Fluctuations of K-band galaxy counts

M. López-Corredoira¹ and J. E. Betancort-Rijo²,³

¹ Astronomisches Institut der Universität Basel, Venusstrasse 7, 4102 Binningen, Switzerland
² Instituto de Astrofísica de Canarias, C/ Vía Láctea, s/n, 38200 La Laguna (Tenerife), Spain
³ Departamento de Astrofísica, Universidad de La Laguna, Tenerife, Spain

Received 15 May 2003 / Accepted 31 October 2003

Abstract. We measure the variance in the distribution of off-plane (|b| > 20°) galaxies with $m_K < 13.5$ from the 2MASS K-band survey in circles of diameter between 0.344° and 57.2°. The use of a near-infrared survey makes the contribution of Galactic extinction to these fluctuations negligible. We calculate these variances within the standard ΛCDM model assuming that the sources are distributed like halos of the corresponding mass, and it reproduces qualitatively the galaxy count variance. Therefore, we conclude that the counts can be explained in terms only of the large scale structure. A second result of this paper is a new method to determine the two point correlation function obtained by forcing agreement between model and data. This method does not need the knowledge of the two-point angular correlation function, allows an estimation of the errors (which are low with this method), and can be used even with incomplete surveys.

Using this method we get $\xi(z = 0, r < 10 h^{-1} \text{Mpc}) = (29.8 \pm 0.5)(r/h^{-1} \text{Mpc})^{-1.79^{+0.03}_{-0.02}}$, which is the first measure of the amplitude of $\xi$ in the local Universe for the K-band. It is more or less in agreement with those obtained through red optical filter selected samples, but it is larger than the amplitude obtained for blue optical filter selected samples.

Key words. cosmology: large-scale structure of Universe – infrared: galaxies – galaxies: statistics – cosmology: theory

1. Introduction

The distribution of the number of galaxies or clusters of galaxies in a certain volume $V$ can be used to test the large-scale structure of the universe. The calculation of probabilities of different structures in a clustering model are sometimes amenable to analytical techniques: for instance, for voids (Betancort-Rijo 1990), density of Abel clusters (Betancort-Rijo 1995), number of galaxies in a randomly placed sphere and cluster density profiles (Betancort-Rijo & López-Corredoira 1996), etc. The calculation of variance of the number of galaxies within the cones corresponding to the circles in the sky, or another 2D figure, (i.e., galaxy counts) is another possibility, and even easier to compare with the observations since we do not need any information about the redshifts of the individual galaxies. This is precisely the purpose of this paper: the calculation from a model of the variance of the number of galaxies in the different regions of the sky, and to see how well they fit real data obtained directly from a sky survey. This will allow us to determine some parameters of the two-point correlation function independent of any other method.

One disadvantage of galaxy counts is that they are affected by Galactic extinction, and it is difficult to separate the fluctuations due to this extinction from the real fluctuation in the galaxy distribution. However, the arrival of near-infrared galaxy surveys such as DeNIS or 2MASS provides a way to avoid this problem, since the extinction in K-band, the one to be used in this paper, is 10 times lower than in V-band. In V-band, the dispersion of galaxy counts due only to the fluctuations of extinction in scales around 1 degree is around 30% (see Eq. (4) and the parameters given in the paragraph before Eq. (4)) at galactic latitudes $|b| > 20°$, while in K-band is only ~3%. Since the fluctuations we measure due to the large-scale structure are 25–80% in the explored scales, calculation of the structure parameters are very sensitive to the exact knowledge of the galactic extinction in visible bands; however, in the near-infrared, the calculation of the parameters is much less sensitive to the extinction and we can be sure that the measured fluctuations correspond to the real distribution of galaxies. Moreover, in the K-band we have an important advantage: the K-correction is nearly independent of galaxy type (Mannucci et al. 2001), and this will allow us to make the K-corrections without knowing the galaxy type of the sources in the survey.

Maps of galaxy counts have been available for quite a long time and have been used for several purposes. Zwicky (1957, p. 84), for instance, produced them from the available catalogues in visible bands, with a smooth correction of galactic extinction as a function of $b$, to explore intergalactic extinction.

