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Large frequency drifts during type I X-ray bursts

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Abstract. We study the spin-down of a neutron star atmosphere during the type I X-ray burst in low mass X-ray binaries. Using polar cap acceleration models, we show that the resulting stellar "wind" torque on the burning shell due to the flowing charged particles (electrons, protons and ions) from the star's polar caps may change the shell's angular momentum during the burst. We conclude that the net change in the angular momentum of the star's atmosphere can account for rather large frequency drifts observed during type I X-ray burst.

Key words. stars: neutron – stars: magnetic fields – X-rays: binaries – X-rays: bursts

1. Introduction

The discovery of high coherence, large modulation amplitudes, and stable frequency oscillations in the range $\nu_0 \sim$ 270–620 Hz during type I X-ray bursts in weakly-magnetic ($B \leq 10^{10}$ G) accreting (at intermediate rate $10^{-11}\ M_\odot\ \mathrm{yr}^{-1} < \dot{M} < 10^{-8}\ M_\odot\ \mathrm{yr}^{-1}$) neutron stars in low mass X-ray binaries (LMXBs) has issued many new puzzles for investigators. An initial puzzle seen in the observations was that the oscillation frequency increases by $\Delta\nu \sim$ a few Hz during the burst. Strohmayer et al. (1997) firstly proposed that this frequency shift is due to the conservation of angular momentum of the decoupled burning shell from the neutron star in which the shell undergoes spin changes as it expands and contracts during the type I X-ray bursts. Motivated by this proposal, Cumming & Bildsten (2000) studied the rotational evolution of the neutron star atmosphere during a thermonuclear burst by considering one-dimensional vertical hydrostatic expansion. By assuming conservation of the angular momentum of the shell, they showed that a hot burning shell might expand hydrostatically by $\Delta R \sim 20$ m (Ayasli & Joss 1982; Hanawa & Fulimoto 1984; Bildsten 1998), and lag behind the neutron star by $\Delta\nu \sim \nu_s(2\Delta R/R) \sim 1\ \mathrm{Hz}(\nu_s/300\ \mathrm{Hz})(\Delta R/20\ \mathrm{m})(10\ \mathrm{km}/R)$ where ν_s is neutron star spin frequency and R the radius. As the shell cools down and contracts, the rising oscillations due to a temperature inhomogeneity, drifts upward as we seen by $\Delta\nu$ of few Hertz. By assuming that the burning shell rotates rigidly, they found rough agreement with the observed values with fractional frequency shifts $\Delta\nu/\nu_0 \leq 0.8\%$.

However, recent observations suggested that purely radial hydrostatic expansion and angular momentum conservation alone cannot account for explaining rather large frequency drifts ($\Delta\nu/\nu_0 \sim 1.3\%$) observed in some bursts (Galloway et al. 2001; Wijnands et al. 2001). As pointed out by Galloway et al. (2001), the required expansion by these frequency drifts is 4–5 times larger than the one predicted by Cumming & Bildsten (2000), see Table 1. In order to achieve the rather large frequency shift, Cumming et al. (2001) improved the calculations done by Cumming & Bildsten (2000) by including the general relativistic corrections for either slowly or rapidly rotating star. They found that the general relativity has small effect on the angular momentum conservation law (~5–10%). Comparing with the data, for a rigidly rotating atmosphere, they obtained that the expected spin-down is a factor of two or three less than the actual observed values. In another attempt, Spitkovsky et al. (2001) considered a two-dimensional hydrodynamics model that the burning spots can spread over the neutron star surface. Due to the combination of the radial expansion of the burning shell and rotation of the star, they proposed that horizontal hydrodynamics flows may arise in the neutron star burning ocean during the type I X-ray burst. By taking into account the action of the Coriolis force due to the rapid rotation of the star, they showed that the horizontal flows may explain many features of observed bursts such as the very short rise time of X-ray bursts and the lack of burst oscillations in several bursts. Further, they argued that during cooling the hot ashes left by burning front, there is temperature gradient between equator and pole, increasing toward the pole, which drives a zonal thermal wind directed *backward* to the neutron star rotation. [1] They found that *only if* the frequency drift due to the radial expansion is combined with the geostrophic drift

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[1] Actually they showed that due to the hydrostatic balance the material will likely ignite sooner at the equator rather than the pole. As a result, one would expect the burning front cools down at the equator firstly.

Table 1. In this table we obtain the lowest radius expansion ΔR (based on hydrostatic expansion models) and the scaled magnetic field ηB (based on the polar cap particle acceleration models) for some X-ray bursts in LMXBs. We calculate these quantities for two chosen values of the star spin frequency, $\nu_s = 300$ Hz and $\nu_s = 600$ Hz, such that the corresponding frequency shift is comparable with observations. ν_0 is the oscillation frequency and $\Delta\nu$ its corresponding shift seen during the burst. The value of ηB is based on the assumed value of $\Delta R \simeq 20$ m for both spin frequencies, see Eq. (9).

