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Two-fluid matter-quintessence FLRW models: Energy transfer and the equation of state of the universe

A. Gromov[1,4], Yu. Baryshev[2,4], and P. Teerikorpi[3]

1 Centre for Advanced Studies of the Saint-Petersburg State Technical University, 195251 St. Petersburg, Russia
2 Astronomical Institute of the Saint-Petersburg University, 198904 St. Petersburg, Russia
3 Tuorla Observatory, University of Turku, 21500 Piikkiö, Finland,
4 Isaac Newton Institute of Chile, St. Petersburg Branch

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Abstract. The dark energy – dark matter interaction has recently been proposed as a solution of the CDM crisis at galactic halo scales. We have made a study of general properties of two-fluid FLRW models with energy transfer between DE and matter, independently of the specific mechanism of interaction. A mixture of quintessence with negative pressure ($p_{\rm Q} = w\epsilon_{\rm Q}$) and matter with positive pressure ($p = \beta\epsilon_{\rm m}$) define the associated one-fluid model ($p = \gamma\epsilon$). It is shown that, for given w and β, the energy transfer defines γ and, therefore, the total gravitating mass and dynamics of the model. The behaviour of the energy content, gravitating mass, pressure, and energy transfer are given as functions of the scale factor for interesting examples from different two-fluid classes, defined by the presence/absence of energy transfer and by the stationarity/non-stationarity of the equations of state. Three characteristic scales $a_{\mathcal{E}}$, $a_{\mathcal{P}}$, $a_{\mathcal{M}}$ separate periods of time in which quintessence energy, pressure and gravitating mass dominate. Sequences of the scales define six evolution types.

Key words. cosmology: theory – cosmology: dark matter

1. Introduction

In studies of cosmological models with dark energy, DE and matter are usually viewed as independent substances so that the energy-momenta of the partial fluids are separately conserved. Recently the discussion of the CDM crisis at galactic halo scales has put forward the question of interaction of CDM particles and in particular that between CDM and DE (Peebles & Ratra 2003; Ostriker & Steinhardt 2003; Farrar & Peebles 2003). As emphasized by Farrar & Peebles there are many possible unexplored interaction mechanisms. Fortunately, the parameters of FLRW models with interacting DE may be restricted from observations (Teerikorpi et al. 2003).

Here we study general properties of two-fluid FLRW models in which the DE component dominates at late epochs and there may be energy transfer between DE and matter. Independently of specific mechanisms of interaction in the complex dark sector, we study the behaviour of the main parameters of two-fluid models, determined by energy transfer between DE and matter. The first attempt to consider an interaction of matter with Λ was made by Bronstein (1933). One can find historical notes in Baryshev & Teerikorpi (2002) and Peebles & Ratra (2003).

Send offprint requests to: P. Teerikorpi,
e-mail: pekkatee@astro.utu.fi

In Sect. 2 we give a brief introduction to two-fluid interacting FLRW models. In Sect. 3 we introduce a new classification of such models and study the general case. Three particular examples of the classification are described in Sect. 4 and our conclusions are given in Sect. 5.

2. The two-fluid model: A summary

The derivation of the FLRW equations contains the following basic elements:
1) The Einstein equations (notations from Landau & Lifshitz 1971).

$$\Re^i_k - \frac{1}{2} g^i_k \Re = \frac{8\pi G}{c^4} T^i_k. \qquad (1)$$

2) The Bianchi identity in the form:

$$T^i_{k;i} = 0. \qquad (2)$$

3) The RW line element in spherical comoving space coordinates (χ, θ, ϕ) and synchronous time t:

$$ds^2 = c^2 dt^2 - S^2(t) d\chi^2 - S^2(t) I^2_k(\chi) \left(d\theta^2 + \sin^2\theta d\phi^2\right). \qquad (3)$$

4) The total energy momentum tensor in comoving coordinates (ordinary matter, vacuum, quintessence):

$$T^i_k = \mathrm{diag}(\varepsilon, -p, -p, -p). \qquad (4)$$

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Here $I_k(\chi) = (\sin(\chi),\ \chi,\ \sinh(\chi))$ for the curvature constant $k = (+1, 0, -1)$, respectively. $S(t)$ is the scale factor, $\varepsilon = \rho c^2$ is the energy density, and p is the pressure.

2.1. The FLRW one-fluid model

For the one-fluid model the (0, 0) and (1, 1) components of Eq. (1) give the FLRW equations

$$\frac{\dot S^2}{S^2} + \frac{kc^2}{S^2} = \frac{8\pi G}{3c^2}\varepsilon, \quad 2\frac{\ddot S}{S} + \frac{\dot S^2}{S^2} + \frac{kc^2}{S^2} = -\frac{8\pi G}{c^2}p, \tag{5}$$

where $\dot{} = \mathrm{d}/\mathrm{d}t$. Equation (2) implies:

$$3\frac{\dot S}{S} = -\frac{\dot\varepsilon}{\varepsilon + p}\cdot \tag{6}$$

The equation of state for the one-fluid model is

$$p = \gamma\,\varepsilon. \tag{7}$$

The characteristic energy content e of the sphere with the radius equal to the proper metric distance $r = S(t)\chi$ is[1]:

$$e(r) = \int \varepsilon \mathrm{d}V = 4\pi\,\varepsilon\, S^3\,\sigma_k(\chi). \tag{8}$$

The gravitating mass M_g is defined by the equation of motion ($\ddot r = -G\,M_\mathrm{g}(r)/r^2$) from Eqs. (5):

$$M_\mathrm{g}(r) = \frac{\varepsilon + 3p}{c^2}\int_0^r \mathrm{d}V = \frac{e}{c^2}\,(3\,\gamma + 1). \tag{9}$$

2.2. Time dependence of energy

Two factors cause a time dependence of energy of each particular fluid and, hence, of the associated one-fluid model: 1) the expansion of the universe, and 2) one fluid may be converted into another.