The purpose of this paper is first to show that the count variances are in qualitative good agreement with those obtained assuming that the source correlation function is equal to that obtained in the standard ΛCDM model for virialized objects.
with circular velocity larger than 120 km s$^{-1}$. Then, by considering a slightly modified power law (the correlation of the those haloes is almost exactly a power law in the most relevant region) and determining the values of the amplitude and exponent leading to best agreement with the actual counts. This have proved to be an accurate way of determining the correlation function of the sources. Cumulative galaxy counts (in optical) versus the limiting magnitude have been used to constrain the departure from homogeneity–fractal distribution–at large scales (Sandage et al. 1972). A log $N(m) \propto 0.6m$ indicates total homogeneity and this is more or less observed in near infrared counts too: with 2MASS data for low redshift (Schneider et al. 1998, Fig. 5) or other surveys. Here, we will not explore further the homogeneity at very large scales but focus on the clustering at small scales in the local Universe ($z < 0.4$). The use of galaxy counts to derive the two-point angular correlation function has been considered by Porciani & Giavalisco (2002); however, we will recover directly (under some assumptions) the two-point correlation function of the sources, and, therefore, their relative biasing with respect to the mentioned halos.

The paper is divided as follows: Sect. 2 describes the observational data and how to measure the variance in the distribution of galaxies; Sect. 3 explains how to calculate this variance within a model of the large scale structure; and Sect. 4 makes the comparison between data and model predictions and derives the parameters of the two-point correlation function necessary to get an agreement between model and data.

2. Galaxy counts

The data used in this work have been taken from the extended sources of the 2MASS-project (Jarrett et al. 2000). All-sky release (http://www.ipac.caltech.edu/2mass/releases/docs.html). Completeness limit: $m_K = 13.5$ (Schneider et al. 1998; Jarrett et al. 2000; Maller et al. 2003, Sect. 2). Assuming an average color of $B-K \approx 4$, this is equivalent to an optical limit of 17.5 (Schneider et al. 1998), deep enough for statistical studies of the large scale structure, the local structures are not too predominant, but shallow enough to exclude high redshift [the galaxies have redshifts $z < 0.3$–0.4 (Cole et al. 2002), and an average ($z \approx 0.083$ (see below for details)]. We do not analyze samples of galaxies below this limit of $m_K$ (for instance, $m_K < 12.0$ or $m_K < 11.0$, etc.) because this would represent the very local Universe rather than the large scale structure.

In Fig. 1, we see a representation of the fluctuations at a scale of $3^\circ$ in the whole sky (with on average around 150 galaxies per area with complete coverage). In the zone of avoidance, $|b| > 20^\circ$, there is a clear deficit of galaxies due to the extinction, which is small compared to the optical but negligible in near plane regions. The reliability is larger than 99% in $|b| > 20^\circ$ (Schneider et al. 1998).

In order to quantify the fluctuations, we count the number of galaxies in each circle of the sky with angular radius $r_0$. In 3D space, we count the galaxies within the corresponding cone in the line of sight. We select only the circular regions in $|b| > 20^\circ$ which were covered more than 90%. The cumulative counts up to magnitude 13.5 are expressed per unit area (we divide the number of galaxies per region by the area, $S$, of the region). We measure the counts in randomly placed circular areas instead of a regular mesh. Since the number of random circles in which we measure the galaxy counts is around 8 times larger than the total area divided by the area of the circle, the information lost is very low (0.033% of the galaxies would not be in any circle if the distribution were Poissonian). Once we have these counts for each of the $n$ regions containing respectively $n_i$ galaxies, we calculate the average,

$$<N> = \frac{1}{S} \sum_{i=1}^{n} n_i \approx 14.4 \text{ deg}^{-2},$$

and the dispersion with respect to the average,

$$\sigma_N(\theta_b) = \sqrt{\frac{1}{S^2} \sum_{i=1}^{n} n_i^2} - \frac{\sum_{i=1}^{n} n_i^2}{S^2}.$$  

This last number gives us information about the amount of structure in the 3D cones with angular radius $\theta_b$. An example of the distribution of counts is given in Fig. 2 and is compared with a negative binomial distribution with the same variance, to which a wide variety of clustering processes lead (Betancort-Rijo 2000).

