

Atomic data from the IRON Project

LII. Electron excitation of Ni⁺²⁴★

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Abstract. This paper reports on the calculation of collision strengths for electron induced transitions in the beryllium-like ion Ni xxv whose ground state is $1s^2 2s^2 1S_0$. We make use of the R -matrix method in conjunction with the intermediate frame coupling transformation (IFCT). The target has 98 fine structure states $1s^2 nl n' l' S L J$ corresponding to $n = 2$ and $n' = 2, 3, 4$. Our calculation extends up to 440 Ry with respect to the ground state. In order to obtain reliable rate coefficients at high temperatures (T above 2×10^7 K) we have extended our collision strengths to much higher energies by using a variety of techniques, including use of Burgess's interactive code OmeUps. The effective collision strength Υ is tabulated against $\log T$ for the 45 transitions linking the lowest 10 levels. We also give results for 5 transitions in the X-ray region which are needed in solar studies. The temperature range $6.3 \leq \log T \leq 8.3$ is centred on $\log T = 7.1$ which is where this ion has its maximum coronal abundance.

Key words. atomic data

1. Introduction

The *raison d'être* of the IRON Project (Hummer et al. 1993) was to get a small group of atomic physicists from several institutes to work with a common objective, that of calculating rate coefficients for electron induced transitions between $S L J$ levels in positive ions. Work has been harmonised by using the same set of computer codes, which arose out of the R -matrix ones described by Berrington et al. (1995). In its ten years existence the project members have generated an enormous amount of data which is described in more than fifty publications. Anyone wishing to have a list of these should visit the web site at <http://www.usm.uni-muenchen.de/people/ip/iron-project.html>

Here we present results for beryllium-like nickel, Ni⁺²⁴. In 1992 there appeared a hefty publication by Zhang & Sampson devoted to collisional excitation of this ion, with earlier work by Sampson and his colleagues being reported by Goett et al. (1980). Another team in the United States also produced tables of effective collision strengths (Bhatia et al. 1986). All these earlier calculations made use of distorted wave methods that take account of electron exchange and relativistic effects.

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* Figures 1 to 8, and Appendix A are only available in electronic form at <http://www.edpsciences.org>

2. Atomic orbitals for Ni⁺²⁴

Clementi & Roetti (1973) is an excellent source of atomic wavefunctions and we have used their 1s, 2s radial orbitals while calculating 2p, 3s, 3p, 3d, 4s, 4p, 4d, 4f ourselves with the configuration interaction program CIV3 (Hibbert 1975; Hibbert et al. 1991). CIV3 uses analytic radial orbitals $P_{nl}(r)$ which are expressed as sums of Slater type orbitals as follows :

$$P_{nl}(r) = \sum_{j=1}^k c_{jnl} \frac{(2\zeta_{jnl})^{l_{jnl} + \frac{1}{2}}}{[(2l_{jnl})!]^{\frac{1}{2}}} r^{l_{jnl}} \exp(-\zeta_{jnl} r).$$

Table 1 shows the values of the coefficients and exponents used in the collision calculation.

3. Target energy levels

We used the program AUTOSTRUCTURE to calculate the energy levels of Ni xxv and found these tallied well (eight figure agreement) with the level energies produced by the R -matrix code. The energies relative to the ground state for the lowest 98 levels are given in rydbergs ($1 \text{ Ry} = 109737.32 \text{ cm}^{-1}$) in Table 2. AUTOSTRUCTURE gives the quantum numbers p , S , L , J of each level as well as the Label (configuration identifier) shown in Table 2.

4. The quality of the target

One way of testing the target is to calculate oscillator strengths with it and see how these compare with those of other

Table 1. Radial function parameters for Ni⁺²⁴.

nl	c_{jnl}	I_{jnl}	ζ_{jnl}
1s	0.98789	1	27.51310
	0.01097	1	48.93450
	-0.00004	2	12.72440
	0.00320	2	21.97850
2s	-0.33875	1	27.51310
	0.00059	1	48.93450
	1.29943	2	12.72440
	-0.30984	2	21.97850
3s	0.36943	1	17.12263
	-3.29172	2	7.29109
	3.76773	3	8.25358
4s	0.25004	1	16.24011
	-3.94079	2	5.78564
	11.62791	3	5.71076
	-8.67396	4	6.16419
2p	0.97880	2	12.84069
	0.02563	2	22.53802
3p	0.93402	2	10.80721
	-1.48168	3	8.15175
4p	0.83985	2	9.55737
	-3.84500	3	6.19587
	3.78083	4	6.20918
3d	1.00000	3	8.46223
4d	1.34954	3	7.16910
	-1.83248	4	6.13833
4f	1.00000	4	6.25668

investigators. Fawcett (1984, 1985) tabulates weighted oscillator strengths (i.e. gf values) for transitions $n = 2 \rightarrow n = 3$ in many beryllium-like ions, including Ni xxv. He made use of Robert D. Cowan's code, which is widely thought to provide benchmark data. It is therefore important to compare our results with Fawcett's. Cowan's code only provides the length gauge oscillator strength, which in general is more reliable than the velocity gauge one. In Table 3 we give length gauge line and oscillator strengths for all the allowed transitions occurring amongst the lowest 10 levels. We also include some additional transitions of astrophysical importance. In Table 4 we compare our gf values with those of Fawcett.

