A&A 410, 289–297 (2003)
DOI: 10.1051/0004-6361:20031301

Astronomy & Astrophysics

Research Note

Rotating stellar models and dynamic tides in close binaries: A first approach

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Received 22 April 2003 / Accepted 8 August 2003

Abstract. Rotating stellar models – homogeneous and evolved ones – were introduced for the first time, in order to investigate the effects of rotationally induced changes in the internal structure of stars on the apsidal motion in close binary systems within the framework of the theory of dynamic tides. The rotating models are calculated in the quasi-spherical approximation under the assumption that the star rotates as a solid body. The models show that the spectrum of resonances between dynamic tides and free oscillation modes is substantially modified by rotation. In particular, the changes in the internal structure of the rotating models yield complex spectra which are more than simple images of the spectra in non-rotating models shifted in the period space. An immediate consequence of the differences between rotating and non-rotating models is that a reconsideration of the global problem of the apsidal-motion may be in order.

Key words. stars: binaries: close – stars: evolution – stars: interiors – stars: rotation

1. Introduction

Until recently, theoretical apsidal-motion investigations were often carried out within the framework of the static-tide approximation in order to avoid the complexity of a time-dependent tidal potential. The most commonly used formulae for the rate of change of the longitude of the periastron established in this framework are due to Cowling (1938) and Sterne (1939). The resulting apsidal motion rates combined with the rotational and relativistic contributions to the secular changes of the longitude of the periastron have been compared with observations by numerous authors, with varying degrees of success (Semeniuk & Paczynski 1968; Cisneros-Parra 1970; Giménez 1981; Claret & Giménez 1993).

The validity of the static-tide approximation has been investigated from a theoretical point of view by Papaloizou & Pringle (1980), Smeyers et al. (1991), Quataert et al. (1996), Smeyers et al. (1998), and Smeyers & Willems (2001). These authors concluded that within the framework of the theory of dynamic tides, the effects of stellar compressibility and of resonances between dynamic tides and free oscillation modes may be important, depending on the coupling between the stellar interior and the harmonic forcing potential. However, none of these studies were ever applied to a large and homogeneous sample of close binary stars showing apsidal motion.

In an attempt to overcome this deficiency a series of systematic studies, aimed at comparing recent theoretical advances regarding the apsidal motion in close binaries with observational data, was initiated by Willems & Claret (2002, 2003) and Claret & Willems (2002). In the latter paper, the authors pointed out the importance of the quality of the observations, and in particular the importance of the accurate knowledge of the masses and the radii of the binary components. They selected a sample of 21 eclipsing binary systems with accurate determinations of their absolute dimensions and computed specific stellar interior models for each component rather than interpolating between existing grids of models. Since the orbital periods of all systems in the sample were known with high accuracy, the authors carefully analysed the effects of stellar compressibility and resonant dynamic tides for each system as a function of the less certain rotational angular velocity. The resulting comparison between theoretically predicted and observationally derived apsidal motion rates showed a satisfactory degree of agreement. However, despite this agreement, some problems still remain: a) some systems do not fit the standard picture of apsidal motion, as mentioned in Claret & Willems (2002), b) the effect of rotation was taken into account only as a correction of the classical apsidal motion constant.

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Concerning point (b), statistical investigations show a tendency of the components of short-period binaries to be synchronized at the periastron; this being an interesting physical constraint since such stars do not rotate necessarely with the same rotational velocities as their single counterparts. Points (a) and (b) justify the introduction of more complexity in the models. In addition, the introduction of rotating models further contributes to the completeness of the problem.

As mentioned, a limitation of the investigation by Claret & Willems (2002) was that the interior stellar configurations were not rotating ones: rotation was only taken into account through the classical term, already included in the Cowling-Sterne formula, and through a correction factor associated with the centrifugal potential, introduced by Claret (1999). However, rotation should be included in the calculations through a) the classical rotational term in the Cowling-Sterne equation, b) the effects of the centrifugal potential on the internal structure models, c) the introduction of rotational terms in the tidal equations, and d) the effects of rotational mixing.

Point (a) has been considered since the first apsidal motion investigations, but often with limited accuracy due to the large error bars on the ratio of the rotational angular velocities of the component stars. In many cases where the accurate determination of the angular velocity ratio proved difficult or even impossible, the contribution of the stars' rotational distortion to the apsidal motion rate was determined under the assumption of pseudo-synchronised rotation. Point (b) was first explored from a theoretical point of view by Stothers (1974), who determined that the apsidal motion constant k_2 should be corrected by a factor of the order of the distortion-parameter f_2 given by Eq. (3). The change in k_2 is basically caused by the effects of the centrifugal potential on the stellar structure equations. Later, Claret & Giménez (1993) obtained similar results using a simplified version of the method described by Kippenhahn & Thomas (1970) to determine the effects of rotation on the internal structure. Claret (1999) followed up on this approach by implementing a more realistic treatment of stellar rotation in his stellar evolution code to study the influence of rotation on the theoretical apsidal motion rate. The resulting corrections on the apsidal motion constant k_2 affect both the rotational and tidal terms in the classical Cowling-Sterne formula. We note that the effects of the Coriolis force (part of point (c)) have been considered by Savonije et al. (1995) to derive the tidal spin-up rate of a massive star but not yet in the apsidal motion context. Point (d), finally, we plan to consider in future works.

The aim of the present investigation is to serve as a first exploratory step to extend the investigation by Claret & Willems (2002) by considering point (b) in the framework of the theory of dynamic tides. Our goal is to isolate the effects on the apsidal motion resulting solely from the changes in the internal structure models induced by stellar rotation and their interactions with an external time-dependent potential. To this end, we determine the apsidal motion rate within the framework of the theory of dynamic tides and compare the rate obtained for standard (non-rotating) models with the rate obtained for rotating models. As a first approximation, we here do not take into account the effects of rotation in the tidal equations (point (c)). In doing so, we aim to assess the importance of changes in the stellar compressibility and the eigenfrequency spectrum of the rotationally distorted star.