The fluctuations due to the intrinsic clustering of galaxies, $\sigma_{\text{ext}}$, will be the total fluctuations minus the other independent sources of fluctuation subtracted quadratically:

$$\langle (\delta N)_{\text{ext}}^2 \rangle = \langle (\delta N)_{\text{Poission}}^2 \rangle - \langle (\delta N)_{\text{ext}}^2 \rangle - \langle \delta N_{\text{other}}^2 \rangle - \langle \delta N_{\text{other}} \rangle.$$  

The galactic extinction rms fluctuations, $\sigma_{\text{ext}}$, may be evaluated using the Schlegel et al. (1998) maps of extinction: in $|b| > 20^\circ$, the mean extinction in $K$-band is $A_K = 0.019$ mag, and the fluctuations of this extinction are $\sigma_{A_K} \approx 0.020$ mag (in the scales with $\theta_b$ around 1 degree; it changes slightly with the scale). Given that log $N \propto \text{constant} + 0.6m_{K}$ (for a homogeneous distribution; see, for instance, Sandage et al. 1972):}

$$\sigma_{\text{ext}} \equiv \sqrt{\langle \delta N_{\text{ext}}^2 \rangle_{\text{ext}}} = 0.6 \langle \ln 10 \rangle \sigma_{A_K} < < N >.$$  

For the $K$-band $\sigma_N >> \sigma_{\text{ext}}$, so the small uncertainties in the knowledge of the statistical properties of the extinction are not important. Note however that in visible bands $\sigma_{\text{ext}}$ will be 10 times larger, comparable to the dispersion due to the large-scale structure and, therefore, it is not possible to obtain accurate information about this last dispersion without an accurate measure of the extinction fluctuations. This is precisely the advantage of infrared surveys.

The Poissonian fluctuations are the most important contribution, other than those due to large scale structure, and it can be exactly determined:

$$\sigma_{\text{Poission}} \equiv \sqrt{\langle (\delta N)_{\text{Poission}}^2 \rangle} = \sqrt{\langle <N> <1/A> \rangle}.$$  

Once the fluctuations due to the structure are obtained by mean of Eq. (3), these can be compared with the predictions of a standard model, as in the following sections.
Fig. 1. Aitoff projection in galactic coordinates (up) and equatorial coordinates (down) of the 2MASS/“All-sky release” galaxy counts with $m_K < 13.5$; average: 14.4 galaxies/deg$^2$. Galaxies were counted in square regions of 3 deg $\times$ 3 deg. A smoothing of the map was carried out. Black color stands for regions with very few galaxies, and high density is represented by white color. Extinction effects are negligible in the fluctuations. Poissonian noise is not negligible but it is relatively small compared to the intrinsic fluctuations due the large scale-structure (see text).

3. Model

The calculation of the fluctuations $\frac{\delta N}{N} \equiv \frac{\sqrt{\langle (\delta N)^2 \rangle}}{\langle N \rangle}$ due to the clustering of the galaxies is carried out in the following way:

$$\langle N \rangle = A\pi \int_{\theta_1 < \theta_0} r^2 \Phi[M_K < M_{K,\text{limit}}(r)] dr,$$

(6)

$$\langle (\delta N)^2 \rangle = \pi^2 \int_{\theta_1 < \theta_0} dr_1 \int_{\theta_2 < \theta_0} \Phi[M_K < M_{K,\text{limit}}(r_1)]$$

$$\times \Phi[M_K < M_{K,\text{limit}}(r_2)] \delta[|r_1 - r_2|, z(r_1)] dr_2,$$

(7)

