A&A 419, 771–776 (2004)
DOI: 10.1051/0004-6361:20034093

Astronomy & Astrophysics

Semi-classical collisional functions in a strongly correlated plasma

H. Ben Chaouacha[1], N. Ben Nessib[1], and S. Sahal-Bréchot[2]

[1] Groupe de Recherche en Physique Atomique et Astrophysique, Faculté des Sciences de Bizerte, 7021 Zarzouna, Tunisia
[2] Laboratoire d'Étude du Rayonnement et de la Matière en Astrophysique, UMR CNRS 8112 – LERMA, Observatoire de Paris, Section de Meudon, 92195 Meudon Cedex, France

Received 18 July 2003 / Accepted 2 February 2004

Abstract. Collisions between atoms (or ions) and electrons play an important role in the interpretation of line spectra and for the modelling of stellar interiors. Plasma shielding effects due to electron and ion correlations are not negligible in the physical conditions of white dwarf atmospheres, owing to their high density. They also play a role in cool stars and for atomic transitions that are quasi-degenerate. In the standard formalism of Stark impact broadening of spectral lines and of cross sections, the electrostatic Coulomb potential is used to describe the interaction between the perturbing electrons and the emitting atom. Electronic correlations (screening effects) are usually taken into account by introducing a cut-off in the interaction when the electron-atom distance exceeds the Debye radius $R_{\rm D}$. A more consistent treatment to describe collective effects is the Debye-Hückel potential: the two-particle Coulomb field is shielded by the ensemble of the surrounding electrons. This is a good approximation only for high temperature and low density plasmas (weakly non-ideal plasmas), while for strongly non-ideal plasmas, the Coulomb cut-off potential or the ion sphere potential are more appropriate. These potentials, which can be written as the Coulomb potential with two correcting terms, are widely used in the literature.
In this paper, we investigate the ion sphere model to describe the electron atom interaction in a strongly coupled plasma. New semi-classical collisional functions are derived for both the transition probability and the cross section, using the classical path approximation.

Key words. atomic processes

1. Introduction

Stark broadening of spectral lines is important for astrophysical modelling. It is found to be a reliable tool for understanding the characteristics of the plasma. This requires, in practice, a detailed knowledge of various processes, especially inelastic electronic collisions exciting and deexciting the level of the studied line in the plasma. The atomic processes in strongly correlated plasma have received particular attention in recent years.

In a previous work (Ben Nessib et al. 1997) analytical expressions of the impact semi-classical functions entering the expression of the electron collisional line width were calculated by considering the cut-off potential valid for correlated plasmas. It is found that the surrounding atoms and ions presenting in the plasma may have strong perturbing effects on the wave functions of the bound electrons. Thereby, they significantly affect the atomic transition probability as well as the atomic cross-section. Unfortunately, the cut-off model is not applicable to very high density plasmas. For a strongly non-ideal plasma the ion sphere model is found to be more suitable (Jung & Yoon 2000a; Salzmann & Szichman 1987).

In this work, we derive analytic expressions of the semi-classical collisional functions using the static screening Coulomb interactions. As we are interested only in neutral atoms emitters for large impact parameters at relatively low energies (temperatures of the order of a few thousands or few ten thousands of degrees), we may neglect the plasma screening effects on the trajectory, and we may also use the semi-classical straight line trajectory. The corrections due to strong correlations plasma effects, even if they are not negligible, will be introduced rather in the transition probability calculation. In fact, dynamic screening Coulomb interactions using the plasma dielectric function in collision processes in non-ideal plasmas have been intensively investigated by several authors (Song & Jung 2003b; Jung 1997, 2002, 2003; Kim & Jung 2001; Jung & Yoon 2000b,c). However, the difference between the static and the dynamic plasma screening effects on the atomic excitation process is found to be significant only for relatively high energy projectiles (Jung & Yoon 2000b). The excitation cross section including screening effects is shown to decrease as the non-ideality plasma parameter increases (Song & Jung 2003a;

Send offprint requests to: S. Sahal-Bréchot,
e-mail: sylvie.sahal-brechot@obspm.fr

Jung 2000). On the other hand, a hyperbolic orbit is reliable for ions emitters owing to the Coulomb ion-electron interaction. Such a trajectory would be also more reliable to describe screening effects on the perturber motion colliding with a neutral atom, especially for very low collision energies in the neighborhood of the threshold (Jung 1993, 1994). However, for the large impact parameter region, the straight line trajectory can be used even when the emitter is an ion (Jung 2000).

