

# Reabsorption of self-generated turbulent energy by pick-up protons in the outer heliosphere

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**Abstract.** It is well known that newly created pick-up ions are first injected into a toroidal distribution in the velocity space which comoves with the solar wind. This distribution is, however, unstable with respect to excitation of resonant MHD waves, and thus by wave-driving some specific amount of the initial kinetic energy of pick-up ions is handed over to the ambient wave fields, while the pick-up ion velocity distribution thereby is rearranged. In this paper we describe the development of the wavenumber spectrum of pick-up proton-generated turbulence and the pick-up proton energy spectrum resulting from resonant reabsorption of this turbulence, by solving simultaneously the coupled system of two differential equations for the spectral turbulence power and the pick-up proton spectral intensity. We show that the majority of pick-up generated wave energy in the range of wavenumbers above the wavenumber at which the waves are produced is reabsorbed by the pick-up protons themselves.

**Key words.** acceleration of particles – waves – solar wind

## 1. Introduction

Plasma measurements by the Voyager spacecraft clearly show that the solar wind at its expansion to large solar distances behaves highly nonadiabatic such that its temperature even increases in the outer heliosphere (e.g. Richardson & Smith 2003). These measurements were challenges for the theory to explain such behaviour of solar wind plasma. The most popular idea on the solar wind heating at present is associated with the dissipation of pick-up proton-generated waves due to the cyclotron resonance interaction with solar wind protons (Williams et al. 1995; Smith et al. 2001; Fahr & Chashei 2002; Chashei et al. 2003). In fact pick-up protons are injected into a toroidal velocity distribution at the event of their production. This distribution since highly unstable (Wu & Davidson 1972) quickly evolves to nearly isotropic bispherical distribution, while free kinetic energy of the initial distribution is released in form of Alfvén waves. Then due to nonlinear cascading the wave energy is transported to smaller spatial scales where it is finally absorbed by solar wind protons.

It, however, follows from the results by Williams et al. (1995) and Smith et al. (2001) that most likely only a small fraction of this free energy (not more than 5%) is needed to explain the observed temperature profiles. The question thus arises why only this small fraction is consumed for solar wind proton heating and where the major portion of the wave energy produced during the process of pick-up proton isotropization is hidden. In the case of the quasi-radial magnetic field typical

for the polar solar wind, the wave energy can be considerably reduced due to large-scale fluctuations of the interplanetary magnetic field (IMF) as was shown by Zank & Cairns (2000). Recently Isenberg et al. (2003) have shown that the required reduction in the wave energy in the case of the azimuthal IMF occurs if effects of wave dispersion are taken into account. In the frame of our model presented here we propose an additional possibility for the reduction in the wave energy available for solar wind protons and show that the majority of pick-up generated wave energy in the range of wavenumbers above the wavenumber at which the waves are produced is reabsorbed by the pick-up protons themselves. The wave damping is connected with the cyclotron resonant interaction of the waves and pick-up protons which as a consequence leads to their stochastic acceleration. In fact, it follows from our calculations that practically no energy is available for solar wind proton heating. However, we show that a small fraction of the wave energy can nevertheless be absorbed by solar wind protons, if we remove some simplifications from our present model.

## 2. Self-consistent description of pick-up protons and pick-up proton-generated turbulence spectra

At large heliocentric distances ( $\geq 10$  AU) the IMF is almost azimuthal (especially near the ecliptic). In these regions velocities of newly created pick-up ions thus are almost perpendicular to the magnetic field lines. According to Huddleston & Johnstone (1992) the free energy density of the ring

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distribution of pick-up ions to the first-order accuracy, ignoring terms of order  $v_A/U$ , in the case of perpendicular pick-up is

$$E = m_i n_i v_A U / 2, \quad (1)$$

where  $m_i$  is the ion mass,  $n_i$  is the number density of the pick-up ions in the ring distribution,  $v_A$  is the Alfvén speed, and  $U$  is the solar wind speed. In this case generated Alfvén waves propagating parallel ( $k > 0$ ) and antiparallel ( $k < 0$ ) to the magnetic field have equal intensities. Then the total energy density injected per unit of time into waves propagating in both directions due to instability of the ring distribution of pick-up protons can be written as

$$Q_T = \beta_i n_H m_p v_A U / 2, \quad (2)$$

where  $\beta_i$  is the ionization frequency of and  $n_H$  is the number density of hydrogen atoms. Since in the solar wind rest frame the initial speed of pick-up protons originating due to ionization of the atoms equals the local speed of the solar wind, the injection of the energy into the turbulent wave field occurs around the wavenumber  $k_{inj} = \Omega_p / U$ , where  $\Omega_p$  is the proton gyrofrequency (see e.g. Huddleston & Johnstone 1992).