5. Effective collision strengths

Our ultimate aim is to tabulate the temperature dependent effective collision strength $\Upsilon(i - j)$, defined by:

$$\Upsilon(i - j) = \int_0^\infty \Omega(i - j) \exp(-E_j/kT) d(E_j/kT)$$

where E_j is the energy of the colliding electron after excitation has occurred and $k = 6.3335 \times 10^{-6}$ Ry deg⁻¹ is the Boltzmann constant. We performed the numerical integration by linearly interpolating the $\Omega(i, j) \exp(-E_j/kT)$ data points as suggested by Burgess & Tully (1992). In order to try and delineate the resonance structure in the collision strengths we calculated each $\Omega(i - j)$ at between six and seven thousand energy points. Results are given in Table 5 for the 45 transitions between the lowest 10 levels. The temperature range is centered on 1.6×10^7 degrees which is close to the temperature at which Ni⁺²⁴ has its maximum abundance under coronal equilibrium conditions (Arnaud & Rothenflug 1985). Our results for $\Upsilon(i, j)$ are given as a function of $\log T$, which is a more convenient choice than T since the temperature in the table varies by two orders of magnitude, from $10^{6.3}$ K to $10^{8.3}$ K.

We determined effective collision strengths for five additional transitions because of their special astrophysical interest. The transitions are 1-14, 1-15, 3-18, 5-12, 5-20 and results are given in Table 6.

6. A close look at the collision strengths

Figures 1 to 6 show the collision strengths for the 45 transitions between the lowest 10 levels. In each case the energy is measured relative to the final state of the transition, i.e. $\Omega(i, j)$ is plotted against E_j where the index j refers to the upper level. The range covers $0 \leq E_j \leq 100$ Ry. The ordinate scale is adjusted so that the highest peak touches the top of its box.

The higher a peak the narrower the resonance is likely to be. In order to avoid the possibility of getting unrealistically large contributions to $\Upsilon(i, j)$ from isolated peaks in $\Omega(i, j)$, we used a small constant step in energy of 0.0196418 rydbergs in the interval $0.0193996 \leq E_2 \leq 132.111$. The computing took nearly 350 hours on a Sun Ultra Enterprise 450 SunOS 5.6, with the output occupying about 15 Mb of computer space after being compressed.

One can see that we have succeeded in catching a few isolated high points, but how many more have succeeded in slipping through the net we cannot tell. One case of special interest is the transition 4-5 for which two peaks are so close to threshold ($E_5 = 0$) that they are indistinguishable from the ordinate axis. We replot both peaks in Fig. 7 where, in order to delineate them, we have increased the number of data points in the two narrow energy intervals where the resonances occur. The inclusion of these additional data points in the collision strength file causes $\Upsilon(4, 5)$ to change from 0.0626 at $\log T = 6.3$ to 0.0382, a decrease of 40 per cent. At $\log T = 7.1$ the decrease is 15%, which is a lot smaller but still not insignificant.

By altering the scale of the ordinate axis the dense "undergrowth" caused by resonances can be made visible. Alternatively one can plot the logarithm of $\Omega(i, j)$, as is done in Fig. 8 for the eight transitions shown in Fig. 1. The complicated structure is very impressive but its effect on the thermally averaged collision strengths is often overshadowed by the effect of the isolated peaks which appear clearly in Figs. 1 to 6.

Since the present investigation began, one of us (NRB) has attended to the thorny problem of how best to deal with the resonances that pepper collision strengths. As a result some

Table 2. Ni xxv level energies in rydberg units relative to the ground state. Theoretical results from AUTOSTRUCTURE.