For the associated one fluid the Bianchi identity is:

$$\frac{\mathrm{d}\left(\varepsilon S^3\right)}{\mathrm{d}t} + p\frac{\mathrm{d}\left(S^3\right)}{\mathrm{d}t} = 0. \tag{10}$$

Representing the total energy density and pressure as a sum of two parts:

$$p = p_\mathrm{Q} + p_\mathrm{m} \qquad \varepsilon = \varepsilon_\mathrm{Q} + \varepsilon_\mathrm{m} \tag{11}$$

one defines, following Davidson (1962), the energy transfer from quintessence to matter u_m and from matter to quintessence u_Q as follows:

$$\frac{\mathrm{d}\left(\varepsilon_\mathrm{Q} S^3\right)}{\mathrm{d}t} + p_\mathrm{Q}\frac{\mathrm{d}\left(S^3\right)}{\mathrm{d}t} = -u_\mathrm{m}$$
$$\frac{\mathrm{d}\left(\varepsilon_\mathrm{m} S^3\right)}{\mathrm{d}t} + p_\mathrm{m}\frac{\mathrm{d}\left(S^3\right)}{\mathrm{d}t} = -u_\mathrm{Q}. \tag{12}$$

[1] The volume element $\mathrm{d}V = S^3\,I_k^2(\chi)\,\sin^2(\theta)\,\mathrm{d}\chi\,\mathrm{d}\theta\,\mathrm{d}\phi$. For $k = +1, 0, -1$: $\sigma_k(\chi) = \chi^2/2 - \sin 2\chi/4, \chi^3/3, \sinh 2\chi/4 - \chi^2/2$.

2.3. The two-fluid model for matter and quintessence

In his pioneering work, Lemaître (1933) studied a model containing dust and the cosmological constant without interaction. Energy transfer was first applied for a two-fluid cosmology by Davidson (1962). We also make use of the results by Chernin (1965), Thomas & Schulz (2000), Coley & Tupper (1986b), Amendola & Tocchini–Valentini (2001).

We study two kinds of substances: 1) a perfect fluid with positive pressure; 2) a fluid with positive energy and negative pressure. The equations of states are:

$$p_\mathrm{m} = \beta\,\varepsilon_\mathrm{m}, \tag{13}$$

where $0 \le \beta \le 1$ and $\beta = \varepsilon_\mathrm{m}/p_\mathrm{m} > 0$, and

$$p_\mathrm{Q} = w\,\varepsilon_\mathrm{Q}, \tag{14}$$

where $-1 \le w \le 0$ and $w = \varepsilon_\mathrm{Q}/p_\mathrm{Q} < 0$. The minimum value for w has been usually accepted as that corresponding to the vacuum ($w = -1$). In a model of phantom energy (Caldwell 2002; Caldwell et al. 2003) a value of $w < -1$ was also regarded as admissible. Our approach does not depend on the particular value of w. An important function

$$\alpha(S) = \varepsilon_\mathrm{Q}(S)/\varepsilon_\mathrm{m}(S) \tag{15}$$

characterizes the relative contributions to the energy density from quintessence and matter (e.g. Wetterich 1995; see for a different approach Coley & Wainwright 1992).

The two-fluid model is defined on two levels: 1) the level of partial one-fluid, with interactions through the common gravity field and by energy transfer from one fluid to the other, and 2) the level of the associated one-fluid (as termed by Coley & Wainwright 1992). For a full description of a two-fluid one has to study both levels simultaneously. The effective one-fluid equation of state is not produced by particles alone, but together with DE.

2.3.1. The associated one-fluid model

We specify the partial quantities and equations of state:

$$\varepsilon_\mathrm{Q} = \rho_\mathrm{Q}c^2, \qquad \varepsilon_\mathrm{m} = \rho_\mathrm{m}c^2, \qquad \varepsilon = \varepsilon_\mathrm{Q} + \varepsilon_\mathrm{m},$$
$$p_\mathrm{Q} = w\varepsilon_\mathrm{Q}, \qquad p_\mathrm{m} = \beta\varepsilon_\mathrm{m}, \qquad p = p_\mathrm{Q} + p_\mathrm{m},$$
$$p = \gamma\varepsilon. \tag{16}$$

Each fluid has an equation of state (Eq. (7)) with different coefficients: $\beta(S)$ for matter and $w(S)$ for quintessence. Thus the equation of state for the associated one-fluid is:

$$\gamma = \frac{p}{\varepsilon} = \frac{w\alpha + \beta}{\alpha + 1}\cdot \tag{17}$$

For the energy ratio $\alpha \ge 0$ one finds that $w \le \gamma \le \beta$. The associated one-fluid description of the two-fluid model includes Eqs. (5)–(7) and Eq. (17). The three functions of the scale factor $w(S)$, $\beta(S)$ and $\alpha(S)$ are assumed given.

2.3.2. The two-fluid model

The energy-momentum tensor of the associated one-fluid is the sum of the partial tensors. From Eq. (2) we have

$$T^i_{k;i} = T_{(\mathrm{m})}{}^i_{k;i} + T_{(\mathrm{Q})}{}^i_{k;i} = 0 \tag{18}$$

which leads to Eq. (6) with $\varepsilon = \varepsilon_\mathrm{Q} + \varepsilon_\mathrm{m}$, $p = p_\mathrm{Q} + p_\mathrm{m}$.

Here we consider two cases in the two-fluid description. First, the EM tensors of the partial fluids are separately conserved:

$$T_{(\mathrm{m})}{}^i_{k;i} = 0 \qquad \text{and} \qquad T_{(\mathrm{Q})}{}^i_{k;i} = 0, \tag{19}$$

so that $u_\mathrm{Q} = 0$ and $u_\mathrm{m} = 0$. In the second case we allow the presence of the energy transfer, when $u_\mathrm{Q} \neq 0$ and $u_\mathrm{m} \neq 0$ but $u_\mathrm{Q} + u_\mathrm{m} = 0$ which is equivalent to Eq. (6). We note that in his pioneering paper Bronstein (1933) considered the third case when the conservation of energy is violated by a $\mathrm{d}\Lambda/\mathrm{d}t$ term.

For two-fluid FLRW models the characteristic energy content within the sphere of a fixed comoving radius χ is

$$e = 4\pi\,\varepsilon S^3 \sigma_k(\chi) = 4\pi\,(\alpha+1)\,\varepsilon_\mathrm{m}\, S^3\,\sigma_k(\chi). \tag{20}$$

The gravitating mass M_g is the sum

$$M_\mathrm{g}(r) = M^\mathrm{m}_\mathrm{g} + M^\mathrm{Q}_\mathrm{g} = \frac{e}{c^2}\,(3\,\gamma+1). \tag{21}$$