The paper is divided into 3 main sections. In Sect. 2, we briefly recall the basic ingredients for the calculation of models for rotating single stars. In Sect. 3, we outline the framework in which these models are used as equilibrium models to calculate the time-dependent tidal distortion and the associated apsidal motion rate in close binaries. Results for homogeneous and evolved rotating main-sequence models of 2 $M_\odot$ and 10 $M_\odot$ are presented in Sect. 4 together with some conclusions and future perspectives.

2. The rotating models

The method used to calculate the rotational models is based on the quasi-spherical approximation introduced by Kippenhahn & Thomas (1970) with some numerical modifications to take into account the contribution of the rotational distortion to the total potential of the star and direct integration of Radau's equation up to the 4th order. At the lowest-order of approximation, the potential can be expressed as

$$\psi = \frac{GM_\psi}{r} + \frac{1}{2}\Omega^2 r^2 \sin^2\theta - \frac{4\pi G}{5r^3} P_2(\cos\theta) \int_0^a \rho \frac{\mathrm{d}}{\mathrm{d}a'}\left(a'^5 f_2\right) \mathrm{d}a', \quad (1)$$

where

$$r = a\,[1 - f_2 P_2(\cos\theta)] \quad (2)$$

and

$$f_2 = \frac{5\Omega^2 a^3}{3GM_\psi(2+\eta_2)} \quad (3)$$

(Kopal 1959). Here, Ω is the rotational angular velocity, $P_2(\cos\theta)$ is the second-degree Legendre polynomial, a is the mean radius of the level surface, and η_2 is a measure of the mass concentration which is directly related to the apsidal motion constant k_2. The remaining symbols have their usual meaning and are defined in, e.g., Claret (1999). Note that there is a typographical error in Eq. (1) of Claret (1999) where GM/r^2 must be replaced by GM/r.

The stellar structure equations governing the conservation of mass and the generation of energy are formally the same as for a non-rotating star. The difference is that the equipotential surfaces are no longer spheres so that the variables r, M, and L must be replaced by the corresponding variables r_ψ, M_ψ, and L_ψ defined on equipotential surfaces. The equations governing the hydrostatic equilibrium and the transport of energy then take the form

$$\frac{\partial P_\psi}{\partial M_\psi} = -\frac{GM_\psi}{4\pi r_\psi^4} f_P, \quad (4)$$

$$\frac{\partial \ln T_\psi}{\partial \ln P_\psi} = \frac{3\kappa L_\psi P_\psi f_T}{16\pi acGM_\psi T_\psi^4 f_P}, \quad (5)$$

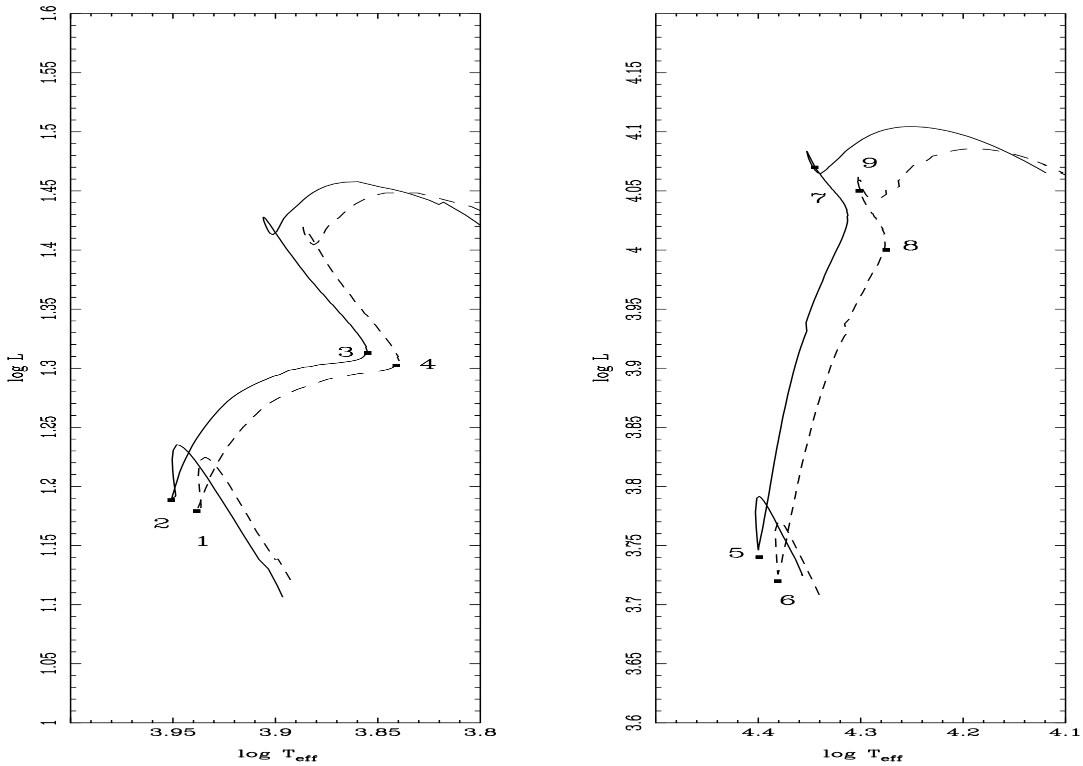

Fig. 1. Evolutionary tracks of rotating (dashed lines) and standard (continuous lines) stellar models of 2 $M_\odot$ (left) and 10 $M_\odot$ (right), chemical composition $(X, Z) = (0.70, 0.02)$, mixing-length parameter $\alpha = 1.52$, and no core overshooting.

while the Schwarzschild criterion is modified as

$$\frac{\partial \ln T_\psi}{\partial \ln P_\psi} = \min\left[\nabla_{\rm ad}, \nabla_{\rm rad}\frac{f_T}{f_P}\right]. \qquad (6)$$

The geometrical factors f_T and f_P can be computed for every shell of the model using the relation between the mean radius a and the radius of the level surface r_ψ up to the third order in the distortion (see Eq. (1) in Claret 1999). For the purpose of this investigation, we assume the star rotates as a solid body and neglect the possible effects of rotationally induced mixing. Although these are somewhat idealised assumptions, we consider them to be a suitable first approximation for the present investigation.