In this paper, we replace the cut-off model by the ion-sphere potential, and we give new analytical expressions, in the semi-classical approximation, for both the collisional functions of the transition probability and the cross-section. We consider the straight trajectory method to study the motion of the projectile electron, as a function of the impact parameter ρ and of the ion sphere radius R_c.

2. Theory

We will study in this part the total inelastic cross-sections entering the expression of the impact width of isolated (non-hydrogen) lines.

2.1. Total inelastic cross-section and collisional impact broadening of isolated lines.

Within the impact approximation the profile is Lorentzian for isolated lines. Overlapping lines are beyond the scope of the present study. For the line corresponding to the transition between the initial level i and the final level f, the half half-width w and the shift d are given by Baranger's formula (1958):

$$w + id = N_p \int_0^\infty v f(v) \mathrm{d}v \int_0^\infty 2\pi\rho \mathrm{d}\rho \times \left\{1 - \left\langle i \left| S \right| i \right\rangle \left\langle f \left| S^{-1} \right| f \right\rangle \right\}_{AV}, \quad (1)$$

where N_p designates the density of the perturber, S the scattering matrix obtained for the atom-perturber interaction corresponding to the impact parameter ρ and the relative velocity v, $f(v)$ the relative atom-perturber Maxwell distribution of velocities, and $\{...\}_{AV}$ the angular average over the magnetic quantum numbers.

For the transition between the level $i(n_i l_i L_i S_i J_i)$ and $f(n_f l_f L_f S_f J_f)$, the total width at half intensity $W = 2w$ can be put in the form (Sahal-Bréchot 1969):

$$W = N_p \int_0^\infty v f(v) \mathrm{d}v \times \left(\sum_{j \neq i} \sigma_{ij}(v) + \sum_{j' \neq f} \sigma_{ff'}(v) + \sigma_{el} \right), \quad (2)$$

where j, j' refers to the perturbing levels.

The elastic contribution to the width σ_{el} is not relevant for the present paper. The inelastic cross-section $\sigma_{ij}(v)$ (respectively $\sigma_{ff'}(v)$) are obtained by an integration over the impact parameter ρ of the transition probabilities $P_{ij}(v,\rho)$ (respectively $P_{ff'}(v,\rho)$) as:

$$\sum_{j \neq i} \sigma_{ij}(v) = \pi R_1^2 \sum_{j \neq i} P_{ij}(v, R_1) + \int_{R_1}^{R_D} 2\pi\rho \mathrm{d}\rho \sum_{j \neq i} P_{ij}(v,\rho). \quad (3)$$

The perturbation theory used for the derivation of the S-matrix leads to a divergence in the integration over the impact parameter. Thus a lower cut-off R_1 is required. In addition, for high densities or for very small energy differences, an upper cut-off

$$R_D = \left(\frac{kT}{4\pi N_e e^2} \right)^{\frac{1}{2}}, \quad (4)$$

is introduced to take into account the shielding.

The expression for P_{ij} (respectively $P_{ff'}$) is given within the first order time-dependent perturbation theory by an average over the initial Zeeman states M_i coupled to a sum over the final states M_j (Griem et al. 1962).

$$P_{ij}(v,\rho) = \frac{1}{2J_i + 1} \sum_{M_i, M_j} \frac{1}{\hbar^2} \times \left| \int_{-\infty}^{+\infty} \left\langle n_i l_i J_i M_i \left| V(t) \right| n_j l_j J_j M_j \right\rangle \exp\left(\frac{\mathrm{i}(E_j - E_i)t}{\hbar}\right) \mathrm{d}t \right|^2, \quad (5)$$

where $V(t)$ designates the interaction potential between the atom and the charged perturber moving along a classical path at time t, and E_i, E_j are the energies of the initial and final levels respectively

The cross-section σ follows by an integration over the impact parameter ρ:

$$\sigma_{ij}(v) = 2\pi \int_0^\infty P_{ij}(v,\rho)\, \rho\, \mathrm{d}\rho. \quad (6)$$

2.2. Non-correlated semi-classical collisional functions for the total cross-section

In a non-correlated plasma the interaction is described by the electrostatic Coulomb potential, which is expressed as:

$$V = \frac{Z Z_p e^2}{r_p} - Z_p e^2 \sum_{i=1}^{N} \frac{1}{r_{ip}}, \quad (7)$$

where Ze and $Z_p e$ are the charges of the interacting particles, r, r_{ip} are the coordinates of the projectile electron and the bound electron, respectively (Ben Nessib et al. 1997).