The waves generated around  $k_{inj}$  suffer diffusion in  $k$ -space due to nonlinear wave-wave interactions. Thus the energy input from pick-up protons to the waves is cascading up and down from the injection wavenumber. A diffusion approximation for spectral transfer of energy in isotropic hydrodynamical turbulence has been developed by Leith (1967). More recently, Zhou & Matthaeus (1990) extended the diffusion theory for application to solar wind turbulence. The transport equation for the wavepower spectral density which they derived describes effects of spatial convection, cascading of energy in wavenumber space, injection and damping of wave energy. The equation has been applied by Miller & Roberts (1995) and Miller et al. (1996) to study stochastic acceleration of solar wind protons and electrons in impulsive solar flares. Also le Roux & Ptuskin (1998) made use of this equation to study stochastic acceleration of pick-up ions by ambient solar wind turbulence taking into account damping of waves due to energization of the pick-up ions. While the original transport equation by Zhou & Matthaeus (1990) has been derived for isotropic turbulence, Miller & Roberts (1995) and le Roux & Ptuskin (1998) applied it to slab Alfvénic turbulence due to lack of any better theory. In our present description for pick-up proton-generated turbulence we will follow the two last-mentioned papers.

Under quasi-radially symmetric conditions the transport equation for the spectral density of the wavepower,  $W_k$ , has the following form:

$$\frac{\partial W_k}{\partial t} + \frac{U}{r^3} \frac{\partial}{\partial r} (r^3 W_k) = \frac{\partial}{\partial k} \left( D_{kk} \frac{\partial W_k}{\partial k} \right) + \gamma_k W_k + Q_k, \quad (3)$$

where  $W_k$  is normalized in such a way that  $\int_{-\infty}^{+\infty} W_k(r, k) dk$  is the total energy density of waves propagating parallel and antiparallel to the magnetic field. The second term on the left-hand side describes the spatial variation of spectral wavepower in the expanding solar wind and, in the absence of other

terms in the above equation, leads to the well-known WKB result. Here and in the following we assume that the solar wind speed  $U$  is constant. The first term on the right-hand side represents diffusion in wavenumber space. In the frame of the Kolmogorov phenomenology, the diffusion coefficient according to Zhou & Matthaeus (1990) has the following form:

$$D_{kk}(r, k) = C^2 v_A |k|^{7/2} \sqrt{\frac{W_k}{B^2/4\pi}}, \quad (4)$$

where  $C^2$  is the Kolmogorov constant which is adopted here to be equal to 1, and  $B$  is the magnetic field magnitude. We prefer to use the Kolmogorov form of the diffusion coefficient, since it describes solar wind turbulence more adequately than the Kraichnan phenomenology in the case when the normalized cross-helicity is small (Miller & Roberts 1995). The second term in the right-hand side of Eq. (3) describes the damping of Alfvén waves due to resonant interaction with pick-up protons. The characteristic time-scale of this process is larger than that of the evolution of the ring velocity distribution to the bispherical distribution and it results in stochastic acceleration of the pick-up protons. The associated damping rate  $\gamma_k$  has the following form (e.g. Miller & Roberts 1995):

$$\gamma_k(r, k) = -2\pi^2 e^2 \left( \frac{v_A}{c} \right)^2 \frac{1}{|k|} \int_{v_0}^{\infty} \frac{4\pi v}{m_p} f(r, v) dv, \quad (5)$$

where  $e$  is the electron charge,  $v_0 = \Omega_p / |k|$ , and  $f(r, v)$  is the isotropic part of the velocity distribution function of pick-up protons. Here and in the following the symmetry of the spectral density  $W_k$  about  $k = 0$  is used. Finally, the source term  $Q_k$  is the energy input in the wave field from pick-up protons during the process of isotropization. As is evident from the foregoing, this term can be written as

$$Q_k(r, k) = \frac{1}{2} Q_T(r) \left[ \delta(k + k_{inj}) + \delta(k - k_{inj}) \right]. \quad (6)$$

In Eq. (6) the  $\delta$ -functions imply that the free energy of the ring distribution of pick-up protons is exclusively injected at the wavenumber  $k_{inj}$  and this energy is redistributed among waves propagating parallel and antiparallel to the magnetic field in equal proportions.