We calculated two evolutionary sequences, one for a 2 $M_\odot$ star and one for a 10 $M_\odot$ star, which are representative for the mass range of stars in binaries showing apsidal motion. The adopted chemical composition was $(X, Z) = (0.70, 0.02)$ and the mixing-length parameter 1.52. For the rotational models, the initial rotational angular velocities were taken to be 1×10^{-4} s^{-1} and 8×10^{-5} for the 2 $M_\odot$ and the 10 $M_\odot$ star, respectively. They are compatible with observed periods for stars of the same spectral type (see Tassoul 1978), although the equatorial velocities are larger in order to emphasize the effects of rotation. The rotational angular velocities of the models we used are not high and the set of adopted equations is adequate. Such equations were used even for faster rotators (Kippenhahn & Thomas 1970; Endal & Sofia 1976; Law 1981; Meynet & Maeder 1997, etc.). Since we are mainly interested in differential effects between rotating and non-rotating models, and given that the interaction between rotation and convection is not well known, we did not consider the possible effects of core overshooting. For more details on the construction of the stellar models, we refer to Claret (1995) and Claret (1999). The resulting evolutionary tracks are shown in the HR diagrams displayed in Fig. 1. Note that, as expected, the rotating models are colder than their non-rotating counterparts. In addition, the inclusion of the centrifugal potential tends to make the mass of the rotating models more concentrated towards the center (see also Claret 1999 and references given therein). However, as we shall see in the following sections, this is a *stationary* effect rather than a dynamical one. In the framework of the theory of dynamic tides, the results are a lot less simple to analyse. For the remainder of the paper, we restrict ourselves to a few selected models along the evolutionary sequences. The position of these models in the HR diagram is indicated by the numbered solid squares in Fig. 1. Some properties of the adopted models relevant to this investigation are summarised in Table 1.

3. The apsidal-motion rate due to the tidal distortion of a binary component

In order to determine the contribution of the tidal distortion of a binary component to the apsidal-motion rate within the framework of the theory of dynamic tides, we assume the star to have a rotation axis perpendicular to the orbital plane and we treat its companion as a point mass. We denote the mass and the radius of the star by M_1 and R_1, respectively, and the mass of the companion by M_2. Furthermore, let $P_{\rm orb}$ be the period, a the semi-major axis, and e the eccentricity of the orbit.

Following Polfliet & Smeyers (1990), we expand the tide-generating potential, $\varepsilon_T\, W(\boldsymbol{r}, t)$, in terms of unnormalised

Table 1. Relevant parameters of the selected non-rotating and rotating stellar models.

Number of model	Mass ($M_\odot$)	age (10^6 years)	log g (cgs)	$\Omega(\mathrm{s}^{-1})$	log k_2	X_c
1	2	15.400	4.300	0.00	−2.337	0.694
2	2	15.575	4.261	1.40×10^{-4}	−2.419	0.693
3	2	908.538	3.792	0.00	−2.583	0.045
4	2	910.392	3.747	6.84×10^{-5}	−2.681	0.064
5	10	0.119	4.238	0.00	−1.923	0.698
6	10	0.106	4.181	1.10×10^{-4}	−2.051	0.698
7	10	20.635	3.695	0.00	−2.404	0.001
8	10	20.640	3.477	4.50×10^{-5}	−2.656	0.037
9	10	21.122	3.537	5.80×10^{-5}	−2.752	0.001

spherical harmonics $Y_\ell^m(\theta,\phi)$ and in Fourier series in terms of multiples of the companion's mean motion $n = 2\pi/P_{\rm orb}$ as

$$\varepsilon_T\, W(\boldsymbol{r},t) = -\varepsilon_T \frac{G M_1}{R_1} \sum_{\ell=2}^{4} \sum_{m=-\ell}^{\ell} \sum_{k=-\infty}^{\infty} c_{\ell,m,k} \left(\frac{r}{R_1}\right)^{\ell} \times Y_\ell^m(\theta,\phi)\, \exp\{ {\rm i}\, [(k\,n + m\,\Omega)\,t - k\,n\,\tau]\}. \quad (7)$$

Here, $\varepsilon_T = (R_1/a)^3\,(M_2/M_1)$ is a small dimensionless parameter representing the ratio of the tidal force to the gravity at the star's equator, G is the Newtonian constant of gravitation, τ is a time of periastron passage, $\boldsymbol{r} = (r,\theta,\phi)$ is a system of spherical coordinates with respect to an orthogonal frame of reference that is corotating with the star, and the factors $c_{\ell,m,k}$ are Fourier coefficients depending on the orbital eccentricity. For a more detailed definition of the coordinate system r,θ,ϕ and a discussion of the Fourier coefficients $c_{\ell,m,k}$, we refer to Smeyers et al. (1998).

Since the tide-generating potential is dominated by the terms associated with the second-degree spherical harmonics, we restrict ourselves to the determination of the apsidal-motion rate resulting from the terms associated with $\ell = 2$. The rate of secular change of the longitude of the periastron ω due to the time-dependent tidal distortion of a component of a close binary is then given by

$$\dot\omega_{\rm dyn} = \left(\frac{R_1}{a}\right)^5 \frac{M_2}{M_1} \frac{2\pi}{P_{\rm orb}} \left[2\,k_2\, G_{2,0,0} + 4 \sum_{k=1}^{\infty} \left(F_{2,0,k}\, G_{2,0,k} + F_{2,2,k}\, G_{2,2,k} + F_{2,-2,k}\, G_{2,-2,k}\right)\right] \quad (8)$$

(Smeyers et al. 1998; Smeyers & Willems 2001). The rate depends on the orbital eccentricity through the factors $G_{2,m,k}$ and on the response of the star to the tidal forcing of the companion through the coefficients $F_{2,m,k}$. For a precise definition and an elaborate discussion of the quantities $G_{2,m,k}$ and $F_{2,m,k}$, we refer to Smeyers & Willems (2001) and Willems & Claret (2002).