2.4. Dimensionless equations

Below we use dimensionless quantities, as defined in Gromov et al. (2002)[2]:

$$a(\tau) = S(t)/l_0, \qquad \tau = t/t_0,$$

$$\mathcal{E}_i = \frac{\varepsilon_i}{\varepsilon_0}, \qquad \mathcal{P}_i = \frac{p_i}{\varepsilon_0}, \qquad E_i = \frac{e_i}{e_0}, \qquad U_i = \frac{u_i}{\varepsilon_0}\,t_0$$

where $i = \mathrm{Q}, \mathrm{m}$, and

$$\mathcal{E} = \mathcal{E}_\mathrm{Q} + \mathcal{E}_\mathrm{m}, P = P_\mathrm{Q} + P_\mathrm{m}, E = E_\mathrm{Q} + E_\mathrm{m}, U = U_\mathrm{Q} + U_\mathrm{m}$$

$$\mu(a) = M_\mathrm{g}(t)/M_0 = 3\sigma_\mathrm{k}(\chi)(3\gamma+1)a^3\mathcal{E}.$$

Then initial equations are rewritten as

$$\frac{\dot a^2}{a^2} + \frac{k}{a^2} = \mathcal{E}, \quad 2\,\frac{\ddot a}{a} + \frac{\dot a^2}{a^2} + \frac{k}{a^2} = -3\,P, \tag{22}$$

$$3\,\frac{\dot a}{a} = -\frac{\dot{\mathcal{E}}}{\mathcal{E}+P}, \tag{23}$$

$$P = \gamma\ \mathcal{E}, \tag{24}$$

$$U_i = \frac{1}{a^3}\left[\frac{\mathrm{d}}{\mathrm{d}\tau}\left(\mathcal{E}_i\,a^3\right) + P_i\,\frac{\mathrm{d}}{\mathrm{d}\tau}\,a^3\right]. \tag{25}$$

The equation for the scale factor now has the form:

$$a^2\,\ddot a = -\Omega_0\,\mu(a), \tag{26}$$

where Ω_0 is the cosmological density parameter.

[2] We use $\dot{} \equiv \mathrm{d}/\mathrm{d}\tau$, and $t_0 = \frac{1}{H_0}$, $l_0 = c\,t_0$, $\varepsilon_0 = \rho_0\,c^2 = \frac{3H_0^2}{8\pi G}$.

3. General properties of two-fluid FLRW models

3.1. The classification

In a two-fluid negative pressure and gravitating components lead to novel behaviour of the total pressure and gravitating mass. It is useful to define four classes (Gromov et al. 2002; Teerikorpi et al. 2003) according to two independent properties: 1) the fluids have a stationary (SES) or at least one has a non-stationary (NSES) equation of state and 2) the presence (ET) or absence (NET) of energy transfer between the components (Table 1 in Teerikorpi et al. 2003). Further, the behaviour of the function $\alpha(a) = \mathcal{E}_\mathrm{Q}/\mathcal{E}_\mathrm{m}$ defines three kinds of models:

1. *coherent model*, when

$$\alpha(a) = \text{const}, \tag{27}$$

which is defined by Eq. (27) at any moment of time;

2. *asymptotically coherent model*, when Eq. (27) is reached at a limit of large time:

$$\lim_{a\to\infty} \alpha'(a) = 0 \tag{28}$$

and

3. *non-coherent model* (Eqs. (27) and (28) invalid).

3.2. The general solution: ET-NSES models

The ET-NSES class is the most general of two-fluid models. There, any fluid (the associated and two particular ones) may have a nonstationary equation of state. Here we fully describe the two-fluid problem in terms of the associated one fluid and two particular fluids simultaneously. This leads us to rewrite the FLRW equations in terms of the coefficients of the three equations of state γ, β, w.

3.2.1. The input equations and their solution

We describe the associated one-fluid model by Eqs. (23, 24) with $\gamma = \gamma(a)$. The solution of these equations, with the initial conditions stated for the present epoch, is:

$$\mathcal{E} = \mathcal{E}(1)\,\exp\left(-3\int_1^a \frac{\gamma+1}{x}\,\mathrm{d}x\right). \tag{29}$$

The two particular fluids are described by Eqs. (25) and the equations of state:

$$P_\mathrm{Q} = w(a)\ \mathcal{E}_\mathrm{Q}, \quad P_\mathrm{m} = \beta(a)\ \mathcal{E}_\mathrm{m}. \tag{30}$$

From $a^3\left(U_\mathrm{m} - \frac{U_\mathrm{Q}}{\alpha}\right)$ and $U_\mathrm{Q} + U_\mathrm{m} = 0$, we find:

$$\frac{U_\mathrm{m}}{E_\mathrm{m}}\,\frac{\alpha+1}{\alpha}\,a^3 = 3\,\frac{\dot a}{a}\,(\beta - w) - \frac{\dot\alpha}{\alpha} \tag{31}$$

where the energy content $E_i = \mathcal{E}_i a^3$. This leads to a general result: *the model can be coherent (*$\alpha = \text{const}$*) only if there is energy transfer*. The critical transfer $\boldsymbol{U}^\mathrm{E}_\mathrm{m}$ which keeps α constant (coherence) is:

$$\boldsymbol{U}^\mathrm{E}_\mathrm{m} = 3\,\frac{\dot a}{a}\,\mathcal{E}(\beta - w)\,\frac{\alpha}{(\alpha+1)^2} = \boldsymbol{U}^\mathrm{E}_\mathrm{m}(w,\beta,\gamma,a). \tag{32}$$

$\boldsymbol{U}_{\rm m}^{\rm E}$ separates the behaviour of the two-fluid with different signs of $\dot\alpha(a)$: Eq. (31) implies that $\dot\alpha(a) > 0$ requires $U_{\rm m} < \boldsymbol{U}_{\rm m}^{\rm E}$; $\dot\alpha(a) < 0$ requires $U_{\rm m} > \boldsymbol{U}_{\rm m}^{\rm E}$. That is, the ratio $\frac{\mathcal{E}_Q}{\mathcal{E}_{\rm m}}$ increases, if $U_{\rm m} < \boldsymbol{U}_{\rm m}^{\rm E}$ and decreases if $U_{\rm m} > \boldsymbol{U}_{\rm m}^{\rm E}$.

Previously the energy transfer has been calculated for some model of interaction between dust and radiation, i.e. for given $w(a)$ and $\beta(a)$ (e.g. McIntosh 1967, 1968; Sistero 1971; Coley & Tupper 1985, 1986a). Now we know that energy transfer is inevitably required for coherence.

