Box simulations of rotating magnetoconvection

Effects of penetration and turbulent pumping

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Abstract. Various effects of penetration in rotating magnetoconvection are studied by means of three-dimensional numerical simulations employing the code NIRVANA. A local, 2-layer model is applied dividing the computational domain (which is a rectangular box placed tangentially on a sphere at latitude 45°) in an unstable polytropic region on top of a stable polytropic region. Different realizations of convection are examined parameterized by Taylor numbers $Ta = \{0, 6 \times 10^4, 6 \times 10^5\}$ and magnetic field strengths $\beta = \{5, 50, 500, 5000, \infty\}$. We find a rather distinctive behavior of the penetration depth Δ on the system parameters (Ta, β) . In non-rotating convection Δ is a monotonically decreasing function of β^{-1} which is due to magnetic quenching effects. Also, penetration is subject to rotational quenching, i.e. Δ is reduced for increasing rotation rate. In the intermediate regime of (Ta, β) , the effects of rotation and magnetic field on Δ do *not* simply add (see Fig. 3). We find, nevertheless, a very strong reduction of the penetration depth of overshooting turbulence by both rotation and magnetism.

Penetrative convection is closely associated with the mixing of a passive scalar quantity advected with the flow. In the long term, the tracer material penetrates significantly deeper into the stable layer than suggested by Δ which is due to the cumulative effect of isolated, fast-moving plumes. In case of a weak magnetic field, penetrative convection also serves to ensure a downward transport of magnetic flux by turbulent pumping with an average rate $\gamma_z \approx -7 \times 10^{-3}$ measured in units of the sound speed at the top z -boundary. For larger magnetic fields the pumping effect is quenched and even changes sign in the convection zone. This effect is suggested as being due to the effect of “turbulent buoyancy” which in density-stratified media transports a given magnetic field upwards if it is not too strong (Kichatinov & Rüdiger 1992).

Key words. convection – magnetohydrodynamics – turbulence – magnetic fields

1. Introduction

Turbulent convection is the dominant mixing process in many stellar interiors. Especially, in stars with convective cores, mixing of chemical elements has a direct impact on the star’s evolution by regulating nuclear fusion processes. Beside the redistribution of chemical, convection also serves to transport energy, momentum and magnetic flux. Of particular interest might be overshooting regions as the natural envelopes of stellar convection zones. This is because of several reasons. In certain A type stars, for example, overshooting motions may enable the merger of the distinct, convectively unstable ionization zones of hydrogen and helium into one coherent, well-mixed region (Latour et al. 1981). In convectively unstable shells of late-type stars like the Sun, penetrative convection is believed to enable the downward transport of the poloidal magnetic field into the overshoot region. Such transport is essential for the overshoot dynamo scenario. In this scenario, the toroidal magnetic field is regenerated out of the poloidal field by shearing motions in the overshoot layer which, however, requires an efficient

mechanism to transport the poloidal field downwards. Whereas the existence of shear flows at the base of the convection zone in the Sun has indeed been deduced from helioseismological measurements (Gough & Toomre 1991; Thompson et al. 1996), our knowledge about the mechanism of magnetic field pumping is still insufficient. The efficiency of pumping is likely correlated with the vigor of overshooting which, in turn, is controlled by the dynamics within the convection zone. Any estimates of the pumping effect therefore require the study of penetrative convection and its dependence on important parameters like the rotation rate and magnetic field strength.

Efforts to grasp the process of convective penetration in a quantitative way come from both analytical investigations and direct numerical simulations. Mixing-length theory has often been used to describe transport phenomena. Although successful in treating some aspects of mixing within convection zones, applied to the problem of convective penetration, mixing-length approaches yield less convincing results, however. For a more detailed discussion of the drawbacks in this context, consult the work of Renzini (1987). Another analytical recipe to treat penetration without falling back on mixing-length approximations makes use of scaling arguments. Such

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methods are based on the assumption that convective penetration is dominated by isolated plumes. Based on a simple model, Schmitt et al. (1984) derived the depth of penetration to be proportional to $V^{3/2}$ where V denotes the velocity of a plume when it enters an adiabatic stable layer bordering the convection zone. Zahn (1991) refined that model, dividing the region of penetration into an adiabatic part where convection can be regarded still as efficient and a thermal adjustment layer thereafter. Zahn derived expressions for the extent of penetration which, as a special case, include the scaling law of Schmitt et al. If rotational effects and magnetic fields come into play, such analytical investigations meet their limits. Direct numerical simulations of convective penetration then is the only alternative to estimate penetration depths. Such calculations have been performed in the past by several authors in two space dimensions (Cattaneo et al. 1990; Xie & Toomre 1993; Hurlburt et al. 1994) and, more recently, in three space dimensions (Singh et al. 1996; Nordlund & Stein 1996; Singh et al. 1998a,b; Saikia et al. 2000).