For comparison, the rate of secular apsidal motion due to the tidal distortion of a binary component derived by Cowling (1938) and Sterne (1939) is given by

$$\dot\omega_{\rm tidal} = \left(\frac{R_1}{a}\right)^5 \frac{M_2}{M_1} \frac{2\pi}{P_{\rm orb}}\, k_2\, 15\, f\left(e^2\right), \quad (9)$$

where k_2 is the apsidal-motion constant, and

$$f\left(e^2\right) = \left(1-e^2\right)^{-5} \left(1 + \frac{3}{2}\,e^2 + \frac{1}{8}\,e^4\right). \quad (10)$$

The determination of the coefficients $F_{2,m,k}$ in Eq. (8) requires the numerical integration of the system of differential equations governing the time-dependent tidal motions of the mass elements in a component of a close binary. As mentioned in the Introduction, we here do not consider any additional terms introduced in the tidal equations by the star's rotation. In particular, we discard the effects of the Coriolis force and the centrifugal force. We furthermore consider the tidal force as a small time-dependent perturbation of the star's hydrostatic equilibrium and neglect the effects of the nonadiabatic energy exchange between the moving mass elements and their surroundings. The system of differential equations then corresponds to the system of equations governing linear, isentropic forced oscillations in a spherically symmetric equilibrium star (see, e.g., Willems & Claret 2002).

For close resonances, the use of the isentropic approximation may lead to indefinitely large amplitudes for the components of the tidal displacement field. We bypass this problem by limiting the determination of the apsidal-motion rates to orbital periods for which the relative difference between the forcing angular frequency of the dynamic tide and the eigenfrequency of the oscillation mode involved in the resonance is larger than $0.1\,\varepsilon_T$ (Smeyers & Willems 2001; Willems & Claret 2002). Since, for main-sequence stars, the eigenfrequencies of the oscillation modes are relatively unaffected by the nonadiabatic effects, the isentropic approximation is adequate to study the position of the resonances as a function of the orbital period.

Although these various assumptions limit the quantitative conclusions we may draw from this investigation somewhat, they allow us to form a first qualitative picture of the differences between rotating models and their non-rotating counterparts without the complications of more difficult input physics. We intend to follow-up on this first exploratory step in future investigations by implementing more sophisticated treatments of nonadiabatic and rotational effects on dynamic tides in close binaries.

4. Results for 2 $M_\odot$ and 10 $M_\odot$ rotating models

The apsidal motion rate resulting from the tidal distortion of a 2 $M_\odot$ ZAMS binary component is shown in Fig. 2 for orbital periods ranging from 2 to 10 days. For illustration, we used a companion mass $M_2 = 2\,M_\odot$ and an orbital eccentricity $e = 0.25$. The dashed line represents the contribution

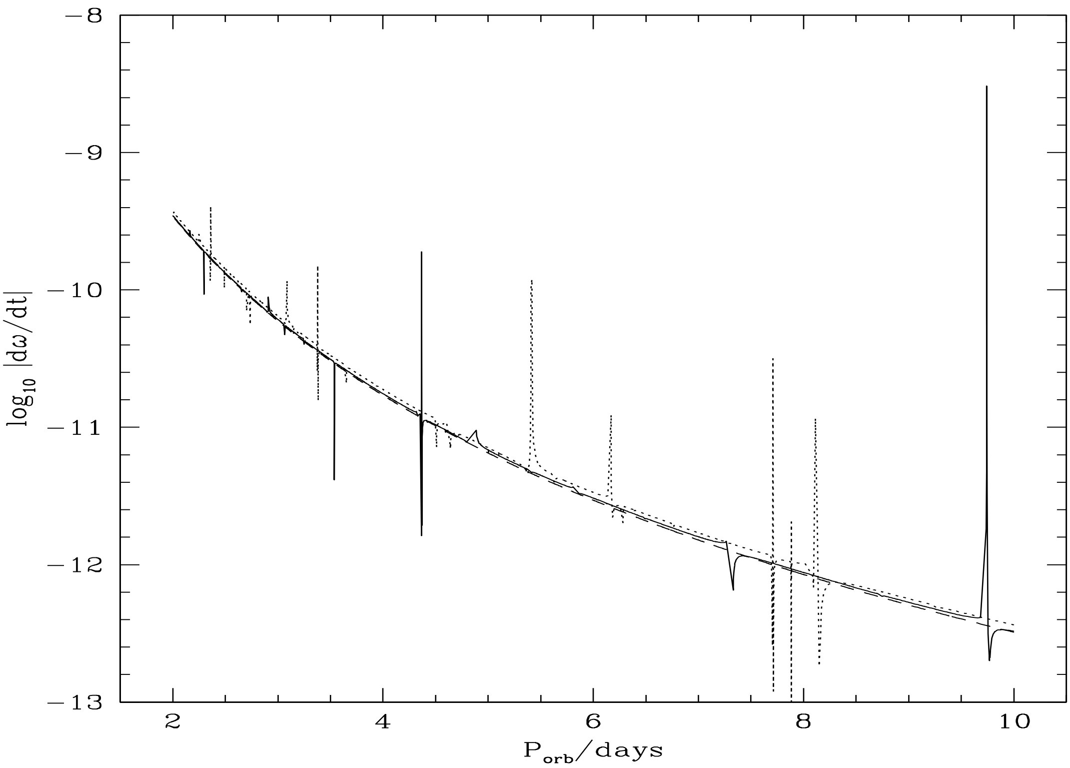

Fig. 2. The logarithm of the apsidal-motion rate (in radians per second) resulting from the tidal distortion of a 2 $M_\odot$ ZAMS model. Solid and dotted lines represent the apsidal-motion rates determined by means of Eq. (8) in the case of a non-rotating and a rotating model, respectively (see Table 1, models 1 and 2). For comparison, the dashed line represents the contribution of the tidal distortion to the classical apsidal-motion rate resulting from Eq. (9) in the case of the non-rotating model. The peaks in the curves correspond to resonances between dynamic tides and the star's free oscillation modes g^+.

