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Aberration in proper motions

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Abstract. Approved space astrometric missions (SIM, GAIA) are aiming at a few microarcseconds per year precision in yearly proper motions and even less than a microarcsecond in the definition of an extragalactic reference frame. At such a level of accuracy, the curvature of stellar orbits around the center of the Galaxy cannot be neglected. The curvature of the Solar system barycentric motion induces a time-dependent component of the aberration, which has the properties of an apparent proper motion of the galaxies. This effect reaches 4 μas per year in some regions of the sky. In the case of stars, it is combined with a similar effect due to the curvature of the circular galactocentric orbit of the star, which may reach 60 μas per year for a star situated at 500 parsecs from the center of the Galaxy, and much larger closer to it. The paper gives the proofs and the formulae permitting one to compute this aberration in proper motions. The conclusion is that, at this high level of accuracy, one should present the astrometric data in a galacto-centric rather than in a barycentric reference frame.

Key words. astrometry – Galaxy: general – Galaxy: kinematics and dynamics – reference systems

1. Introduction

A few years from now, two major space astromery missions will be launched, which aim at a few microarcsecond (μas) accuracies. The Space Astrometry Mission (SIM) will observe over 3000 quasars in order to construct an extragalactic reference frame with an accuracy of at least 4 μas (Danner & Unwin 1999). GAIA mission will observe over 100 000 extragalactic objects for the same objective, but with an expected accuracy of a fraction of a microarcsecond (see ESA 2000).

In addition, it will determine yearly proper motions of a few million stars with an uncertainty better than 4 μas and 25 million with an uncertainty better than 10 μas. This will provide a very large amount of data for kinematic and dynamical studies within our Galaxy. It is fundamental that no significant systematic residual error remains present in the data, which would result in biases in global statistical results and in the use of particular pieces of data. Any statistical result is generally significantly more precise than the individual uncertainties. This means that all possibly known disturbing effect must be corrected, at least at the level of a μas, and preferably significantly better. In this paper, we present one of such effects that may introduce important biases in the dynamics of stars in the Galaxy, especially in its central parts.

2. Aberration of a star in the Galaxy

Currently, the celestial reference frame (ICRF) is based upon the directions of remote extragalactic objects, assumed to be kinematically fixed, to the accuracy of VLBI observations. The ICRF is actually a realization of the International Celestial Reference System (ICRS) as defined by the IAU-2000 resolutions (IAU 2002) and which is also called Barycentric Celestial Reference System (BCRS). This system is used for stellar and galactic astronomy, as well as for all applications in the solar system. Since observations are usually referred to the geocenter, one must transform the observed quantities to become barycentric. The transformation includes, in particular, the effects of the non-linear motion of the Earth around the barycenter of the solar system. The most important is the well-known aberration that moves yearly the apparent positions approximately on an ellipse with a semi-major axis equal to $\kappa = 20.496''$. It is much larger than the stellar parallax, but it is a function only of the motion of the Earth and of the direction of the star, and therefore can be corrected in full. Another effect is the geodesic precession, which is a rotation of the geocentric reference frame with respect to the barycentric one. The formulæ in a relativistic environment for all these corrections are well established, and are in general use (Kovalevsky & Seidelmann 2003).

However, a fundamental assumption in the barycentric ICRS is that it is dynamically fixed. This means that the motion of the barycenter of the solar system is assumed to be linear. With the advent of the microarcsecond astrometry, the curvature of this motion around the center of the Galaxy is no longer negligible. Brumberg (1991) has shown that the corresponding geodetic precession is 0.85 μas per century, and is still unobservable. But, this is not the case for the aberration. It is well known that the linear motion of the barycenter is of the order of 220 kilometers per second, which produces an aberrational displacement of the position of stars of about 150″. However,

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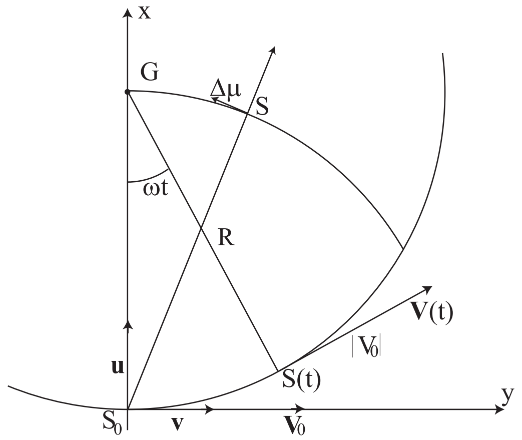

Fig. 1. The evolution of the velocity vector ***V***.

this displacement has no consequence on studies of kinematics and dynamics within the Galaxy and this fixed aberration is ignored.