Object	Time (UT)	ν_0 (Hz)	$\Delta\nu/\nu_0$ (10^{-3})	ΔR_{300} (m)	ΔR_{600} (m)	$(\eta B)_{300}$ (10^{10} G)	$(\eta B)_{600}$ (10^{10} G)
4U 1636-54	1996 Dec. 28 (22:39:22)[1,2]	580.5	~2–4	≥40	≥20	≥3	≥1
	1996 Dec. 29 (23:26:46)[2,3]	581.5	~3	≥40	≥20	~2	≤0.01
	1996 Dec. 31 (17:36:52)[2,3]	581.	~3	≥40	≥20	~2	≤0.01
4U 1702-43	1997 Jul. 26 (14:04:19)[4]	329.85 ± 0.1	7.7 ± 0.3	≥50	–	≥6	–
	1997 Jul. 30 (12:11:58)[4]	330.55 ± 0.02	4.8 ± 0.3	≥30	–	≥2	–
4U 1728-34	1996 Feb. 16 (10:00:45)[4,5]	364.23 ± 0.05	6.6 ± 0.1	≥50	–	≥5	–
	1997 Sep. 9 (06:42:56)[4]	364.10 ± 0.05	5.9 ± 0.2	≥45	–	≥4	–
Aql X-1	1997 Mar. 1 (23:27:39)[6,7]	549.76 ± 0.04	4.3	≥50	≥25	≥3	~1
4U 1916-053	1998 Aug. 1 (18:23:45)[8]	269–272	13.0	≥70	–	~12.5	–
MXB 1658-298	1999 Apr. 14 (11:44:52)[9]	567	9.0	≥100	≥50	~10	~7.5

References: (1) Strohmayer et al. (1998); (2) Miller (2000); (3) Strohmayer (1999); (4) Strohmayer & Markwardt (1999); (5) Strohmayer et al. (1996); (6) Zhang et al. (1998); (7) Fox et al. (2000); (8) Galloway et al. (2001); (9) Wijnands et al. (2001).

caused by backward zonal flows, one may expect to observe the rather large frequency drifts of burst oscillations in tails of some bursts.

Because of no coherent pulsations seen in persistent emission from the majority of neutron stars in LMXBs, it is believed that they are weakly magnetized ($B \leq 10^9$ G). So, in previous studies the effect of magnetic field of the neutron star was ignored in the rotational evolution of burning atmosphere during the X-ray bursts. In this paper we study the change in the angular momentum of the burning shell during the type I X-ray burst. This study is based on polar cap particle acceleration models in the pulsar polar cap regions. Due to the star's rotation and magnetic field an electric field must be induced in magnetosphere, i.e. $\boldsymbol{E} = -(\boldsymbol{\Omega}_s \times \boldsymbol{r}) \times \boldsymbol{B}/c$, that has non-zero component along the magnetic field lines ($E_\parallel = \boldsymbol{E} \cdot \boldsymbol{B}$) (see for recent review Mestel 1998). $\boldsymbol{\Omega}_s$ is the angular velocity of the star, and r is a distance outside of the star. The parallel electric field accelerates charged particles along the open magnetic field lines above the polar caps up to ultrarelativistic energies toward the star's light cylinder ($r_{lc}\Omega_s = c$). Here, we argue that, for typical magnetic field $B \sim 10^8$ G, the net charged particles flowing from star surface to infinity in the pulsar polar caps, would exert a stellar wind torque on the burning shell that may cause the angular momentum of the shell to change during the burst. We show that the net change in the angular momentum of the shell during the burst can account for rather large frequency drifts of burst oscillations.

2. The model

The theory of particle acceleration in pulsar magnetosphere has been studied for three decades, after pioneering work by Goldreich & Julian (1969) on pulsar electrodynamics. The charged particles accelerated by the parallel electric field (relative to the magnetic field lines) escape from the surface along the open field lines which extended beyond the star's light cylinder (of radius $r_{lc} \sim 5\times10^9/\nu_s$ cm, where ν_s in Hertz), and form a relativistic stellar wind. Several kind of acceleration mechanisms has been developed (Mestel 1998), however, the one proposed by Arons (1979, 1981) may apply likely to X-ray burst pulsars that we will consider here. Arons assumed that charged particles (electrons and ions) flow freely from the neutron star surface due to the thermal activities. Freely escaping particles from the neutron star surface depends on the binding energy (referred as the cohesive energy for ions and as the work function for electrons) and the surface temperature. For typical magnetic field of a neutron star in LMXBs ($\sim 10^8$ G), the threshold temperature for thermionic emission of electrons is $T_e \sim 10^4$ K and for ions is $T_i \sim 10^3$ K (Luo et al. 2000) that are 10^4–10^5 lower that surface temperature of the neutron star in LMXBs. Further, due to the thermonuclear activities on the surface of the star during the X-ray burst, the particles' kinetic energy would increase significantly. Therefore, freely flowing particles is very likely from the surface of such a pulsar.