of the tides to the apsidal motion rate within the framework of the classical Cowling-Sterne formula in the case of a non-rotating 2 $M_\odot$ ZAMS stellar model. On the scale of the figure this rate is almost indistinguishable from the rate derived for the rotating 2 $M_\odot$ ZAMS stellar model (not shown). The solid and dotted lines in Fig. 2 represent the contribution of the tides to the apsidal-motion rate within the framework of the theory of dynamic tides in the case of a non-rotating and a rotating 2 $M_\odot$ ZAMS stellar model, respectively. In the case of the non-rotating model, the forcing frequencies $k\,n + m\,\Omega$ with respect to the corotating frame of reference (see Eq. (7)), were determined using the same rotational angular velocity Ω as that of its rotating counterpart. The induced forcing frequencies are then the same for both models, so that any differences arising can be expected to be related to differences in the internal structure.

It is not straightforward to analyse the results shown in Fig. 2, but it is interesting to note that resonances between dynamic tides and free oscillation modes may produce both negative and positive deviations from the apsidal motion rate predicted by the Cowling-Sterne formula (see also Smeyers & Willems 2001). The changes in the internal structure of the rotating models cause a different pattern of resonances due to the different frequency spectrum of the model's free modes of oscillation. In particular, the decrease of the size of the radiative envelope of a rotating model with respect to a non-rotating one yields higher eigenfrequencies for the rotating models. The changes in the internal structure also affect the spacing between two consecutive eigenfrequencies and the behaviour of the eigenfunctions of the oscillation modes involved in the resonances. These two effects respectively alter the density of the resonances and the strength of the coupling between the tidal force and the oscillation modes involved in the resonances. Because of these complex dependencies, the pattern of resonances that emerges for the rotating model is more than just a simple shift of the pattern found for the non-rotating model.

In order to emphasize the differences between the apsidal-motion rate predicted by the static-tide approximation in the classical Cowling-Sterne formula and the apsidal-motion rate taking into account the effects of dynamic tides, we introduce the relative difference Δ between the two rates as

$$\Delta_{\rm dyn} \equiv \frac{\dot{\omega}_{\rm tidal} - \dot{\omega}_{\rm dyn}}{\dot{\omega}_{\rm dyn}}. \qquad (11)$$

The variations of Δ as a function of the orbital period are displayed in Fig. 3 for the non-rotating and rotating 2 $M_\odot$ ZAMS models by solid and dotted lines, respectively. The relative differences make the occurrence of resonances and the different patterns in the rotating and the non-rotating models even more apparent than in Fig. 2. Away from any resonances between dynamic tides and free oscillation modes, the relative differences are furthermore seen to be systematically more negative for the rotating models than for the non-rotating ones. The observation by Smeyers & Willems (2001) that the classical Cowling-Sterne formula tends to underestimate the contribution of a

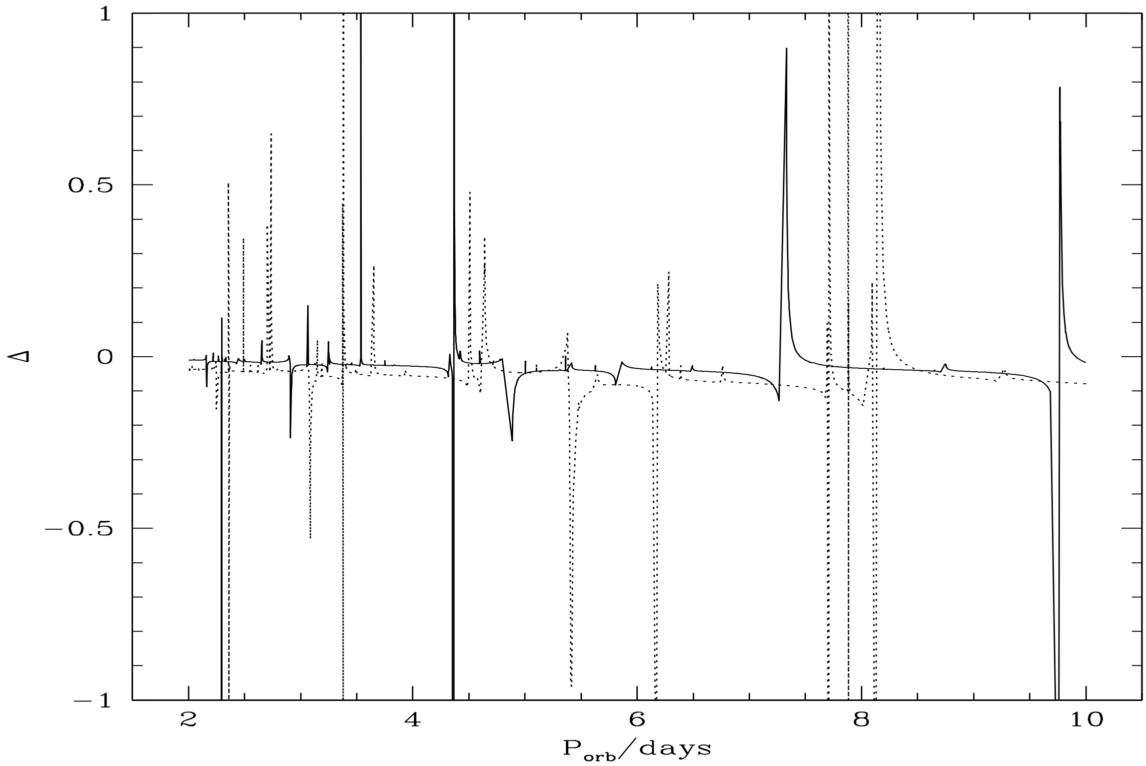

Fig. 3. The relative differences Δ (see Eq. (11)) for the same non-rotating (solid lines) and rotating (dotted lines) 2 $M_{\odot}$ ZAMS models as in Fig. 2.