$\frac{1}{r_{ip}}$ is expanded in multipolar components and only the long-range part is retained in the perturbation theory:

$$V = \frac{Z Z_p e^2}{r_p} - \sum_{\lambda=1}^{N} \frac{4\pi Z_p e^2}{2\lambda + 1} \times \frac{1}{r_p^{\lambda+1}} \times \sum_{\mu=-\lambda}^{+\lambda} \sum_{i=1}^{N} r_i^\lambda Y_{\lambda\mu}(\widehat{r_p}) Y^*_{\lambda\mu}(\widehat{r_i}). \quad (8)$$

The first term in this expression is the Coulomb term (it is null for the neutral perturbers), and it does not play a role in the calculation of the inelastic cross-sections due to its spherical symmetry. For the calculation of the cross-section between the levels having dipolar electric transition, we have to retain only the dipole term ($\lambda = 1$), i.e.:

$$V_{\mathrm{dip}} = -\frac{4\pi Z_p e^2}{3} \frac{1}{r_p^2} \sum_{\mu=0,\pm1} Y_{1\mu}(\widehat{r_p}) \times \sum_{i=1}^{N} r_i \times Y^*_{1\mu}(\widehat{r_i}). \quad (9)$$

Here, we shall adopt the usual semi-classical description of the collision process. The projectile moves along a straight path with a velocity v and the radiator is localized at the origin. The impact parameter is ρ, and t (= 0) is the time of the closest approach. The transition probability from an initial state i to a final state j from the first order perturbation theory is given by (Sahal-Bréchot 1969; Griem et al. 1962).

$$P_{ij}(v,\rho) = \frac{1}{3}\frac{Z_{\mathrm{p}}^2 e^4}{\hbar}\frac{4\pi}{3}R_{\mathrm{line}}^2 R_{\mathrm{mult}}^2\ l_{>}\ I^2 \sum_{\mu=0;\pm1}\left|J_{1\mu}\right|^2, \tag{10}$$

where

$$J_{1\mu} = \int_{-\infty}^{+\infty} \mathrm{e}^{\mathrm{i}\omega_{ij}t} \times \frac{Y_{1\mu}(\widehat{r_{\mathrm{p}}})}{r_{\mathrm{p}}^2}\mathrm{d}t \tag{11}$$

is the collisional term.

R_{line} and R_{mult} are defined in Shore & Menzel (1968) as:

$$R_{\mathrm{line}}(SLJ, SL'J') = [(2J+1)\times(2J'+1)]^{\frac{1}{2}} \times \begin{Bmatrix} J & 1 & J' \\ L' & S & L \end{Bmatrix}, \tag{12}$$

where R_{mult} in the case of one electron above a closed shell is:

$$R_{\mathrm{mult}}(L_c lSL, L_c l'SL') = [(2L+1)\times(2L'+1)]^{\frac{1}{2}} \times \begin{Bmatrix} L & 1 & L' \\ l' & L_{\mathrm{c}} & l \end{Bmatrix}. \tag{13}$$

Also, the terms $l_{>}$ and ω_{ij} are defined as:

$$l_{>} = \max(l_i, l_j),$$
$$\omega_{ij} = \frac{E_j - E_i}{\hbar},$$

and the radial integral I is defined as:

$$I = \int_0^{\infty} R_{n_i l_i}(r) R_{n_j l_j}(r) r \mathrm{d}r. \tag{14}$$

If we apply the parametric representation of a straight line trajectory (Sahal-Bréchot et al. 1996):

$$\begin{cases} r_{\mathrm{p}} = \sqrt{\rho^2 + v^2t^2} = \dfrac{\rho}{\sin(\theta_{\mathrm{p}})}, \\ x_{\mathrm{p}} = \rho\cos(\phi_{\mathrm{p}}), \\ y_{\mathrm{p}} = \rho\sin(\phi_{\mathrm{p}}), \\ z_{\mathrm{p}} = r_{\mathrm{p}}\cos(\theta_{\mathrm{p}}). \end{cases} \tag{15}$$