Substituting $\frac{\dot a}{a}$ from Eq. (22) into Eq. (31) shows how the energy transfer depends on curvature ($k \neq 0$):

$$\frac{U_{\rm m}}{\mathcal{E}}\frac{(\alpha+1)^2}{\alpha} = \pm\,3\,\sqrt{\mathcal{E}-\frac{k}{a^2}}\,(\beta-w) - \frac{\dot\alpha}{\alpha}\cdot \quad (33)$$

Using the definition of α, Eq. (15), and its time derivate, $\dot\alpha\,(\gamma-w)^2 = (\gamma-w)\dot\beta + (\beta-\gamma)\,\dot w - (\beta-w)\,\dot\gamma$, Eq. (31) leads to the expression for the energy transfer:

$$U_{\rm m}(\gamma,a) = \mathcal{E}(\gamma,a)\,\sqrt{\mathcal{E}(\gamma,a)-\frac{k}{a^2}}\left[\pm 3\,\frac{(\beta-\gamma)(\gamma-w)}{\beta-w} \mp\frac{a}{(\beta-w)^2}\Big((\gamma-w)\beta' + (\beta-\gamma)\,w' - (\beta-w)\,\gamma'\Big)\right] \quad (34)$$

where $' = \frac{\rm d}{{\rm d}a}$ and $\mathcal{E}(\gamma,a)$ is defined by Eq. (29). The sign "+" corresponds to expansion while the sign "−" corresponds to collapse. Thus the energy transfer $U_{\rm m}$ depends on γ, β, w, their derivates and, also, on the scale factor a (as an independent variable) and curvature. For a more realistic situation with given k, $\beta(a)$ and $w(a)$, Eq. (34) shows that $\gamma(a)$ is completely defined by $U_{\rm m}(a)$.

Equations (31)–(34) represent in different forms how a convertion of one-fluid into another ($U_{\rm m} \neq 0$) affects the properties of particular fluids (equations of state, energy transfer rates) and the expansion of space $a(t)$. Equation (34) gives the dependence of the energy transfer on γ and a as an independent variable. Excluding γ from Eqs. (34), (29) and (26) we find that energy transfer completely defines the gravitating mass as a function of the scale factor. Thus, *the total gravitating mass and, consequently, the dynamics of the expansion, is determined by the energy transfer*.

The general (ET-NSES) solution for particular fluids is represented by the function $\alpha(a)$ and two coefficients of the equation of state $\beta(a)$ and $w(a)$:

$$\mathcal{E}_{\rm Q} = \frac{\mathcal{E}\,\alpha}{\alpha+1}, \qquad \mathcal{E}_{\rm m} = \frac{\mathcal{E}}{\alpha+1},$$
$$E_{\rm Q} = \mathcal{E}_{\rm Q}\,a^3, \qquad E_{\rm m} = \mathcal{E}_{\rm m}\,a^3,$$
$$P_{\rm Q} = w\ \mathcal{E}_{\rm Q}, \qquad P_{\rm m} = \beta\ \mathcal{E}_{\rm m}, \quad (35)$$

$$\mu = \mu_{\rm Q} + \mu_{\rm m},$$

where

$$\mu_{\rm Q} = 3\,\sigma_k(\chi)\,(3\beta+1)\,a^3\ \mathcal{E}_{\rm Q},$$
$$\mu_{\rm m} = 3\,\sigma_k(\chi)\,(3\,w+1)\,a^3\ \mathcal{E}_{\rm m}.$$

3.2.2. Three characteristic scales

Depending on the initial conditions and the class of a model, there is one (or, possibly, more) moment of time when the one-fluid is effectively dust, i.e. $\gamma(a_{\rm ed}) = 0$. The general expression for the "effective dust scale" $a_{\rm ed}$ is:

$$\alpha(a_{\rm ed}) = -\beta/w. \quad (36)$$

The evolution of a two-fluid from initial conditions at $a_{\rm min}$ to an asymptotic state with $a \to \infty$ has a complex character, as shown by the contributions of energy, pressure and gravitating mass of each component to the total quantities. Equations (35) give the relative contributions:

$$\alpha = \frac{\mathcal{E}_{\rm Q}}{\mathcal{E}_{\rm m}} = \frac{E_{\rm Q}}{E_{\rm m}}$$
$$\mathcal{P} = \frac{|P_{\rm Q}|}{P_{\rm m}} = \frac{\alpha}{\alpha_{\mathcal{P}}}, \quad \mathcal{M} = \frac{|\mu_{\rm Q}|}{\mu_{\rm m}} = \frac{\alpha}{\alpha_{\mathcal{M}}}, \quad (37)$$

where $\alpha_{\mathcal{P}}$ and $\alpha_{\mathcal{M}}$ depend on a:

$$\alpha_{\mathcal{P}}(a) = \frac{\beta(a)}{|w(a)|}, \quad \alpha_{\mathcal{M}}(a) = \frac{3\,\beta(a)+1}{|3\,w(a)+1|}\cdot \quad (38)$$

Three characteristic scales $a_{\mathcal{E}}$, $a_{\mathcal{P}}$ and $a_{\mathcal{M}}$ appear so that

$$a = a_{\mathcal{E}} \quad \text{implies} \quad \alpha = 1,$$
$$a = a_{\mathcal{P}} \quad \text{implies} \quad \mathcal{P} = 1, \qquad a_{\rm P} = a_{\rm ed},$$
$$a = a_{\mathcal{M}} \quad \text{implies} \quad \mathcal{M} = 1, \quad (39)$$

where $a_{\rm ed}$ is defined by Eq. (36) These are the solutions of the equations:

$$\alpha(a_{\mathcal{E}}) = 1,$$
$$\frac{|w(a_{\mathcal{P}})|}{\beta(a_{\mathcal{P}})}\,\alpha(a_{\mathcal{P}}) = 1,$$
$$\frac{|3\,w(a_{\mathcal{M}})+1|}{3\beta(a_{\mathcal{M}})+1}\,\alpha(a_{\mathcal{M}}) = 1. \quad (40)$$

$\beta(a)$, $w(a)$ and $\alpha(a)$ are model-dependent, so there is no possibility to study the general case.

Equation (31) shows that the energy transfer is required for coherence (α = const). One may also substitute $\alpha = \alpha_{\mathcal{M}}\,\mathcal{M}$ into Eq. (31), and find for *constant* $\mathcal{M}$ the critical value for energy transfer:

$$\boldsymbol{U}_{\rm m}^{\rm M} = \left[3\,\frac{\dot a}{a}\,(\beta-w) - \frac{{\rm d}\,\ln\alpha_{\mathcal{M}}}{{\rm d}\tau}\right]\mathcal{E}\frac{\mathcal{M}\alpha_{\mathcal{M}}}{(\mathcal{M}\,\alpha_{\mathcal{M}}+1)^2}\cdot \quad (41)$$

A similar formula may be derived for the critical energy transfer $\boldsymbol{U}_{\rm m}^{\rm P}$ producing constant $\mathcal{P}$.