Outward flowing charged particles above the pulsar polar caps along the open magnetic field lines causes the Goldreich-Julian (corotation) charge density $\rho_{GJ} \sim \boldsymbol{\Omega}_s \cdot \boldsymbol{B}/2\pi c$ cannot be kept balanced (Harding & Muslimov 1998). As a result, a strong electric field develops along the magnetic field, $E_\parallel$, above the magnetic poles due to the departure of total charge density ρ_e from the Goldreich-Julian charge density, starts from zero at the surface and grows with distance above the surface. Even though $\rho_e = \rho_{GJ}$, and therefore $E_\parallel = 0$ at the surface, the curvature of the field lines causes the area of the open-field region to increase, so that ρ_e increases faster than ρ_{GJ}, and then a charge deficit grows with distance (Harding & Muslimov 1998). Calculation of the parallel electric field is so complicated and depends on several mechanisms that work to enhance or screen the field. In a series of papers Harding & Muslimov (1998, 2001, 2002); and Harding et al. (1998) have extensively studied the influence of pair production, inertial frame-dragging effect, curvature radiation, and inverse Compton scattering on the $E_\parallel$. They showed that for the unsaturated region with altitude in range $0 < z \ll (\Omega_s R/c)^{1/2} \simeq 0.1$ (for $\Omega_s = 600\pi(\nu_s/300\ \mathrm{Hz})$ rad/s and $R \sim 10^6$ cm) above the stellar surface, the parallel electric field grows linearly with height as $E_\parallel(\theta,\phi,z) = E_0(\theta,\phi)z$, while for the saturated region with $z > (\Omega_s R/c)^{1/2}$, the parallel electric field that is nearly constant respect to the altitude, drops by three order of magnitudes. Here θ and ϕ are spherical polar and azimuthal angles, and z is the altitude in units of stellar radius. The amplitude $E_0(\theta,\phi)$ depends on magnetic field strength B, spin frequency ν_s, and orientation of the magnetic field symmetric axis relative to the star spin axis χ. In a simple form, one can estimate $E_\parallel$ when the charged particles flow freely from the neutron star surface (see Arons 1981; Usov & Melrose 1995; Harding & Muslimov 2001) as

$$E_\parallel \simeq 8.2\times10^5\ \mathrm{V/m}\ \left(\frac{\nu_s}{300\ \mathrm{Hz}}\right)^{5/2}\left(\frac{R}{10^6\ \mathrm{cm}}\right)^{5/2} \times\left(\frac{B}{10^8\ \mathrm{G}}\right)\left(\frac{z}{10^{-4}}\right). \quad (1)$$

Note we calculated $E_\parallel$ here for a typical neutron star in LMXBs with nearly perpendicular field orientation relative to the spin axis. The relative fraction of electric force to the gravity force above the burning front during the burst, i.e. $z \sim 10^{-3}$, for electrons and protons are $\sim 8\times10^4$ and ~ 42, respectively. As a result, electrons, protons, and even ionized helium atoms can be accelerated easily along the magnetic field lines by parallel electric field presented in the polar caps of neutron stars in LMXBs. Since the magnetic field lines in the polar cap regions are not closed and extended beyond the star's light cylinder, the accelerated particles in these regions may leave the neutron star magnetosphere. The resulting flow of particles from the polar caps to infinity produces a relativistic "stellar wind", that exerts a "stellar wind torque" (Michel 1991) on the shell, and then causes the net angular momentum of the shell to change during the burst. The net wind torque exerted on the shell due to the out flowing charged particles from the star polar caps is estimated by Michel (1991) for the Goldreich-Julian charge density to be

$$T_\mathrm{W} = I^2/4c\Omega_s \quad (2)$$

where $I = \eta n_\mathrm{GJ} e v A_\mathrm{pc}$ is the total current of the charged particles flowing from the polar caps with the area $A_\mathrm{pc} \approx 1.6\pi R^3/R_c$ that is 10% of total star's area ($4\pi R^2$). Here $n_\mathrm{GJ} \simeq 0.36\Omega_s B/2\pi ec \simeq 7.5\times10^8(\nu_s/300\ \mathrm{Hz})(B/10^8\ \mathrm{G})\ \mathrm{cm}^{-3}$ is the Goldreich-Julian charge number density, $e = 4.8\times10^{-10}$ esu, $R_c = (GM_s/\Omega_s^2)^{1/3}$ is the corotation distance for a star with mass M_s, and $v \geq 0.1c$ is average speed of particles at $z \sim 10^{-3}$. The velocity of particle grows up quickly to $\sim c$ at $z \sim 0.1$ as it is accelerated by $E_\parallel$. $\eta \geq 1$ is a dimensionless factor (free parameter of the model) such that the quantity ηn_GJ represents the actual number density of particles that accelerated and left the star due to the parallel electric field. The value of $\eta \simeq 1$ for old and isolated neutron stars (with no observed bursts), i.e. the space charge density is nearly Goldreich-Julian charge density for these stars. But in neutron stars in LMXBs with an ocean of accumulated hot matters, the value of η must be greater than 1, due to the free ejection of particles from the front surface of the ocean. Further, during bursting the accumulated materials with $T \geq 10^8$ K on the neutron star surface, the kinetic energy of particles inside the ocean abruptly increases and then, the space charge density would increase significantly in regions with non-zero parallel electric field. As a result, one would expect that the number particles that may leave the shell would increase during the burst.