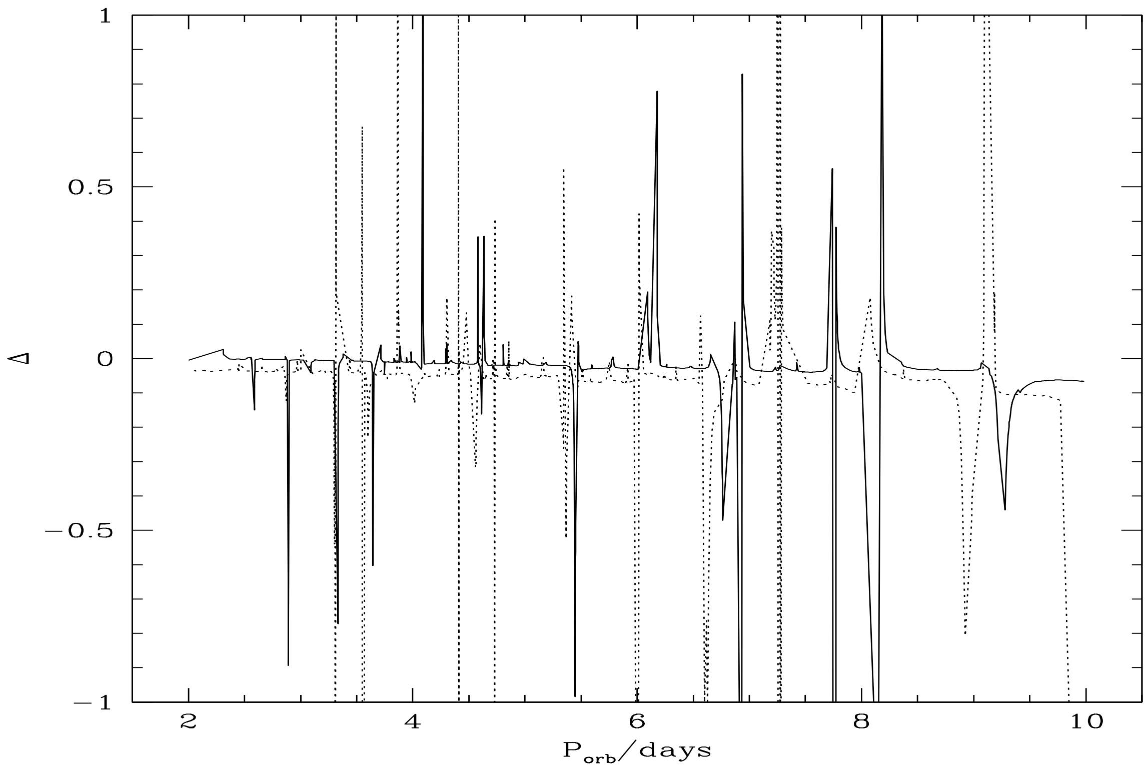

Fig. 4. Same as Fig. 3, but for evolved standard and rotating 2 $M_{\odot}$ main-sequence models (Table 1, models 3 and 4).

star's tidal distortion to the apsidal motion is therefore emphasised even more by the structural changes induced by stellar rotation. In Fig. 4, the relative differences Δ are shown in the case of evolved standard and rotating 2 $M_{\odot}$ main-sequence models. The main difference with the 2 $M_{\odot}$ ZAMS models is the larger number of resonances occurring in the considered period range. The increase is caused by the growth of the radiative envelope which causes the eigenfrequency spectrum of a more evolved