3.2.3. The evolution types

We study only models fully dominated 1) by matter in the very early epoch ($\alpha < 1, \mathcal{P} < 1, \mathcal{E} < 1$), while 2) by dark energy at large times ($\alpha > 1, \mathcal{P} > 1, \mathcal{E} > 1$). Then the characteristic scales allow 6 types of nonequalities:

$$a_{\mathcal{E}} < a_{\mathcal{P}} < a_{\mathcal{M}}, \ \text{EPM}; \quad a_{\mathcal{E}} < a_{\mathcal{M}} < a_{\mathcal{P}}, \ \text{EMP}$$
$$a_{\mathcal{P}} < a_{\mathcal{M}} < a_{\mathcal{E}}, \ \text{PME}; \quad a_{\mathcal{P}} < a_{\mathcal{E}} < a_{\mathcal{M}}, \ \text{PEM}$$
$$a_{\mathcal{M}} < a_{\mathcal{E}} < a_{\mathcal{P}}, \ \text{MEP}; \quad a_{\mathcal{M}} < a_{\mathcal{P}} < a_{\mathcal{E}}, \ \text{MPE}. \quad (42)$$

Table 1. Detailed description of the EPM evolution type.

$a_{\min} < a < a_{\mathcal{E}}$	$\mathcal{E}_M > \mathcal{E}_Q$ $P_M > \|P_Q\|$ $\mu_M > \|\mu_Q\|$
$a_{\mathcal{E}} < a < a_{\mathcal{P}}$	$\mathcal{E}_M < \mathcal{E}_Q$ $P_M > \|P_Q\|$ $\mu_M > \|\mu_Q\|$
$a_{\mathcal{P}} < a < a_{\mathcal{M}}$	$\mathcal{E}_M < \mathcal{E}_Q$ $P_M < \|P_Q\|$ $\mu_M > \|\mu_Q\|$
$a_{\mathcal{M}} < a < \infty$	$\mathcal{E}_M < \mathcal{E}_Q$ $P_M < \|P_Q\|$ $\mu_M < \|\mu_Q\|$

Each nonequality system defines an "evolution type": a unique sequence of the scales $a_{\mathcal{E}}$, $a_{\mathcal{P}}$ and $a_{\mathcal{M}}$, which border the periods of time when dark energy dominates in total energy, pressure and gravitating mass, respectively. The ratio between energies E_Λ and E_m was used by Chernin et al. (2002) to make a division between the growth and suppression of structure formation. In Table 1 the EPM evolution type is described in detail. Equation (31) leads to a dependence of α on U_m for a given $w(a)$ and $\beta(a)$. Equations (37)–(40) lead to a dependence of the evolution type on α for a given $w(a)$ and $\beta(a)$. So, for a given $w(a)$ and $\beta(a)$ the evolution type is connected to the energy transfer (which defines the gravitating mass). Thus, *there exists a correspondence between the dynamics of a model, its evolution type and energy transfer*.

4. Classifying two-fluid FLRW models: Examples

4.1. NET-SES models

Such two-fluid models have been much studied because the input equations are simple. We discuss the evolution types for $w = -1, -2/3, -1/3$, 0 and $\beta = 0, 1/3, 2/3, 1$.

4.1.1. Input equations and their solutions

The input equations are given by Eq. (23) for each fluid:

$$3\,\frac{\dot a}{a} = -\frac{\dot{\mathcal{E}}_Q}{\mathcal{E}_Q + P_Q}, \quad P_Q = w\ \mathcal{E}_Q, \quad w = \text{const.},$$
$$3\,\frac{\dot a}{a} = -\frac{\dot{\mathcal{E}}_m}{\mathcal{E}_m + P_m}, \quad P_m = \beta\ \mathcal{E}_m, \quad \beta = \text{const.},$$
$$U_Q = 0, \qquad U_m = 0. \tag{43}$$

Equation (31) is written in the form:

$$3\,\frac{\dot a}{a}\,(\beta - w) - \frac{\dot\alpha}{\alpha} = 0 \tag{44}$$

and has a solution

$$\alpha = \alpha(1)\,a^{3\,(\beta - w)}. \tag{45}$$

Table 2. NET-SES models: ($w = -1$, $\beta = 0$) give a constant pressure, ($w = -1/3$, $\beta = 0$) a constant gravitating mass.

w	β	E	P	μ
-1	0	$\mathcal{E}_m(1)\,a^3 + \mathcal{E}_Q(1)$	$-\mathcal{E}_m(1)$	$-2\,\mathcal{E}_m(1)\,a^3$
$-1/3$	0	$\mathcal{E}_m(1)\,a + \mathcal{E}_Q(1)$	$-\mathcal{E}_m(1)/(3\,a^2)$	$\mathcal{E}_Q(1)$

The solution of Eqs. (43) is well known. The interesting quantities have the following dependencies on a:

$$\mathcal{E}_Q(a) = \frac{\mathcal{E}_Q(1)}{a^{3(w+1)}}, \qquad \mathcal{E}_m(a) = \frac{\mathcal{E}_m(1)}{a^{3(\beta+1)}},$$
$$E_Q(a) = \frac{\mathcal{E}_Q(1)}{a^{3\,w}}, \qquad E_m(a) = \frac{\mathcal{E}_m(1)}{a^{3\beta}},$$
$$P_Q(a) = w\,\frac{\mathcal{E}_Q(1)}{a^{3\,(w+1)}}, \qquad P_m(a) = \beta\,\frac{\mathcal{E}_m(1)}{a^{3\,(\beta+1)}}, \tag{46}$$
$$\mu_Q(a) = 3\sigma_k(\chi)\mathcal{E}_Q(1)\,\frac{3\,w + 1}{a^{3\,w}}$$

and analogously for $\mu_m(a)$. We comment on two models in this class: ($w = -\frac{1}{3}$, $\beta = 0$) has a constant total gravitating mass and a time-dependent energy content. ($w = -1$, $\beta = 0$) produces a total constant pressure and variable energy content and gravitating mass (Table 2).

4.1.2. The condition of coherence

Assuming coherence (see Eq. (27)) Eq. (45) leads to

$$\frac{\dot a}{a}\,(\beta - w) = 0, \tag{47}$$

which for the non-static solution reduces to the simplest case of a one-fluid dust model:

$$\beta = w = 0. \tag{48}$$

Assuming asymptotic coherence (Eq. (28)) Eq. (45) gives

$$\alpha' = 3\,\alpha\,\frac{\beta - w}{a} = 3\,\alpha(1)\,(\beta - w)\,a^{3\,(\beta - w)-1}, \tag{49}$$

so, $\lim_{a\to\infty} \alpha' = 0$ only for $\beta - w < \frac{1}{3}$.

