A&A 407, L9–L12 (2003)
DOI: 10.1051/0004-6361:20030976

Astronomy & Astrophysics

Letter to the Editor

Limits on dark energy–matter interaction from the Hubble relation for two-fluid FLRW models

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Received 19 May 2003 / Accepted 25 June 2003

Abstract. In the standard model ($\Omega_{\rm m}^0 \approx 0.3$, $\Omega_\Lambda^0 \approx 0.7$) it is assumed that there is no convertion (energy transfer) between the two components. However, this hypothesis requires observational tests. A general approach to multicomponent FLRW models comes from a new classification (Gromov et al. 2002) which naturally follows from two independent model properties: 1) stationarity (or not) of equations of state of component substances and the presence (or absence) of energy transfer between these. The associated one-fluid model becomes characterized by $\gamma = P_{\rm total}/\mathcal{E}_{\rm total}$. For the case γ = constant (which for a stationary equation state of dark energy means coherent evolution and is caused by energy transfer), a general integral expression for the analogue of the Mattig equation is derived. For flat geometry, the integral reduces to a simple analytic expression. Comparison with the magnitude-redshift relation for the standard model shows that $\gamma \approx -0.55$ gives the same relation within 0.05 mag in the range $0 < z < 1.2$. The observable γ gives a robust upper limit to w in the equation of state of dark energy.

Key words. cosmology: theory – dark energy – cosmological parameters

1. Introduction

A number of observations reveal cosmological dark energy. The measurements, most recently by WMAP, of the density parameter Ω^0 from the angular power spectrum of the CBR (Bennet et al. 2003), show that $\Omega^0 = \Omega_{\rm m}^0 + \Omega_\Lambda^0 = 1.02 \pm 0.02$. Constrained by such a flat universe, type Ia Supernovae (see e.g. Tonry et al. 2003) give the result that $\Omega_\Lambda^0 \approx 0.7$. Also, the data on luminous quasars agree with such a model (Teerikorpi 2003). The smooth Hubble flow around our Local Group suggests still other evidence for a large Λ component (Chernin 2001) and its variation with time (Baryshev et al. 2001).

The term "dark energy" means a substance having the equation of state $p_{\rm Q} = w\varepsilon_{\rm Q}$ ($-1 \le w < 0$), which may be time variable (Peebles & Ratra 1988). Usually one considers the "ordinary" matter and dark energy as interacting by mutual gravitation only. Then, e.g. Schuecker et al. (2003) concluded that $w = -0.95(+0.30, -0.35)$. But the problem of the interaction of the dark energy with other matter components was emphasized by Peebles & Ratra (2003). A phenomenological study of the properties of two-fluid cosmologies, with and without energy transfer, within a new classification which naturally arises in the two-fluid problem, was made by Gromov et al. (2002). Here we discuss an important class with energy transfer, the coherently evolving model, and derive for it the magnitude-redshift relation. This gives a prospect to test observationally dark energy–dark matter interaction.

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2. Two-fluid models with matter and dark energy

2.1. The classification

A two-fluid with dark energy together with some mixture of matter much differs from a two-fluid with "ordinary" matter: negative pressure and gravitating components give rise to a new behaviour of the total pressure and gravitating mass. To facilitate the study, Gromov et al. (2002) divided all two-fluid models into four classes according to two independent properties: 1) the fluids have a stationary (SES) or at least one has a non stationary (NSES) equation of state and 2) the presence or absence (ET, NET) of energy transfer $U_{\rm Q}$ and $U_{\rm m}$ between the components (see Table 1). Futhermore, we separate three different kinds of two-fluid models depending on the behaviour of the function $\alpha(a) = \mathcal{E}_{\rm Q}/\mathcal{E}_{\rm m}$:
1) the coherent model, when

$$\alpha(a) = \text{constant}, \tag{1}$$

2) the asymptotically coherent model, when

$$\lim_{a\to\infty} \alpha'(a) = 0, \tag{2}$$

3) the non-coherent model, when Eqs. (1), (2) are invalid.

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Table 1. Classification of two-fluid FLRW models. ET means "energy transfer"; NET is "no energy transfer"; SES is "stationary equation of state"; NSES is "non stationary equation of state". Associated one-fluid models become characterized by $\gamma = P_{\rm total}/\mathcal{E}_{\rm total}$. SM is "standard model".

