

H⁺ + H(1s) collisions at intermediate impact velocities as a new source of UV and VUV radiation

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Abstract. The process of radiative charge exchange in H⁺ + H(1s) collisions at the intermediate ion-atom impact velocities are treated in this work as a source of continuous EM emission in the UV and VUV range. The spectral intensity of this emission is determined, within the semiclassical method developed in previous works, for the ion-atom impact energies (in the center of mass reference frame) from 0.5 keV to 12.5 keV. The results obtained show that the spectral intensity of the examined EM emission increases for several orders of magnitude when passing from the visible to the VUV range of wavelength, and that the position of the maximum of this spectral intensity drifts with increase of collision energy from $\lambda \cong 51$ nm to $\lambda \cong 18$ nm. These results imply that considered radiation processes may be of interest in astrophysics as a new sources of continuous short-wave EM emission.

Key words. atomic processes – radiation mechanisms: general – stars: atmospheres

1. Introduction

A long time ago it has been found that some processes of the radiative charge exchange and photo-ionization in ion-atom collisions could be important in astrophysical spectroscopic phenomena. In Boggess (1959), the processes of radiative charge exchange and photo-association in H⁺ + H(1s) collisions have been treated as a source of continuous electromagnetic (EM) emission in some nebulae with temperatures $T \sim 10\,000$ K. Later, the influence of these processes were confirmed in cases of Sun photosphere ($T \lesssim 6000$ K) and some DA white dwarfs ($T \lesssim 25\,000$ K). The corresponding results were presented in Mihajlov et al. (1993, 1994a) and Stancil (1994). The processes of radiative charge exchange and photo-association in He⁺(1s) + He(1s²) collisions have also been considered in the case of the DB white dwarfs photosphere ($T \lesssim 50\,000$ K). The results were presented in Mihajlov & Dimitrijevic (1992), Stancil (1994) and Mihajlov et al. (1994b, 1995). The most important result regarding the optical characteristics of these stars' photospheres is the processes' contributions to the continuum emission/absorption in optical domain of EM spectra. For example, the contribution is about 10% in the case of Sun photosphere, and 10%–40% in the case of DB white dwarfs photosphere at $T_{\text{eff}} \lesssim 20\,000$ K, and it decreases as T_{eff} increases. Although these radiative collisions are much slower than radiative electron-ion collisions in photo-recombination

and bremsstrahlung processes, their role becomes significant and even dominant when the ionization degree in observed astrophysical plasmas is less than 10^{-4} . Because of this, the processes of radiative charge exchange and photo-ionization in He⁺(1s)+He(1s²) collisions have been taken into account in the investigation of EM radiation transport in the DB white dwarfs photosphere (Beauchamp et al. 1997).

The common feature of all mentioned astrophysical objects is that the influence of their relative movement on radiative ion-atom collision processes, important in plasmas with $T \sim 10\,000$ K, can be neglected. The typical impact energy in these processes is around 1 eV, which corresponds to collision velocity much less than v_0 (atomic unit velocity, 2.188×10^8 cm s⁻¹). Hence the simple quasi-static approach can be used (Bates 1951).

However, there are some astrophysical object where the relative movement of their plasma greatly exceeds the thermal velocities. The most significant example are the giant streams of the hydrogen plasma (jets), ejected from the central parts of active galaxies. The typical velocities of such jets can be 1000–2500 km s⁻¹, as in the case of Akn 120 galaxy (Popovic et al. 2001). Another characteristic example are streams of weakly ionized hydrogen plasma (outflows), produced during the creation of young stars (Eisloffel et al. 2000). The velocity of these streams can be greater than 200 km s⁻¹. Also, some stars can release the outer layers of their atmospheres. Typical example is the yellow hyper giant ρ -Cassiopeiae, with velocity of weakly ionized layers around 100 km s⁻¹ (Lobel et al. 2003).