where $r$ stands for the radial distance in comoving coordinates, $A = 2\pi(1 - \cos \theta_0)$ is the area (in steradians) of the circular regions, $\pi$ is the mean space density of galaxies (which is irrelevant in this context since it cancels when we calculate $\delta N/N$), $\Phi[M_K < M_{K,\text{limit}}(r)]$ is the cumulative normalized luminosity function up to absolute magnitude $M_{K,\text{limit}}[r_{\text{phys}} = r/(1 + z)] = m_{K,\text{limit}} + 5 - 5 \log r_{\text{phys}} - CorrK(r_{\text{phys}})$. The K-correction is $CorrK[r_{\text{phys}} = r/(1 + z)] = -2.955z(r_{\text{phys}}) + 3.321z(r_{\text{phys}})^2$ for $z < 0.40$ (beyond $z = 0.40$ we see almost no galaxy), for any galaxy type (obtained from the fit of data from Mannucci et al. 2001). The redshift is $z(r_{\text{phys}})$ corresponds to the velocity $v(r_{\text{phys}}) = 100r_{\text{phys}}(h^{-1} \text{Mpc}) \text{ km s}^{-1}$ (linear Hubble law taken as an approximation for low redshift galaxies). The luminosity function $\Phi$ is taken from Kochanek et al. (2001). We neglect the evolution of the luminosity function, which is mild ($\frac{dM_k}{dz} \approx 0.5$, Pozzetti et al. 2003) and very small at the mean redshift of our galaxies ($\langle z \rangle \approx 0.083$); a shift of $\sim 0.04$ mag will produce a variation of $\sim 2\%$ in both $\langle \delta N \rangle$ and $\langle N \rangle$ in the same direction; since we calculate the rate $\frac{\delta N}{N}$, this small variations cancel out each other to first order, and only second order variations remain, which are negligible. The errors associated with the uncertainty in the luminosity itself affect the normalization, which is also irrelevant for the calculation of $\delta N/N$ since the
normalization factor cancels out. Finally, the two-point correlation function is taken to be (for comoving coordinates):

\[
\xi(r, z) = \xi(r, z = 0)(1 + z)^{-3(\gamma + \epsilon)},
\]

(8)

\[
\xi(r, z = 0) = \begin{cases} 
-1, & r < 0.01 \\
\left(\frac{r}{r_0}\right)^\gamma, & 0.01 < r < 10 \\
FT[P(K)], & r \geq 10 
\end{cases}
\]

(9)

where \(FT[\ldots]\) stands for Fourier transform, and \(r\) is given in units \(h^{-1}\) Mpc. The truncation of the two-point correlation function is set at 10 \(h^{-1}\) kpc, but the exact value does not matter; our results do not depend on this minimum scale, it is introduced to avoid computer algorithm problems at very small scales. More exactly, the change of regime should occur at the matching point but, for the interesting range of parameters, this takes place around 10 \(h^{-1}\) Mpc. The power spectrum for the linear part \((r \geq 10 h^{-1}\) Mpc), \(P(K)\), is given for a CDM scenario with initial Harrison-Zeldovich power spectra (Bardeen et al. 1986):

\[
P(K) = \frac{A \ln(1 + 2.34q)^2}{K \sqrt{1 + 3.89q + (16.1q)^2 + (5.46q)^3 + (6.71q)^4}}
\]

\[
q = \frac{K}{\Omega_{m0}h \text{ (h/Mpc)}},
\]

(10)

and we adopt \(A = 1.984 \times 10^4\) (\(r_0 = 1\)) and \(\Omega_{m0}h = 0.21\). Within small scales \((\theta_0 \leq 0.040\) rad\), the non-linear part is predominant, so slight variations of the parameter \(\Omega_{m0}h\) or the normalization \(A\) will not lead to changes in our results. For instance, numerical experiments have shown that a variation of 10% in the amplitude \(A\) leads to variations of \(\approx 2\%\) in \(\delta N/N(\theta_0 = 0.040\) rad\) or \(\approx 0.7\%\) in \(\delta N/N(\theta_0 = 0.010\) rad\) and much less for lower \(\theta_0\); since the error bars of \(\delta N/N\) are of this order, we consider as negligible these small variations due to the change of the parameters in \(P(K)\). In scales larger than \(\theta_0 = 0.040\) rad, the errors bars are too large compared to the possible variations in \(P(K)\). In all ranges, even variations up to 10–20% in \(P(K)\) will not affect \(\delta N/N\) compared to the errors.