	$U_{\rm Q} = U_{\rm m} = 0$	$U_{\rm Q} = -U_{\rm m} \neq 0$
both w, β	NET-SES	ET-SES
are constant		
	$\gamma(a) \neq$ const.	$\gamma(a) \neq$ const. or
	(SM: $\beta = 0$, $w = -1$)	$\gamma(a) =$ const.
w and/or β	NET-NSES	ET-NSES
are not constant		
	$\gamma(a) \neq$ const. or	$\gamma(a) \neq$ const. or
	$\gamma(a) =$ const.	$\gamma(a) =$ const.

2.2. The input equations and energy transfer

We use the FRLW equations in a dimensionless form

$$\frac{\dot a^2}{a^2} + \frac{k}{a^2} = \mathcal{E}, \tag{3}$$

$$2\,\frac{\ddot a}{a} + \frac{\dot a^2}{a^2} + \frac{k}{a^2} = -3\,P, \tag{4}$$

$$3\,\frac{\dot a}{a} = -\frac{\dot{\mathcal{E}}}{\mathcal{E} + P}, \tag{5}$$

$$P = \gamma\ \mathcal{E}, \tag{6}$$

$$U_i = \frac{1}{a^3}\left[\frac{\rm d}{{\rm d}\tau}\left(\mathcal{E}_i\, a^3\right) + P_i\,\frac{\rm d}{{\rm d}\tau}\,a^3\right], \tag{7}$$

with dimensionless variables ($\dot{}\equiv {\rm d}/{\rm d}\tau$):[1]

$$a(\tau) = S(t)/l_0, \qquad \tau = t/t_0,$$
$$\mathcal{E}_i = \frac{\varepsilon_i}{\varepsilon_0}, \qquad \mathcal{P}_i = \frac{p_i}{\varepsilon_0}, \qquad E_i = \frac{e_i}{e_0}, \qquad U_i = \frac{u_i}{\varepsilon_0}\,t_0$$

where $i = Q, m$, and

$$\mathcal{E} = \mathcal{E}_{\rm Q} + \mathcal{E}_{\rm m}, \quad P = P_{\rm Q} + P_{\rm m}, \quad E = E_{\rm Q} + E_{\rm m},$$
$$U = U_{\rm Q} + U_{\rm m}.$$

The two component fluids are described by Eq. (7) and the equations of state:

$$P_{\rm Q} = w(a)\ \mathcal{E}_{\rm Q}, \quad P_{\rm m} = \beta(a)\ \mathcal{E}_{\rm m}. \tag{8}$$

Following these definitions, the coefficient of the equation of state for the associated one-fluid model is

$$\gamma = \frac{P}{\mathcal{E}} = \frac{w\alpha + \beta}{\alpha + 1}\cdot \tag{9}$$

We describe the associated one-fluid model by Eqs. (5), (6) which have the solution with the initial conditions stated for the present epoch:

$$\mathcal{E} = \ \mathcal{E}(1)\ \exp\left(-3\int_1^a \frac{\gamma+1}{x}\,{\rm d}x\right). \tag{10}$$

[1] Here: $t_0 = \frac{1}{H_0}$, $l_0 = c\,t_0$, $\rho_0 = \frac{3\,H_0^2}{8\pi G}$, $\varepsilon_0 = \rho_0\,c^2$.

Calculating $a^3\ (U_{\rm m} - U_{\rm Q}/\alpha)$ and keeping in mind that $U_{\rm Q} + U_{\rm m} = 0$, we find the equation

$$\frac{U_{\rm m}}{\mathcal{E}_{\rm m}}\,\frac{\alpha+1}{\alpha} = 3\,\frac{\dot a}{a}\,(\beta - w) - \frac{\dot\alpha}{\alpha}\cdot \tag{11}$$

Equation (11) implies that a model can be coherent ($\alpha =$ const.) only if there is energy transfer. Also, it gives the condition for no energy transfer, $U_{\rm Q} = U_{\rm m} = 0$, in the form

$$\alpha = \alpha(1)\ \exp\left(3\int_1^a \frac{\beta - w}{x}{\rm d}x\right). \tag{12}$$

Any other function $\alpha(a)$ leads to non-zero energy transfer.