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Another very interesting case is the formation of two nearly spherically symmetrical layers of hydrogen plasma, an outer (slow wind) and inner (fast wind), moving from the central star with velocities of 20 km s^{-1} and 2000 km s^{-1} , respectively (Icke 2003, 1991; Kwok 1982). A great difference in velocities results in strong interaction of these layers, which manifests in non-thermal atom-ion collisions with typical impact velocities at $100\text{--}1000 \text{ km s}^{-1}$.

The atom-ion collisions at non-thermal impact velocities can also be important in interactions of the solar wind or comets' tails with earth (and some other planet) atmosphere. Since the particles' velocities in solar's wind are in domain $100\text{--}2000 \text{ km s}^{-1}$, the protons and helium ions collide with atoms and molecules in atmosphere with impact velocities at $\sim 1000 \text{ km s}^{-1}$. The well known consequence of these interactions, for example is aurora borealis in Earth's atmosphere, and some other consequences reported for example in Kharchenko et al. (2003). However, the role of radiative ion-atom processes has not yet been thoroughly taken into account. Thus, for UV and VUV could be important the radiative charge exchange processes of protons from the solar wind with oxygen atoms (almost resonant case), as well as with nitrogen atoms in the upper layers of Earth's atmosphere. Similar example is the interaction of the upper layers of Iovian atmosphere with streams of weakly ionized plasma, produced by volcanic activities of its satellite Io (Michael 2003; Michael & Bhardwaj 2000). The obtained results suggest that the ion-atom collision processes with impact velocities at 100 km s^{-1} can be important.

With regard to all these examples, our primary objective is to show the influence of ion-atom collisions at impact velocities $100\text{--}2000 \text{ km s}^{-1}$ in corresponding astrophysical plasmas. These processes are sources of the UV and VUV electromagnetic emission and are of interest in various diagnostics.

We will consider the following ion-atom collision processes:



in the middle range of above mentioned impact velocities. Here $H = H(1s)$ denotes a hydrogen atom in its ground state, and ε_λ is the energy of the photon with the wavelength λ . The main problem in this case is the non-thermal impact velocity, so the methods for solving similar problems with thermal collision velocities cannot be used.

Two previous papers are of importance for us here, one by Drukarev & Mihajlov (1974), another Ermolaev & Mihajlov (1991) (henceforth referred to as E&M), where the process of type (1) were treated in significantly different way than in the papers mentioned above. In the first, a dynamical semiclassical theory of the process (1) was evolved, and in the second, a method (based on this theory) was worked out for calculating spectral characteristics of a group of processes of that type in the range of intermediate collision energies. In E&M this method was tested just in the example of the process (1) at impact energies of 10 keV , in the infra-red and visible region. However, the results of later estimations (Mihajlov & Ermolaev 1998) showed that the intensity of the examined EM emission

at these impact energies should increase for several orders of magnitude at transition from the visible into the VUV region. In view of the fact that a proof of such results would imply that the process (1) in the case of the intermediate $H^+ + H$ impact energies may also be interesting in astrophysical aspect, we have performed corresponding calculations of spectral intensity of the EM emission generated in this process. Here we present the results of the calculations of this spectral intensity for wavelengths from visible up to the soft X-ray region, which refer to the $H^+ + H$ impact energies in the center of mass reference frame from 0.5 keV to 12.5 keV .

2. Theoretical remarks

Since above mentioned calculation has been done with help of the method described in details in previous work (E&M), we will briefly consider just its basic features. Quantum electrodynamical theory, on which this method is based, was developed for process (1) on a qualitative level (Drukarev & Mihajlov 1974), presuming that adiabatic and impact parameter approximation could be applied at the same time. Later (E&M), theory has been extended to all processes $A^+ + A$, where A is an atom with only one or two s -electrons outside of closed shells ($A = H, He, Li$ etc.), A^+ is its single positive ion, where both A^+ and A are in their ground states. The main assumption of the theory is that electronic component of each such a system could be described by superposition of two adiabatic electronic states of corresponding molecular ion A_2^+ , the ground and the first excited state, which exactly complete the group of electronic states of A_2^+ which are adiabatically correlated with electronic states of $A^+ + A$ system in separate limit, i.e. when the internuclear distance $R \rightarrow \infty$. These two molecular states will be here denoted by $|1, R\rangle$ and $|2, R\rangle$, and the corresponding molecular terms by $U_1(R)$ and $U_2(R)$, respectively. In order to apply both adiabatic and impact-parameter approximation together, the processes of type (1) in E&M were treated under conditions