Two questions arise as to the suitability of Eq. (9): 1) is the assumption of a power-law for non-linear scales appropriate? 2) could we apply a power-law for all scales instead of a \(\Lambda CDM\) model for the linear regime? Both questions are answered in other papers but they will also be answered by the result of the fit of the counts itself, shown in Sect. 4 (see Fig. 3). As to the first question, the fit of the power law with \(r_0 = 6.66 h^{-1}\) Mpc, \(\gamma = 1.79\) is remarkably good for low \(\theta_0\) (which is nearly independent of the linear part of \(\xi\)), a power law in the non-linear regime gives a very good fit. The answer to the second question is provided by the following consideration: a power-law in all scales gives more structure at \(r > 40 h^{-1}\) Mpc than the power-law in the non-linear regime + \(\Lambda CDM\) model in the linear regime. The first option gives further fluctuations at large \(\theta_0\); the difference is not high enough to reject the first option (we have already said that the fit is not very sensitive to the parameters in the linear power spectra), but the \(\Lambda CDM\) model in the linear regime (solid line in Fig. 3) is considerably better than the power-law at all scales (long dashed line in Fig. 3).

There are also theoretical reasons to use expression (9) for \(\xi(r)\). \(\Lambda CDM\) simulations lead to a correlation function for virialized halos with circular velocity \(>120\) km/s with this form and \(\gamma = 1.7\) (Primack 2001, Fig. 1). This function turns out to be almost exactly equal to that for APM galaxies. For \(K\)-selected galaxies the correlations do not need to be equal to that for halos; there may exist some biasing. However, the bias \(b(r) = \left(\frac{\xi(r)}{\xi_{\text{halo}}(r)}\right)^{\frac{1}{2}}\) is expected to be a mild function of \(r\). So, it seems plausible to use expression (9) with different amplitudes and slightly different \(\gamma\) with respect to the halos (or APM galaxies).
With this in mind, we shall see in next section that with the assumed shape (that for the mentioned haloes) in Eq. (9), there is qualitative agreement with the observed variances. We consider this result as a confirmation of the assumption that the measured variances are due to the large scale structure. We consider that large scale structure should explain not only the main part of the variances, but the whole of them. Forcing this by choosing the appropriate values for $r_0$, $\gamma$ leads us to an alternative method for determining the correlation function.

The evolution of the correlation function depends on $\Omega_{\text{mat}}$ and $\Omega_\Lambda$ through $\epsilon$. In our case, since the average redshift $(z) = 0.083$ (for $m_K < 13.5$ using the aforementioned luminosity function and K-correction), the exact value of $\epsilon$ is not so important, and small variations will not significantly affect the results of the model. The evolutionary corrections in this small range of redshifts are also negligible, especially in K-band (Carlberg et al. 1997). We take the value $\epsilon = -0.1$, which comes from the approximation of $\xi \propto D(z)^2$, for comoving coordinates (which holds provided that the shape of $\xi$ does not evolve, i.e. $\gamma$ is constant with respect to $z$, proved by Carlberg et al. 1997), where $D$, the growing factor of the linear density fluctuations, is given by (Heath 1977; Carroll et al. 1992)

$$D(z) = \frac{5\Omega_{\text{mat}}(z)(1+z)}{2/[R(z)]} \int_0^R \frac{f(R)\,dR}{R}$$

$$R = 1 + \frac{1}{1+z}$$

$$f(R) = \left[1 + \Omega_{\text{mat}}(z) \left(\frac{1}{R} - 1\right) + \Omega_\Lambda \left(R^2 - 1\right)\right]^{-1/2}.$$  

This leads to a value of $\epsilon \approx -0.1$ for $\Omega_{\text{mat}}(z) = 0.3$, $\Omega_\Lambda = 0.7$. Although, as we have said before, this value changes with other cosmological parameters, our outcome will be practically independent of these small changes. $\epsilon$ was measured by Carlberg et al. (1997) using a K-band deep survey, and although the accuracy was not very high, they obtained a value ($\epsilon = 0.2 \pm 0.5$) which is compatible with $\epsilon = -0.1$. Also, Roche et al. (1999) obtained $\epsilon \approx 0$.

4. Comparison between model and data. Fit of $r_0$ and $\gamma$

In the previous section, the only non-specified parameters are $\gamma$ and $r_0$, on which the fluctuations strongly depend. These will be fitted in this section. Through the comparison between model and data, we have a new method to obtain the two point correlation function, which does not explicitly require the two-point angular correlation function, does not suffer from edge problems, allows an estimation of the errors, and can be used even with incomplete surveys. Porciani & Giavalisco (2002) have also used galaxy counts for this purpose, but they derive the two-point angular correlation function and then insert it in the Limber equation (which assumes power law two point correlation function in all ranges), so it is halfway between our method and the standard one.

