mean error in metallicity of the stars in the Galactic plus MC sample is about 0.08 dex. By repeating the Monte Carlo simulations it was derived that for a uniform error bar of 0.05 dex the two sources of error become comparable, and that for a uniform error bar of 0.03 dex or less the error in the metallicity becomes insignificant compared to the fit error.

Although the present results seem to indicate a significant metallicity effect, more data are needed to quantify this effect better. An important issue is an internally consistent metallicity scale. The simulations even suggest an accuracy to a level of 0.05 dex or better. However, this seems very difficult to achieve in practice as the error in the metallicity not only depends on the S/N in the spectra and the number of lines, but also on the uncertainties in the adopted effective temperature, gravity and model atmospheres (e.g. Fry & Carney 1997). This implies that an improvement in the present result must come from an increased sample of stars with periods  $\lesssim 45$  d spread over a range in metallicities as large as possible.

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