For the popular flat model Eq. (11) becomes:

$$U_{\rm m}\,\frac{(\alpha+1)^2}{\alpha} = 3\ \mathcal{E}^{3/2}\,(\beta - w) -\ \mathcal{E}\,\frac{\dot\alpha}{\alpha}\cdot \tag{13}$$

The flat coherent ET-SES model with $\gamma =$ const. has special interest in view of the coupled quintessence models (see e.g. Amendola & Toccini-Valentini 2001). For it we find the non-zero energy transfer

$$U_{\rm m}\,\frac{(\alpha+1)^2}{\alpha} = 3\,\frac{\beta - w}{a^{9/2(\gamma+1)}} \neq 0. \tag{14}$$

In terms of the in principle observable quantities γ and α, the energy transfer in case of dust may be written as

$$U_{\rm m} = -3\frac{\gamma}{\alpha+1}\,\frac{1}{a^{9/2(\gamma+1)}} \neq 0. \tag{15}$$

3. The $m - z$ relation for models with constant γ

The metric (proper) distance–redshift (Mattig) relation $r(z)$ and the magnitude–redshift relation $m(z)$ are well-known for the NET-SES models (including the flat standard model), with the density parameter $\Omega^0_{\rm m}$ and $\Omega^0_{\rm Q}$. In the case of flat geometry the normalized $r(z)$-relation is (e.g. Carrol & Press 1992):

$$r(z) = \int_0^z \frac{{\rm d}x}{(1+x)^{3/2}\,\sqrt{\Omega^0_{\rm Q}(1+x)^{3w} + \Omega^0_{\rm m}(1+x)^{3\beta}}}\cdot \tag{16}$$

This familiar class of models has $\gamma(a)\ \neq$ const., as all NET-SES models. Note that the case $\gamma(a) =$ const. appears in three classes, giving it special interest. Here we derive the $r(z)$ relation for the coherent ET-SES models discussed above. The essential property of the model, $\gamma(a) =$ const., allows for the relation a surprisingly simple expression.

The general expression for $r(z)$ is obtained by the following steps. The radial motion equation has the form:

$$2a\ddot a + \dot a^2 + k = -3a^2 P(a) \tag{17}$$

with initial conditions

$$a(1) = 1, \qquad \dot a(1) = \pm\sqrt{\Omega^0 - k}, \tag{18}$$

where

$$P(a) = \sum_{i=1}^{n} P_i(a) \tag{19}$$

Table 2. Difference $\triangle m(z) = m(z) - m_{\rm SM}(z)$ between models with γ = constant and the SM ($\Omega_{\rm Q} = 0.70$, $\Omega_{\rm m} = 0.30$).

z	$\gamma = -0.75$	$\gamma = -0.70$	$\gamma = -0.65$	$\gamma = -0.60$	$\gamma = -0.55$	$\gamma = -0.50$	$\gamma = -0.45$	$\gamma = -0.40$	$\gamma = -0.35$
0.20	0.03	0.01	−0.00	−0.02	−0.03	−0.05	−0.06	−0.08	−0.09
0.40	0.07	0.04	0.02	−0.01	−0.04	−0.07	−0.10	−0.12	−0.15
0.60	0.13	0.09	0.05	0.01	−0.03	−0.07	−0.11	−0.15	−0.19
0.80	0.19	0.14	0.09	0.04	−0.01	−0.06	−0.11	−0.16	−0.21
1.00	0.25	0.19	0.13	0.08	0.02	−0.04	−0.10	−0.16	−0.21
1.20	0.32	0.25	0.18	0.12	0.05	−0.02	−0.08	−0.15	−0.21
1.40	0.39	0.31	0.23	0.16	0.08	0.01	−0.06	−0.14	−0.21
1.60	0.45	0.37	0.28	0.20	0.12	0.04	−0.04	−0.12	−0.20
1.80	0.52	0.42	0.33	0.24	0.15	0.07	−0.02	−0.11	−0.19
2.00	0.58	0.48	0.38	0.29	0.19	0.10	0.00	−0.09	−0.18

is useful for description of n-fluid models while for one-fluid associated models a more proper expression is

$$P(a) = \gamma(a)\,\mathcal{E}(1)\,\exp\left(-3\int_1^a \frac{\gamma(x)+1}{x}{\rm d}x\right). \tag{20}$$

The solution of Eqs. (17), (18) and (20) is:

$${\rm d}\tau = \pm\sqrt{\frac{a}{F(a)}}\,{\rm d}a, \tag{21}$$

where

$$F(a) = \Omega^0 - ka - 3\int_1^a x^2 P(x){\rm d}x. \tag{22}$$

The normalized metric distance $r(z)$ is:

$$r(z) = \int_0^z \frac{{\rm d}x}{H(x)} = \int_0^z \frac{{\rm d}x}{(1+x)^{3/2}\sqrt{F(x)}}\cdot \tag{23}$$

Here $a = 1/(1+z)$ and $H(z) = \dot{a}/a$.