The Y_{lm} functions can be written as:

$$Y_{10} = \sqrt{\frac{3}{4\pi}}\cos(\theta_{\mathrm{p}}) = \sqrt{\frac{3}{4\pi}}\frac{vt}{\sqrt{\rho^2+v^2t^2}}, \tag{16}$$

and

$$Y_{1\pm1} = \mp\sqrt{\frac{3}{8\pi}}\sin(\theta_{\mathrm{p}})\,\mathrm{e}^{\pm\mathrm{i}\phi} = \mp\sqrt{\frac{3}{8\pi}}\frac{\rho}{\sqrt{\rho^2+v^2t^2}}\,\mathrm{e}^{\pm\mathrm{i}\phi}. \tag{17}$$

By introducing these expressions in Eq. (11) we obtain the following expressions:

$$|J_{10}| = \sqrt{\frac{3}{4\pi}}\frac{2}{\rho v}\ zK_0(z), \tag{18}$$

$$|J_{1\pm1}| = \sqrt{\frac{3}{8\pi}}\frac{2}{\rho v}\ zK_1(z), \tag{19}$$

where $z = \rho\ \omega_{ij}/v$, and K_0, K_1 are the modified Bessel functions. By introducing these two expressions in Eq. (10), the probability transition becomes (Sahal-Bréchot 1969; Griem et al. 1962):

$$P_{ij}(v,\rho) = \frac{4I_{\mathrm{H}}^2}{E(E_j-E_i)}\frac{m}{m_{\mathrm{e}}} \times f\left(n_i l_i J_i \rightarrow n_j l_j J_j\right) \times \frac{a_0^2}{\rho^2}A(z), \tag{20}$$

where a_0 designates the Bohr radius, m the electron mass, m_{e} the reduced electron mass, E the perturber energy, $f(n_i l_i J_i \rightarrow n_j l_j J_j)$ the oscillator strength, I_{H} the ionization energy of hydrogen and $A(z)$ the collision function:

$$A(z) = z^2\left[K_0^2(z) + K_1^2(z)\right]. \tag{21}$$

This can be written also as:

$$A(z) = A_0(z) + 2A_{\pm}(z), \tag{22}$$

where

$$A_0(z) = z^2 K_0^2(z), \tag{23}$$

and

$$A_{\pm}(z) = \frac{1}{2}z^2 K_1^2(z). \tag{24}$$

Then, the total inelastic cross-section in a non-correlated plasma $\sigma_{ij}(v)$ is obtained after integration of the transition probability over the impact parameter, where a lower cut-off R_1 is required. An upper cut-off at the Debye radius R_{D} is also introduced to take into account the shielding due to the ensemble of the surrounding electrons:

$$\sigma_{ij}(v) = \pi R_1^2 P_{ij}(v, R_1) + \pi a_0^2 Z_{\mathrm{P}}^2 \frac{8I_{\mathrm{H}}^2}{E\left(E_j - E_i\right)}\frac{m}{m_{\mathrm{e}}} \times f\left(n_i l_i J_i \rightarrow n_j l_j J_j\right)\ [a(z_1) - a(z_{\mathrm{D}})], \tag{25}$$

where $z_1 = \dfrac{R_1\omega_{ij}}{v}$ and $z_{\mathrm{D}} = \dfrac{R_{\mathrm{D}}\omega_{ij}}{v}$.

The integration of $\dfrac{A(z)}{z}$ over the impact parameter leads to the $a(z)$ function:

$$a(z) = \int_z^{\infty}\frac{A(z')}{z'}\mathrm{d}z' = zK_0(z)K_1(z). \tag{26}$$

As for $A(z)$, $a(z)$ can be put in the following form:

$$a(z) = a_0(z) + 2a_{\pm}(z), \tag{27}$$

where

$$a_0(z) = \frac{1}{2}z^2\left[K_1^2(z) - K_0^2(z)\right], \tag{28}$$

and

$$a_\pm(z) = \frac{1}{4}\left\{z^2\left[K_0^2(z) - K_1^2(z)\right] + 2zK_0(z)K_1(z)\right\}. \tag{29}$$