In the present work we will use 3D modeling in order to estimate penetration depths for various realizations of convection including the effects of rotation and magnetic field. In contrast to Tobias et al. (1998, 2001) and Dorch & Nordlund (2001), who are also working in three dimensions, the turbulent advection velocity γ is explicitly computed in our paper. It is – not surprisingly – mainly directed downwards in both the convectively unstable zone and the overshooting region beneath this zone. In contrast to the results of Ossendrijver et al. (2002), for strong magnetic fields the pumping velocity in the convection zone is upwards (see Table 2 in Sect. 3). Note that the magnetic field strength in Ossendrijver et al. remains sub-equipartitioned in the simulations so that effects nonlinear in the mean field are small in this study.

The objective of a previous paper on rotating magnetoconvection (Ziegler 2002, hereafter Paper I) was to explore the spatiotemporal evolution for a non-homogeneous toroidal magnetic field and the generation of mean flows. These prior results provide the basis of the present study which analyzes the properties of convective penetration for these simulations. The paper is organized in the following manner. Section 2 reviews the convection model described in more detail in Paper I. Section 3 presents the results starting with a brief discussion of the numerical outcome of Paper I. Our conclusions are finally given in Sect. 4.

2. Problem formulation

Penetrative convection is studied for a fully compressible, viscous gas obeying the ideal gas equation of state. The model considers a local Cartesian box placed tangentially on a rotating sphere at latitude 45° and involves a 2-layer planar polytropic stratification. Coordinate axes are along the box edges with unit vectors $\hat{x}$ (equatorward), $\hat{y}$ (azimuthal direction) and $\hat{z}$ (radial direction) (see also Paper I for the geometrical settings). The upper layer of extent d is convectively unstable whereas the lower layer of the same extent is stable and serves to brake overshooting motions. The depth of the stable layer is sufficient to avoid any impingement of downward-directed plumes on the

lower boundary. The computational domain spans a volume $(x, y, z) \in [0, 4d] \times [0, 4d] \times [-2d, 0]$ discretized by $96 \times 96 \times 128$ mesh points. The grid points are uniformly distributed in each coordinate direction. The aspect ratio of grid spacings is $1:1:\frac{3}{8}$ where the smaller grid spacing in z is chosen to account for the vertical stratification.

The problem is described by the visco-resistive MHD equations supplemented by a heat conduction term in the energy balance equation and by the equation of state

$$\partial_t \varrho = -\nabla \cdot (\varrho \mathbf{u}), \quad (1)$$

$$\begin{aligned} \partial_t (\varrho \mathbf{u}) = & -\nabla \cdot (\varrho \mathbf{u} \mathbf{u}) - \nabla p + \nabla \cdot \boldsymbol{\tau} - 2\varrho \boldsymbol{\Omega} \times \mathbf{u} \\ & + \frac{1}{\mu_0} (\nabla \times \mathbf{B}) \times \mathbf{B} + \varrho \mathbf{g}, \end{aligned} \quad (2)$$

$$\partial_t \mathbf{B} = \nabla \times (\mathbf{u} \times \mathbf{B} - \eta \nabla \times \mathbf{B}), \quad (3)$$

$$\begin{aligned} \partial_t e = & -\nabla \cdot (e \mathbf{u}) - p \nabla \cdot \mathbf{u} + Q_{\text{vis}} + \nabla \cdot (\kappa \nabla T) \\ & + \frac{\eta}{\mu_0} |\nabla \times \mathbf{B}|^2 \end{aligned} \quad (4)$$

$$p = \frac{k}{\mu m_u} \varrho T. \quad (5)$$

ϱ , p , e , T , $\mathbf{u}$, $\mathbf{B}$ denote the mass density, pressure, thermal energy density, temperature, velocity and magnetic field. The remaining quantities are the viscous stress tensor $\boldsymbol{\tau}$,

$$\tau_{ij} = \varrho \nu \left(\nabla u_{ij} + \nabla u_{ji} - \frac{2}{3} \nabla u \delta_{ij} \right) \quad (6)$$

where ν is the kinematic viscosity coefficient, the viscous heating term

$$Q_{\text{vis}} = \sum_{ij} \tau_{ij} \nabla u_{ij}, \quad (7)$$

the (constant) gravitational field $\mathbf{g} = -g\hat{z}$, $g > 0$, the magnetic permeability $\mu_0 = 4\pi \times 10^{-7} \text{ VsA}^{-1}\text{m}^{-1}$, the conductivity coefficient κ , Boltzmann's constant k , the atomic mass unit m_u , the mean molecular weight $\bar{\mu} = 1$, the ratio of specific heats $\gamma = C_p/C_v = 5/3$ and the magnetic diffusivity $\eta = 1/\mu_0\sigma$ where σ denotes the electrical conductivity. Rotation of the sphere is accounted for by the Coriolis force term in the momentum Eq. (2).