Immediately after their injection pick-up protons have a highly anisotropic ring velocity distribution which is unstable and results in generation of Alfvén waves as discussed above. Simultaneously, the pick-up protons undergo pitch-angle scattering in the self-generated wave field, redistributing them from the ring velocity distribution onto a nearly isotropic bispherical distribution. In addition to pitch-angle scattering the pick-up protons experience energy diffusion due to the interaction with turbulent fluctuations. We assume in this paper that processes of wave generation and pick-up proton isotropization are essentially faster than any other processes under consideration and hence will consider further only isotropic velocity distributions of pick-up protons. The Fokker-Planck type transport equation for the isotropic velocity distribution function has the following form:

$$\frac{\partial f}{\partial t} + U \frac{\partial f}{\partial r} = \frac{1}{v^2} \frac{\partial}{\partial v} \left( v^2 D_{vv} \frac{\partial f}{\partial v} \right) + \frac{2Uv}{3r} \frac{\partial f}{\partial v} + S, \quad (7)$$

where  $D_{vv}(r, v)$  is the pitch-angle-averaged velocity diffusion coefficient and  $S(r, v)$  is the source of pick-up ions. The velocity distribution function is normalized such that  $\int_0^\infty 4\pi v^2 f(r, v) dv$  is the number density of pick-up protons. The velocity diffusion coefficient can be written as (see e.g. Miller & Roberts 1995)

$$D_{vv}(r, v) = 2 \frac{\pi^2 e^2}{m_p^2} \left( \frac{v_A}{c} \right)^2 \frac{1}{v} \int_{k_0}^\infty \left[ 1 - \left( \frac{k_0}{k} \right)^2 \right] \frac{W_k}{k} dk. \quad (8)$$

In Eq. (8)  $k_0 = \Omega_p/v$  is the minimum resonant wavenumber for a pick-up proton of speed  $v$ . The source term in Eq. (7) describing production of pick-up protons due to ionization of interstellar hydrogen atoms is

$$S(r, v) = \frac{\beta_i n_H}{4\pi U_{sh}^2} \delta(v - U_{sh}), \quad (9)$$

where  $U_{sh} = U(1 - v_A/2U)$  is the radius of the pick-up proton shell distribution reduced by the losses of their energy during the process of wave generation (e.g. Fahr & Chashei 2002). Along the upwind direction the number density of interstellar hydrogen atoms is sufficiently well given by (Fahr 1971)

$$n_H(r) = n_{H\infty} \exp\left(-\frac{\beta_{iE} r_E^2}{V_{H\infty} r}\right). \quad (10)$$

The system of Eqs. (3) and (7) is closed and makes it possible to determine  $W_k$  and  $f$  self-consistently. These equations are coupled both by the damping rate  $\gamma_k$  given by Eq. (5) and by the velocity diffusion coefficient  $D_{vv}$  given by Eq. (8). Only time-independent solutions of Eqs. (3) and (7) are considered here. In order to find the solutions the following iterative procedure has been applied. First we solve Eq. (7) taking  $D_{vv} = 0$ . Then  $\gamma_k$  is calculated from Eq. (5). On this basis a solution of Eq. (3) for  $W_k$  is found. This solution is then used to calculate  $D_{vv}$  from Eq. (8). Finally, we again solve Eq. (7) but now taking into account the calculated diffusion coefficient  $D_{vv}$ . The new velocity distribution function is the base for calculations of new values of  $\gamma_k$  and  $W_k$ . The iterative process continues until the velocity distribution function and the wave energy spectral density are essentially independent on the iteration number.

To solve Eq. (7), the method of transforming this partial differential equation of second order into a set of stochastic differential equations (SDEs) was used (see e.g. Chalov et al. 1995, 1997). Each solution of the SDEs is a stochastic trajectory in phase-space. In order to find the velocity distribution function we simulate a statistically relevant set of stochastic trajectories and determine the corresponding density of these trajectories in phase-space. Unlike for Eq. (7), the finite-difference method, based on a splitting of physical processes, was used to solve Eq. (3). For the convective part of the equation the explicit counterflow scheme was applied. The diffusive part of the equation was treated with an implicit scheme in which the Newton method was used to solve the nonlinear system of equations. To achieve the convergence of numerical solutions, the source term with the  $\delta$ -function was approximated by a set of bounded functions. As the boundary conditions for Eq. (3) we adopt that  $W_k = 0$  at the smallest,  $k_{\min}$ , and the largest,  $k_{\max}$ , values of the wavenumber. In our calculations  $k_{\min} = 2 \times 10^{-3} k_{\text{inj}}$ , which