Now, if we consider that this motion is not linear, the velocity vector rotates with time, and, consequently, the barycentric position of the observed objects changes with time, in such a way that it produces an effect similar to a proper motion. It is not a real motion, but introduces a systematic error in the proper motions, which may corrupt kinematic and dynamical results derived from them. Here we present this effect in detail and establish the formulae that describe it.

3. Aberration due to the circular motion of a star

Let us consider a star S revolving on a circular orbit around the center G of the Galaxy at a distance of R parsecs. Let V_0 be the circular velocity of S. For a large region of the Galaxy around the galactic plane, V_0 is of the order of 200 km s^{-1} (Fich & Tremaine 1991). Expressed in parsecs per year, the circular velocity V_c is

$$V_\mathrm{c} = 1.0227 \times 10^{-6}\, V_0,$$

where V_0 is expressed in kilometers per second. The angular velocity ω, expressed in radians is

$$\omega = V_\mathrm{c}/R. \qquad (1)$$

Let S_0 be the position of S at time $t = t_0$, and set the axes of coordinates S_0x (unit vector, $\boldsymbol{u}$), along S_0G, and S_0y (unit vector, $\boldsymbol{v}$), perpendicular in such a way that the rotation is direct (Fig. 1). At time t_0, the velocity $\boldsymbol{V}_\mathrm{c}$ is parallel to Gy. At time t in years, it has turned by ωt, so that the components of the velocity vector $\boldsymbol{V}(t)$ are

$$V_x(t) = V_\mathrm{c} \sin \omega t = V_\mathrm{c}\omega t + ...$$
$$V_y(t) = V_\mathrm{c} \cos \omega t = V_\mathrm{c} - \frac{1}{2}\omega^2 t^2 + ... \qquad (2)$$

Each component produces an aberration on the position of S as seen from an outer fixed point. Classically, the displacement $\Delta\boldsymbol{S}$ is

$$\Delta\boldsymbol{S} = -\boldsymbol{V}(t)/c, \qquad (3)$$

where c is the speed of light. Each component in Eq. (2) produces a component of the aberration:

1. $\Delta\boldsymbol{S}_0 = (-V_\mathrm{c}/c)\boldsymbol{v}$ is the constant galactic aberration, in the Gy direction, mentioned in the introduction.
2. $\Delta\boldsymbol{S}_1 = (-V_\mathrm{c}\omega t/c)\boldsymbol{u}$ is along the direction $\boldsymbol{GS}$. It has the form of a proper motion. The yearly aberration in proper motion is, therefore:

$$\Delta\boldsymbol{\mu} = \left(-\frac{V_\mathrm{c}\omega}{c}\right)\boldsymbol{u}. \qquad (4)$$

3. $\Delta\boldsymbol{S}_2 = (-\frac{1}{2}\frac{V_\mathrm{c}\hat{E}}{c}\omega^2 t^2)\boldsymbol{v}$ is an additional displacement along $\boldsymbol{V}_0$. Being proportional to the square of ω, it is negligible (for the Sun its value is 0.00022 μas per year) and will not be considered here.

From now on, we shall deal only with the aberration $\Delta\boldsymbol{S}_1$.

4. Non-circular orbits

In general, the velocity of a star is not perpendicular to GS. We shall consider, locally, that it is a part of an elliptic orbit with its focus at G. This approximation is valid to the extent that the perturbations due to an asymmetry of the galactic field or the presence of nearby stars are negligible during the observations used to determine the proper motion. The equation of motion under these conditions is:

$$\frac{\mathrm{d}^2\boldsymbol{GS}}{\mathrm{d}t^2} = \frac{\mathrm{d}\boldsymbol{V}}{\mathrm{d}t} = -\frac{kM}{R^3}\boldsymbol{GS} = -\frac{kM}{R^2}\boldsymbol{u}, \qquad (5)$$

where k is the constant of gravitation and M is the equivalent mass that controls the motion of S. One can see that the derivative of the velocity is, like in the case of the circular motion, along GS. Calling again V_c the circular velocity at the point S, and applying Eq. (1), we can use Kepler's third law and write,

$$kM = \omega^2 R^3 = V_\mathrm{c} R$$

and, substituting this expression into (5), one gets

$$\frac{\mathrm{d}\boldsymbol{V}}{\mathrm{d}t} = -\frac{V_\mathrm{c}^2}{R}\boldsymbol{u} = -\omega V_\mathrm{c}\boldsymbol{u}. \qquad (6)$$

The corresponding aberration effect is therefore,

$$\Delta\boldsymbol{\mu} = -\frac{V_\mathrm{c}\omega}{c}\boldsymbol{u},$$

which is exactly Eq. (4). It results, that, whatever is the local shape of the orbit, the aberration on proper motion is the same as if the motion was circular. So, to compute it, it is sufficient to know the circular velocity of the Galaxy at the point S. Let us point out that the circular motion of the Galaxy is determined from radial velocities that are not affected by aberration.