3. Strongly correlated plasma

3.1. The collision functions in the cut-off model

Multiparticles correlation effects caused by simultaneous interactions of a large number of particles have been taken into account according to different criteria to introduce the cut-off in the interaction when the electron-atom distance exceeds a certain radius. The potential must depend on the plasma properties. In a high density and low temperature plasma, the cut-off potential is reliable to describe the interaction process. In this case, the impact approximation is expressed as $TN_e^{\frac{-1}{3}} > 4.2\times10^{-3}$ where the temperature T is expressed in Kelvin and the density in cm^{-3}. The plasma screening parameter

$$\gamma = 2.6\times10^{-3}\, z^{\frac{5}{3}}\,\frac{N_e^{\frac{1}{3}}}{T}, \tag{30}$$

must be also greater than 0.5 (Ben Nessib et al. 1997). If only the dipolar long range part of the perturbation theory is considered, the Coulomb cut-off potential, for the atom-perturber interaction, is written as follows:

$$\begin{cases} V_c(t) = -\dfrac{4\pi Z_p e^2}{3}\displaystyle\sum_{\mu=0;\pm1}\left[\frac{Y_{1\mu}}{r_p^2} - \frac{Y_{1\mu}}{R_c r_p}\right] \\ \qquad\times\sum_{i=1}^{N} r_i Y_{1\mu}^*(\widehat{r_i}),\ \text{if } r_p < R_c, \\ V_c(t) = 0,\ \text{if } r_p > R_c, \end{cases} \tag{31}$$

where

$$R_c = \left(\frac{3Z}{4\pi N_e}\right)^{\frac{1}{3}} \tag{32}$$

designates the ion sphere radius. To obtain the transition probability, the $J_{1\mu}$ functions in Eq. (11) have to be replaced by the correlated functions $J_{1\mu}^c$ given by Ben Nessib et al. (1997):

$$J_{1\mu}^c = \int_{-\infty}^{+\infty} e^{i\omega_{ij}t}\left[\frac{Y_{1\mu}(\widehat{r_p})}{r_p^2} - \frac{Y_{1\mu}(\widehat{r_p})}{R_c r_p}\right] dt. \tag{33}$$

After some calculations, the preceding $A(z)$ function becomes the correlated function $A^c(z)$:

$$A^c(z) = A(z) - \pi\frac{z^2}{z_c}e^{-z}\,\left[K_0(z) + K_1(z)\right] + \frac{\pi^2}{2}\frac{z^2}{z_c^2}e^{-2z}, \tag{34}$$

where $z_c = \dfrac{R_c\omega_{ij}}{v}$.

The integration of $\dfrac{A^c(z)}{z}$ over z involves the correlated function, which is denoted by $a^c(z)$:

$$a^c(z) = \int_z^\infty \frac{A^c(z')}{z'}dz'. \tag{35}$$

This leads to the following expression:

$$a^c(z) = a(z) - \pi\frac{z}{z_c}e^{-z}K_1(z) + \frac{\pi^2}{8z_c^2}(1+2z)e^{-2z}. \tag{36}$$

3.2. The collision functions in the ion sphere model

The plasma is considered to be strongly correlated if $\gamma \geq 0.5$. In this condition, we replace the cut-off potential by the ion sphere model, which seems to be a quite reliable framework. In practice it corresponds to the Coulombian potential corrected by two terms (Jung & Yoon 2000a; Salzmann & Szichman 1987; Gutierrez 1994):

$$\begin{cases} V_{SI}(t) = \dfrac{1}{r_{ip}}\left(1 - \dfrac{3}{2}\dfrac{r_p}{R_c} + \dfrac{r_p^3}{R_c^3}\right);\ r \leq R_c, \\ V_{SI}(t) = 0;\ r > R_c. \end{cases} \tag{37}$$

As mentioned above R_c is the ion sphere radius. Accordingly, in the expression of the transition probability, the $J_{1\mu}$ functions in Eq. (33) have to be changed by the correlated functions $J_{1\mu}^{SI}$.

$$J_{1\mu}^{SI} = \int_{-\infty}^{+\infty} e^{i\omega_{ij}t}\left[\frac{Y_{1\mu}(\widehat{r_p})}{r_p^2} - \frac{3}{2}\frac{Y_{1\mu}(\widehat{r_p})}{R_c r_p} + \frac{r_p Y_{1\mu}(\widehat{r_p})}{R_c^3}\right]dt. \tag{38}$$