It is necessary to note that for neutron stars in LMXBs with intermediate accretion rate, $10^{-11}\ M_\odot\ \mathrm{yr}^{-1} < \dot{M} < 10^{-8}\ M_\odot\ \mathrm{yr}^{-1}$, the existence of pulsar wind torque might not be consistent with accretion. One would expect the motion of the accreting materials to oppose and shut off the radio pulsar wind, and contribute to the angular momentum of the system. This scenario is unlikely "during the X-ray burst" due to the following reasons: before the burst the accretion disk extends down to the neutron star's surface. When the burst stars, a huge thermonuclear explosion on the surface of the star causes the explosive eruption of the accumulated materials from the surface during the burst with bulk velocity close to the speed of light. The resulting luminosity due to the explosion exceeds the Eddington limit at this time. As a result, the inner part of the accretion disk is swept away by the burst radiation pressure. As a result, one would expect the action of accretion to more or less shut off in this period due to the strong interaction of the expanded shell with the inner accretion disk. Interestingly, Shaposhnikov et al. (2003) used the latter mechanism to explain why the emitted flux from neutron star in the X-ray source 4U 1728-34 is not constant during the expansion stage. Furthermore, it has been suggested the action of accretion torque is balanced by the gravitational radiation torque in neutron stars in LMXBs, see Bildsten (1998a, 2002) for details. Therefore, the increased angular momentum due to accretion is lost to gravitational radiation. The latter mechanism provides a natural explanation for the rotation of stars in a narrow range of frequencies, $\nu_s \sim 300$ Hz. Finally, the magnitude of accretion torque $T_a \sim \dot{M}(GM_s R)^{1/2} \simeq 8.6\times10^{32}$ dyne cm $(\dot{M}/10^{-9}\ M_\odot\ \mathrm{yr}^{-1})$ (for $M_s = 1.4\ M_\odot$ and $R = 10$ km) is one order of magnitude smaller than that of the wind torque $T_w \sim 9.32\times10^{33}$ dyne cm for $\eta \sim 1000$ and $B \sim 10^8$ G, see Eq. (3) and the discussion below Eq. (9).

By replacing I in Eq. (2) we obtain

$$T_w \simeq 9.32\times10^{27}\ \mathrm{dyne\ cm}\ \eta^2\left(\frac{M_s}{1.4\ M_\odot}\right)^{-2/3} \times\left(\frac{\nu_s}{300\ \mathrm{Hz}}\right)^{7/3}\left(\frac{R}{10\ \mathrm{km}}\right)^{6}\left(\frac{B}{10^8\ \mathrm{G}}\right)^{2}. \quad (3)$$

Equation (3) gives the exerted torque on the neutron star surface due to the flowing charged particles from the pulsar polar caps. Therefore, the burning shell of the neutron star will experience this torque as it expands during the burst. The angular momentum of the shell at a distance R from the center of the star in Newtonian dynamics can be written as

$$\ell = \kappa M R^2 \Omega, \quad (4)$$

where constant $\kappa = (I/MR^2)_{\rm shell}$ depends on the equation of state and $\kappa \leq 2/3$ (an uniform spherical shell). Here M and I are the mass and moment inertia of the shell. Equation (4) may be corrected by $\ell = \kappa MR^2 \sin^2\theta\Omega_{\rm s}/(1-2GM_{\rm s}/R)^{1/2}$ for a relativistic spherical star and by $\ell = \kappa MR^2 \sin^2\theta(\Omega_{\rm s}-\omega)/(1-2GM_{\rm s}/R)^{1/2}$ for a rotating relativistic star (see Cumming et al. 2001). Here $\omega = J_{\rm s}/R^3$ where $J_{\rm s}$ is the total angular momentum of the star. However, as shown by Cumming et al. (2001), the relativistic corrections have small contributions to the final results, and so we ignore them. The time evolution of the angular momentum of the shell will be

$$\frac{\Delta}{\Delta t}(R^2\Omega)_{\rm shell} = -T_{\rm w}/\kappa M, \tag{5}$$

or

$$(R^2\Omega)_{\rm shell}(2\Delta R/R + \Delta\Omega/\Omega)_{\rm shell} = -(T_{\rm w}/\kappa M)\Delta t. \tag{6}$$

Therefore, the change in the angular velocity of the burning shell will be

$$(\Delta\Omega/\Omega)_{\rm shell} = -2\Delta R/R - (T_{\rm w}/\kappa MR^2\Omega)\,\Delta t. \tag{7}$$

Here we assumed that the change in the mass of the shell is order of or less than $(\Delta R/R)^2$ and then we neglect it (see Cumming et al. 2001). Equation (7) represent the change in angular velocity of the shell during the burst. Now we discuss the change in angular velocity in three stages, before the X-ray burst or the recurrence period, the rising period, and after the burst.

