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Letter to the Editor

Towards the monitoring of atmospheric turbulence model

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Abstract. A new method is proposed for the verification of the atmospheric turbulence model. It is based on the reconstruction of the phase structure function from simultaneous measurements of the Angle-of-Arrival longitudinal covariance at different baselines with the Generalized Seeing Monitor (GSM). In addition, with this new technique we obtain the first non-model dependent estimates of the outer scale with the GSM. Preliminary results of the reconstructed phase structure function with this technique are presented and compared to those obtained theoretically with the most known atmospheric turbulence models.

Key words. atmospheric turbulence

1. Introduction

Atmospheric turbulence reduces severely the resolution of the ground-based telescopes and degrades the performances of high angular resolution (HAR) techniques (Interferometry and Adaptive Optics (AO)). These observing methods require a better understanding of the behavior of the perturbed wavefronts, more exactly a better knowledge of the atmospheric turbulence model and the associate parameters. This is very crucial for the modelization in the domain of HAR techniques. Indeed, it is well-known that the performance of an AO system depends upon the seeing conditions, the outer scale $\mathcal{L}_0$, the isoplanatic angle θ_0 and the wavefront coherence time τ_0. The definition of most of these parameters is given using the Kolmogorov model which is valid only in the inertial range (spatial range between inner and outer scales). Other models (a review is given in Voitsekhovich 1995) have been proposed for a better understanding of the atmospheric turbulence effects beyond the limit scales. These models are empirical and up to now nothing could decide in favour of one of them. In addition, the small values found for the outer scale (Coulman et al. 1988) reduce significantly the inertial range and therefore the Kolmogorov model field to take into account the turbulence effects in the performance of large telescopes and long baseline interferometers.

In this paper, we describe a new technique of reconstruction of the phase structure function (PhSF) which is characteristic of the atmospheric turbulence model. The principle of this technique is based on simultaneous measurements of the Angle-of-Arrival (AA) longitudinal covariance at different baselines with the GSM. The first attempts of this PhSF reconstruction have been performed by Mariotti et al. (1984) and later by Davis et al. (1995) obtained respectively with the I2T and SUSI interferometers. With these instruments, the PhSF have been measured at only 3 different baselines which have been performed sequentially. On the other hand, the technique suggested in this paper allows a continuous reconstruction of this PhSF from AA longitudinal covariances measured simultaneously at several baselines with the GSM instrument. In addition, the GSM is user-friendly and runs faster than an interferometer which allows a monitoring of this PhSF (every 4 min, see Ziad et al. 2000) and, therefore, the atmospheric turbulence model verification. Indeed, the shape of the PhSF is characteristic of the atmospheric turbulence model (Voitsekhovich 1995). In addition, with this new technique, the first non-model-dependent estimates of the outer scale are provided with the GSM (currently the von Kármán model is used, see Ziad et al. 2000).

A theoretical description of this new technique is given in Sect. 2 in the context of optical astronomy. In Sect. 3, we briefly describe the GSM instrument and its particular configuration in regards to this new suggested technique. The data analysis and the associated results of the PhSF are presented in Sect. 4 and compared to those obtained theoretically from the most known models. The conclusions are given in Sect. 5.

2. Theoretical background

In the context of optical astronomy, the study of the atmospheric turbulence effects can be analyzed by means of the air refractive index and therefore by means of wavefront phase. The structure function of these parameters has a specific behavior for each model (Voitsekhovich 1995). The phase structure function D_ϕ is defined as the mean-squared

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difference in phase ϕ measured at 2 points separated by baseline $\boldsymbol{B}$,

$$D_\phi(\boldsymbol{B}) = \left\langle [\phi(\boldsymbol{r}) - \phi(\boldsymbol{r}+\boldsymbol{B})]^2 \right\rangle \quad (1)$$

where $\langle\rangle$ represents an ensemble average.

The general expression of this quantity is deduced from the phase spectrum $W_\phi(\boldsymbol{f})$ as,

$$D_\phi(\boldsymbol{B}) = 4\pi \int_0^{+\infty} \mathrm{d}f f W_\phi(\boldsymbol{f}) \Big[1 - J_0(2\pi B f)\Big] \left[\frac{2J_1(\pi D f)}{\pi D f}\right]^2 \quad (2)$$

where f is the modulus of the spatial frequency and D the aperture diameter measuring the phase ϕ. J_0 and J_1 represent the Bessel functions.

In the case of Kolmogorov model and if one neglects the telescope spatial filtering, integrating Eq. (2) gives the well-known expression,

$$D_\phi(\boldsymbol{B}) = 6.88(B/r_0)^{5/3} \quad (3)$$

where r_0 represents the Fried parameter.