4.1.3. Asymptotic behaviour and evolution type

Substituting (45) into (17) one finds $\gamma(a)$:

$$\gamma(a) = \beta - \frac{\beta - w}{1 + \frac{1}{\alpha(a)}}, \tag{50}$$

which has the following asymptotical properties:

$$\lim_{a\to 0}\gamma(a) = \beta, \qquad \lim_{a\to\infty}\gamma(a) = w, \tag{51}$$

so, *any matter-quintessence NET-SES model is effectively matter at early times and quintessence at later times*.

The unique moment when the total pressure = 0 and the associated one-fluid is dust-like defines the scale a_{ed}:

$$a_{ed} = \left[\frac{-w}{\beta}\,\frac{\mathcal{E}_Q(1)}{\mathcal{E}_m(1)}\right]^{\frac{1}{3(\beta-w)}}. \tag{52}$$

Table 3. Evolution types for NET-SES models with various values of w and β. The numbers are given by Eqs. (39) and (40) for $\mathcal{E}_Q(1) = 0.7$ and $\mathcal{E}_m(1) = 0.3$ (**ME**) means $\alpha_{\mathcal{M}} = \alpha_{\mathcal{E}}$ etc.

	$w = -1$	$w = -2/3$	$w = -1/3$
$\beta = 0$	$a_{\mathcal{E}} = 0.754$	$a_{\mathcal{E}} = 0.655$	$a_{\mathcal{E}} = 0.429$
	$a_{\mathcal{P}} = 0.00$	$a_{\mathcal{P}} = 0.00$	$a_{\mathcal{P}} = 0.00$
	$a_{\mathcal{M}} = 0.598$	$a_{\mathcal{M}} = 0.655$	$a_{\mathcal{M}} = \infty$
	PME	**P(ME)**	**PEM**
	$a_{\rm ed} = \infty$	$a_{\rm ed} = \infty$	$a_{\rm ed} = \infty$
$\beta = 1/3$	$a_{\mathcal{E}} = 0.809$	$a_{\mathcal{E}} = 0.754$	$a_{\mathcal{E}} = 0.655$
	$a_{\mathcal{P}} = 0.615$	$a_{\mathcal{P}} = 0.598$	$a_{\mathcal{P}} = 0.655$
	$a_{\mathcal{M}} = 0.809$	$a_{\mathcal{M}} = 0.949$	$a_{\mathcal{M}} = \infty$
	P(ME)	**PEM**	**(PE)M**
	$a_{\rm ed} = 1.07$	$a_{\rm ed} = 0.950$	$a_{\rm ed} = 0.655$
$\beta = 2/3$	$a_{\mathcal{E}} = 0.844$	$a_{\mathcal{E}} = 0.809$	$a_{\mathcal{E}} = 0.754$
	$a_{\mathcal{P}} = 0.778$	$a_{\mathcal{P}} = 0.809$	$a_{\mathcal{P}} = 0.949$
	$a_{\mathcal{M}} = 0.915$	$a_{\mathcal{M}} = 1.07$	$a_{\mathcal{M}} = \infty$
	PEM	**(PE)M**	**EPM**
	$a_{\rm ed} = 0.915$	$a_{\rm ed} = 0.809$	$a_{\rm ed} = 0.598$
$\beta = 1$	$a_{\mathcal{E}} = 0.868$	$a_{\mathcal{E}} = 0.844$	$a_{\mathcal{E}} = 0.809$
	$a_{\mathcal{P}} = 0.868$	$a_{\mathcal{P}} = 0.915$	$a_{\mathcal{P}} = 1.07$
	$a_{\mathcal{M}} = 0.975$	$a_{\mathcal{M}} = 1.11$	$a_{\mathcal{M}} = \infty$
	(PE)M	**EPM**	**EPM**
	$a_{\rm ed} = 0.868$	$a_{\rm ed} = 0.778$	$a_{\rm ed} = 0.615$

To find the evolution types of NET-SES models, we use the function α to calculate two functions $\mathcal{P}$ and $\mathcal{M}$:

$$\alpha(a) = \frac{\mathcal{E}_Q(1)}{\mathcal{E}_m(1)}\, a^{3(\beta-w)},$$
$$\mathcal{P}(a) = \frac{|w|}{\beta}\,\alpha(a), \quad \mathcal{M}(a) = \frac{|3\,w+1|}{3\beta+1}\,\alpha(a)\cdot \tag{53}$$

The characteristic scales (Eq. (39)) are simply calculated:

$$a_{\mathcal{E}} = \left(\frac{\mathcal{E}_m(1)}{\mathcal{E}_Q(1)}\right)^{\frac{1}{3(\beta-w)}}, \quad a_{\mathcal{P}} = \left(\frac{\beta}{|w|}\frac{\mathcal{E}_m(1)}{\mathcal{E}_Q(1)}\right)^{\frac{1}{3(\beta-w)}},$$
$$a_{\mathcal{M}} = \left(\frac{3\beta+1}{|3\,w+1|}\frac{\mathcal{E}_m(1)}{\mathcal{E}_Q(1)}\right)^{\frac{1}{3(\beta-w)}}. \tag{54}$$

Table 3 illustrates the complex evolution from the matter-dominated model to quintessence-dominance. Different physical quantities pass through "boundaries" ($\alpha = 1$; $\mathcal{P} = 1$; $\mathcal{M} = 1$) between the matter and quintessence stages at different times. Note that $a_{\rm ed}$ cannot fully define the evolution type and dynamics.

4.1.4. The time dependence of the gravitating mass

The total gravitating mass in any NET-SES model has its extremum at

$$a_* = \left(\frac{\beta}{-w}\,\frac{3\beta+1}{3\,w+1}\right)^{\frac{1}{3(\beta-w)}}. \tag{55}$$

As a_* should be real, so $-1/3 < w < 0$. The requirement of positive $\mu''(a_*)$ is reduced to $\beta > w$ (always valid). So, in NET-SES the gravitating mass has a minimum for $-1/3 < w < 0$. No extrema exist for $-1 \le w \le -1/3$.