$$v \lesssim v_{e,A}, \quad \varepsilon_\lambda \ll E(v), \quad (2)$$

where v is the impact velocity of atom A and ion A^+ , $E(v)$ is the corresponding impact energy of these particles in their center of mass coordinate system, and $v_{e,A}$ is the orbital velocity of s -electron in the outer shell of the atom A .

In agreement with the basic theory (E&M), the process (1) have been considered as an result of one photon radiative transitions $|2, R\rangle \rightarrow |1, R\rangle$, caused by interaction of electronic subsystem of $A^+ + A$ with the vacuum EM field. Cross-section density (per unit energy of photon) for such processes is

$$\frac{d\sigma_\omega(v)}{d\varepsilon_\omega} = 2\pi \int_0^\infty \frac{dP_\omega(\rho, v)}{d\varepsilon_\omega} \rho d\rho, \quad (3)$$

where ε_ω denotes energy of photon with angular frequency ω , ρ denotes impact-parameter, and $dP_\omega(\rho, v)/d\varepsilon_\omega$ – adequately defined total probability density for emission of photon during $A^+ + A$ collision with given v , ω and ρ . The applied procedure for determination of probability density requires just that the term splitting $U_{12}(R)$ and modulus of the corresponding dipole

matrix element, i.e. $D_{12}(R)$, are known as functions of R . Here we have in mind the quantities defined as

$$U_{12}(R) = U_2(R) - U_1(R), \quad D_{12}(R) = |\langle 1; R | \mathbf{D} | 2; R \rangle|, \quad (4)$$

where \mathbf{D} is the operator of the total electronic dipole moment of molecular ion A_2^+ . It has been important for us that this procedure allows formal application for all impact parameters, velocities and photon energies.

One should draw attention that the above mentioned treatment of processes of type (1) in the case of $A^+ + A$ system is justifiable only in the range of R where $U_{12}(R) > 0$. However, it is well known that this condition, which is always satisfied at large R (owing to definition of states $|1, R\rangle$ and $|2, R\rangle$), could be unsatisfied in the range of relative small R . On example, in the case of $A = \text{He}$, the terms $U_1(R)$ and $U_2(R)$, in accordance with Gupta & Matsen (1967), are crossing when R is close to $0.5a_0$, where a_0 is the atomic length unit. The possibility of such crossings causes that presented method could be applied to above described ion-atom systems, just under a presumption that the range $R < a_0$ gives neglectable account to EM radiation. Due to conceptual reasons, this method in E&M has been treated just in connection with infrared and visible range, where the last presumption is always satisfied.

However, it follows from just mentioned that in the case of $A = \text{H}$, when $U_{12}(R) > 0$ is satisfied everywhere, the method developed in E&M could be directly applied at once in infrared and visible and in the range of smaller wavelengths, until that is in agreement with adiabatic and impact parameter approximation. Having all this in mind, we have applied the method to calculation of spectral intensity of EM radiation generated by process (1), taking into account condition (2). The spectral intensity is defined by

$$S(\lambda, v) = \varepsilon_\lambda \cdot v \cdot \frac{d\sigma_\lambda(v)}{d\lambda}, \quad (5)$$