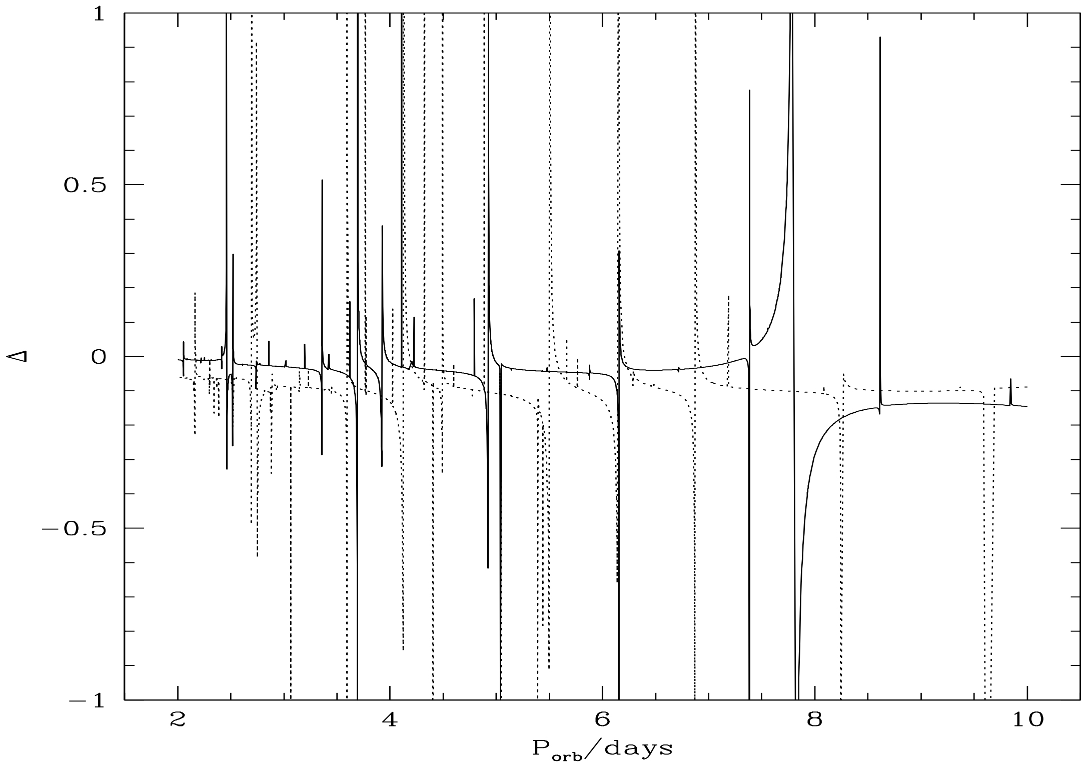

Fig. 5. Same as Fig. 3, but for standard and rotating 10 $M_{\odot}$ ZAMS models (Table 1, models 5 and 6).

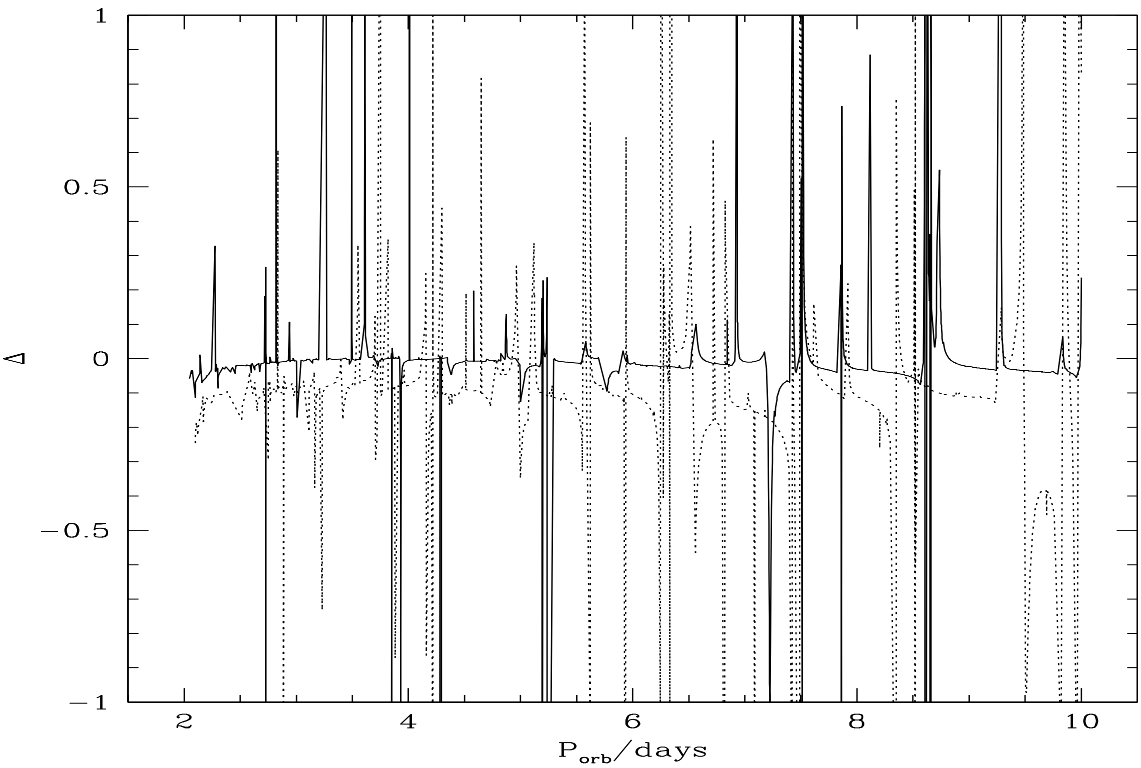

Fig. 6. Same as Fig. 3, but for evolved standard and rotating 10 $M_{\odot}$ main-sequence models (Table 1, models 7 and 8).

stellar model to be denser than that of a less evolved model. A similar behaviour is found in the case of 10 $M_{\odot}$ stellar models, which is illustrated in Figs. 5 and 6.

In the absence of resonances, there is another important effect due to the interaction between the stellar compressibility and rotation for more evolved models (see Figs. 4 and 6). The differences in the apsidal-motion rate of rotating models and non-rotating models can still reach about 10%. However, for evolved models, an additional difficulty arises from the the comparison between rotating and non-rotating models