Thus, we obtain after integration the collision function $A^{SI}(z)$:

$$A^{SI}(z) = A_0^{SI}(z) + 2A_\pm^{SI}(z), \tag{39}$$

where

$$A_0^{SI}(z) = \left[zK_0(z) - \left(\frac{3}{2}\right)\frac{\pi z}{2z_c}e^{-z} + \left(\frac{z}{z_c}\right)^3\left(\frac{\sin(x_c z)}{z^2} - \frac{x_c\cos(x_c z)}{z}\right)\right]^2, \tag{40}$$

and

$$A_\pm^{SI}(z) = \frac{1}{2}\left[zK_1(z) - \left(\frac{3}{2}\right)\frac{\pi z}{2z_c}e^{-z} + \frac{z^2}{z_c^3}\sin(x_c z)\right]^2. \tag{41}$$

Here $x_c = \dfrac{vt_c}{\rho}$, where t_c designates the time cut-off. We notice that x_c is introduced to solve the divergence problem originating from the third term of the potential expression.

To obtain the $a^{SI}(z)$ function, we have to integrate $\dfrac{A^{SI}(z)}{z}$ over z:

$$a^{SI}(z) = \int_z^\infty \frac{A^{SI}(z')}{z'}dz'. \tag{42}$$

The cross-section collisional function is obtained finally by a simple integration of Eq. (42):

$$a^{SI}(z) = a_0^{SI}(z) + 2a_\pm^{SI}(z), \tag{43}$$

where

$$a_0^{SI}(z) = a_0(z) + \alpha_1(z) + \alpha_2(z) + \alpha_3(z) + \alpha_4(z). \tag{44}$$

Here $a_0(z)$ designates the collisional function for the cross section corresponding to an ideal plasma. In the $\alpha_1(z)$ function we find the modified Bessel functions K_1 and K_2:

$$\alpha_1(z) = -\frac{3}{2}\pi\frac{z}{z_c}e^{-z}\left\{\left(1+\frac{z}{3}\right)K_1(z)-\frac{z}{3}K_2(z)\right\} + \left(\frac{3}{2}\right)^2\frac{\pi^2}{16}\frac{(2z+1)}{z_c^2}e^{-2z}. \quad (45)$$

The $\alpha_2(z)$, $\alpha_3(z)$ and $\alpha_4(z)$ are sinusoïdal functions from the third term in the potential expression, i.e.:

$$\alpha_2(z) = -\frac{z^2}{4z_c^6}\left\{1+\frac{x_c z^2}{2}\right\} + \frac{z}{2x_c z_c^6}\left\{\frac{11}{4}-\frac{z^2x_c^2}{2}\right\}\sin(2x_c z) + \frac{1}{2x_c z_c^6}\left\{\frac{11}{8x_c}-\frac{7z^2x_c}{4}\right\}\cos(2x_c z). \quad (46)$$

$$\alpha_3(z) = \left\{\frac{z^2}{5}K_3(z)-\frac{4z}{3}K_2(z)+zK_0(z)+K_1(z)\right\}\times\frac{2z\sin(x_c z)}{z_c^3} - \left\{\left(2+\frac{2z}{3}\right)K_1(z)+\left(\frac{z^2}{5}-\frac{2z}{3}\right)K_2(z)\right\}\times\frac{2zx_c\cos(x_c z)}{z_c^3}. \quad (47)$$

$$\alpha_4(z) = -\frac{3\pi}{2z_c^4}\frac{ze^{-z}}{1+x_c^2}\left\{\left(1+zx_c^2\right)\sin(x_c z) + x_c(1-z)\cos(x_c z)\right\} - \frac{3\pi}{2z_c^4}\frac{e^{-z}}{\left(1+x_c^2\right)^2}\times\left\{\left(1+x_c^2(4z-1)\right)\sin(x_c z) + 2x_c\left(1-z\left(1-x_c^2\right)\right)\cos(x_c z)\right\} + \frac{3\pi}{2z_c^4}\frac{x_c e^{-z}}{\left(1+x_c^2\right)^3}\times\left\{2x_c\left(x_c^2-3\right)\sin(x_c z)+2\left(1-3x_c^2\right)\cos(x_c z)\right\}. \quad (48)$$