where $d\sigma_\lambda(v)/d\lambda$ is the corresponding density of cross section per unit of photon wavelength. We would like to emphasize that, owing to practical reasons, this definition is different from the definition in E&M, where the spectral intensity was expressed in terms of $d\sigma_\omega(v)/d\varepsilon_\omega$. Namely, here we took into account that, from applicative point of view, the definition of spectral characteristics as functions of wavelength is more suitable in UV (even close to the soft X-rays) region than the definition where the photon energy is used. Besides, we have had in mind that this difference can not disturb of application of the method developed in E&M, because the density of cross section $d\sigma_\lambda(v)d\lambda$ in Eq. (5) could be taken in oblique

$$\frac{d\sigma_\lambda(v)}{d\lambda} = \left[\frac{d\sigma_\omega(v)}{d\varepsilon_\omega} \right]_{A=\text{H}} \cdot \left| \frac{d\varepsilon_\omega}{d\lambda} \right|, \quad (6)$$

where $[d\sigma_\omega(v)/d\varepsilon_\omega]_{A=\text{H}}$ is the quantity given by Eq. (3) for $A = \text{H}$, and $\omega = 2\pi c/\lambda$, c being the speed of light.

3. Spectral intensity: Final expressions

Even though from Eqs. (5) and (6) follows that the spectral intensity is obtained on the base of procedure in E&M, the final

expressions, with help of which $S(\lambda, v)$ is determined here, are different from the corresponding expressions in E&M in many details. Because of that we thought it is necessary to write these expression:

$$S(\lambda, v) = \frac{2\pi^2 e^2}{3\hbar^2} \cdot \frac{\sum_{i=1,2} J_i(\lambda, v)}{v \lambda^4}, \quad (7)$$

$$J_i(\lambda, v) = 2\pi \int_0^{+\infty} I_i^2(\rho, \lambda, v) \rho d\rho, \quad (i = 1, 2), \quad (8)$$

$$I_1(\rho, \lambda, v) = \int_{-\infty}^{+\infty} \frac{2D_{12}(R(x, \rho))}{eR(x, \rho)} \cdot U_{12}(R(x, \rho)) \cdot x \sin(\Phi(x, \rho)) dx, \quad (9)$$

$$I_2(\rho, \lambda, v) = \int_{-\infty}^{+\infty} \frac{2D_{12}(R(x, \rho))}{eR(x, \rho)} \cdot U_{12}(R(x, \rho)) \cdot \rho \cos(\Phi(x, \rho)) dx, \quad (10)$$

$$\Phi(x, \rho) = \frac{1}{\hbar v} \int_0^x [\varepsilon_\lambda - U_{12}(R(x', \rho))] dx'. \quad (11)$$

where variable x has introduced instead of the time t , with help of relation $t = x/v$. In these expressions e and \hbar are absolute value of electron charge and Planck's constant respectively, $\varepsilon_\lambda = 2\pi\hbar c/\lambda$, $R(x, \rho) = (x^2 + \rho^2)^{1/2}$, and the quantities $U_{12}(R)$ and $D_{12}(R)$ are defined by relations (4). The values of these quantities, which have been treated as outer parameters of theory, are obtained from Bates et al. (1954) Greenland (1982), Ramaker & Peek (1972, 1973).

4. Results and discussion

Using Eqs. (5)–(11) we have done calculation of the spectral intensity $S(\lambda, v)$ for process (1), in the ranges: λ from $\cong 2$ nm to $\cong 453$ nm, and velocity v from $0.2v_0$ to $1.0v_0$. Higher border of velocity range is caused by the first of conditions (2), where the velocity $v_{e,A}$ for $A = \text{H}$ is very close to v_0 . Higher border of wavelength range is chosen so that to prolong results from E&M. Namely, the corresponding photon energy is closed to 2.75 eV and, accordingly to that, a little bit smaller than upper bound of photon energies in the mentioned paper, which was 0.107807 au, i.e. approximately 2.93 eV.