<i>i</i>	<i>E(i)</i>	Label	<i>i</i>	<i>E(i)</i>	Label	<i>i</i>	<i>E(i)</i>	Label	<i>i</i>	<i>E(i)</i>	Label
1	0.000000	2s2s ¹ S ₀	26	101.7399	2p3p ³ D ₂	51	129.0688	2s4p ³ P ₂ ^o	76	133.9455	2p4s ¹ P ₁ ^o
2	3.422734	2s2p ³ P ₀ ^o	27	101.8460	2p3s ¹ P ₁ ^o	52	129.1154	2s4p ¹ P ₁ ^o	77	134.4221	2p4p ¹ P ₁
3	3.799246	2s2p ³ P ₁ ^o	28	101.8472	2p3p ³ P ₀	53	129.4405	2s4d ³ D ₁	78	134.4791	2p4p ³ P ₂
4	4.973346	2s2p ³ P ₂ ^o	29	102.2933	2p3d ³ F ₂ ^o	54	129.4609	2s4d ³ D ₂	79	134.5170	2p4p ³ D ₃
5	7.744101	2s2p ¹ P ₁ ^o	30	102.7873	2p3d ³ F ₃ ^o	55	129.4991	2s4d ³ D ₃	80	134.5573	2p4p ³ S ₁
6	9.525816	2p2p ³ P ₀	31	102.8615	2p3p ³ P ₁	56	129.6919	2s4d ¹ D ₂	81	134.7526	2p4p ¹ D ₂
7	10.46280	2p2p ³ P ₁	32	102.8920	2p3d ³ D ₂ ^o	57	129.7341	2s4f ³ F ₂ ^o	82	134.9096	2p4d ¹ D ₂ ^o
8	10.98964	2p2p ³ P ₂	33	103.0009	2p3p ³ D ₃	58	129.7434	2s4f ³ F ₃ ^o	83	134.9177	2p4d ³ F ₄ ^o
9	12.54472	2p2p ¹ D ₂	34	103.0477	2p3d ³ D ₃ ^o	59	129.7633	2s4f ³ F ₄ ^o	84	134.9882	2p4d ³ D ₃ ^o
10	14.67026	2p2p ¹ S ₀	35	103.0965	2p3p ³ P ₂	60	129.8058	2s4f ¹ F ₃ ^o	85	135.0184	2p4p ¹ S ₀
11	95.54350	2s3s ³ S ₁	36	103.1393	2p3p ³ S ₁	61	132.2852	2p4s ³ P ₀ ^o	86	135.0608	2p4d ³ P ₁ ^o
12	96.18294	2s3s ¹ S ₀	37	103.7968	2p3p ¹ D ₂	62	132.3318	2p4s ³ P ₁ ^o	87	135.0686	2p4d ³ P ₂ ^o
13	97.17785	2s3p ³ P ₀ ^o	38	103.9686	2p3d ³ F ₄ ^o	63	132.7700	2p4p ³ D ₁	88	135.0703	2p4d ³ P ₀ ^o
14	97.18699	2s3p ³ P ₁ ^o	39	104.0304	2p3d ¹ D ₂ ^o	64	133.0536	2p4p ³ P ₁	89	135.1501	2p4f ¹ F ₃
15	97.57083	2s3p ¹ P ₁ ^o	40	104.2863	2p3d ³ D ₃ ^o	65	133.0597	2p4p ³ P ₀	90	135.1806	2p4f ³ F ₄
16	97.60345	2s3p ³ P ₂ ^o	41	104.4852	2p3d ³ P ₁ ^o	66	133.0738	2p4p ³ D ₂	91	135.2126	2p4f ³ D ₂
17	98.49155	2s3d ³ D ₁	42	104.4955	2p3d ³ P ₂ ^o	67	133.2977	2p4d ³ F ₂ ^o	92	135.2257	2p4f ³ F ₃
18	98.55255	2s3d ³ D ₂	43	104.5069	2p3d ³ P ₀ ^o	68	133.4893	2p4d ³ D ₂ ^o	93	135.2428	2p4f ³ G ₅
19	98.65170	2s3d ³ D ₃	44	104.5617	2p3p ¹ S ₀	69	133.5172	2p4d ³ F ₃ ^o	94	135.2557	2p4f ¹ G ₄
20	99.27112	2s3d ¹ D ₂	45	105.0979	2p3d ¹ F ₃ ^o	70	133.5563	2p4d ³ D ₁ ^o	95	135.2627	2p4d ¹ F ₃ ^o
21	99.86928	2p3s ³ P ₀ ^o	46	105.2054	2p3d ¹ P ₁ ^o	71	133.6197	2p4f ³ G ₃	96	135.2822	2p4f ³ D ₁
22	100.0568	2p3s ³ P ₁ ^o	47	128.2668	2s4s ³ S ₁	72	133.6505	2p4f ³ F ₂	97	135.3136	2p4d ¹ P ₁ ^o
23	100.9957	2p3p ³ D ₁	48	128.4768	2s4s ¹ S ₀	73	133.6662	2p4f ³ D ₃	98	135.3284	2p4f ¹ D ₂
24	101.4106	2p3s ³ P ₂ ^o	49	128.8985	2s4p ³ P ₀ ^o	74	133.6681	2p4f ³ G ₄			
25	101.7374	2p3p ¹ P ₁	50	128.9166	2s4p ³ P ₁ ^o	75	133.8636	2p4s ³ P ₂ ^o			

Table 3. Ni xxv: line strengths *S*, oscillator strengths *f*, *A*(s⁻¹) values and wavelengths λ (Å) from AUTOSTRUCTURE ($1.655^{-3} = 1.655 \times 10^{-3}$).

<i>S</i>	<i>f</i>	<i>A</i>	λ	Transition
1.655 ⁻³	2.096 ⁻³	8.106 ⁺⁷	2.398 ⁺²	1–3
5.763 ⁻²	1.488 ⁻¹	2.376 ⁺¹⁰	1.177 ⁺²	1–5
7.325 ⁻³	2.373 ⁻¹	6.002 ⁺¹²	9.376 ⁺⁰	1–14
1.320 ⁻²	4.293 ⁻¹	1.094 ⁺¹³	9.339 ⁺⁰	1–15
2.621 ⁻²	6.150 ⁻²	8.170 ⁺⁹	1.294 ⁺²	2–7
2.696 ⁻²	1.715 ⁻²	1.356 ⁺¹⁰	1.591 ⁺²	3–6
1.891 ⁻²	1.400 ⁻²	4.999 ⁺⁹	1.367 ⁺²	3–7
3.425 ⁻²	2.736 ⁻²	6.824 ⁺⁹	1.267 ⁺²	3–8
2.058 ⁻³	2.000 ⁻³	7.371 ⁺⁸	1.042 ⁺²	3–9
6.960 ⁻⁵	8.407 ⁻⁵	2.395 ⁺⁸	8.382 ⁺¹	3–10
5.183 ⁻²	5.457 ⁻¹	2.361 ⁺¹³	9.617 ⁺⁰	3–18
3.239 ⁻²	1.185 ⁻²	4.782 ⁺⁹	1.660 ⁺²	4–7
6.592 ⁻²	2.644 ⁻²	7.682 ⁺⁹	1.515 ⁺²	4–8
3.198 ⁻²	1.614 ⁻²	7.443 ⁺⁹	1.203 ⁺²	4–9
1.402 ⁻³	2.775 ⁻⁴	2.123 ⁺⁷	5.115 ⁺²	5–6
6.763 ⁻⁴	2.043 ⁻⁴	1.213 ⁺⁷	3.352 ⁺²	5–7
2.955 ⁻²	1.066 ⁻²	5.408 ⁺⁸	2.808 ⁺²	5–8
9.541 ⁻²	5.089 ⁻²	5.654 ⁺⁹	1.898 ⁺²	5–9
4.253 ⁻²	3.273 ⁻²	3.781 ⁺¹⁰	1.316 ⁺²	5–10
9.793 ⁻⁴	9.624 ⁻³	1.816 ⁺¹²	1.030 ⁺¹	5–12
5.473 ⁻²	5.566 ⁻¹	2.247 ⁺¹³	9.956 ⁺⁰	5–20

Table 4. Ni xxv: comparing the present wavelengths $\lambda(\text{AS})$ and weighted oscillator strengths $gf(\text{AS})$ from AUTOSTRUCTURE with those of Fawcett (1984), $\lambda(\text{F})$ and $gf(\text{F})$. Wavelengths are in Å.