Figure 3 shows the data and some model predictions. The numbers are given in Table 1. The best fit is for $r_0 = 6.66$ h$^{-1}$ Mpc, $\gamma = 1.79$. Lower values of $r_0$ for the same $\gamma$, such as $r_0 = 4.9$ h$^{-1}$ Mpc, give considerably less structure than observed for the present sample of galaxies. For example, the two-point correlation function derived from the APM optical survey (equal to that for the mentioned halos): $\gamma = 1.7$, $r_0 = 4.5$ h$^{-1}$ Mpc (Baugh 1996), shows substantially smaller values than that obtained for the blue band. This leads us to agree with Carlberg et al. (1997), that optically (blue) selected surveys appear to be significantly less correlated than K-selected galaxies.

The values of the parameters $\gamma$ and $r_0$ which are compatible with the data are shown in Fig. 4, derived from a modified $\chi^2$-test for correlated data (Rubíno-Martín & Betancort-Rijo 2003, Sect. 4) applied to the data for $\theta_1 > 7^\circ$ (the other points have very large error bars and are not useful for constraining the parameters; moreover, they are more dependent on the values of $\xi$ in the linear regime).

In the simplest version of this test ($P_i = 0 \text{ } \forall i$, see Rubíno-Martín & Betancort-Rijo 2003), which for the present type of problem cannot be far from the best one (determined by an optimal set of $P_i$), the ordinary uncorrelated $\chi^2$, $\chi_{\text{ord}}$, and the number of degrees of freedom are rescaled by a certain factor, $A$, which is a function of the correlations:

$$\chi_{\text{mod}}^2 = A(C_{ij})\chi_{\text{ord}}^2,$$

and for the modified degrees of freedom we have:

$$n_{\text{mod}} = A(C_{ij})n_{\text{ord}}.$$

where $A$ depends on the correlations among data for all pairs of angular distances (see Eq. (22) of Rubíno-Martín & Betancort-Rijo 2003). To obtain the confidence levels we proceed with $\chi_{\text{mod}}^2$, $n_{\text{mod}}$ as in an ordinary uncorrelated test with $\chi_{\text{ord}}^2$, $n_{\text{ord}}$. The correlations between the data, $C_{ij} = \langle \delta N(\theta_i)\delta N(\theta_j) \rangle$ are

![Figure 4](image-url)
amplitude. For not related to the rms errors as in a normal distribution. The sizes of the confidence regions are
It must be noticed that the sizes of the confidence regions are
\[ r_0 = 6.66, \gamma = 1.79 \]
Table 1. \( \delta N/N \) as a function of \( \theta_0 \) for the 2MASS-data and the models (same as Fig. 3).