The (non-magnetic) hydrostatic 2-layer polytropic state is realized by a piecewise constant conductivity coefficient which, in effect, controls the stability of the stratification:

$$\kappa(z) = \begin{cases} \kappa_s & -2d < z < -d \quad (\text{stable region}) \\ \kappa_c & -d < z < 0 \quad (\text{unstable region}) \end{cases} \quad (8)$$

where the subscript “s” (“c”) denotes the stable (unstable) region, and

$$T = \begin{cases} \frac{F_0}{\kappa_s} (z_1 - z) + T_1 & -2d < z < -d \\ \frac{F_0}{\kappa_c} (z_2 - z) + T_2 & -d < z < 0 \end{cases} \quad (9)$$

$$\rho = \begin{cases} \rho_1 \left(\frac{T}{T_1} \right)^{m_s} & -2d < z < -d \\ \rho_2 \left(\frac{T}{T_2} \right)^{m_c} & -d < z < 0 \end{cases} \quad (10)$$

$$p = \begin{cases} \frac{k}{m_u} \rho_1 T_1 \left(\frac{T}{T_1} \right)^{m_s+1} & -2d < z < -d \\ \frac{k}{m_u} \rho_2 T_2 \left(\frac{T}{T_2} \right)^{m_c+1} & -d < z < 0 \end{cases} \quad (11)$$

where the subscript 0 (1, 2) refers to values at vertical position $z_0 = -2d$ ($z_1 = -d$, $z_2 = 0$) and $F_0 = \kappa_s(dT/dz)_0$ is the heat flux prescribed at z_0 . Continuity of the solution at z_1 requires

$$T_1 = \frac{F_0}{\kappa_c} d + T_2, \quad (12)$$

$$\rho_1 = \rho_2 \left(\frac{T_1}{T_2} \right)^{m_c}. \quad (13)$$

The polytropic indices of the layers are given by

$$m_s = \frac{m_u}{k} \frac{\kappa_s}{F_0} g - 1, \quad (14)$$

$$m_c = \frac{m_u}{k} \frac{\kappa_c}{F_0} g - 1. \quad (15)$$

The magnetoconvection simulations start with initial conditions taken from evolved states of hydrodynamic convection in the background atmosphere (9)–(11) superimposed with a non-homogeneous, toroidal magnetic field of profile

$$\mathbf{B}(z) = \frac{B_1}{9} \left(10 \exp \left(-4 \frac{(z - z_1)^2}{d^2} \ln 10 \right) - 1 \right) \hat{\mathbf{y}} \quad (16)$$

where B_1 is the maximum field strength prevailing at the stable/unstable interface ($z = z_1$).

The problem is characterized by the dimensionless numbers Ra (Rayleigh number), Pr (Prandtl number), Pm (magnetic Prandtl number), Ta (Taylor number) and $\beta = 2\mu_0 p_1/B_1^2$. The initial polytropic indices are $m_s = (d \ln p/d \ln T)_s - 1 = 9$ and $m_c = (d \ln p/d \ln T)_c - 1 = 1$. Furthermore, the same dimensionless notation is introduced as in Paper I. That is, coordinates are measured in units of d , time in units of the sound crossing time $\tau_{S2} = d/c_{S2}$ where $c_{S2} = (p_2/\rho_2)^{1/2}$ is the isothermal sound speed at z_2 , velocity in units of c_{S2} , density in units of ρ_2 , pressure in units of p_2 , temperature in units of T_2 and the magnetic field in units of B_1 . Periodic boundary conditions are assumed in the horizontal directions. Boundary conditions in z are:

$$\partial_z T = -2 \text{ at } z = -2, \quad (17)$$

$$T = 1 \text{ at } z = 0, \quad (18)$$

$$\left. \begin{aligned} \partial_z u_x &= \partial_z u_y = u_z = 0 \\ \partial_z B_x &= \partial_z B_y = 0, B_z \text{ from } \nabla \cdot \mathbf{B} = 0 \end{aligned} \right\} \text{ at } z = -2, 0. \quad (19)$$

Condition (17) effectively fixes F_0 . For the magnetic field a kind of “open” condition is used described in more detail in Paper I.