This expression diverges for large baselines and therefore doesn't show the saturation observed by the interferometers (Davis et al. 1995; Mariotti et al. 1984). This saturation leads to an attenuation of the phase spectrum at low frequencies limited by a finite outer scale which is explicitly indicated in different models,

von Kármán model:

$$W_\phi(\boldsymbol{f}) = 0.0229 r_0^{-5/3} \left(f^2 + \frac{1}{\mathcal{L}_0^2}\right)^{-11/6}. \quad (4)$$

Greenwood-Tarazano model:

$$W_\phi(\boldsymbol{f}) = 0.0229 r_0^{-5/3} \left(f^2 + \frac{f}{\mathcal{L}_0}\right)^{-11/6}. \quad (5)$$

One can remark that when the outer scale $\mathcal{L}_0$ tends to infinity these expressions converge toward the Kolmogorov one.

In the case of these models, the integral in Eq. (2) has no analytical expression but approximate solutions exist under some assumptions (Voitsekhovich 1995; Conan et al. 2000).

Figures 3 and 4 of Voitsekhovich (1995) show the PhSF behavior for these models without spatial filtering of the telescope aperture. This spatial filtering is taken into account in whole of this paper. One can remark the significant difference between the different models for the same turbulence conditions (r_0, $\mathcal{L}_0$) pointing the fact that the PhSF is an interesting criterion to check the model validity.

In this paper, we suggest a new method for monitoring the PhSF from AA longitudinal covariances measured with the GSM. Indeed, the AA covariance C_α is related to the PhSF (Roddier 1981) by,

$$C_\alpha(x, y) = \frac{\lambda^2}{8\pi^2} \frac{\partial^2 D_\phi(x, y)}{\partial x^2}. \quad (6)$$

By integrating 2 times this expression, one can reconstruct the PhSF. Indeed, the integral of Eq. (6) over x-direction leads to

$$D_\phi(x, y) = \frac{8\pi^2}{\lambda^2} \int \mathrm{d}x \left[\int \mathrm{d}x C_\alpha(x, y)\right] + Ex + F \quad (7)$$

Fig. 1. The GSM configuration at Calern Observatory near the GI2T interferometer. Only four of the six GSM modules are presented here.

where E and F are integration constants. F is determined by the fact that $D_\phi(x, y)$ is equal to zero at the origin.

A direct determination of the second integration constant E requires the knowledge of the PhSF at one baseline B preferably large (saturation range). This is possible with an interferometer; the standard deviation of the optical path difference (OPD) between the 2 arms separated by a baseline $B = \sqrt{x^2 + y^2}$ is given by,

$$\sigma_{\mathrm{OPD}}(B) = \frac{\lambda}{2\pi} \sqrt{D_\phi(B)}. \quad (8)$$

Another way to obtain this second integration constant E is to use the Kolmogorov phase structure function in Eq. (3) for small baselines ($B \ll \mathcal{L}_0$). This is justifiable because the Kolmogorov model is only valid in the inertial range which is limited by the outer scale $\mathcal{L}_0$. Indeed, by fitting our reconstructed PhSF model with Eq. (3) for small baselines ($B < 1$ m) should lead to the constant E determination.

3. The GSM instrument

The GSM consists of evaluating the optical parameters of the perturbed wavefront by measuring AA fluctuations. Indeed, the GSM uses the same principle than a Shack-Hartmann, i.e., measuring AA at different points of the wavefront. Computing spatio-temporal correlations of these measured AA leads to estimates of the seeing ϵ_0, outer scale $\mathcal{L}_0$, isoplanatic angle θ_0 and coherence time τ_0.

The instrument consists of different identical units equipped with 10 cm telescopes installed on equatorial mounts and pointing at the same star (Fig. 1). Each telescope measures the AA fluctuations by mean of flux modulation which is produced by the displacement of the observed star image over a Ronchi grating.

The AA fluctuations are measured with 5 ms resolution time during 2 min acquisition time. Data are processed immediately after each acquisition, allowing a quasi real-time monitoring of ϵ_0, $\mathcal{L}_0$, θ_0, τ_0 and of the covariances. The data acquisition is repeated typically every 4 min. A detailed description of the GSM instrument is given by Ziad et al. (2000).

In the framework of this paper, a special set-up of the GSM has been performed. Two modules were installed on a

Table 1. The GSM configurations used during the night of June 06, 2001 at Calern Observatory. The units 1, 2 and 3, 4, 5, 6 were synchronized in a manual way.