$\lambda(\text{AS})$	$\lambda(\text{F})$	$gf(\text{AS})$	$gf(\text{F})$	Transition
9.376	9.381	0.2373	0.21	1–14
9.339	9.341	0.4293	0.45	1–15
9.892	9.917	0.0235	0.027	2–11
9.339	9.347	0.0534	0.084	2–23
9.269	9.172	0.0144	0.014	2–31
9.933	9.957	0.0703	0.077	3–11
9.623	9.632	0.5430	0.54	3–17
9.375	9.382	0.1155	0.16	3–23
9.304	9.206	0.0688	0.082	3–25
9.304	9.312	0.3809	0.51	3–26
9.294	9.302	0.1113	0.12	3–28
9.177	9.187	0.0140	0.01	3–35
10.061	10.082	0.1274	0.13	4–11
9.744	9.748	0.0366	0.036	4–17
9.738	9.743	0.5434	0.53	4–18
9.728	9.734	3.0395	2.98	4–19
9.417	9.421	0.0047	0.01	4–26
9.417	9.314	0.0764	0.096	4–31
9.296	9.300	0.5602	0.75	4–33
9.287	9.294	0.3957	0.45	4–35
10.304	10.321	0.0289	0.032	5–12
9.956	9.966	1.6698	1.82	5–20
10.231	10.233	0.0856	0.077	8–22
10.078	10.080	0.1738	0.16	8–24
9.767	9.770	2.9566	2.95	8–40
10.192	10.194	0.0536	0.048	7–21
10.171	10.174	0.0304	0.028	7–22
10.020	10.023	0.1164	0.11	7–24
10.066	10.073	0.0563	0.055	6–22
10.204	10.204	0.1380	0.13	9–27
9.846	9.853	4.6880	4.88	9–45
10.453	10.454	0.0605	0.054	10–27

guidelines have been proposed by Badnell & Griffin (2001) and Badnell et al. (2001). The steplength we used is close to $4 \times 10^{-5} z^2 = 0.02304$ Ry, the largest steplength recommended by Badnell et al. (2001). In retrospect we see that a smaller value than this would have been preferable in certain energy intervals covered by our calculation.

7. Some minor surgery

A graphical examination of collision strengths showed that $\Omega(1, 14)$ and $\Omega(1, 15)$ are negative over narrow intervals of energy. Since the cause of this is entirely numerical – a collision strength cannot be negative – we simply cut out the unphysical sections and let the collision strengths be constant across the resulting energy gaps. Before the operation the thermally averaged collision strengths are of course smaller, but only by a negligible amount. For example, at $\log(T) = 6.5$ $\Upsilon(1, 14)$ is 4 per cent down and 1.5 per cent at $\log(T) = 7.1$.

8. How far to go in energy?

By choosing 22 continuum orbitals to span the spherical R -matrix box, we are able to obtain collision strengths for electrons incident on the ground state with energies not exceeding about 440 Ry. We ran the code at 5 additional points in order to span the interval $132.111 < E_2 < 440$ Ry in which there are no resonances. However one needs to go to even higher energies than this in order to calculate reliable effective collision strengths at temperatures which are more than about 2×10^7 K. In order to see how far to go in energy we did a simple model calculation of $\Upsilon(i, j)$ assuming $\Omega(i, j) = 1$ from $E_j = 0$ up to $E_j = E_{\text{max}}$ and zero beyond. By letting $E_{\text{max}} = 6000$ Ry the error at $\log T = 8.3$ is about 1 per cent. From this simple model we see that it is essential to calculate, or estimate, the value of $\Omega(i, j)$ for energies up to several thousand rydbergs. We did this by using the methods proposed by Burgess & Tully (1992). For optically allowed transitions oscillator strengths are needed and these we obtained from AUTOSTRUCTURE, and the results are given along with line strengths S and A values in Table 3. For optically forbidden transitions between levels with the same parity and spin we calculated the high energy Born limits in the manner described by Burgess et al. (1997), hereafter BCT. For optically forbidden intersystem transitions we make use of a comparable method developed by one of us (MCC), see Appendix. NRB has included this method in his well known AUTOSTRUCTURE code. We consider simple examples in Appendix A, showing how to perform the Racah algebra needed when just a few mixed target configurations are used to describe the target. Table 7 shows the results obtained with AUTOSTRUCTURE, in which the method described in Appendix A has been made automatic and therefore includes *all* the mixed target configurations present in our target. The numerical values of the collision strength given in Appendix A are not as accurate as the results presented in Table 7. Finally, by using Alan Burgess’s graphical interactive program OmeUps (see Burgess & Tully 1992), it is possible to make spline fits to the last five data points of $\Omega(i - j)$ and the high energy limit point. This way we obtain values of $\Omega(i, j)$ at 8 energies ranging from 440 Ry to 10^9 Ry which we deem sufficient for the purpose of thermal averaging.

9. Comparing present and past

Bhatia et al. (1986), hereafter BFS, give collision strengths for transitions between the lowest 20 levels, but only at one energy,

Table 5. Ni⁺²⁴ effective collision strengths $\Upsilon(i-j)$ for $6.3 \leq \log T \leq 8.3$ ($1.837^{-3} = 1.837 \times 10^{-3}$).