2.1. Before the X-ray burst

In this period the neutron star accrets materials from its companion star and accumulates them in the ocean that more or less rigidly corotates with the star (Cumming et al. 2001). The parallel electric field inside the ocean is nearly neutralized due to redistribution of charge particles of the ocean. Further, due to the large plasma density of infalling matters, the electric field outside of the ocean would be shorted out. Therefore, the polar cap acceleration is more or less suppressed due to strongly accreting plasma in this period. As a result, the pulsar wind torque would be negligible.

Furthermore, because of nearly rigid rotation of the ocean with star the net wind torque (if exists) during this period will be acted more or less on the star itself. Therefore, Eq. (7) should be corrected for this period as

$$\begin{aligned}\left(\frac{\Delta\Omega}{\Omega}\right)_{\rm shell} &= -(T_{\rm w}/\kappa' M_{\rm s}R^2\Omega)\,\Delta t,\\ &\simeq -4.44\times10^{-17}\left(\frac{\eta}{100}\right)^2\left(\frac{M_{\rm s}}{1.4\,M_\odot}\right)^{-5/3}\\ &\quad\times\left(\frac{R}{10\ {\rm km}}\right)^4\left(\frac{\nu_{\rm s}}{300\ {\rm Hz}}\right)^{4/3}\left(\frac{B}{10^7\ {\rm G}}\right)^2\left(\frac{\Delta t_{\rm rec}}{10^4\ {\rm s}}\right),\end{aligned} \tag{8}$$

that is extremely small. Here $\Delta t_{\rm rec}$ is the burst recurrence time and $\kappa' = 2/5$. As is clear, the net wind torque on the shell/star in this period and so the change in angular velocity of the shell/star is negligible.

2.2. The rising time

This period starts with abruptly thermonuclear ignition of the accumulated hydrogens and heliums during the recurrence time. The burning shell expands in less than $\sim$1 s up to $z \simeq 2\times10^{-3}$ (Ayasli & Joss 1982; Hanawa & Fulimoto 1984; Bildsten 1998). In this period, as in the hydrostatic models, we assume that the rotational evolution of the expanding shell is nearly independent of the star itself.

As we mentioned before, in this period due to the huge thermonuclear explosion on the surface of the star, the accumulated materials are explosively erupted out of the star with a mass bulk velocity close to the speed of light. The outgoing plasma density within period of time $\Delta t \sim 1$ s, is $\sim[(A_{\rm pc}/4\pi R^2)M_{\rm shell}]/A_{\rm pc}$ that is 10 times (or more) larger than infalling plasma density $m_{\rm acc}/A_{\rm pc}$ due to the accretion flow at the same period of time (note that as mentioned by Strohmayer & Bildsten (2003), the bursts occur when the accretion rate is clearly less than $10^{-10}\ M_\odot\ {\rm yr}^{-1}$.). Here $m_{\rm acc} = \dot{M}\Delta t$ is the average mass of accreting materials during Δt. As a result, the expanding shell sweeps away the inner part of accretion disk and then one would expect the action of accretion to more or less shut off during this period.

The parallel electric force at height $z \simeq 2\times10^{-3}$ is 80 times bigger than gravitational force for hydrogen atoms, see Eq. (1). Therefore, the charged particles that rise with the expanding shell will be accelerated toward the outside of light cylinder by the strong parallel electric field along the open magnetic field lines. Further, the sudden thermonuclear explosion of the accumulated materials in the ocean causes the thermal energy of the charged particles to increase. Consequently, we expect that the number density of particles that may leave the magnetosphere $\eta n_{\rm GJ}$ in this period increases significantly. The resulting wind torque for $\eta \sim 10^3$ will give a net change in angular velocity of the shell as

$$\begin{aligned}\left(\frac{\Delta\Omega}{\Omega}\right)_{\rm shell} &= -2|\Delta R|/R - (T_{\rm w}/\kappa MR^2\Omega)\,\Delta t,\\ &\simeq -2\times10^{-3}\left(\frac{z}{10^{-3}}\right)\\ &\quad -7.42\times10^{-3}\left(\frac{\eta}{10^3}\right)^2\left(\frac{M_{\rm s}}{1.4\,M_\odot}\right)^{-2/3}\left(\frac{M}{10^{21}\ {\rm g}}\right)^{-1}\\ &\quad\times\left(\frac{R}{10\ {\rm km}}\right)^4\left(\frac{\nu_{\rm s}}{300\ {\rm Hz}}\right)^{4/3}\left(\frac{B}{10^8\ {\rm G}}\right)^2\left(\frac{\Delta t_{\rm rise}}{1\ {\rm s}}\right),\end{aligned} \tag{9}$$

where $\kappa = 2/3$ and $\Delta R \simeq 10$ m is the average change in thickness of the burning shell during a burst. For $\eta \sim 10^3$ and $B \sim 10^8$ G ($\eta B \sim 10^{11}$ G), the net change in angular velocity of the shell given by Eq. (9) is in good agreement with large frequency shifts seen in observations. We note that for $\eta \sim 10^3$, the number of particles that left the star $\Delta N \sim 4\pi\eta n_{\rm GJ}R^2\Delta R$ is much smaller than total number of particles in the ocean $N \sim M/m_{\rm p}$, i.e. $\Delta N/N \simeq 10^{-17}$. As a result, the change in the mass of the shell $(\Delta M/M)$ is negligible. In Table 1 we calculate the quantity ηB for both 300 Hz and 600 Hz spin frequencies (for those which applicable), to produce the frequency shifts observed in various systems. As a result, the suggested magnetic fields are in range $1{-}10\times10^8$ G for $\eta \sim 100{-}1000$. This