3. Numerical results

A series of runs has been made in the parameter space (Ta, β) for fixed $Ra = 3 \times 10^5$, $Pr = 0.1$ and $Pm = 1$, respectively. A summary of all simulations is given in Table 1 including the Coriolis number, $Co = 2\tau\Omega$ where τ is the measured convective turnover time, and other interesting quantities that are referred to in the text.

Table 1. The sample of simulations.

model	Ta	Co	β	$\overline{\langle u_z^2 \rangle}^{1/2}(z_1)$	Δ	Δ_m
M00	0	0	∞	26.6	0.44	–
M01	0	0	5000	26.2	0.44	0.55
M02	0	0	500	22.2	0.33	0.53
M03	0	0	50	17.3	0.23	0.39
M04	0	0	5	18.9	0.17	0.32
M10	6×10^4	1.3	∞	23.4	0.35	–
M11	6×10^4	1.3	5000	23.6	0.36	0.48
M12	6×10^4	1.4	500	22.6	0.37	0.45
M13	6×10^4	1.8	50	13.5	0.25	0.35
M14	6×10^4	2.8	5	10.8	0.11	0.27
M20	6×10^5	4.2	∞	19.1	0.27	–
M21	6×10^5	4.2	5000	19.0	0.28	0.39
M22	6×10^5	4.4	500	15.6	0.23	0.34
M23	6×10^5	6.0	50	10.0	0.17	0.29
M24	6×10^5	9.1	5	6.5	0.05	0.23

3.1. Structure of magnetoconvection

We start by summarizing some main results of Paper I. The very different picture of weak and strong field magnetoconvection emanating from the simulations is captured in Fig. 1 presenting snapshots of the velocity field and magnetic energy distribution exemplary for the models M01 with parameter set ($Ta = 0$, $\beta = 5000$) and M24 with parameter set ($Ta = 6 \times 10^5$, $\beta = 5$). These models in the suite of simulations turn out to be those which give the extremes in penetration properties. In the weak field case, convection essentially shows the same behavior as its non-magnetic counterpart, with the magnetic field treated more or less as a passive ingredient advected with the flow. Such convection is dominated by narrow regions of strong, downward-directed plumes embedded in broader regions of upflowing material. This spatial asymmetry between narrow downflow regions and broader upflow regions is a typical feature of compressible convection in stratified media observed before in numerical simulations (Cattaneo et al. 1991; Weiss et al. 1996; Brummell et al. 1996). The downflows are connected to form a quasi-laminar, cellular network of vertical extent ≈ 0.2 comparable to the thickness of the upper thermal boundary layer. With increasing depth the network becomes disintegrated and merges with the moderately turbulent interior. The flow is highly time-dependent, occasionally generating fast plumes which are able to penetrate deeply into the stable zone. Such an event is seen in the slice representation of Fig. 1 showing the projected velocity in the plane $y = 2.5$. The turbulent nature of convection also manifests in the topology of the magnetic field. Magnetic flux accumulates in small-scale structures distributed over the whole convection zone and over large parts of the stable layer where it has been transported by overshooting motions.

As the magnetic field strength is increased, the structure of convection evolves towards two-dimensionality with the direction of invariance being the direction of the mean magnetic field. If the magnetic field dominates over convection,

Fig. 1. Structure of magnetoconvection for the models M01 (top part) and M24 (bottom part). The velocity structure is shown as grey-scale image of u_z along the box walls and at depth $z = -0.9$ overlotted by velocity vectors (left panels) and illustrated as a projected velocity field in the plane $y = 2.5$ (middle panels). The right panels show the distribution of magnetic energy in terms of isosurface plots overlaid by contour plots at $y = 2$.

the resulting solution is best described as laminar throughout the domain with the convective motions occurring in the form of cylindrical rolls aligned in the y -direction – the primary direction of the mean magnetic field with $\langle B_x \rangle \ll \langle B_y \rangle$ where $\langle \cdot \rangle$ denotes averaging over the horizontal directions (note that $\langle B_z \rangle = 0$ because of the boundary conditions). The vigor of convection is substantially reduced as expressed by a rms velocity of only 30% of that of weak field convection.

The presence of rotation generates larger-scale horizontal flows via a linear coupling process. Namely, the Coriolis force provides an effective mechanism to transform vertical momentum into horizontal momentum. This effect probably has direct influence on the penetration properties controlled by vertical motions. The detailed structure of the velocity field crucially depends on the parameters Ta and β . In weak field convection, the presence of rotation reduces the scales of motion and inertial oscillations of the larger-scale flows enhance the mobility of the network. In strong field convection, rotation favors streaming motions in $\pm y$ -direction which contain a large fraction of the total kinetic energy, in contrast to the corresponding non-rotating case. A relatively small percentage of the kinetic energy is stored in the (horizontally-averaged) mean velocity having non-vanishing components in all coordinate directions – vertical, meridional (equatorwards) and zonal (in azimuthal direction).