$i-j$	6.3	6.5	6.7	6.9	7.1	7.3	7.5	7.7	7.9	8.1	8.3
1-2	1.837 ⁻³	2.018 ⁻³	2.068 ⁻³	1.942 ⁻³	1.681 ⁻³	1.360 ⁻³	1.046 ⁻³	7.747 ⁻⁴	5.575 ⁻⁴	3.929 ⁻⁴	2.726 ⁻⁴
1-3	1.631 ⁻²	1.685 ⁻²	1.731 ⁻²	1.756 ⁻²	1.761 ⁻²	1.764 ⁻²	1.776 ⁻²	1.802 ⁻²	1.843 ⁻²	1.894 ⁻²	1.954 ⁻²
1-4	1.144 ⁻²	1.162 ⁻²	1.132 ⁻²	1.031 ⁻²	8.750 ⁻³	6.985 ⁻³	5.318 ⁻³	3.910 ⁻³	2.803 ⁻³	1.976 ⁻³	1.376 ⁻³
1-5	2.801 ⁻¹	2.908 ⁻¹	3.048 ⁻¹	3.226 ⁻¹	3.444 ⁻¹	3.705 ⁻¹	4.006 ⁻¹	4.341 ⁻¹	4.699 ⁻¹	5.068 ⁻¹	5.450 ⁻¹
1-6	1.844 ⁻⁴	2.110 ⁻⁴	2.369 ⁻⁴	2.477 ⁻⁴	2.366 ⁻⁴	2.102 ⁻⁴	1.790 ⁻⁴	1.499 ⁻⁴	1.259 ⁻⁴	1.074 ⁻⁴	9.374 ⁻⁵
1-7	1.943 ⁻⁴	2.312 ⁻⁴	2.591 ⁻⁴	2.627 ⁻⁴	2.375 ⁻⁴	1.946 ⁻⁴	1.481 ⁻⁴	1.069 ⁻⁴	7.451 ⁻⁵	5.062 ⁻⁵	3.380 ⁻⁵
1-8	6.326 ⁻⁴	7.018 ⁻⁴	7.508 ⁻⁴	7.585 ⁻⁴	7.250 ⁻⁴	6.703 ⁻⁴	6.156 ⁻⁴	5.721 ⁻⁴	5.422 ⁻⁴	5.239 ⁻⁴	5.140 ⁻⁴
1-9	8.529 ⁻⁴	9.245 ⁻⁴	9.668 ⁻⁴	9.678 ⁻⁴	9.335 ⁻⁴	8.850 ⁻⁴	8.410 ⁻⁴	8.103 ⁻⁴	7.932 ⁻⁴	7.866 ⁻⁴	7.867 ⁻⁴
1-10	6.353 ⁻⁴	7.062 ⁻⁴	7.548 ⁻⁴	7.569 ⁻⁴	7.124 ⁻⁴	6.415 ⁻⁴	5.650 ⁻⁴	4.948 ⁻⁴	4.355 ⁻⁴	3.880 ⁻⁴	3.515 ⁻⁴
2-3	2.749 ⁻²	2.439 ⁻²	2.172 ⁻²	1.887 ⁻²	1.569 ⁻²	1.243 ⁻²	9.458 ⁻³	6.960 ⁻³	4.991 ⁻³	3.509 ⁻³	2.432 ⁻³
2-4	1.890 ⁻²	1.770 ⁻²	1.657 ⁻²	1.509 ⁻²	1.333 ⁻²	1.159 ⁻²	1.010 ⁻²	8.969 ⁻³	8.150 ⁻³	7.590 ⁻³	7.226 ⁻³
2-5	5.371 ⁻³	6.122 ⁻³	6.401 ⁻³	6.049 ⁻³	5.198 ⁻³	4.137 ⁻³	3.112 ⁻³	2.249 ⁻³	1.579 ⁻³	1.087 ⁻³	7.388 ⁻⁴
2-6	1.232 ⁻³	1.245 ⁻³	1.242 ⁻³	1.174 ⁻³	1.032 ⁻³	8.483 ⁻⁴	6.603 ⁻⁴	4.932 ⁻⁴	3.573 ⁻⁴	2.531 ⁻⁴	1.765 ⁻⁴
2-7	1.316 ⁻¹	1.367 ⁻¹	1.434 ⁻¹	1.517 ⁻¹	1.619 ⁻¹	1.740 ⁻¹	1.878 ⁻¹	2.031 ⁻¹	2.191 ⁻¹	2.355 ⁻¹	2.522 ⁻¹
2-8	3.042 ⁻³	3.413 ⁻³	3.595 ⁻³	3.477 ⁻³	3.079 ⁻³	2.529 ⁻³	1.963 ⁻³	1.461 ⁻³	1.055 ⁻³	7.454 ⁻⁴	5.182 ⁻⁴
2-9	1.454 ⁻³	1.888 ⁻³	2.034 ⁻³	1.900 ⁻³	1.587 ⁻³	1.221 ⁻³	8.875 ⁻⁴	6.199 ⁻⁴	4.218 ⁻⁴	2.820 ⁻⁴	1.864 ⁻⁴
2-10	2.966 ⁻⁴	3.674 ⁻⁴	3.873 ⁻⁴	3.585 ⁻⁴	2.983 ⁻⁴	2.288 ⁻⁴	1.656 ⁻⁴	1.151 ⁻⁴	7.791 ⁻⁵	5.177 ⁻⁵	3.398 ⁻⁵
3-4	7.408 ⁻²	6.853 ⁻²	6.331 ⁻²	5.674 ⁻²	4.891 ⁻²	4.098 ⁻²	3.400 ⁻²	2.844 ⁻²	2.430 ⁻²	2.133 ⁻²	1.927 ⁻²
3-5	2.751 ⁻²	2.582 ⁻²	2.416 ⁻²	2.160 ⁻²	1.816 ⁻²	1.446 ⁻²	1.108 ⁻²	8.326 ⁻³	6.236 ⁻³	4.725 ⁻³	3.667 ⁻³
3-6	1.353 ⁻¹	1.412 ⁻¹	1.492 ⁻¹	1.596 ⁻¹	1.722 ⁻¹	1.868 ⁻¹	2.031 ⁻¹	2.206 ⁻¹	2.386 ⁻¹	2.567 ⁻¹	2.751 ⁻¹
3-7	9.960 ⁻²	1.037 ⁻¹	1.089 ⁻¹	1.150 ⁻¹	1.222 ⁻¹	1.306 ⁻¹	1.403 ⁻¹	1.510 ⁻¹	1.625 ⁻¹	1.744 ⁻¹	1.868 ⁻¹
3-8	1.799 ⁻¹	1.874 ⁻¹	1.963 ⁻¹	2.065 ⁻¹	2.183 ⁻¹	2.322 ⁻¹	2.486 ⁻¹	2.672 ⁻¹	2.877 ⁻¹	3.091 ⁻¹	3.316 ⁻¹