The $a_{\pm}^{SI}$ function can be written as:

$$a_{\pm}^{SI}(z) = a_{\pm}(z) + 0.5[\beta_1(z)+\beta_2(z)+\beta_3(z)+\beta_4(z)], \quad (49)$$

where the first term $a_{\pm}(z)$ designates the collisional function for the cross-section corresponding to an ideal plasma, $\beta_1(z)$ a function of the modified Bessel functions K_1 and K_2, i.e.:

$$\beta_1(z) = \frac{\pi}{2}\frac{z^2}{z_c}e^{-z}\left\{K_1(z)-K_2(z)\right\} + \left(\frac{3}{2}\right)^2\frac{\pi^2}{16}\frac{(2z+1)}{z_c^2}e^{-2z}. \quad (50)$$

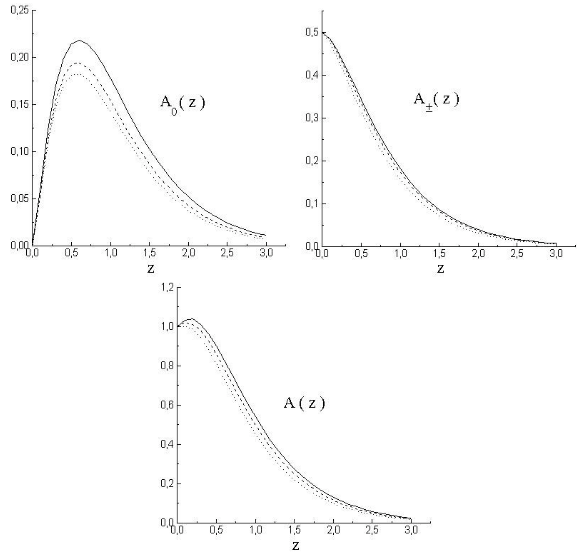

Fig. 1. Collision functions for the transition probability. Full lines: non-correlated functions $A_0(z)$, $A_{\pm}(z)$ and $A(z)$. Dotted lines: correlated functions $A_0^c(z)$, $A_{\pm}(z)$ and $A(z)$. Dashed lines: strongly correlated functions $A_0^{SI}(z)$, $A_{\pm}^{SI}(z)$ and $A^{SI}(z)$ for $z_c = 20$.

$\beta_2(z)$, $\beta_3(z)$ and $\beta_4(z)$ are sinusoïdal functions derived from the third term in the potential expression:

$$\beta_2(z) = \frac{1}{4z_c^6}\left\{\frac{z^4}{2}-\frac{z}{x_c}\left(z^2-\frac{3}{2x_c^2}\right)z\sin(2x_c z) - \frac{3}{2x_c^2}\left(z^2-\frac{1}{2x_c^2}\right)\cos(2x_c z)\right\}, \quad (51)$$

$$\beta_3(z) = \frac{2z^2}{z_c^3}\left\{(K_2(z)-\frac{z}{5}K_3(z))\sin(x_c z) - \frac{x_c z}{5}K_2(z)\cos(x_c z)\right\}, \quad (52)$$

$$\beta_4(z) = -\frac{3\pi}{2z_c^4}\frac{z^2e^{-z}}{1+x_c^2}\left\{\sin(x_c z)+x_c\cos(x_c z)\right\} - \frac{3\pi}{z_c^4}\frac{ze^{-z}}{\left(1+x_c^2\right)^2}\left\{\left(1-x_c^2\right)\sin(x_c z)+2x_c\cos(x_c z)\right\} + \frac{3\pi}{z_c^4}\frac{e^{-z}}{\left(1+x_c^2\right)^3}\left\{\left(3x_c^2-1\right)\sin(x_c z) + x_c\left(3-x_c^2\right)\cos(x_c z)\right\}. \quad (53)$$

We have compared the effects of the Coulomb, cut-off and ion sphere potentials on the different collisional functions for a typical value $z_c = 20$. The curves relative to the transition probability functions $A_0(z)$, $A_{\pm}(z)$ and $A(z)$ are represented in Fig. 1, for these three potentials. It can be seen that both the ion sphere and cut-off models do not alter the standard functions, but they introduce a decrease in their values, especially for the lower energies. However, as expected, such a decrease seems to be more pronounced for the ion sphere model. Similar effects are observed for the three cross-section functions $a_0(z)$,