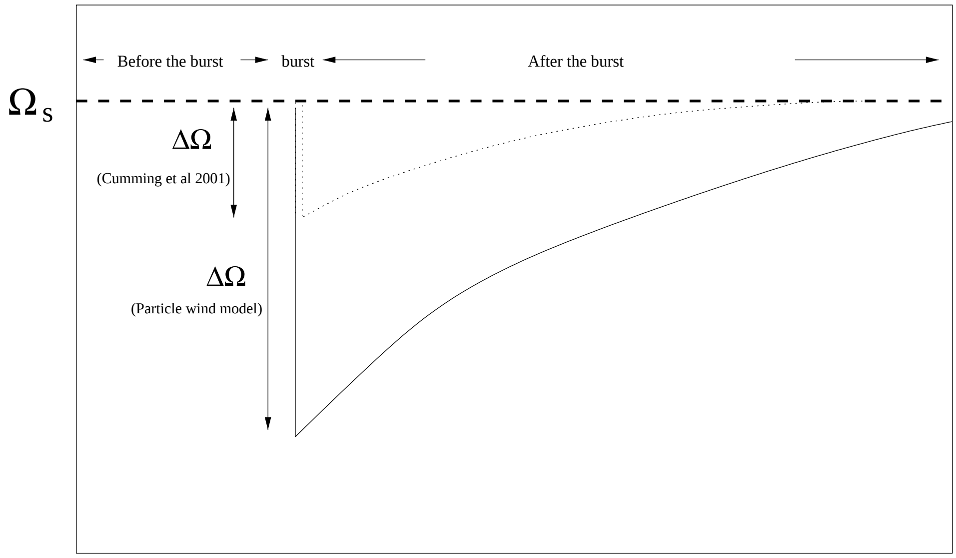

Fig. 1. In this figure we schematically compare the change in angular velocity of the expanding shell and then oscillations' frequency during the type I X-ray burst suggested by Cumming et al. (2001) (dotted line) and the one proposed here (solid line). Due to the exerted torque on the shell by the particle wind from the magnetic polar cap regions during the burst, we expect a larger depth in $\Delta\Omega$ rather than the model discussed by Cumming et al. (2001). As a result, the larger drift would be expected in the burst tail. The solid and dotted curve after the burst are drawn schematically to show star-shell recoupling in this period.

is consistent with other observational evidences about the magnetic field of neutron stars in LMXBs.

2.3. After the X-ray burst

At final stage, after the thermonuclear flash, the expansion will be stopped by the star's gravity force, and then the expanded shell starts to contract as they cool down. Because of the charge redistributions during the expansion, the original induced electric field is neutralized inside the expanded shell. So the magnitude of the parallel electric field inside the ocean will drop to zero, and then the electrostatic acceleration will cease inside the ocean. Due to the lack of outward electrostatic acceleration, the ocean's particles are mostly accelerated downward by strong gravity. Furthermore, the action of accretion plasma is starting to recover, and as a result, the polar cap acceleration will shut down in this period. Although, because of new charge distribution, a new parallel electric field may develop above the burned front we expect that the polar cap particle acceleration mechanism is more or less unlikely after the burst. As a result, the number of particles that may leave the star will decrease as well, i.e. $\eta \leq 10$. Further as the shell cools down, it gets closer and closer to the star, and then its coupling to the star gets stronger and stronger. This can be understood by noting the magnetic field near the star's surface opposes the differential rotation arising between shell and surface of the star during the burst (otherwise the magnetic field inside of the shell will be wound up and intensified due to the shearing layers – the winding of the magnetic field). Therefore, the net wind torque will be acting more and more on the whole star again rather than shell itself, and so we can neglect it, see Sect. 2.1.

The magnetic recoupling of the shell forces the shell to spin up until it achieves the star's spin frequency. The shell gains its angular momentum deficit $(MR^2\Delta\Omega)_{\rm shell}$, from the star through the various mechanisms such as magnetic field winding, see for more detail Cumming & Bildsten (2000). The angular velocity of the star changes by $\delta\Omega_{\rm s} = (M/M_{\rm s})\Delta\Omega \simeq 10^{-12}\Delta\Omega$ which is negligible. As a result, one would expect that the oscillations' frequency will increase as the shell spins up by $\Delta\nu/\nu_0 \sim \Delta\Omega/\Omega$. Interestingly, the magnetically recoupling of the star-shell system may explain why the burst oscillations, as seen in the observations, reach to their maximum frequencies (i.e. the star spin frequency) before the shell completely contracted.