3-9	1.560 ⁻²	1.726 ⁻²	1.813 ⁻²	1.824 ⁻²	1.791 ⁻²	1.755 ⁻²	1.741 ⁻²	1.758 ⁻²	1.804 ⁻²	1.871 ⁻²	1.957 ⁻²
3-10	1.441 ⁻³	1.678 ⁻³	1.764 ⁻³	1.689 ⁻³	1.500 ⁻³	1.273 ⁻³	1.066 ⁻³	9.092 ⁻⁴	8.037 ⁻⁴	7.425 ⁻⁴	7.156 ⁻⁴
4-5	3.822 ⁻²	3.679 ⁻²	3.482 ⁻²	3.120 ⁻²	2.615 ⁻²	2.063 ⁻²	1.554 ⁻²	1.135 ⁻²	8.129 ⁻³	5.771 ⁻³	4.103 ⁻³
4-6	1.297 ⁻³	1.471 ⁻³	1.578 ⁻³	1.534 ⁻³	1.345 ⁻³	1.080 ⁻³	8.136 ⁻⁴	5.860 ⁻⁴	4.097 ⁻⁴	2.810 ⁻⁴	1.904 ⁻⁴
4-7	1.690 ⁻¹	1.766 ⁻¹	1.867 ⁻¹	1.992 ⁻¹	2.142 ⁻¹	2.315 ⁻¹	2.507 ⁻¹	2.714 ⁻¹	2.928 ⁻¹	3.143 ⁻¹	3.359 ⁻¹
4-8	3.425 ⁻¹	3.576 ⁻¹	3.769 ⁻¹	4.005 ⁻¹	4.286 ⁻¹	4.613 ⁻¹	4.982 ⁻¹	5.385 ⁻¹	5.806 ⁻¹	6.233 ⁻¹	6.669 ⁻¹
4-9	1.774 ⁻¹	1.851 ⁻¹	1.932 ⁻¹	2.015 ⁻¹	2.107 ⁻¹	2.217 ⁻¹	2.348 ⁻¹	2.497 ⁻¹	2.658 ⁻¹	2.823 ⁻¹	2.992 ⁻¹
4-10	3.018 ⁻³	3.470 ⁻³	3.595 ⁻³	3.368 ⁻³	2.882 ⁻³	2.293 ⁻³	1.728 ⁻³	1.254 ⁻³	8.850 ⁻⁴	6.126 ⁻⁴	4.184 ⁻⁴
5-6	7.162 ⁻³	7.917 ⁻³	8.767 ⁻³	9.685 ⁻³	1.074 ⁻²	1.195 ⁻²	1.327 ⁻²	1.461 ⁻²	1.590 ⁻²	1.712 ⁻²	1.827 ⁻²
5-7	9.157 ⁻³	9.627 ⁻³	1.001 ⁻²	1.008 ⁻²	9.838 ⁻³	9.443 ⁻³	9.076 ⁻³	8.825 ⁻³	8.707 ⁻³	8.703 ⁻³	8.793 ⁻³
5-8	1.747 ⁻¹	1.844 ⁻¹	1.962 ⁻¹	2.096 ⁻¹	2.249 ⁻¹	2.423 ⁻¹	2.616 ⁻¹	2.822 ⁻¹	3.032 ⁻¹	3.241 ⁻¹	3.449 ⁻¹
5-9	5.264 ⁻¹	5.493 ⁻¹	5.781 ⁻¹	6.132 ⁻¹	6.553 ⁻¹	7.045 ⁻¹	7.598 ⁻¹	8.193 ⁻¹	8.803 ⁻¹	9.407 ⁻¹	1.000
5-10	2.145 ⁻¹	2.227 ⁻¹	2.335 ⁻¹	2.468 ⁻¹	2.625 ⁻¹	2.806 ⁻¹	3.006 ⁻¹	3.222 ⁻¹	3.445 ⁻¹	3.669 ⁻¹	3.898 ⁻¹
6-7	1.731 ⁻²	1.959 ⁻²	2.051 ⁻²	1.955 ⁻²	1.705 ⁻²	1.384 ⁻²	1.065 ⁻²	7.886 ⁻³	5.684 ⁻³	4.023 ⁻³	2.813 ⁻³
6-8	1.584 ⁻²	1.708 ⁻²	1.764 ⁻²	1.721 ⁻²	1.594 ⁻²	1.431 ⁻²	1.273 ⁻²	1.141 ⁻²	1.040 ⁻²	9.679 ⁻³	9.174 ⁻³
6-9	4.859 ⁻³	6.210 ⁻³	6.779 ⁻³	6.432 ⁻³	5.450 ⁻³	4.248 ⁻³	3.125 ⁻³	2.213 ⁻³	1.530 ⁻³	1.045 ⁻³	7.106 ⁻⁴
6-10	1.615 ⁻³	2.142 ⁻³	2.415 ⁻³	2.347 ⁻³	2.013 ⁻³	1.572 ⁻³	1.148 ⁻³	8.022 ⁻⁴	5.436 ⁻⁴	3.612 ⁻⁴	2.370 ⁻⁴
7-8	4.631 ⁻²	5.113 ⁻²	5.305 ⁻²	5.096 ⁻²	4.559 ⁻²	3.871 ⁻²	3.195 ⁻²	2.618 ⁻²	2.165 ⁻²	1.828 ⁻²	1.585 ⁻²
7-9	3.054 ⁻²	3.478 ⁻²	3.642 ⁻²	3.479 ⁻²	3.064 ⁻²	2.542 ⁻²	2.037 ⁻²	1.612 ⁻²	1.282 ⁻²	1.040 ⁻²	8.683 ⁻³
7-10	6.308 ⁻³	7.518 ⁻³	7.909 ⁻³	7.410 ⁻³	6.285 ⁻³	4.932 ⁻³	3.656 ⁻³	2.604 ⁻³	1.803 ⁻³	1.224 ⁻³	8.198 ⁻⁴
8-9	7.110 ⁻²	8.133 ⁻²	8.555 ⁻²	8.247 ⁻²	7.390 ⁻²	6.313 ⁻²	5.284 ⁻²	4.432 ⁻²	3.784 ⁻²	3.318 ⁻²	2.993 ⁻²
8-10	1.114 ⁻²	1.281 ⁻²	1.338 ⁻²	1.278 ⁻²	1.140 ⁻²	9.779 ⁻³	8.302 ⁻³	7.134 ⁻³	6.285 ⁻³	5.701 ⁻³	5.311 ⁻³
9-10	2.010 ⁻²	2.186 ⁻²	2.282 ⁻²	2.277 ⁻²	2.200 ⁻²	2.101 ⁻²	2.017 ⁻²	1.963 ⁻²	1.936 ⁻²	1.927 ⁻²	1.928 ⁻²