corresponds to the correlation wavenumber of slab ambient solar wind turbulence, and it means that this turbulence is not taken into account here. The largest value of the wavenumber,  $k_{\max}$ , has been varied in the range  $(10^3 - 10^4) k_{\text{inj}}$  to make sure that the obtained solutions of Eq. (3) do not depend on  $k_{\max}$  in a vicinity of the solar wind proton dissipation scale  $k_{\text{dis}} = \Omega_p/v_A$  (e.g. Leamon et al. 1998). The knowledge of  $W_k$  near  $k_{\text{dis}}$  will allow to estimate the diffusive flux of wave energy through  $k_{\text{dis}}$  which can be absorbed by solar wind protons.

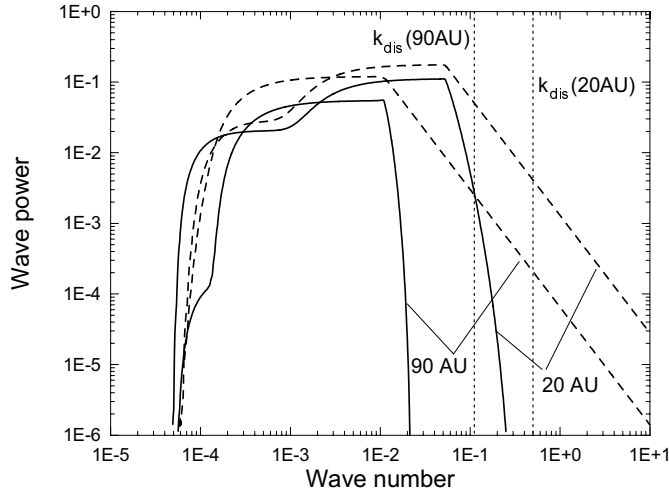
At the inner spatial boundary a stationary solution of Eq. (3) without the convective term was found and it was then used as the inner boundary condition for the complete version of Eq. (3). This boundary condition is appropriate for large values of  $k$  when the wave energy diffusion is the dominant process as compared with convection, but it can be incorrect at small  $k$ . In order to check the influence of this boundary condition on solutions of Eqs. (3) and (7), we placed the inner boundary sequentially at 5 and 10 AU. It turns out that  $W_k$  does not depend on the inner boundary condition practically in the whole range of  $k$  under consideration (except for  $k \sim k_{\min}$ ) at  $r \geq 20$  AU and in the range  $k \geq 10 k_{\min}$  at  $r > 10$  AU.

### 3. Results of numerical calculations

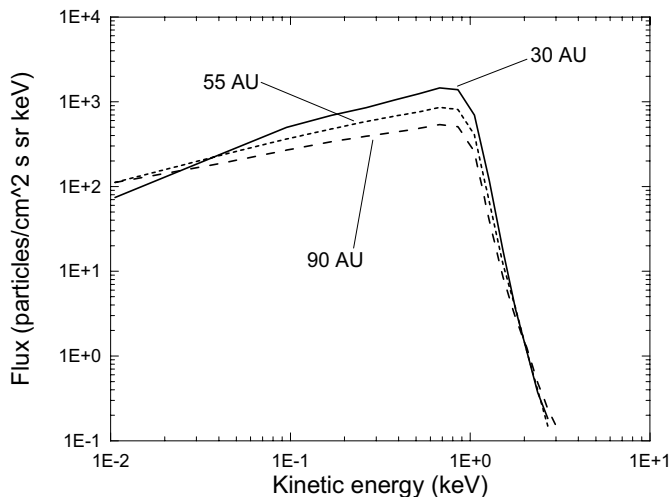
We take the solar wind and Alfvén speed to be  $450 \text{ km s}^{-1}$  and  $45 \text{ km s}^{-1}$ , respectively. At the orbit of the Earth we adopt  $\Omega_{pE} = 0.5 \text{ s}^{-1}$  and  $\beta_{iE} = 0.63 \times 10^{-6} \text{ s}^{-1}$ . The parameters of interstellar neutral hydrogen are  $V_{H\infty} = 20 \text{ km s}^{-1}$  and  $n_H(TS) = 0.1 \text{ cm}^{-3}$ . Figure 1 shows normalized spectral wave powers  $W_k^* = (er_E^{1/2}/m_p c U)^2 W_k$  as functions of the normalized wavenumber  $k^* = kU/\Omega_{pE}$  at distances of 20 and 90 AU. The solid lines are calculated spectral powers for the case when all terms in Eq. (3) are taken into account, while the dashed lines are spectral powers for the case when  $\gamma_k = 0$ , i.e. when the wave damping owing to resonant interactions between the waves and pick-up protons is not taken into account. The maxima of  $W_k^*$  correspond to the injection wavenumbers  $k_{\text{inj}}$ . They shift towards small  $k$  as the distance from the Sun increases due to decrease in  $\Omega_p$ . The dashed lines at  $k \geq k_{\text{inj}}$  have the Kolmogorov slope ( $W_k \propto k^{-5/3}$ ). In contrast, there is a plateau distribution below  $k_{\text{inj}}$  appearing at both distances which is formed due to the decrease of  $D_{kk}$  with decreasing  $k$ . It is evident from Fig. 1 that pick-up proton-generated turbulence is essentially absorbed by the pick-up protons. Especially dramatic changes in the wave spectra occur at large wavenumbers since pick-up protons interact resonantly with waves with  $k > (U/v)k_{\text{inj}}$ , where  $v \lesssim U$ . The vertical dotted lines in Fig. 1 show the solar wind proton dissipation wavenumbers at 20 and 90 AU. It is seen that the wave energy density at  $k_{\text{dis}}$  is many orders of magnitude smaller in the case when the wave damping by pick-up protons is taken into account as compared with the case when it is ignored.