3.2. Penetration into the stable layer

Convective motions entering the stable layer are braked and, finally, stopped or turned horizontally, thereby transferring energy to the stable layer. While plumes are decelerated, a certain amount of energy is also provoked to generate gravity waves (Hurlburt et al. 1994, hereafter HTMZ). These waves interact with the penetrating plumes and modify their dynamics. The amount of energy deposited in such waves depends on the properties of the stratification and are small in our case. Because of this we dispense with an analysis of gravity waves.

In accord with HTMZ, the extent of penetration is estimated with the help of the averaged vertical kinetic energy flux $\overline{\langle F_{\text{kin}} \rangle}(z)$ where $F_{\text{kin}} = \rho u_z^2$ and the bar (brackets) denotes time (horizontal) averaging. The average over time typically covers 10 turnover times and is taken in the later stages of evolution at which statistical equilibrium already has been established. The penetration depth is then defined as $\Delta = -1 - z_{(0)}$ where $z_{(0)} < -1$ is the vertical position for which $\overline{\langle F_{\text{kin}} \rangle}(z_{(0)}) = 0$. Figure 2 shows the z -profile of $\overline{\langle F_{\text{kin}} \rangle}$ for various models. Clearly, $\overline{\langle F_{\text{kin}} \rangle}$ is negative throughout the convection zone. At some depth, $\overline{\langle F_{\text{kin}} \rangle}$ becomes zero for the first time which provides a measure for the degree of convective penetration. The resulting Δ is depicted in Fig. 3 as a function of the initial problem parameters (Ta, β) . Without rotation and without

Fig. 2. Averaged kinetic energy flux, $\overline{\langle F_{\text{kin}} \rangle}$, for several models.

Fig. 3. Penetration depth Δ as a function of initial parameters Ta and β . Calculated values are denoted by symbols: $\beta = \infty, 5000$ (triangle), $\beta = 500$ (asterisks), $\beta = 50$ (diamond), $\beta = 5$ (box). The dashed line corresponds to $\beta = \infty$.

Fig. 4. Penetration depth as a function of $\overline{\langle u_z^2 \rangle}^{1/2}(z_1)$. Calculated values are denoted by symbols: $\beta = \infty, 5000$ (triangle), $\beta = 500$ (asterisks), $\beta = 50$ (diamond), $\beta = 5$ (box).

a magnetic field we find $\Delta = 0.44$. HTMZ studied convective penetration by means of 2D simulations with the dependence on the relative stability parameter S ,

$$S = -\frac{\frac{1}{\gamma-1} - m_s}{\frac{1}{\gamma-1} - m_c}, \quad (20)$$

of a 2-layer polytropic configuration similar to the one used here. Applying definition (20), our 2-layer system has stability $S = 15$ which characterizes it as relatively stiff. HTMZ found that Δ monotonically decreases with S which has been confirmed numerically in 3D experiments by Singh et al. (1996). For $S = 15$, HTMZ state a value of $\Delta = 0.31$ which is smaller than ours. This is due to differences in the initial conditions, dimensionality of the problem and different Ra used in the simulations so that the results are not directly comparable. Note that the relative stiffness of the 2-layer system changes over time because the final convective state approaches an isentropic state. This means that the effective m_c increases whereas m_s practically remains constant. The role of the parameter S in the study of penetration depths then appears a bit dubious.

In the absence of magnetic fields, Δ is reduced with increasing Ta . The penetration depths for $Ta = 6 \times 10^4$ and $Ta = 6 \times 10^5$ are $\Delta = 0.35$ and $\Delta = 0.27$, respectively. This result is an expression of rotational quenching and can be understood as a consequence of the Coriolis force which serves to transform vertical momentum into horizontal momentum through linear coupling. This mechanism lowers the vertical kinetic energy flux and, in effect, leads to a smaller penetration depth. For weak magnetic field convection ($\beta = 5000$), one essentially finds the same functional behavior with Ta . In the case of strong field convection ($\beta = 5$), however, the extent of penetration is significantly reduced. This can be associated with magnetic quenching effects of convection due to the mean toroidal field present in the solution. We observe the same dependence of Δ on Ta as found for $\beta = 5000$ i.e. Δ is a monotonically decreasing function of Ta . The penetration depth is smallest for model M24 with parameter set ($Ta = 6 \times 10^5, \beta = 5$) where rotational- and magnetic quenching effects seem to work in cooperation to yield $\Delta = 0.05$. It must be noted, however, that this value has an uncertainty of roughly the same amount. This is because $\langle F_{\text{kin}} \rangle$ is rather small for this model and, therefore, the position of its first zero in the stable layer fluctuates considerably over time. The situation turns out to be a bit more complex for the models with intermediate magnetic field strengths ($\beta = 500, 50$). For both cases it is found that Δ first *increases* when going from $Ta = 0$ to $Ta = 6 \times 10^4$. Obviously, for some regimes in (Ta, β) -space the generic effects of rotation and magnetic fields on Δ do not simply add.