Table 6. Ni⁺²⁴ effective collision strengths for further transitions of astrophysical importance.

$\log T$	$\Upsilon(1, 14)$	$\Upsilon(1, 15)$	$\Upsilon(3, 18)$	$\Upsilon(5, 12)$	$\Upsilon(5, 20)$
6.3	4.138 ⁻³	6.108 ⁻³	5.272 ⁻²	5.685 ⁻³	6.329 ⁻²
6.5	4.102 ⁻³	6.265 ⁻³	5.443 ⁻²	4.098 ⁻³	6.511 ⁻²
6.7	4.204 ⁻³	6.626 ⁻³	5.696 ⁻²	2.979 ⁻³	6.782 ⁻²
6.9	4.502 ⁻³	7.304 ⁻³	6.083 ⁻²	2.236 ⁻³	7.208 ⁻²
7.1	5.056 ⁻³	8.419 ⁻³	6.654 ⁻²	1.785 ⁻³	7.845 ⁻²
7.3	5.922 ⁻³	1.007 ⁻²	7.450 ⁻²	1.558 ⁻³	8.736 ⁻²
7.5	7.140 ⁻³	1.234 ⁻²	8.499 ⁻²	1.504 ⁻³	9.901 ⁻²
7.7	8.734 ⁻³	1.527 ⁻²	9.814 ⁻²	1.583 ⁻³	1.134 ⁻¹
7.9	1.068 ⁻²	1.883 ⁻²	1.137 ⁻¹	1.765 ⁻³	1.305 ⁻¹
8.1	1.296 ⁻²	2.297 ⁻²	1.315 ⁻¹	2.025 ⁻³	1.497 ⁻¹
8.3	1.559 ⁻²	2.774 ⁻²	1.517 ⁻¹	2.353 ⁻³	1.715 ⁻¹

Table 7. Ni⁺²⁴ collision strengths for forbidden transitions in the high energy limit from AUTOSTRUCTURE.

$\Omega(1, 6) = 5.901^{-5}$	$\Omega(1, 8) = 5.197^{-4}$	$\Omega(1, 9) = 8.311^{-4}$	$\Omega(1, 10) = 2.596^{-4}$	$\Omega(2, 4) = 6.847^{-3}$	$\Omega(3, 4) = 1.488^{-2}$
$\Omega(3, 5) = 1.481^{-3}$	$\Omega(4, 5) = 5.579^{-4}$	$\Omega(6, 8) = 8.134^{-3}$	$\Omega(6, 9) = 5.693^{-5}$	$\Omega(6, 10) = 1.369^{-6}$	$\Omega(7, 8) = 1.023^{-2}$
$\Omega(7, 9) = 4.879^{-3}$	$\Omega(8, 9) = 2.321^{-2}$	$\Omega(8, 10) = 4.601^{-3}$	$\Omega(9, 10) = 1.958^{-2}$		

Table 8. $\Upsilon(i, j)$ for $T = 1.2 \times 10^7$. CBT, present results; BFS, Bhatia et al. (1986); ZS, Zhang & Sampson (1992).