Possibly the main simplification which we presently have used in our model is connected with the fact that the sources of wave energy are considered as  $\delta$ -functions centered at the injection wavenumber  $k_{\text{inj}}$ . This enforces that the wave energy is more or less completely absorbed by pick-up protons before it





**Fig. 1.** Normalized spectral wave powers  $W_k^*$  as functions of the normalized wavenumber  $k^*$  at 20 and 90 AU. The solid lines are spectral powers in the case when the wave damping owing to resonant interactions between the waves and pick-up protons is taken into account, while the dashed lines are spectral powers at  $\gamma_k = 0$ . The vertical dotted lines show the solar wind proton dissipation wavenumbers.



**Fig. 2.** Differential fluxes of pick-up protons (in the solar wind rest frame) at different distances from the Sun.

can diffuse up to  $k_{\text{dis}}$ . In fact, however, the injected energy has a smooth distribution with maxima near  $\pm k_{\text{inj}}$  and this distribution spreads above  $k_{\text{dis}}$  (Huddleston & Johnstone 1992). Based on Eq. (37) by Huddleston & Johnstone (1992) we have estimated that the wave power above  $k_{\text{dis}}$  is about 10% of the total free energy power injected due to isotropization of pick-up protons. It means that at least a minor part of this energy can serve as energy input to solar wind protons.

Figure 2 shows differential fluxes of pick-up protons (in the solar wind rest frame) at different distances from the Sun. The absorption of wave energy by pick-up protons results in their acceleration and formation of the high energy tails, as shown in Fig. 2. Although particles with energies above the injection energy  $\approx 1$  keV are clearly seen, the high energy tails are not highly pronounced, suggesting that stochastic acceleration of pick-up protons in self-generated wave fields is not very

efficient. Note that the pick-up proton spectra at energies  $>1$  keV calculated by Chalov et al. (1997) are more populated by high energy particles. In contrast to the present paper, Chalov et al. (1997) did consider ambient solar wind turbulence ignoring possible wave damping, and, what is even more important, in addition to acceleration by small-scale Alfvénic turbulence, they took into account acceleration by large-scale fluctuations in the solar wind speed and magnetic field magnitudes. It was shown in that paper that acceleration by the large-scale fluctuations is the more important mechanism in the outer heliosphere to produce particles of higher energies.

#### 4. Conclusions

We have presented a selfconsistent description of the evolution of pick-up proton spectra interacting with the power spectrum of purely pick-up driven MHD waves. As we show the resulting power spectrum shows an extended plateau below the injection wave number  $k_{\text{inj}}$  and a sharp decrease at  $k > k_{\text{inj}}$ . At the solar wind proton dissipation wavenumber  $k_{\text{dis}}$  the wave energy density is many orders of magnitude smaller in the case when the wave damping by pick-up protons is taken into account as compared with the case when it is ignored. The wave energy input into solar wind protons thus turns out to be negligible in the frame of our model. However, we show that up to 10% of the total free energy is still available to heat the solar wind background if we remove some simplifications from the model. The pick-up proton spectra even at large distances (90 AU) do still show high energy shoulders populated by preaccelerated particles which can be promising candidates for undergoing a shock-drift acceleration at the termination shock to eventually become anomalous cosmic ray particles.

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