Units combination	Baseline (m)	Baseline (m)
1–2	0.25	0.25
3–4	0.80	0.80
3–5	2.4	3.0
3–6	6.0	10.0
4–5	1.6	2.2
4–6	5.2	9.2
5–6	3.6	7.0

common mount on a central pier working as a differential image motion monitor (DIMM) with a 25 cm baseline (for r_0 estimation). These 2 modules were managed by a first computer PC1. Four other modules having different mounts were interfaced to a second computer PC2. The modules were located at 1.5 m above the ground on different platforms installed on rails allowing a fast baseline change. Two configurations have been chosen with the 6 modules (see Table 1) for a better sampling of the AA longitudinal covariance function. The starting time of the acquisitions of the two PCs was made in a manual way.

4. Results

During the night of July 06, 2001 measurements of the AA covariances have been performed with the GSM at the Calern Observatory near the GI2T interferometer (Mourard et al. 2001). Two GSM configurations have been used successively during these observations (see Table 1). At 0h22 UT the GSM switched from the first configuration to the second one (see Table 1). In order to increase the sampling of the AA covariance we combined the results of the 2 nearest acquisitions in the 2 configurations. These acquisitions have been performed at 0h16 and 0h22 UT on the same source δ Cyg. The value at the origin is an average of the different variances obtained with the different modules. As 0.25 m and 0.8 m baselines are present in both configurations (see Table 1), an average of the 2 acquisitions has been adopted. These results are shown in Fig. 2. One can remark that the AA fluctuations present negative covariances which is specific to the longitudinal case. A theoretical fit has been done combining a polynomial function for the oscillations and a decreasing exponential to attenuate these oscillations for long baselines and also to avoid the divergence of the polynomial fit. The best fit is shown in Fig. 2 obtained with a polynomial of degree 9. The coefficients of the polynomial and the exponential are obtained using the Mathematica fit function "NonLinearFit" (see Mathematica Web Site at `http://www.wolfram.com/`). This function which uses the Levenberg-Marquardt method searches for a least-squares fit to a list of data according to a model containing unknown parameters (Bates et al. 1988). The only limitation of this method is when the data set is smaller than the number of parameters which is not the case of the fit presented in Fig. 2. The degree of the polynomial has been chosen according to this limitation and also to reduce the residual error.

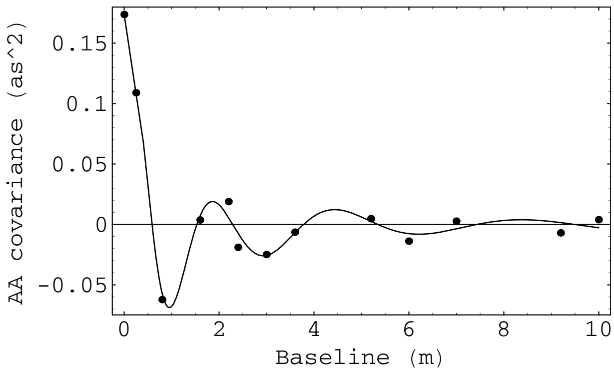

Fig. 2. AA longitudinal covariances measured with the GSM for different baselines. These data are well-fitted by a model (full line) described in Sect. 4.

The model obtained with this fit is easily integrable and therefore make easier the reconstruction of the PhSF. Indeed, including this fitting model in the first term of Eq. (7), leads to a similar form with different coefficients. The value of the integration constant F in Eq. (7) is given by the integral value at the origin.

As explained in Sect. 1 the second integration constant E could be obtained either by interferometric measurements or by fitting the reconstructed model of the PhSF with the Kolmogorov expression in Eq. (3) for small baselines ($B \ll \mathcal{L}_0$). Only this last method has been used because the poor seeing conditions during the simultaneous observations between the GSM (Fig. 2) and the GI2T interferometer on July 06, 2001, didn't allow the GI2T running and therefore the fringe acquisitions. Indeed, during the period of the GSM data presented in Fig. 2 the seeing was of 1.63″ at 0.5 μm.

Thus, the best fit of this reconstructed PhSF model to the Kolmogorov definition in Eq. (3) for baselines less than 1 m gives a value of the integration constant E of 5.6 rad^2 m^{-1}. This constant value leads to a standard deviation of the OPD of 9.5λ for a baseline $B = 15.2$ m. This $\sigma_{\rm OPD}$ value is rather close to those measured directly with the GI2T interferometer during the night of June 22, 2001. Indeed, for the same baseline $B = 15.2$ m, the GI2T interferometer obtained between 23h18 and 23h25 values of $\sigma_{\rm OPD}$ between 11.63λ and 17.28λ for a measured seeing less than 1″ at 0.5 μm.