Results of our calculation are presented in Figs. 1–3. The curves in these figures show the behaviour of the spectral intensity $S(\lambda, v)$, as a function of λ , for several values of v :

$$v = 0.2v_0, 0.2828v_0, 0.4472v_0, 0.6325v_0, \\ 0.7746v_0, 0.8944v_0, 1.0v_0.$$

Expanding of these curves to short wave region has been restricted regarding to the second of conditions in Eq. (2). To enumerated impact velocities correspond impact energies from $E(v) \cong 0.5$ keV, for $v = 0.2v_0$, to $E(v) \cong 12.5$ keV, for $v = 1.0v_0$. The values of $S(\lambda, v)$ in Figs. 1–3 are given in

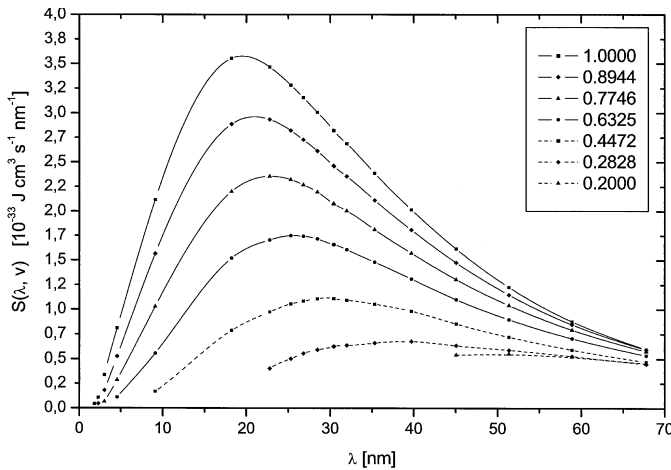


Fig. 1. Spectral intensity $S(\lambda, v)$ as a function of wavelength λ , in the region $\lambda \leq 67.79$ nm, for impact velocities v in the range $0.2v_0 \leq v \leq 1.0v_0$.

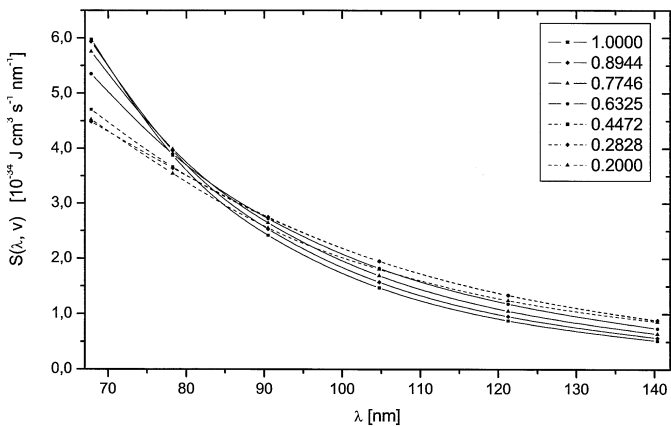


Fig. 2. Same as on the Fig. 1, but in the wavelength range 67.79 nm $\leq \lambda \leq 140.363$ nm.

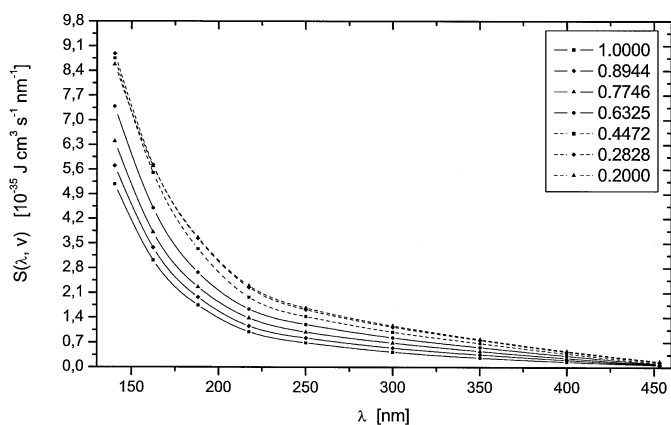


Fig. 3. Same as on the Fig. 1, but in the wavelength range $\lambda \geq 140.363$ nm.

the unit $J\text{ cm}^3\text{ s}^{-1}\text{ nm}^{-1}$, in order to be easily compared with the values from Mihajlov et al. (1993, 1994a) where the spectral intensity of EM emission due to process (1) has been calculated for weakly ionized layers in Solar atmosphere.