i, j	CBT	BFS	ZS	i, j	CBT	BFS	ZS	i, j	CBT	BFS	ZS
1, 2	1.71 ⁻³	5.99 ⁻⁴	7.98 ⁻⁴	2, 9	1.62 ⁻³	1.62 ⁻⁴	2.08 ⁻⁴	5, 6	1.06 ⁻²	1.45 ⁻²	1.29 ⁻²
1, 3	1.76 ⁻²	1.46 ⁻²	1.39 ⁻²	2, 10	3.05 ⁻⁴	2.70 ⁻⁵	3.50 ⁻⁵	5, 7	9.87 ⁻³	7.46 ⁻³	7.06 ⁻³
1, 4	8.93 ⁻³	2.83 ⁻³	3.85 ⁻³	3, 4	4.98 ⁻²	2.03 ⁻²	2.58 ⁻²	5, 8	2.23 ⁻¹	5.68 ⁻²	2.22 ⁻¹
1, 5	3.42 ⁻¹	3.45 ⁻¹	3.43 ⁻¹	3, 5	1.86 ⁻²	4.85 ⁻³	6.50 ⁻³	5, 9	6.51 ⁻¹	6.98 ⁻¹	6.59 ⁻¹
1, 6	2.39 ⁻⁴	7.70 ⁻⁵	9.61 ⁻⁵	3, 6	1.71 ⁻¹	1.76 ⁻¹	1.74 ⁻¹	5, 10	2.61 ⁻¹	2.84 ⁻¹	2.70 ⁻¹
1, 7	2.41 ⁻⁴	5.20 ⁻⁵	8.68 ⁻⁵	3, 7	1.21 ⁻¹	1.19 ⁻¹	1.21 ⁻¹	6, 7	1.74 ⁻²	6.09 ⁻³	8.56 ⁻³
1, 8	7.30 ⁻⁴	4.10 ⁻⁴	3.77 ⁻⁴	3, 8	2.17 ⁻¹	2.06 ⁻¹	2.17 ⁻¹	6, 8	1.61 ⁻²	9.24 ⁻³	1.10 ⁻²
1, 9	9.38 ⁻⁴	6.04 ⁻⁴	4.97 ⁻⁴	3, 9	1.80 ⁻²	1.28 ⁻²	1.24 ⁻²	6, 9	5.57 ⁻³	8.66 ⁻⁴	1.27 ⁻³
1, 10	7.19 ⁻⁴	3.08 ⁻⁴	2.59 ⁻⁴	3, 10	1.52 ⁻³	4.19 ⁻⁴	5.49 ⁻⁴	6, 10	1.97 ⁻³	1.62 ⁻⁴	2.44 ⁻⁴
2, 3	1.60 ⁻²	5.52 ⁻³	8.16 ⁻³	4, 5	3.10 ⁻²	7.19 ⁻³	1.04 ⁻²	7, 8	4.63 ⁻²	2.06 ⁻²	2.67 ⁻²
2, 4	1.35 ⁻²	6.29 ⁻³	7.34 ⁻³	4, 6	1.37 ⁻³	2.29 ⁻⁴	3.00 ⁻⁴	7, 9	3.11 ⁻²	1.16 ⁻²	1.53 ⁻²
2, 5	5.30 ⁻³	1.36 ⁻³	1.94 ⁻³	4, 7	2.13 ⁻¹	2.32 ⁻¹	2.13 ⁻¹	7, 10	6.42 ⁻³	1.36 ⁻³	2.03 ⁻³
2, 6	1.04 ⁻³	4.00 ⁻⁴	5.39 ⁻⁴	4, 8	4.26 ⁻¹	4.57 ⁻¹	4.27 ⁻¹	8, 9	7.50 ⁻²	3.11 ⁻²	3.79 ⁻²
2, 7	1.61 ⁻¹	1.68 ⁻¹	1.63 ⁻¹	4, 9	2.10 ⁻¹	2.01 ⁻¹	2.01 ⁻¹	8, 10	1.16 ⁻²	4.72 ⁻³	5.60 ⁻³
2, 8	3.13 ⁻³	1.14 ⁻³	1.52 ⁻³	4, 10	2.94 ⁻³	7.74 ⁻⁴	1.04 ⁻³	9, 10	2.21 ⁻²	1.48 ⁻²	1.64 ⁻²

namely $E_1 = 110$ Ry. The temperature of a thermal plasma with this mean energy is given by $110/(1.5 \times 6.3335 \times 10^{-6}) = 1.2 \times 10^7$ degrees Kelvin. This is the temperature at which BFS give level populations in their TABLE IF. Presumably they obtained these results using $\Upsilon(i, j) = \Omega(i, j)$.

Zhang & Sampson (1992), hereafter ZS, give collision strengths for $E_j = 19.5075, 52.02, 130.05, 273.105, 520.20, 910.35$ Ry. We used linear extrapolation to estimate the value of $\Omega(i, j)$ at $E_j = 0$ and then thermally averaged the data in order to obtain $\Upsilon(i, j)$ at $T = 1.2 \times 10^7$ K. Table 8 is a comparison of the present results (CBT) with those of BFS and ZS.

From Table 8 we see that for optically allowed transitions between states with the same spin multiplicity the agreement

between CBT and ZS is excellent. For optically allowed transitions between singlet and triplet states the agreement is almost perfect in the case of (5, 8) where CBT/ZS = 1.004 but otherwise varies from between CBT/ZS = 0.82 for (5, 6) up to CBT/ZS = 2.77 for (3, 10). The excellent agreement between CBT and ZS for (5, 8) is not duplicated by BFS since their result, curiously enough, is almost a factor of 4 smaller.

For the remaining transitions $\Upsilon(\text{CBT})$ always exceeds $\Upsilon(\text{BFS})$ and $\Upsilon(\text{ZS})$ by a factor not exceeding 5, except for the transitions (2, 9), (2, 10) and (6, 10). The reason for this is undoubtedly the neglect by BFS and ZS of resonances, which in our calculation occur in great profusion and have a pronounced effect on the collision strengths of optically forbidden transitions.

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