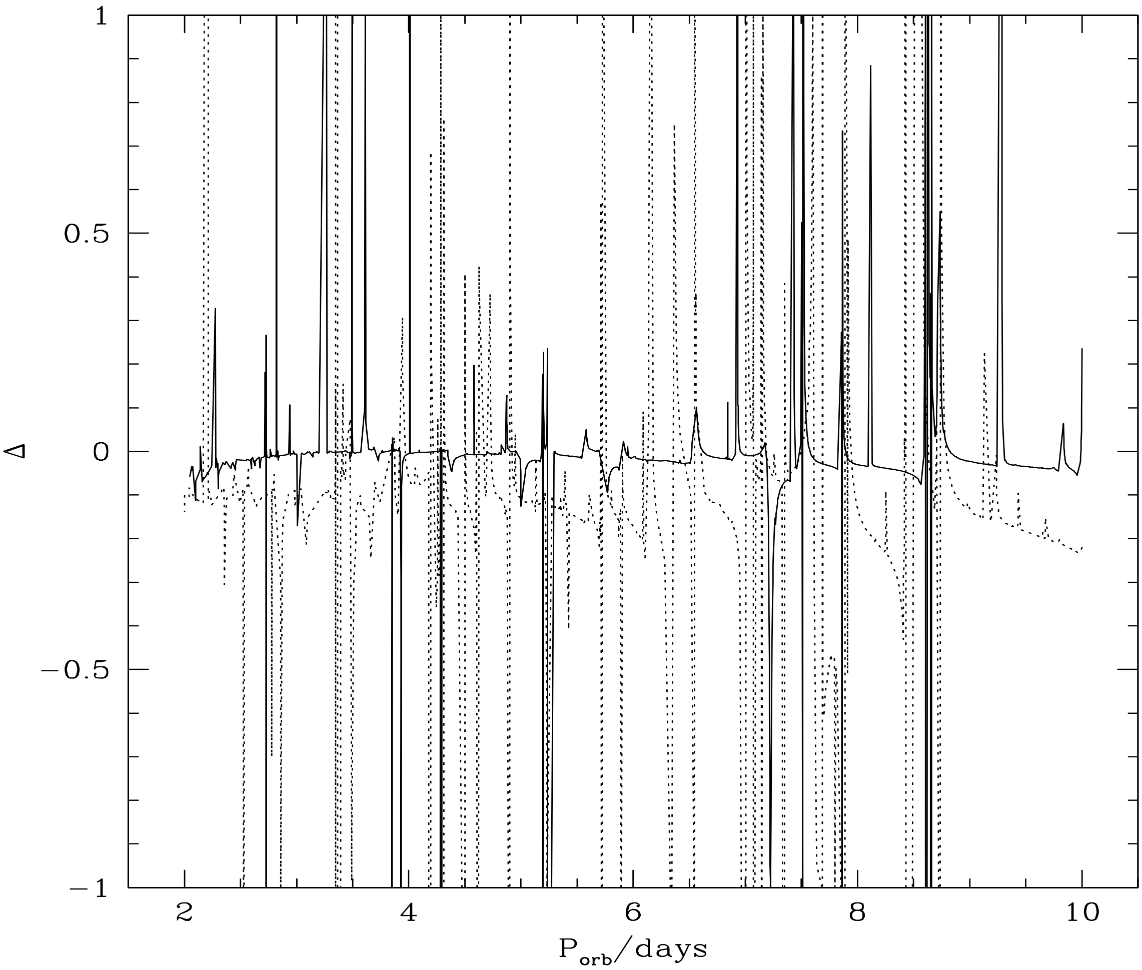

Fig. 7. Same as Fig. 3, but for evolved standard and rotating 10 $M_{\odot}$ main-sequence models with a slightly smaller central hydrogen content for the rotating model than in Fig. 7 (Table 1, models 7 and 9).

itself: once a standard model is selected, with which rotating model should we compare it? The equivalent radius is often different (recall that in the rotating case, we are dealing with a distorted configuration). We here selected models 3, 4, 7, and 8 because they represent the evolutionary stage on the main-sequence where the stellar radius is at its largest, so that the effects of dynamic tides on the apsidal-motion rate are expected to be maximal (Willems & Claret 2002). However, inspection of Table 1 shows that this choice of models does not necessarily correspond to comparable evolutionary stages in terms of the amount of hydrogen burnt in the center. In Fig. 7, we therefore plotted the relative differences Δ for a somewhat further evolved 10 $M_{\odot}$ rotating model than that presented in Fig. 6. The difference in radius between the rotating models 8 (Fig. 6) and 9 (Fig. 7) is only 7 per cent, but the resulting spectra of resonances are quite different. This numerical exercise serves as a clear example of the intrinsic difficulties to determine equivalent rotating and non-rotating stellar models. Consequently, as long as the choice of models is somewhat arbitrary, it is not straightforward to compare the respective spectra of resonant dynamic tides. We expect a similar effect if we compare rotating and non-rotating models that share the same position in the HR diagram, the rotating one being more massive than its counterpart. This important question will be treated in future works.

A direct consequence of these results is that the possible effects of dynamic tides must be considered even more seriously than before since the comparison with observational data may depend critically on the position of the resonances as a function of the orbital period and/or the rotational angular velocity. Until now, the possibility of resonances has only been considered for the eigenmodes of non-rotating models which clearly have a different eigenfrequency spectrum than the modes of rotating models. The inclusion of rotation in the calculation of the equilibrium models may therefore prove to be vital to correctly interpret any comparison between theoretically predicted and observationally determined apsidal-motion rates. However, the effects of stellar rotation on the internal structure of stars are still far from understood so that the additional uncertainties resulting from the calculation of rotating stellar models increases the difficulties of comparing theory with observations rather than solving some of the outstanding issues. Such theoretical limitations should always be considered before more exotic solutions are called upon.

Although a re-analysis of the observational data is outside the scope of the present investigation, a few words on the observed apsidal motion rates in the context of the present results are appropriate. The introduction of rotating models changes significantly the spectrum of resonances between dynamic tides and free oscillation modes. As we have seen, the predicted deviations with respect to the classical Cowling-Sterne formula can be positive or negative, so that resonances may yield smaller as well as larger apsidal motion rates. Such predictions could in principle be reflected in the observational data. In particular, the problematic systems DI Her, As Cam and V451 Cyg (Claret 1997), which present large deviations with respect to the standard theory, would be suitable subjects to study by the introduction of rotating models in the framework of the theory of dynamic tides.

However, as explained in the Introduction, the present models are only exploratory ones. As a logical step forward, we plan to extend the present investigation by implementing the effects of the Coriolis force in the tidal equations and by investigating the non-adiabatic effects in order to evaluate more accurately the resonant amplitudes. Concerning the interior structure models, we are working to introduce the effects of rotational induced mixing. The main contribution of the present numerical experiments is to show that is worth continuing to investigate this theme in more detail.

Acknowledgements. We are grateful to G. Meynet, who has helped to improve the final version of the paper with his comments and suggestions. The Spanish DGESIC (project PB98-0499) is acknowledged for the support received. BW acknowledges the support of the British Particle Physics and Astronomy Research Council (PPARC). This research has made use of the Simbad database, operated at CDS, Strasbourg, France.

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