In E&M it was found that $S(\lambda, v)$ in the velocity range wider than here, uniformly increases when λ decreases, in infrared and visible wavelength range. Looking at the Figs. 1–3 one can conclude that such behaviour of $S(\lambda, v)$ continues through UV region, down to VUV region ($\lambda < 52$ nm), where the spectral intensity reaches its maximum. Apart of that, the position of maximum moves when v increases towards to smaller λ , from $\lambda \cong 51.4$ nm, at $v = 0.2v_0$, to $\lambda \cong 18.2$ nm, at $v = 1.0v_0$.

Presented figures demonstrate another feature of $S(\lambda, v)$. Namely, this quantity, as a function of v , changes its behaviour when λ decreases. Thus, one can see from Fig. 3 that for $\lambda > 215$ nm the spectral intensity uniformly decreases when v increases from $0.2v_0$ to $1.0v_0$. However, one can see in Figs. 2 and 3, that when λ decreases from 215 nm to 70 nm, the tendency of increasing of $S(\lambda, v)$ exists when v increases, at once in lower and in higher part of considered region of impact velocities. At last, Fig. 1 shows that $S(\lambda, v)$ uniformly increases when v increases for $\lambda < 60$ nm.

The described change in behaviour of $S(\lambda, v)$ is caused by the fact that, based on Eqs. (7)–(11), the impact velocity v influences to the spectral intensity in two ways. The first one, direct way, by the factor $1/v$ which multiplies whole right side of Eq. (7). The second one, indirect way, again through $1/v$, which multiplies the right side of Eq. (11). Namely, through the phase $\Phi(x, \rho)$, are immediately expressed the quantities $I_1(\rho, \lambda, v)$ and $I_2(\rho, \lambda, v)$ and, regarding to Eqs. (7) and (8), the spectral intensity $S(\lambda, v)$. Because of the structure of these expressions, the impact velocity influences to $S(\lambda, v)$ primary on the first, direct way, in long wave region, and on the second, indirect way, in the short wave region.

One should consider as the main result of this paper establishing of two facts. The first, in impact velocity range $0.2v_0 \leq v \leq 1.0v_0$, the values of $S(\lambda, v)$ increase more than 10^3 times when wave length changes from $\lambda > 450$ nm (see Fig. 3), to $18\text{ nm} < \lambda < 52$ nm where they are maximal (see Fig. 1). The other fact establishes by comparing values $S(\lambda, v)$, presented in Fig. 1, with the values of the total spectral intensity of EM emission determined by Mihajlov et al. (1993, 1994a). We would like to remind that in these papers, together with process (1), the process of photoassociation has been considered as a source of EM emission from weakly ionized layers of the solar atmosphere, and the total spectral intensity has been defined as a sum of contributions of both processes. Here we have in mind the photosphere and lower part of the chromosphere of the Sun, where the temperature of plasma is between 4500 K and 6000 K, and where the ionization degree is less than 0.001. This comparison shows that the values of $S(\lambda, v)$ in the region $18\text{ nm} < \lambda < 52$ nm, are more than 10^4 times larger than the values of the mentioned total spectral intensity in the region $365\text{ nm} \leq \lambda \leq 820$ nm, where it is maximal (see Mihajlov et al. 1994a).

5. Conclusions

One may conclude on the base of the presented results that the radiative process (1) for intermediate impact energies could be of interest from astrophysical point of view, as a new source of EM emission in the UV region and up to the X-rays region,

with a conspicuous maximum in the VUV region. First of all, we would like to emphasize the necessity of taking into account the process (1), in situation of two weakly ionized hydrogen plasma layers penetrating one into each other and moving with respect to the other with macroscopic velocity $\approx 10^6$ m/s, since this process can very strongly affect the shape of the considered continuous emission spectra in the UV, and especially in the VUV region. It is clear that all presented here could be applied to the interaction of weakly ionized hydrogen plasma layers with beams of intermediate energy hydrogen ions.