5. Aberration in proper motion due to the motion of the barycenter

This result shows that, in order to determine the aberration in proper motion, it is sufficient to represent the motion of the barycenter of the solar system as a circular motion around the

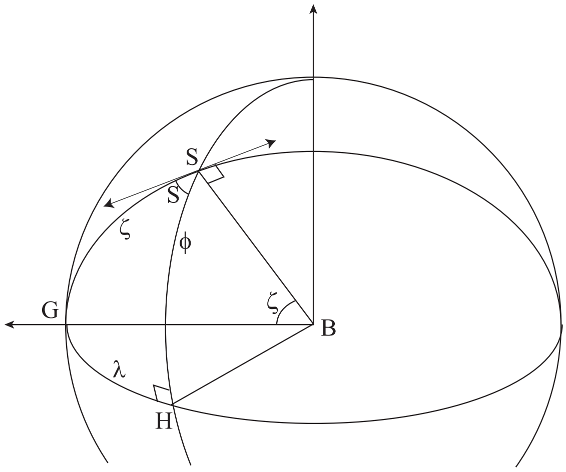

Fig. 2. Projection of the aberration in galactic coordinates.

center of the Galaxy, and neglect its particular motion toward the apex. Assuming $R = 8500$ parsecs, and a circular velocity $V_0 = 220$ km s^{-1}, one obtains, along $\boldsymbol{u}$,

$$\Delta\boldsymbol{\mu} = \frac{\Delta V_0}{c}\boldsymbol{u} = \sigma_0\boldsymbol{u} = 4,0\ \mu\text{as}\ \boldsymbol{u} \text{ per year.}$$

The aberration in the direction of a star S, as seen from the barycenter of the solar system B is the projection of this vector on the sky in the direction of S. If ζ is the angle ($\boldsymbol{GS}$) in Fig. 2, its value is therefore

$$\Delta\mu = \frac{\Delta V}{c}\sin\zeta. \quad (7)$$

$\Delta\mu$ depends only on the angle ζ, so that the effect is the same on a circle centered on G. In galactic coordinates, (λ and ϕ), with an origin in the direction of the center of the Galaxy (Fig. 2), the values of the aberration corrections are

$$\mu_\lambda\cos\phi = \Delta\mu\sin\zeta\sin S \quad\text{and}\quad \mu_\phi = \Delta\mu\sin\zeta\cos S.$$

In the spherical triangle GSH, one has

$$\sin\lambda = \sin\zeta\sin S \quad\text{and}\quad \cos S\,\sin\zeta = \cos\lambda\sin\phi.$$

From these, it finally results that

$$\mu_\lambda\cos\phi = -\sigma_0\sin\lambda\ \mu\text{as per year,}$$
$$\mu_\phi = -\sigma_0\sin\phi\cos\lambda\ \mu\text{as per year.} \quad (8)$$

This effect is mapped on the sky for any ϕ and $0° < \lambda < 180°$ in Fig. 3. The lines of equal aberration are actually circles on the celestial sphere centered at G.

6. Construction of an extragalactic reference frame

This equation, applied to observations of an extragalactic object, produces a compression towards the center of the Galaxy, G, and from its anticenter, AG, proportional to time and $\sin\zeta$. This is shown in Fig. 3. However, there is no rotation of the reference frame. The reduction procedures foreseen for GAIA (ESA 2000) take this into account. It is to be emphasized that, once this is corrected, the reference system is transformed into a galactocentric celestial reference frame (GalCRS). The

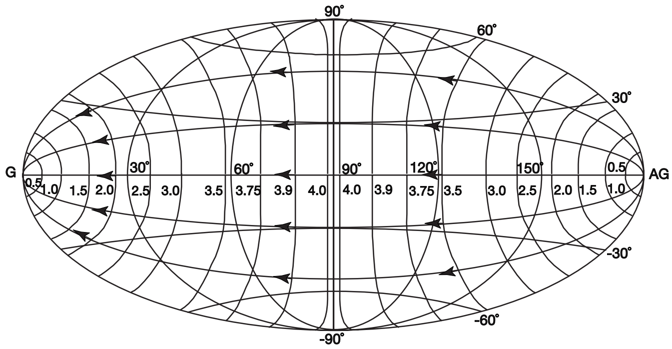

Fig. 3. Magnitude of the yearly aberration in proper motions due to the motion of the barycenter around the center of the Galaxy.