<table>
<thead>
<tr>
<th>( \theta_0 ) (deg.)</th>
<th>2MASS-data</th>
<th>( r_0 = 6.66, \gamma = 1.79 )</th>
<th>( r_0 = 6.66, \gamma = 1.79 ) ( \forall r )</th>
<th>( r_0 = 4.5, \gamma = 1.7 )</th>
<th>( r_0 = 4.9, \gamma = 1.8 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.172</td>
<td>0.869 ± 0.004</td>
<td>0.870</td>
<td>0.873</td>
<td>0.574</td>
<td>0.678</td>
</tr>
<tr>
<td>0.229</td>
<td>0.785 ± 0.004</td>
<td>0.777</td>
<td>0.780</td>
<td>0.521</td>
<td>0.607</td>
</tr>
<tr>
<td>0.286</td>
<td>0.714 ± 0.004</td>
<td>0.712</td>
<td>0.715</td>
<td>0.483</td>
<td>0.557</td>
</tr>
<tr>
<td>0.43</td>
<td>0.599 ± 0.005</td>
<td>0.606</td>
<td>0.609</td>
<td>0.422</td>
<td>0.477</td>
</tr>
<tr>
<td>0.573</td>
<td>0.536 ± 0.006</td>
<td>0.540</td>
<td>0.544</td>
<td>0.383</td>
<td>0.427</td>
</tr>
<tr>
<td>0.859</td>
<td>0.461 ± 0.007</td>
<td>0.457</td>
<td>0.462</td>
<td>0.336</td>
<td>0.368</td>
</tr>
<tr>
<td>1.146</td>
<td>0.413 ± 0.009</td>
<td>0.406</td>
<td>0.411</td>
<td>0.306</td>
<td>0.330</td>
</tr>
<tr>
<td>1.719</td>
<td>0.353 ± 0.011</td>
<td>0.342</td>
<td>0.348</td>
<td>0.270</td>
<td>0.287</td>
</tr>
<tr>
<td>2.292</td>
<td>0.312 ± 0.013</td>
<td>0.302</td>
<td>0.309</td>
<td>0.247</td>
<td>0.259</td>
</tr>
<tr>
<td>3.44</td>
<td>0.254 ± 0.015</td>
<td>0.253</td>
<td>0.261</td>
<td>0.217</td>
<td>0.224</td>
</tr>
<tr>
<td>4.58</td>
<td>0.222 ± 0.018</td>
<td>0.221</td>
<td>0.231</td>
<td>0.195</td>
<td>0.200</td>
</tr>
<tr>
<td>5.73</td>
<td>0.197 ± 0.020</td>
<td>0.198</td>
<td>0.210</td>
<td>0.178</td>
<td>0.182</td>
</tr>
<tr>
<td>6.88</td>
<td>0.179 ± 0.022</td>
<td>0.180</td>
<td>0.194</td>
<td>0.164</td>
<td>0.167</td>
</tr>
<tr>
<td>8.59</td>
<td>0.156 ± 0.024</td>
<td>0.158</td>
<td>0.176</td>
<td>0.146</td>
<td>0.148</td>
</tr>
<tr>
<td>11.46</td>
<td>0.131 ± 0.027</td>
<td>0.132</td>
<td>0.154</td>
<td>0.124</td>
<td>0.125</td>
</tr>
<tr>
<td>14.32</td>
<td>0.106 ± 0.027</td>
<td>0.114</td>
<td>0.139</td>
<td>0.107</td>
<td>0.108</td>
</tr>
<tr>
<td>17.19</td>
<td>0.0895 ± 0.028</td>
<td>0.0997</td>
<td>0.127</td>
<td>0.0946</td>
<td>0.0955</td>
</tr>
<tr>
<td>22.9</td>
<td>0.0662 ± 0.029</td>
<td>0.0798</td>
<td>0.110</td>
<td>0.0762</td>
<td>0.0767</td>
</tr>
<tr>
<td>28.6</td>
<td>0.0535 ± 0.030</td>
<td>0.0663</td>
<td>0.0984</td>
<td>0.0636</td>
<td>0.0640</td>
</tr>
</tbody>
</table>

Fig. 5. Value of \( A(C_{ij}) \) as a function of the number of resamplings.

calculated using resampling (jackknife resampling, Scranton et al. 2002; Maller et al. 2003; 13 resamplings are generated and \( \frac{\Delta N}{N} \) calculated in each one). We find \( A(C_{ij}) = 0.190 \) (in Fig. 5 we see how this value changes very little with the number of resamplings).

The inferred values of the two parameters are:
\[ \gamma = 1.79 ± 0.02 \text{ (68\% C.L.)} \quad [±0.07 \text{ (95\% C.L.)}] \],
\[ r_0^2 = 29.8 ± 0.3 \text{ (68\% C.L.)} \quad [±1.1 \text{ (95\% C.L.)}] \].

It must be noticed that the sizes of the confidence regions are not related to the rms errors as in a normal distribution. The actual rms errors are: 0.03 for the exponent and 0.5 for the amplitude. For \( \gamma = 1.79 \), we derive \( r_0(z = 0) = 6.66 ± 0.04 \text{ (68\% C.L.)} \, h^{-1} \text{Mpc} \) (which is represented in Fig. 3).