Taking guidance from past analytical work the process of penetration is further analyzed with the aim of probing the existence or non-existence of a simple scaling relationship between the penetration depth and velocity of overshooting motions. According to Schmitt et al. (1984) the penetration depth of an isolated plume into an adiabatic stable layer should scale as $V^{3/2}$ where V is the velocity of the plume when it enters the stable layer. Zahn (1991) argued that, based on the assumption of a smoothly varying conductivity coefficient, the zone

of penetration can be described by two distinct parts. Namely, a nearly adiabatic part where convection is still vigorous, and a thermal adjustment layer beneath. For the case of penetration into a nearly adiabatic stable layer, Zahn verified the $V^{3/2}$ -dependence found earlier by Schmitt et al. (1984). In a similar analysis, HTMZ demonstrated that for a piecewise constant conductivity coefficient, as in our case, adiabatic penetration should scale as V^3 and not as $V^{3/2}$ derived for a smooth conductivity coefficient. We check whether our data follows a simple scaling law of the form $\Delta \propto V^\xi$ where ξ is some exponent. The velocity V must be assessed by some characteristic velocity of penetrating motions for which we choose the rms vertical velocity, $V \equiv \langle u_z^2 \rangle^{1/2}(z_1)$, measured at the stable/unstable interface $z = z_1$. Figure 4 displays the logarithm of Δ as a function of the logarithm of $\langle u_z^2 \rangle^{1/2}(z_1)$ computed for each parameter pair (Ta, β) . For comparison the slopes associated with the scaling laws $\Delta \propto V^{3/2}$ and $\Delta \propto V^3$ are also indicated. Obviously, the full data set match none of the simpler scaling laws for adiabatic penetration but shows a more complicated behavior. This has several reasons. First, the thermal structure of the penetration region is not adiabatic. HTMZ argued that for larger S both the thermal adjustment layer and adiabatic layer of penetration may play a significant role and, hence, both provide contributions to the total extent of penetration. If this is indeed true, the V^3 -dependence no longer holds. Second, the presence of a non-homogeneous mean magnetic field in the vicinity of the stable/unstable interface makes penetration a highly non-local process. The braking of overshooting motions is influenced by the Lorentz force so that a local measure of V (here at the stable/unstable interface) may not be sufficient to describe the properties of penetration correctly. Therefore, one cannot expect to find a simple (Δ, V) -relationship for a suite of quite different realizations of convection. Such a non-local character of penetration may also be responsible for the non-monotonic behavior of Δ with Ta found for intermediate magnetic field strengths.

3.3. Mixing of a passive scalar quantity

A natural consequence of penetrative convection is the transport of chemical species from the convection zone into the stable layer. To mimic that process we have followed the evolution of an additional scalar quantity C (the concentration of a hypothetical specimen) which is just passively advected with the flow. With initial conditions

$$C = \begin{cases} 1 & z > -1 \\ 0 & z < -1 \end{cases} \quad (21)$$

we solve the equation

$$\partial_t C + \nabla \cdot (C \mathbf{u}) = 0 \quad (22)$$

which is formally identical to the equation of conservation of mass. Therefore, we can apply the same conservative finite-volume scheme as for the continuity equation. Periodic boundary conditions are imposed in the horizontal directions whereas at the impermeable top and bottom walls $\partial_z C = 0$. Note that integral C is exactly conserved within machine accuracy.

Fig. 5. Time evolution of concentration for the models M01 (top) and M23 (bottom). The grey-scale image displays $\langle C \rangle(z, t)$ and is overplotted by the function $\langle \langle C \rangle \rangle_s(t)$ (right axis). The smaller plots illustrate the time history of concentration in the form of snapshots of the z -profile of $\langle C \rangle$ separated by $\Delta t = 1$.