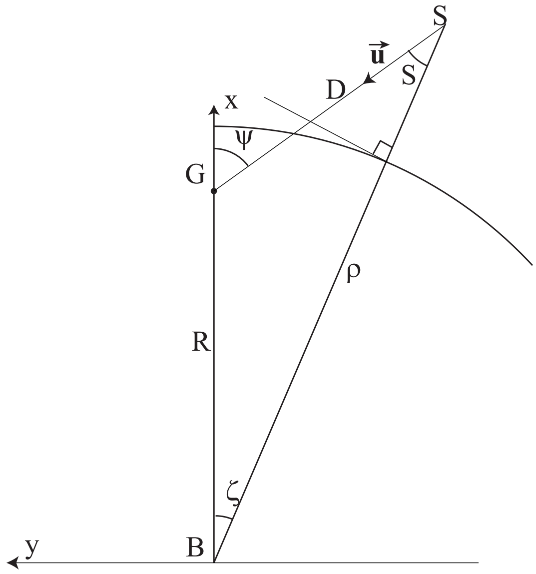

Fig. 4. Projection on the celestial sphere of the vector $\boldsymbol{u}$.

properties of such a reference frame are described in Klioner (1993). Later, Klioner & Soffel (1998) developped the relations between it and the kinematically non-rotating Geocentric Celestial Reference Frame (GCRS). Here, we consider only the transformation from the BCRS and the GalCRS. This frame is inertial for kinematic and dynamical studies of the Galaxy, provided that similar corrections are made on the proper motions of stars, as they are described in the following section.

7. Time-dependent aberration of stars in the Galaxy

Until now, we have considered only the time-dependent aberration produced by the motion of the Sun in the Galaxy. However, every star S revolves more or less around the center of the Galaxy. The results of Sects. 3 and 4 apply also to them and, in particular, the Eq. (4):

$$\Delta\boldsymbol{\mu} = \left(-\frac{V_c\omega}{c}\right)\boldsymbol{u}.$$

$\Delta\boldsymbol{\mu}$ is on the celestial circle GS. The observable part is the projection of $\Delta\boldsymbol{\mu}$ on the celestial sphere. In the plane BGS (Fig. 4),

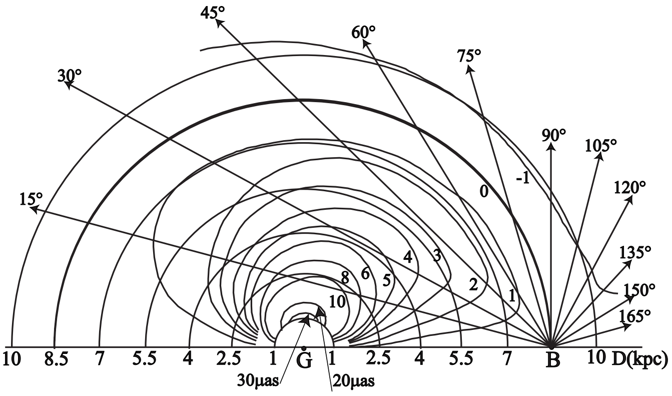

Fig. 5. Magnitude of the total yearly aberration in proper motions in a plane containing G and B, in μas.

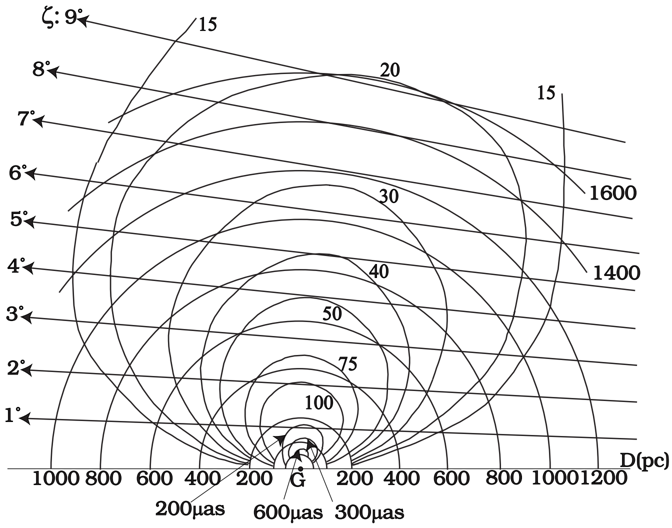

Fig. 6. Enhancement of Fig. 5 around the center of the Galaxy. Arrows give the angular distance ζ from the direction of the center of the Galaxy as seen from the Sun.