4.2. A NET-NSES case: Generalizing Chernin (1965)

Chernin (1965) obtained the solution for radiation plus dust, which we use as the matter component. For quintessence we take w = const. A new property due to complex matter is that the effective equation of state is non-stationary. The input equations are given by Eq. (23):

$$3\,\frac{\dot a}{a} = -\frac{\dot{\mathcal{E}}_Q}{\mathcal{E}_Q + P_Q}, \qquad P_Q = w\ \mathcal{E}_Q, \qquad w = \text{const.},$$
$$3\,\frac{\dot a}{a} = -\frac{\dot{\mathcal{E}}_R}{\mathcal{E}_R + P_R}, \qquad P_R = \frac{\mathcal{E}_R}{3},$$
$$3\,\frac{\dot a}{a} = -\frac{\dot{\mathcal{E}}_D}{\mathcal{E}_D}, \qquad P_D = 0,$$
$$\mathcal{E}_m = \mathcal{E}_R + \mathcal{E}_D \qquad P_m = P_D + P_R = P_R,$$
$$U_Q = 0, \qquad U_R = 0, \qquad U_D = 0. \tag{56}$$

The solutions of Eqs. (56) are well-known .We write:

$$\mathcal{E}_Q = \frac{\mathcal{E}_Q(1)}{a^{3(w+1)}}, \qquad \mathcal{E}_D = \frac{\mathcal{E}_D(1)}{a^3}, \quad \mathcal{E}_R = \frac{\mathcal{E}_R(1)}{a^4},$$
$$E_Q = \frac{\mathcal{E}_Q(1)}{a^{3w}}, \qquad E_D = \mathcal{E}_D(1), \quad E_R = \frac{\mathcal{E}_R(1)}{a},$$
$$P_Q = w\,\frac{\mathcal{E}_Q(1)}{a^{3(w+1)}}, \qquad P_D = 0, \qquad P_R = \frac{1}{3}\,\frac{\mathcal{E}_R(1)}{a^4},$$
$$\mu_Q = \mathcal{E}_Q(1)\,\frac{3\,w+1}{a^{3w}}, \quad \mu_D = \mathcal{E}_D(1), \quad \mu_R = \frac{2\,\mathcal{E}_R(1)}{a}\cdot \tag{57}$$

For α and $\beta = \mathcal{P}_m/E_m$ we find:

$$\alpha(a) = \frac{\mathcal{E}_Q(1)\,a^{1-3w}}{\mathcal{E}_D(1)\,a + \mathcal{E}_R(1)}, \quad \beta(a) = \frac{\mathcal{E}_R(1)}{\mathcal{E}_D(1)\,a + \mathcal{E}_R(1)} \tag{58}$$

Eq. (31) has the form:

$$3\,\frac{\dot a}{a}\,(\beta(a) - w) - \frac{\dot\alpha}{\alpha} = 0. \tag{59}$$

4.2.1. The conditions of coherence

From Eqs. (28), (58) and (59) we find

$$\alpha' = \frac{3\,\mathcal{E}_R(1)}{a}\,\frac{a^{1-3w}}{\mathcal{E}_m(1)\,a + \mathcal{E}_Q(1)}\cdot \tag{60}$$

$$\lim_{a\to\infty}\alpha' = -3\,w\,\frac{\mathcal{E}_R(1)}{\mathcal{E}_m(1)}\,a^{-(3\,w+1)} = 0, \text{ for } \quad w > -\frac{1}{3}$$
$$= -3\,w\,\frac{\mathcal{E}_R(1)}{\mathcal{E}_m(1)}, \qquad \text{for} \quad w = -\frac{1}{3}$$
$$= \infty \qquad \text{for} \quad w \le -\frac{1}{3}\cdot \tag{61}$$

4.2.2. The asymptotic behaviour and evolution types

Equations (58) and (17) give the effective coefficient $\gamma(a)$:

$$\gamma(a) = \frac{\mathcal{E}_Q(1)\,w\ + \frac{\mathcal{E}_R(1)}{3}\,a^{3w-1}}{\mathcal{E}_Q(1) + \mathcal{E}_D(1)\,a^{3w} + \mathcal{E}_R(1)\,a^{3w-1}}\cdot \tag{62}$$

Table 4. The numbers are obtained from Eqs. (39) and (65) with $\mathcal{E}_{\rm Q}(1) = 0.7$ and $\mathcal{E}_{\rm m}(1) = 0.3$; here $\mathcal{E}_{\rm D} = 0.05$, $\mathcal{E}_{\rm R} = 0.25$.

$w = -1$	$w = -2/3$	$w = -1/3$
$a_{\mathcal{E}} = 0.803$	$a_{\mathcal{E}} = 0.743$	$a_{\mathcal{E}} = 0.634$
$a_{\mathcal{P}} = 0.587$	$a_{\mathcal{P}} = 0.563$	$a_{\mathcal{P}} = 0.598$
$a_{\mathcal{M}} = 0.788$	$a_{\mathcal{M}} = 0.921$	$a_{\mathcal{M}} = \infty$
PME	**PEM**	**PEM**
$a_{\rm ed} = 0.587$	$a_{\rm ed} = 0.563$	$a_{\rm ed} = 0.598$

Because $\beta - w > 0$, the associated one-fluid model has the following asymptotic properties:

$$\lim_{a\to 0}\gamma(a) = 1/3, \qquad \lim_{a\to\infty}\gamma(a) = w. \tag{63}$$

So the one-fluid model, associated with the considered NET-NSES two-fluid is effectively radiation at early times and quintessence at late times. Its effective dust scale is

$$a_{\rm ed} = [-3\,w\,\mathcal{E}_{\rm Q}(1)/\mathcal{E}_{\rm R}(1)]^{\frac{1}{3w-1}}\,. \tag{64}$$

The evolution types are given by functions α, $\mathcal{P}$ & $\mathcal{M}$:

$$\mathcal{P} = \frac{3\,w}{\mathcal{E}_{\rm R}(1)}\,(\mathcal{E}_{\rm D}(1)\,a + \mathcal{E}_{\rm R}(1))\;\alpha(a),$$
$$\mathcal{M} = \frac{\mathcal{E}_{\rm Q}(1)\;(3\,w+1)}{\mathcal{E}_{\rm D}(1)\,a + 2\,\mathcal{E}_{\rm R}(1)}\,a^{1-3w} = (3\,w+1)\;\alpha(a). \tag{65}$$

In Table 4 three characteristic scales represent two evolution types for this model: PME and PEM.