Figure 5 presents the time evolution of $\langle C \rangle(z, t)$ and $\langle \langle C \rangle \rangle_s(t)$ ($\langle \langle \cdot \rangle \rangle_s$ means volume-averaging over the stable region) exemplary for two selected models, namely, M01 and M23. In the early stages of evolution, material is rapidly transported downwards by penetrative plumes. Indeed, this occurs on a timescale of the order of the convective turnover time. In the further course of evolution more and more material accumulates in the stable region. At late stages the mean concentration approaches a quasi-steady state where the $\langle C \rangle$ -profile is characterized by a nearly linear increase from the top of the convection zone up to the point where $\langle C \rangle$ reaches its maximum and a sharp decline after. The development of a steep gradient in $\langle C \rangle$ is a consequence of the large relative stability of the 2-layer polytropic stratification (recall in this context that the stability parameter $S = 15$) which causes a hard braking of overshooting convective motions. The total amount of material stored in the stable layer at the end of calculation at t_c^* depends on the problem parameters Ta and β . This is about 60% in model M01 and about 40% in model M23.

The location of the steep gradient of $\langle C \rangle$ can be used to define a penetration depth of the tracer material. We will call this Δ_m which, in general, is a function of time. To allow a comparison with the penetration depth Δ obtained from the averaged kinetic energy flux $\langle F_{kin} \rangle$ we assume in the following that Δ_m is evaluated at the end of the simulation. Figure 6 shows Δ_m as a function of the initial parameters Ta and β . Note first that $\Delta_m > \Delta$, which is due to their different definitions. We find a clear correlation between both quantities in the case of weak field convection indicated by a systematic shift between Δ_m and Δ i.e. $\Delta_m \approx \Delta + 0.11$. A less close but similar

Fig. 6. Penetration depth Δ_m of the tracer material as a function of initial parameters Ta and β . Calculated values are denoted by symbols: $\beta = 5000$ (triangle), $\beta = 500$ (asterics), $\beta = 50$ (diamond), $\beta = 5$ (box).

Fig. 7. Δ_m as a function of time for the extended models M01 and M21.

correlation exists in the case of strong field convection ($\beta = 5$). For intermediate magnetic field strength ($\beta = 500, 50$), Δ_m does not behave in accordance with Δ . Note, for instance, that Δ increases as Ta goes from 0 to 6×10^4 whereas Δ_m decreases at the same time (see Fig. 6).

Actually, Δ_m slightly increases with time caused by the cumulative effect of sporadic fast-moving plumes which are able to penetrate deeper into the stable layer than suggested by Δ . In effect, tracer material is gradually transported further downwards. This process is possibly supported by transport through gravity waves present in the stable layer. To work out the longer-term evolution of penetrative mixing more clearly we have conducted two additional runs which extend the time evolution of models M01 and M21 by a factor of 9. The resulting time-dependence of Δ_m is plotted in Fig. 7. The step-like appearance of the curves is due to the fact that the location of Δ_m on the grid can be determined only with an accuracy given by the grid spacing $\delta z \approx 0.015$. For the non-rotating model M01 the extent of penetration of tracer material has increased to $\Delta_m = 0.66$ at $t = 240$ compared to $\Delta_m = 0.55$ at $t = 27$. In the long term, the passive tracer thus enters the stable layer a distance almost comparable to the total dimension of the convection zone! For the rotating model M21 we find an

increase to $\Delta_m = 0.49$ compared to $\Delta_m = 0.39$ before. Since the rate of increase of Δ_m continuously decreases it seems plausible to assume that the influence of individual strong plumes on Δ_m gradually diminishes so that its longer-term evolution might be dominated by the gravity wave field.

3.4. Mean magnetic field pumping

During the course of evolution, the initial magnetic field is restructured by convective transport and magnetic buoyancy effects. Magnetic buoyancy is of importance in the unstable layer but it is unimportant in the stable layer (see also Paper I). In the stable region, penetrative convection is the dominant non-diffusive transport mechanism for the magnetic field. Figure 8 reveals the time history of $\langle B_y \rangle(z)$ for the two models M01 and M04. When convection dominates over the magnetic field (model M01) there is clear evidence for a net transport of magnetic flux from the convection zone into the stable layer. A similar transport phenomenon has been noted by Tobias et al. (1998) studying the interaction of a thin magnetic slab with turbulent convection. For increasing magnetic field strength non-diffusive transport becomes more and more suppressed. In the strong field model M04 the evolution of $\langle B_y \rangle$ in the stable layer is indeed dominated by magnetic diffusion. This can be checked by comparing the $\langle B_y \rangle$ -profile with the pure diffusion solution represented by dashed lines in Fig. 8. Note that $\langle B_y \rangle$ is smaller than suggested by diffusive decay which might be an indication for the presence of “turbulent” diffusion.