let R be the distance BG (roughly 8500 parsecs), D, the distance SG and ρ the distance of the star, namely BS, which is known since the parallax is determined by space astrometry missions, together with the proper motion. The projection of $\boldsymbol{u}$ on the perpendicular to BS is given by

$$\sin S = \frac{R \sin\zeta}{D},$$

where

$$D = \sqrt{R^2 + \rho^2 - 2R\rho\cos\zeta}.$$

Since $\omega = V_c/D$, one finally gets

$$\Delta\mu_s = \frac{-V_c^2 R \sin\zeta}{c(R^2 + \rho^2 - 2R\rho\cos\zeta)}. \qquad (9)$$

One can see that the aberration in proper motion depends on the distance of the star to the center of the Galaxy and increases for stars close to it. V_c, is practically independent of D, being of the order of 200−220 km per second.

This effect must be combined with the one described in Sect. 5. Actually, they must be subtracted because they are both directed towards G but in one case, the observer in B is moving and in the other, it is the observed object that moves. The total magnitude of the aberration in proper motion is given in Fig. 5 in which we have used the circular velocities given by Fich & Tremaine (1991). The figure represents the result on the galactic plane as a function of the position of the star. Since both effects have a cylindrical symmetry around the BG axis, in any other plane the description would be exactly the same. Note that the effect disappears on the GS axis and on the sphere centered at G and passing by B. This is easy to show considering the isosceles triangle GBS. The effect is from G inside the sphere, and towards it outside.

For clarity, Fig. 6 shows the magnitude of the effect within 1500 parsecs from the center of the Galaxy. I reaches 0.65 mas at a distance of 50 parsecs from G and is larger within that sphere, where it represents a 30 km s^{-}1 correction for the velocity.

The basic assumption of these calculations is that one can describe the local forces as a disturbed central Newtonian force directed towardsthe center of the Galaxy. This may not necessarily be true for halo stars for which the attraction by the galactic plane may be large. However, the magnitude of the effect is small at such distances, and the errors are not larger than a few μas.

8. Conclusions

The magnitude of the aberration in proper motions described in this paper is to be considered in the light of the accuracy that will be achieved by the next generation of astrometric satellites. In the case of GAIA, 4 μas accuracy is very much smaller than the magnitude of the aberration shown in Fig. 6. In this region, proper motions of stars of magnitude brighter than 12 will be observed with that accuracy. Neglecting the interstellar absorption, this would mean stars with absolute magnitudes brighter than −2 or −3, that is all class I and II giants and supergiants, as well as O to B3 main sequence stars. These stars will the best markers of the kinematic and dynamical properties of the nucleus, the bulge and the bar. The correction to be applied to observations range from 2 to 100 or more times the rms of the observations. Even the stars down to magnitude 15, that is of absolute magnitude 0 at 10 000 parsecs, observed with accuracies ranging from 4 to 10 μas will need, in this region, to have their proper motions corrected by quantities larger than the rms.

A limitation in accuracy of the correction is the uncertainty of the determination of the parallax of a remote star and its transformation into a distance. It is suggested that use of the probability distribution function (pdf) described in Kovalevsky (1998), in which the uncertainty is reduced, gives a more probable value of the distance and a Gaussian error distribution is obtained.

Outside a sphere of radius 5000 parsecs from the center of the Galaxy, and for fainter stars, the correction becomes smaller than the rms. However, for consistency, it should be applied for all observed stars, as well as extragalactic objects for which only the correction described in Sect. 5 must be applied.

Applying this correction corresponds to changing from the Barycentric Celestial Reference Frame (BCRS) to a Galactocentric Celestial Reference Frame (GalCRS). It is not the only correction to be applied: the first order aberration (about 150″ for the motion of B minus a quantity depending on the velocity of the star) should also be applied, but it modifies only the positions of the stars, not on their motion. It is not possible to determine it with comparable precision, because the velocities of the Sun and the stars expressed in kilometers per second cannot be sufficiently well known, nor can the third component of the position due to the error in parallax. Actually, it may not be so important for kinematical and dynamical studies, which use a statistical approach over large fields.

It is also to be noted, that the correction for the apparent proper motions of extragalactic objects places automatically the origin of the reference frame at the center of the Galaxy. In conclusion, the choice of GalCRS as the reference frame for galactic studies, is to be seriously considered, but implies the application of the corrections developed in this paper.

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