Introducing the E value in Eq. (7) leads to the PhSF in Fig. 3 (full line). This result is in excellent agreement with the empirical von Kármán model for an outer scale $\mathcal{L}_0$ of 25 m and Fried parameter r_0 of 6.2 cm corresponding to the value measured with the GSM at this time. For comparison, the Kolmogorov and Greenwood-Tarazano models are given for the same conditions (r_0, $\mathcal{L}_0$). The exponential model has been ignored because of its similar behavior to the von Kármán model (Voitsekhovich et al. 1995). As it is predictable, if the $\sigma_{\rm OPD}$ (or integration constant E) estimation had given a stronger value, the most suitable model would be that of Kolmogorov. This is possible in the case of strong values of the outer scale $\mathcal{L}_0$ which have been observed sometimes with the GSM in the most sites (Ziad et al. 2000).

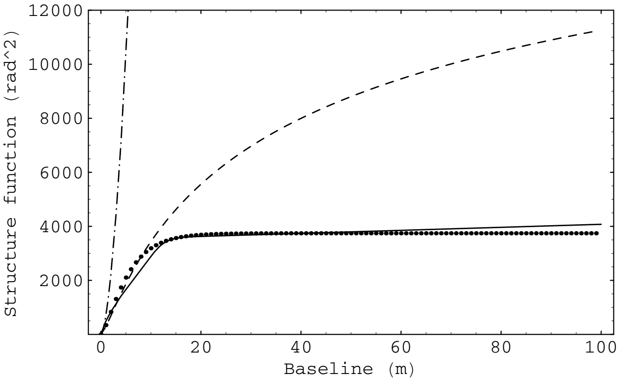

Fig. 3. Phase structure function reconstructed (full line) from GSM data in Fig. 2. An excellent agreement is found with the von Kármán model (dotted line) for $\mathcal{L}_0 = 25$ m and $r_0 = 6.2$ cm. The Kolmogorov (dashed-dotted) and Greenwood-Tarazano (dashed) models are given for the same conditions.

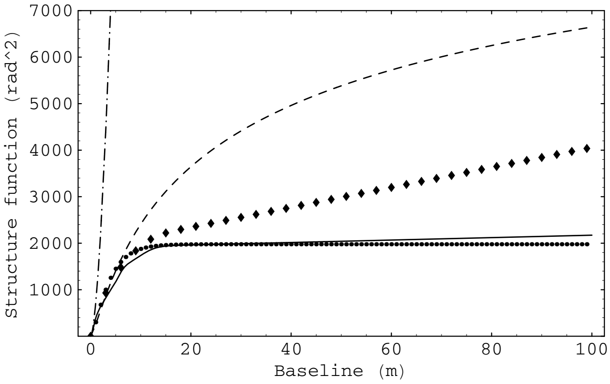

Fig. 4. Same than Fig. 3. The diamond plot correspond to an artificial increase of 0.5λ of the $\sigma_{\rm OPD}$ for a baseline of 15.2 m (see text). The von Kármán and Greenwood-Tarazano models have been obtained with $\mathcal{L}_0 = 17$ m and $r_0 = 6.23$ cm.

In addition, Fig. 4 shows another result obtained 10 mn later than the one presented in Fig. 3. The corresponding integration constant E value of this new result is of 2.62 rad^2 m^{-1} leading to a $\sigma_{\rm OPD}$ value of 7.01λ for a baseline $B = 15.2$ m. The von Kármán model is once again the most convenient for this result with $\mathcal{L}_0 = 17$ m and $r_0 = 6.23$ cm. One can remark that this technique is very sensitive to the outer scale variation even if the seeing remains the same (the r_0 is almost similar for results in Figs. 3 and 4). Indeed, the saturation level and beginning of the reconstructed PhSF in Fig. 4 are lower than those in Fig. 3 leading to different values of the outer scale in the case of the von Kármán model. In addition, if we increase artificially of 0.5λ the $\sigma_{\rm OPD}$ value of the last result (corresponding to a relative difference of 6.6%), this leads to a new E value of 21.48 rad^2 m^{-1} instead of 2.62 rad^2 m^{-1}. The result corresponding to this case is shown in Fig. 4 as asterisk plot. One can remark that this reconstructed phase function deviates from the von Kármán model and tends to Greenwood-Tarazano one. This means that in the case of simultaneous measurement with an interferometer an excellent precision of the OPD measurement is desirable.

5. conclusion

A new method has been proposed for the reconstruction of the phase structure function which is characteristic of the model describing the atmospheric turbulence. This technique allows, therefore, the first non-model dependent estimates of the outer scale $\mathcal{L}_0$ from the GSM with a good sensitivity to $\mathcal{L}_0$ variation. The preliminary results show that the von Kármán model is more convenient for large scales. But the Kolmogorov model could be also suitable in the case of large $\mathcal{L}_0$ values. We plan long observations campaigns in the future to bring a confirmation of these GSM's $\sigma_{\rm OPD}$ with direct and simultaneous measurements of the GI2T interferometer and to bring a better statistical knowledge on the atmospheric turbulence model.

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