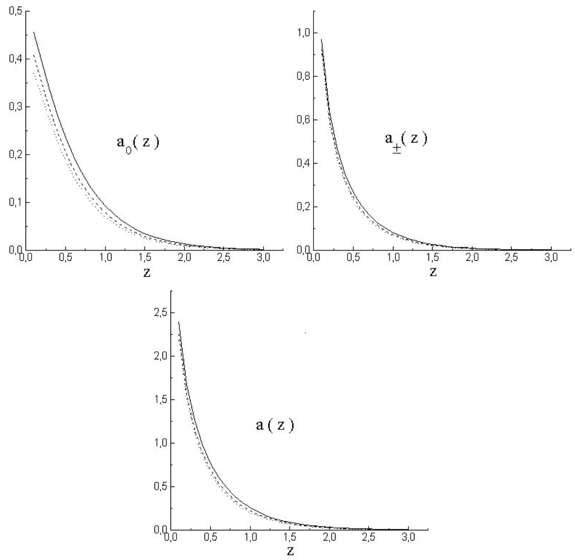

Fig. 2. Collision functions for the cross section. Full lines: non-correlated functions $a_0(z)$, $a_\pm(z)$ and $a(z)$. Dotted lines: correlated functions $a_0^c(z)$, $a_\pm(z)$ and $a(z)$. Dashed lines: strongly correlated functions $a_0^{SI}(z)$, $a_\pm^{SI}(z)$ and $a^{SI}(z)$ for $z_c = 20$.

$a_\pm(z)$, and $a(z)$, which are represented in Fig. 2 using the same value of z_c.

The different corrections become insignificant as z_c increases, since the approximation of strongly coupled plasma has no sense when the R_c exceeds a certain threshold value. On the other hand, the low z_c values yield non-physical SI-curves that may exceed the Coulomb ones, which may be attributed to the fact that the R_c in this range becomes somewhat problematic.

4. Conclusion

The ion sphere model is expected to be well adapted to describe strongly coupled plasmas, by adding two correction terms to the standard one. We use it to calculate new semi-classical collisional functions for both the transition probability and the cross section. We have compared these collisional functions for the Coulomb, the cut-off and the ion sphere potentials. The numerical results show that the increase in the screening leads to a decrease in these functions, especially for the lower values of the impact parameter. In the next step, we will include these functions in the computer code calculating widths and shifts in the impact approximation. This will be the object of a further paper where an application to a helium line will be given.

Acknowledgements. This work has been supported by the cooperation between the French CNRS and the Tunisian DGRSRT.

References

Baranger, M. 1958, Phys. Rev., 112, 855
Ben Nessib, N., Sahal-Bréchot, S., & Ben Lakhdar, Z. 1997, A&A, 324, 799
Griem, H. R., Kolb, A. C., & Oertel, G. 1962, Phys. Rev., 125, 177
Gutierrez, F. A. 1994, JQSRT, 51, 665
Jung, Y.-D. 1993, ApJ, 409, 841
Jung, Y.-D. 1994, Phys. Rev. A, 50, 3895
Jung, Y.-D. 1997, Phys. Plasmas, 4, 21
Jung, Y.-D. 2000, Eur. Phys. J. D, 11, 291
Jung, Y.-D. 2002, Phys. Plasmas, 9, 1460
Jung, Y.-D. 2003, Appl. Phys. Lett., 82, 2395
Jung, Y.-D., & Yoon, J.-S. 2000a, ApJ, 530, 1085
Jung, Y.-D., & Yoon, J.-S. 2000b, Phys. Scripta, 62, 46
Jung, Y.-D., & Yoon, J.-S. 2000c, Phys. Plasmas, 7, 3917
Kim, C.-G., & Jung, Y.-D. 2001, Phys. Scripta, 92, 335
Sahal-Bréchot, S. 1969, A&A, 1, 91
Sahal-Bréchot, S., Vogt, E., Thoraval, S., & Diedhiou, I. 1996, A&A, 309, 317
Salzmann, D., & Szichman, H. 1987, Phys. Rev. A, 35, 807
Shore, B. W., & Menzel, D. H. 1968, Principles of Atomic Spectra (New York: Wiley and Sons)
Song, M.-Y., & Jung, Y.-D. 2003a, J. Phys. B., 36, 2119
Song, M.-Y., & Jung, Y.-D. 2003b, Phys. Plasmas, 10, 3051