In Fig. 1, we compare the corresponding changes suggested by previous studies (Strohmayer et al. 1997; Cumming et al. 2001) and the one we discussed here. As is clear, due to the exerted torque by the particle wind in polar cap regions, the shell's spin frequency, and then the oscillations' frequency decrease more than in the calculation by Cumming et al. (2001) during the rising time. Therefore, we expect to observe larger change in oscillations' frequency during the burst tail.

3. Discussion

Among the ~50 known Galactic LMXBs, the highly coherent burst oscillations with large modulation amplitudes and stable frequencies in range $\nu_0 \sim 270{-}620$ Hz are seen in ten LMXBs during type I X-ray bursts. These oscillations are most commonly seen during tails of bursts, when the burning is thought to have spread over the whole surface and obviously asymmetry is no longer present, are not observed in all type I X-ray bursts from the same source. While it is believed that the oscillation frequency is closely related to the neutron star spin frequency due to the observation of the kHz quasi-periodic oscillations in the persistent emission (van der Klis 2000), there is still an ambiguity that whether the oscillation frequency is the spin frequency or twice the spin frequency. Further, as seen in observations, the oscillation frequency increases by $\Delta\nu$

a few Hz during the burst. This frequency shift is firstly explained by Strohmayer et al. (1997) that the burning shell decouples from the star, and undergoes spin changes due to the conservation of angular momentum of the shell as it expands and contracts during the type I X-ray bursts. Further observations and studies suggested that purely radial hydrostatic expansion and angular momentum conservation alone cannot explain rather large frequency drifts ($\Delta\nu/\nu \sim 1.3\%$) observed in some bursts (Cumming et al. 2001; Galloway et al. 2001; Wijnands et al. 2001). For recent review on the thermonuclear bursts and their properties see Strohmayer & Bildsten (2003).

In this paper, we addressed the latter problem by studying the evolution of angular momentum of the burning shell during the type I X-ray bursts in LMXBs. Based on particle acceleration models near a pulsar polar cap region (Mestel 1998), we studied the change in the angular momentum of the burning shell during the type I X-ray burst. The net charged particles that accelerated by parallel electric field in star's polar caps, flow from the star surface to infinity through open magnetic field lines, would exert a torque on the burning shell (called wind torque) and cause the angular momentum of the shell changes during the burst. We introduced a dimensionless factor η that the quantity $\eta n_{\rm GJ}$ represent the number density of the ejected particles from the burning shell above the pulsar polar caps. We showed that for $\eta \sim 100{-}1000$ (during X-ray bursts) and a typical magnetic field $B \sim 1{-}10 \times 10^7$ G in the pulsar polar caps, the rather large observed frequency drifts of burst oscillations can be explained by the resulting wind torque exerted on the burning shell. In Table 1, we obtained the values for ηB for both 300 Hz and 600 Hz spin frequencies that causes the corresponding frequency drift observed in each burst. As is clear, the resulting magnetic fields' strength is in a good agreement with other observations from the neutron stars in LMXBs, such as the lack of coherence pulsations in persistent emission.

Above the neutron star's polar caps particles flow outward along the open magnetic field lines and a steady charge density cannot be maintained at the Goldreich-Julian density everywhere. Consequently, a strong electric field develops along the magnetic field, extracts the "primary" charged particles with density $n \sim n_{\rm GJ}$ (for a star with no burst activity, see Sect. 2) from the surface and accelerate them to high Lorentz factor ($\sim 10^7$). According to the polar cap models, the predicted radiation luminosity for a typical millisecond pulsar would be $\mathcal{E} \sim 10^{33}(\nu_{\rm s}/300\ {\rm Hz})^{5/28}(B/10^8\ {\rm G})^{-1/7}$ ergs/s that is comparable to γ-ray luminosities. So, one may expect to observe γ-rays in millisecond pulsars that has been detected only in PSR J0218+4232 by EGRET (Kui et al. 2000). However, polar cap particle acceleration is limited by various energy loss mechanisms such as curvature radiation, resonant and nonresonant inverse Compton scattering, and pair production. The accelerated particles along the magnetic field lines suffer energy loss and emit photons through the curvature radiation. Further, these particles scattered by soft thermal X-ray photons emitted from the hot surface and loss energy via inverse Compton scattering. In addition, the high energy photons resulted from the curvature radiation and/or inverse Compton scattering produce a cascade of "secondary" $e^{\pm}$ pair particles with a number density $n_{e^{\pm}} \sim 10^5 n$ above polar cap region that change the charge distributions and screen the electric field (see for more detail Harding & Muslimov 2001). In millisecond pulsars (with no burst activities) the polar cap particle acceleration mechanism was studied by Luo et al. (2000). They showed that in these pulsars this mechanism, which is relatively efficient in accelerating charged particles in the star's polar caps, mostly suffers energy loss from the radiation of accelerated particles along the field lines (curvature radiation). So the maximum energy attainable by the particle is limited by energy loss through the radiation reaction. They found that the maximum Lorentz factor that might be achieved by an individual particle at $z \sim 0.1$ is $\Gamma = (6\pi\varepsilon_0 E_{\|}\rho_{\rm c}^2/e)^{1/4} \sim 10^7$, however, the Lorentz factor for the bulk pair plasma is much smaller $\sim$100. The latter may explain the lack of detection of high energy γ-rays in these stars. Here $E_{\|}$ is given by Eq. (1) and $\rho_{\rm c} \sim (4/3)(cR/2\pi\nu_{\rm s})^{1/2} \sim 5 \times 10^4(300\ {\rm Hz}/\nu_{\rm s})^{1/2}(R/10^6\ {\rm cm})$ m is the curvature radius of the field lines.