These values are in excellent agreement with the estimations of \( \gamma \) in near-infrared surveys by other means: the angular correlation function is at small scales proportional to \( \theta^{-0.8} \) in \( K \)-band (Baugh et al. 1996; Carlberg et al. 1997), which through Limber’s equation gives \( \gamma = 1.8 \). Kümmel & Wagner (2000) give an angular correlation in \( K \)-band proportional to \( \theta^{-0.98±0.15} \) which would give through Limber’s equation \( \gamma = 1.98 ± 0.15 \), again consistent with our value. Maller et al. (2003) with 2MASS \( K \)-band data derive an angular correlation proportional to \( \theta^{-0.76±0.04} \) which gives \( \gamma = 1.76 ± 0.04 \), very close to our preferred value. The value of \( \gamma \) is also in agreement with some simulations of \( \Lambda CDM \) (Primack 2001, Fig. 1: \( \gamma = 1.6–1.7 \)) and the exponents in optical surveys such as APM (\( \gamma = 1.7 \), Baugh 1996).

We get an amplitude \( r_0(z = 0) = 29.8 ± 0.3 \text{ (68\% C.L.)} \), more or less in agreement with the R-selected Las Campanas Redshift Survey which finds \( r_0(z = 0.1) = 20.5 \) (Jing et al. 1998), equivalent (through Eq. (8) with \( t_{\text{conoving}} = t_{\text{phys}}(1 + z) \)) to \( r_0(z = 0) = 27.0 \). This is also in agreement with the extrapolation down to \( z = 0 \) of the evolution of \( r_0^2 \) in \( K \)-band selected galaxies at high redshift (see Fig. 8 of Carlberg et al. 1997). However, the blue band selected surveys give a lower amplitude of the correlations by a factor of two (Carlberg et al. 1997, Sect. 6 and references therein). At high redshifts (\( z \approx 3 \)), this difference of amplitude between \( K \)-band selected galaxies and blue-optical selected galaxies is even larger: a factor 3–4 (Daddi et al. 2003). This is consistent because the correlation length \( r_0 \) is not independent of the range of luminosities of the selected sample (Colombo & Bonometto 2001). Locally it follows roughly a dependence \( r_0 \approx 2.2 \, h^{-1} \text{Mpc} + 0.4 D_L \) with the scale \( D_L = \left[ < n > L \Phi(> L) \right]^{1/3} \) (Colombo & Bonometto 2001).

Figure 3 shows that the fluctuations of galaxy counts may be accurately predicted by an expression of the form given in (9) for the correlation. It must be noted that our method uses a frequentist statistical analysis that not merely determines the best correlation function but also tells us how good the best one is. The fact that the best values (of \( r_0, \gamma \)) are within the
68% C.L. region tells us that expression (9) is as good as it can possibly be.

No effect other than the large scale structure itself, for instance intergalactic extinction, is necessary to explain the variances. When Zwicky (1957, chap. 3) observed that $\sigma_N(\theta_0)/\sigma_{\text{Poisson}}$ increased with $\theta_0$ even for large angles, he interpreted this result as proof of the presence of significant intergalactic extinction. However, as we see in Fig. 3, the fact that $\delta N/N$ decreases slowly with increasing $\theta_0$ is due to the large scale structure itself, that produces extra fluctuations in large scales over those expected from a Poissonian distribution for scales of several degrees or tens of degrees.

We have tested that the $\Lambda$CDM model of large scale structure is consistent with the observed galaxy count variance. This consistency is obtained for particular values of the parameters of the two point correlation function, giving a new method to obtain it indirectly without making use of the two-point angular correlation function. In the application of this method to 2MASS $K$-band galaxies with $m_K < 13.5$, we get $\xi(z = 0, r < 10 h^{-1} \text{Mpc}) = (29.8 \pm 0.3)r^{-1.79_{-0.02}^{+0.02}}$ (68% C.L.).

Acknowledgements. We thanks the referee, A. Maller, for rather helpful comments and suggestions. Thanks are also given to N. Sambhus for proof-reading help. This publication makes use of data products from: 2MASS, which is a joint project of the Univ. of Massachusetts and the Infrared Processing and Analysis Center (IPAC), funded by the NASA and the NSF. This research has made use of the NASA/IPAC Extragalactic Database (NED) which is operated by the Jet Propulsion Laboratory, California Institute of Technology, under contract with NASA. We thank the Swiss National Science Foundation for support under grant 20-64856.01.

References

Zwicky, F. 1957, Morphological Astronomy (Berlin: Springer-Verlag)