For $-1/3 < w < 0$ the total gravitating mass has a unique minimum at a scale a_*

$$a_* = \left(\frac{3}{2}\,\frac{\mathcal{E}_{\rm Q}(1)}{\mathcal{E}_{\rm R}(1)}\,(-w)\,(3\,w+1)\right)^{\frac{1}{3w-1}}. \tag{66}$$

4.3. An ET-SES example: Coherence

ET-SES models allow coherence. An important class of dark energy models is the coupled quintessence (e.g. Wetterich 1995; Amendola 2000), where the total energy momentum tensor is conserved. For coupled quintessence, a stationary DE model was found (Amendola & Tocchini-Valentini 2001) which predicts coherent behaviour of matter and DE at late cosmic epochs. Such a relation was independently studied by Baryshev et al. (2001) to solve the problem of the smooth local Hubble flow. Teerikorpi et al. (2003) discussed the Hubble relation for coherent models. The coherence (α = const), produced by energy transfer, implies γ = const and the model is much simplified.

The associated one-fluid model is described by

$$3\,\frac{\dot a}{a} = -\frac{\dot{\mathcal{E}}}{\mathcal{E}+P}, \quad P = \gamma\;\mathcal{E}, \quad \gamma = \text{const}. \tag{67}$$

Separating its solutions (e.g. $\mathcal{E} = \mathcal{E}(1)/a^{3(\gamma+1)}$) into matter and quintessence we find:

$$\mathcal{E}_{\rm Q} = \frac{\mathcal{E}(1)}{a^{3(\gamma+1)}}\,\frac{\alpha}{\alpha+1}, \quad E_{\rm Q} = \frac{\mathcal{E}(1)}{a^{3\gamma}}\,\frac{\alpha}{\alpha+1},$$
$$P_{\rm Q} = \frac{\mathcal{E}(1)\,w}{a^{3(\gamma+1)}}\,\frac{\alpha}{\alpha+1}, \tag{68}$$

$$\mu_{\rm Q} = 3\sigma_k(\chi)\,\mathcal{E}(1)\,\frac{\alpha}{\alpha+1}\,\frac{3\,w+1}{a^{3\gamma}}$$
$$\mathcal{E}_{\rm m} = \frac{\mathcal{E}(1)}{a^{3(\gamma+1)}}\,\frac{1}{\alpha+1}, \quad E_{\rm m} = \frac{\mathcal{E}(1)}{a^{3\gamma}}\,\frac{1}{\alpha+1},$$
$$P_{\rm m} = \frac{\mathcal{E}(1)\beta}{a^{3(\gamma+1)}}\,\frac{1}{\alpha+1}, \tag{69}$$
$$\mu_{\rm m} = 3\sigma_k(\chi)\,\mathcal{E}(1)\,\frac{1}{\alpha+1}\,\frac{3\beta+1}{a^{3\gamma}}.$$

From Eq. (31) we find the energy transfer

$$U_{\rm m} = 3\,\frac{\alpha}{(\alpha+1)^2}\,\frac{\mathcal{E}(1)}{a^{3(\gamma+1)}}\,\frac{\dot a}{a}\,(\beta - w). \tag{70}$$

The asymptotic behaviour much depends on the sign and value of γ. The functions usually defining evolution type ($\alpha = \mathcal{E}_{\rm Q}/\mathcal{E}_{\rm m}$ = const, $\mathcal{P} = (|w|/\beta)\alpha$ = const, $\mathcal{M} = [(|3\,w+1|)/(3\beta+1)]\alpha$ = const) are now fixed ratios. Which component dominates during evolution depends on the value of α.

For the Amendola & Tocchini-Valentini (2001) solution, reached at late times ($w = -0.7$, $\beta = 0$, $\alpha = 7/3$), the equation of state of an associated one-fluid is given by $\gamma = (w\,\alpha + \beta)/(\alpha+1) = -49/100 \approx -1/2$, which gives

$$\mathcal{E} = \frac{\mathcal{E}(1)}{a^{153/100}} \approx \frac{\mathcal{E}(1)}{a^{3/2}}, \quad E = \mathcal{E}(1)\,a^{147/100} \approx \mathcal{E}(1)\,a^{3/2}$$
$$\mathcal{P} = -\frac{49}{100}\,\frac{\mathcal{E}(1)}{a^{153/100}} \approx -\frac{1}{2}\,\frac{\mathcal{E}(1)}{a^{3/2}}$$
$$\mu = -\frac{47}{100}\,\mathcal{E}(1)\,a^{147/100} \approx -\frac{1}{2}\,\mathcal{E}(1)\,a^{3/2}. \tag{71}$$

The energy transfer for the flat ($k = 0$) model is:

$$U_{\rm m} = 3\,\frac{\alpha}{(\alpha+1)^2}\,\frac{\mathcal{E}^{3/2}(1)}{a^{9/2\,(\gamma+1)}}\,(\beta - w). \tag{72}$$

This gives for the Amendola & Tocchini-Valentini solution

$$U_{\rm m} = \frac{441}{1000}\,\frac{\mathcal{E}^{3/2}(1)}{a^{459/200}} \approx \frac{2}{5}\,\frac{\mathcal{E}^{3/2}(1)}{a^{2.3}}. \tag{73}$$

Thus this model is asymptotically NET-SES.

5. Conclusions

In studies of FLRW models with Λ, Λ and matter are usually viewed as independent substances so that the energy momentum tensors of the partial fluids are separately conserved.

As a solution of the CDM crisis at galactic halo scales, Peebles & Ratra (2003) and Ostriker & Steinhardt (2003) suggested an interaction between dark energy and matter. One may find in the literature several models of DE-DM interaction, but the complexity of the dark sector still leaves a huge unexplored area.

This makes it necessary to study on a phenomenological level the general case when a one-fluid converts into another and the equations of state may be non-stationary. This is possible thanks to the remarkable fact that the FLWR models are defined at the level of the total energy-momentum conservation and are not directly sensitive to the specific mechanism producing the energy transfer. This is one reason why the internal physics of CDM and dark energy are still largely unknown.

We analyzed four classes of models defined by the presence/absence of energy transfer and by the stationarity/non-stationarity of the equations of state.

The equations of state with β for positive and w for negative pressure components and γ for the associated one-fluid, together with the energy transfer (a fluid converts into another), determine the type of evolution of the model depending on the class of the model. From the behaviour of the energy content, gravitating mass, and pressure versus the scale factor, we defined three characteristic scales, $a_{\mathcal{E}}$, $a_{\mathcal{P}}$ and $a_{\mathcal{M}}$. These border time intervals when quintessence energy, pressure and gravitating mass dominate. Sequences of the scales define 6 evolutionary types. It was shown that

- for given w and β, energy transfer defines γ and, hence, the total gravitating mass and dynamics of the model.
- a model can be coherent only if there is energy transfer.
- the dynamics of a model, its evolution type and energy transfer are interrelated.

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