For neutron stars in LMXBs with intermediate accretion rate, however, the polar cap acceleration mechanism mostly suffers from the action of accretion plasma. Because of the large plasma density ($m_{\rm acc}/A_{\rm pc}$) of the accretion flow toward star's polar caps, the parallel electric field in the polar cap regions would be shorted out and the polar cap acceleration mechanism would be suppressed. We note that, this scenario is not likely during X-ray bursts due to the explosive eruption of materials from the surface and possible evacuation of part of inner disk. In this period with $\Delta t \sim 1$ s, the eruptive plasma density is 10 times larger that the accreting plasma density, i.e. $\sim[(A_{\rm pc}/4\pi R^2)M_{\rm shell}]/A_{\rm pc} \sim 10\ m_{\rm acc}/A_{\rm pc}$.

It is necessary to note that the polar cap acceleration mechanism requires the observation of radio pulsed signals from neutron stars in LMXBs. Recently Burgay et al. (2003) carried out observations, searching for radio pulsed emission at 1.4 GHZ from six soft X-ray transient sources (weakly magnetic accreting neutron stars) during their X-ray quiescent phase. No such a signal was detected. However, they discussed several mechanisms such as free-free absorption from the ejected materials during the burst that can hamper the detection and explain the null result. In particular, they showed that during the quiescent, the amount of the gas lost by the neutron star's companion is insufficient to quench the radio emission, but is sufficient to completely absorb the radio signal. Therefore, one cannot generaly rule out the polar cap acceleration mechanism (and its resulting pulsar wind) based on the lack of the radio pulsed signals from these sources. However, further observartions, particularly during the burst activity, are needed to be done in order to decide such a mechanism is irrelavent or not.

Finally, it is interesting to mention that the very recent observation done by Chakrabarty et al. (2003) form the transient X-ray source SAX J1808.4-3658 shows that, as expected, the burst oscillation frequencies are directly related to the star spin frequency. However, it reveals some unusual behaviors too. They observed a rather large frequency drift ($\Delta\nu \sim 5$ Hz, $\Delta\nu/\nu \sim 1\%$) with a time scale one order of magnitude faster than in other neutron stars. Surprisingly, the oscillation reaches to its maximum frequency ($\sim$ star spin frequency) during the burst rise. This is completely inconsistent with conservation of angular momentum of a expanding/contracting shell which requires the oscillation slows down (drifts from star spin frequency) during the burst rise.

Although the magnitude of magnetic field of the neutron star in SAX J1808.4-3658 source is not clear, one may explain the observations by assuming a larger magnetic field and using both polar cap acceleration model and winding of the magnetic field. The winding of magnetic field lines due to the shearing shell would halt the differential rotation of the decoupled shell in a period comparable to the burst duration (1–2 s), see Cumming & Bildsten (2000). This is also in a good agreement with most observations that the oscillation frequencies reach to their maximum sooner than shell contracted completely. In somehow one may say that the magnetic field transports angular momentum from the star to the shell. However, this mechanism (wrapping of the field lines) more or less suffers from the several instabilities (with chaotic behaviors, of course) imposed by nuclear explosion/burning of the accreted materials during the burst in the shell. These instabilities would prevent from the field wrapping process (e.g. by tearing/recombining field lines) unless they are more or less settled down by decreasing nuclear fuels (at the end of the burst rise period) or by a rather large magnetic field ($B \geq 10^{10}$ G, Cumming & Bildsten 2000). The former that is more suitable for other observed bursts, is not applicable to the case SAX J1808.4-3658.

Not only may a larger magnetic field overcome those chaotic instabilities, but also it will provide a consistent explanation for the rather large drift observed in SAX J1808.4-3658 through the pulsar wind mechanism. As is clear from Eq. (9), the stronger magnetic field the larger frequency drift. Therefore, qualitatively, one may say: at the time of shell-star decoupling, the shell will be experienced a larger torque and then will be spun down faster (in a shorter time than the rising time, say 0.01–0.1 s). But, because of the field winding, the risen toroidal field $B_\phi \sim 2\Omega_s tB$ grows very fast. By considering a larger magnetic field seed, the new toroidal field may have enough energy to halt the differential rotation, spin up the shell to catch the star spin frequency during the burst rise. Thereafter, the pulsar wind torque will be acting more or less on the whole star rather than shell itself, and so we can neglect it. However, further theoretical/observational investigations are needed to be done in order to get better understanding of physical processes that are involved.

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