Presented results could be directly used when the relative velocity of the radiation source with respect to the observer, v_r , is much smaller than the speed of light. Namely, our estimations show that in the range $0 \leq v_r \leq 0.01c$ the spectral intensity changes for less than 1% due to Doppler effect. If the source velocity increases up to $0.1c$, the values of $S(\lambda, v)$ might change 10%–15%. This is due to the fact that the spectrum of EM emission generated in the processes (1) at intermediate impact energies is broad and does not contain sharp maxima and minima. On the other hand, if the velocity of the source increases up to $0.1c$, the values of spectral intensity would change for 10% to 15% from the ones presented here, which would in general require additional elaboration. However, one should keep in mind that for practical purposes such an accuracy of determination of the spectral intensity of EM emission is often quite acceptable.

In addition to what has been said so far, we would like to draw readers' attention to the fact that the process (1) at intermediate impact velocities may be of interest also from laboratory aspect. Namely, based on the results presented in this paper it is clear that the process (1) could be used for diagnostic purposes for probing hydrogen plasmas by ion or atom beams.

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References

- Bates, D. R. 1951, MNRAS, 111, 303
 Bates, D. R., Ledsham, K., & Stewart, A. L. 1954, Phil. Trans. R. Soc. A, 206
 Beauchamp, A., Wezemael, F., & Bergeron, P. 1997, ApJ, 108, 559
 Boggess, A. 1959, ApJ, 129, 432
 Drukarev, G. P., & Mihajlov, A. A. 1974, Opt. Spektrosk., 37, 384
 Eisloffel, J., Smith, M. D., & Davis, C. J. 2000, A&A, 359, 1147
 Ermolaev, A. M., & Mihajlov, A. A. 1991, J. Phys. B: At. Mol. Opt. Phys., 24, 155 (referred to in the present paper as E&M)
 Greenland, P. T. 1982, Phys. Rep., 81, 133
 Gupta, B. K., & Matsen, F. A. 1967, J. Chem. Phys., 47, 4860
 Icke, V. 1991, A&A, 251, 369
 Icke, V. 2003, Europhysicsnews, 34/1, 12
 Lobel, A., Dupree, A. K., Stefanik, R. P., & Torres, G. 2003, ApJ, 583, 923
 Kharchenko, V., Rigazio, M., Dalgarno, A., & Krasnopolsky, V. A., ApJ, 585, L73
 Kwok, S. 1982, ApJ, 258, 280
 Michael, M. 2003, Europhysicsnews, 34/3, 85
 Michael, M., & Bhardwaj, A. 2000, Geophys. Res. Lett., 27, 3137
 Mihajlov, A. A., & Dimitrijevic, M. S. 1992, A&A, 256, 305
 Mihajlov, A. A., Dimitrijevic, M. S., & Ignjatovic, Lj. M. 1993, A&A, 276, 187
 Mihajlov, A. A., Dimitrijevic, M. S., Ignjatovic, Lj. M., & Djuric, Z. 1994a, A&AS, 103, 57
 Mihajlov, A. A., Dimitrijevic, M. S., & Ignjatovic, Lj. M. 1994b, A&A, 287, 1026
 Mihajlov, A. A., Dimitrijevic, M. S., Ignjatovic, Lj. M., & Djuric, Z. 1995, ApJ, 454, 420
 Mihajlov, A. A., & Ermolaev, A. M. 1998, IV s'ezd astronomicheskogo obscestva, Trudy, 41, Moskva
 Popovic, L. C., Stanic, N., Kubicela, A., & Bon, E. 2001, A&A, 367, 780
 Ramaker, D., & Peek, J. 1972, J. Phys. B: At. Mol. Phys., 5, 2175
 Ramaker, D., & Peek, J. 1973, Atom. Data, 5, 167
 Stancil, P. C. 1994, ApJ, 430, 360