For the case of $\gamma(a)$ = const. we find

$$H(z) = (1+z)\sqrt{\Omega^0(1+z)^{3\gamma+1} - k} \tag{24}$$

and

$$r(z) = \int_0^z \frac{{\rm d}x}{(1+z)\sqrt{\Omega^0(1+z)^{3\gamma+1} - k}}\cdot \tag{25}$$

This integral may be easily calculated for $\gamma = -1$, $-2/3$ and $-1/3$ for geometries with $k = -1$ and $k = 1$. For the flat model, $k = 0$, the integral gives a simple analytical expression for any γ = const.. Now $H(z) = (1+z)^{\frac{3}{2}(\gamma+1)}$ and

$$r(z) = \frac{2}{3\gamma+1}\left(1 - \frac{1}{(1+z)^{(3\gamma+1)/2}}\right)\cdot \tag{26}$$

This formula reduces for dust ($\gamma = 0$):

$$r(z) = 2\left(1 - \frac{1}{\sqrt{1+z}}\right) \tag{27}$$

and for vacuum ($\gamma = -1$):

$$r(z) = z. \tag{28}$$

Now we compare the predicted Hubble relation with the standard model ($\Omega^0_\Lambda = 0.7$, $\Omega^0_{\rm m} = 0.3$) prediction that best describes the high-z Hubble diagram. For the matter component as dust, $\beta = 0$. The (normalized) luminosity distance $r_{\rm L} = (1+z)r$. Then the difference $\triangle m(z) = m(z) - m_{\rm SM}(z)$ between the SM and an ET-SES model with γ = const. is

$$\triangle m(z) = m(z) - m_{\rm SM}(z) = 5\log\frac{r(z)}{r_{\rm SM}(z)}\cdot \tag{29}$$

The results of the calculation are shown in Table 2. It is interesting that the model with $\gamma \approx -0.55$ gives the $m-z$ relation which is within 0.05 mag from the standard model prediction in a range of redshift $0 < z < 1.2$, where the Hubble diagram for almost all supernovae Ia is currently limited. The age of the model is

$$T_0 = \frac{2}{3}\frac{1}{\gamma+1}\frac{1}{H_0}\cdot \tag{30}$$

With $\gamma < -1/3$ this becomes longer than the Hubble time. Of course, as has been discussed by Amendola & Toccini-Valentini (2001) for their "coherent at late times" model, in order to allow structure formation the coherence cannot exist at arbitrarily large redshifts. Hence, T_0 should be regarded as an upper limit.

4. Conclusions

In spite of the absence of a physical theory of convertion between "ordinary" matter and any kind of dark energy, it is possible to get observational limits on such interaction within a phenomenological approach.

- A new view by Gromov et al. (2002) on the classification of two-fluid FLRW models is based on two independent properties: 1) stationarity (or not) of equations of state of the components and the presence (or absence) of energy transfer between them. Interestingly, the case of a stationary equation of state for the associated one-fluid model appears in 3 out of 4 classes. In two of these classes this property γ = constant

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is due to energy transfer. The associated one-fluid model becomes characterized by $\gamma = P_{\rm total}/\mathcal{E}_{\rm total}$.

• For the new situation γ = constant (meaning coherent evolution in the case of constant w) we have obtained a general integral expression (an analogue of the Mattig equation) in three types of geometry. For the flat geometry, the integral reduces to a simple analytical expression with known limiting cases for pure dust and vacuum.

• Comparison with the $m(z)$ relation for the standard model ($\Omega_\Lambda = 0.7$, $\Omega_{\rm m} = 0.3$) shows that $\gamma \approx -0.55$ gives the same relation within 0.05 mag in the redshift range $0 < z < 1.2$. This means that energy transfer is not excluded by existing $m(z)$ data. From deeper SNIa data, where $m(z)$ starts to deviate from the SM for $\gamma < -0.5$, it will be possible to estimate the value of the energy transfer between the DE and CDM components.

• The negative value of γ gives a robust upper limit to w in the equation of state: $w = \gamma(1 + 1/\alpha) < \gamma$.

In future we will analyze the value of γ directly from observations and discuss it together with the dark energy – matter ratio α and other observational constraints. Note that for $\gamma \approx -0.5$ such an upper limit of w is consistent with that derived by Amendola & Quercellini (2003) for the tracking and coupled dark energy from WMAP background radiation observations.

Acknowledgements. This work has been supported by the Academy of Finland (the project: "Cosmology from the local to the deep galaxy universe") We like to thank the referee P. Schuecker for very useful remarks.

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