Non-diffusive vertical transport of the mean magnetic field consists of two parts, namely, transport by the mean flow $\langle u_z \rangle$ and transport due to small-scale motions. Within mean field theory the latter is expressed by the so-called γ -effect (turbulent pumping) where γ is an average transport velocity due to turbulent motions. Whereas $\langle u_z \rangle$ can be determined directly from the numerical data, γ must be computed from the relation

$$\mathcal{E} \approx \alpha^{\text{symm}} \langle \mathbf{B} \rangle + \gamma \times \langle \mathbf{B} \rangle - \eta_T \nabla \times \langle \mathbf{B} \rangle \quad (23)$$

where $\mathcal{E} = \langle \mathbf{u}' \times \mathbf{B}' \rangle$ is the mean electromotive force given by the cross product of the fluctuating fields $\mathbf{u}'$ and $\mathbf{B}'$, α^{symm} is the symmetric part of the (rank-2) α -tensor and η_T is the (rank-3) tensor of turbulent diffusivity. Terms involving higher-order derivatives of $\langle \mathbf{B} \rangle$ have been ignored in Eq. (23).

We are in particular interested in the z -component of γ responsible for turbulent pumping in the vertical direction.

As known, there are two different sources for the γ -effect. Both an inhomogeneous turbulence intensity (or also eddy diffusivity η_T) and/or a global density stratification lead to turbulent pumping, hence,

$$\gamma = \mathbf{U}^{\text{dia}} + \mathbf{U}^{\text{dens}}. \quad (24)$$

$\mathbf{U}^{\text{dia}}$ is the well-known turbulent diamagnetism which transports the magnetic field towards the regions of lower turbulent intensity so that it can be written as

$$\mathbf{U}^{\text{dia}} = -\Phi(\Omega, \mathbf{B}) \nabla \eta_T. \quad (25)$$

The function Φ depends on the rotation and the magnetic field. It is always positive and it does not vanish for vanishing

Fig. 8. Evolution of $\langle B_y \rangle$ for the models M04 (top) and M01 (bottom). Time instances are $t = 0, 4.0, 8.0, 16.0, 27.3$ measured relative to t_e . The dashed lines represent the diffusion solution at t_e^* .

Fig. 9. Horizontally- and time averaged vertical flow for the models M01 (solid line), M02 (dotted line), M03 (dashed line) and M04 (dash-dotted line).

rotation and vanishing magnetic field. It is thus clear that in the overshoot region the transport is always downwards.

There is, however, also a transport along the density gradient, i.e.

$$\mathbf{U}^{\text{dens}} = -\Psi(\Omega, \mathbf{B})\eta_T \nabla \log \rho \quad (26)$$

(Kichatinov & Rüdiger 1992). Also, Ψ depends on the angular velocity of the basic rotation and on the magnetic field but it vanishes for a vanishing magnetic field in general. The effect only results from the interaction of density stratification and magnetic field so that the sign of Ψ is not obvious.

For fast rotation and strong magnetic field both the functions Φ and Ψ approach zero, an effect which is called (rotational or magnetic) quenching. Even in the frame of quasilinear theories only approximations are known for Φ and Ψ . For

Table 2. Transport velocities of mean magnetic field. All quantities are given in units of 10^{-3} .

model	Ta	β	$\gamma_{z,s}$	$\overline{\langle u_z \rangle}_s$	$\gamma_{z,c}$	$\overline{\langle u_z \rangle}_c$
M01	0	5000	-7.1	-0.9	-1.5	6.5
M02	0	500	-6.8	-0.8	-3.9	6.2
M03	0	50	-2.3	-0.5	5.5	3.9
M04	0	5	-1.0	-0.2	8.0	3.4

a highly simplified turbulence model (“mixing-length approximation”, Kichatinov 1991) both the quenching functions are known for rotation *or* for magnetism, i.e. $\Phi(0, B)$ and $\Phi(\Omega, 0)$. The quenching of the diamagnetic effect Φ is given in Fig. 10 and the functions $\Psi(0, B)$ and $\Psi(\Omega, 0)$ are given in Fig. 11. The latter functions both vanish for vanishing B and/or Ω which means that the density stratification only produces a turbulent advection in rotating or in magnetized turbulences. Note that the function $\Psi(0, \hat{B})$ has no definite sign: it is only positive for very strong magnetic fields¹ while for small and high fields the function is negative. This means that for our magnetic fields which are in equipartition with the turbulent energy the transport is always upwards (“turbulence buoyancy”). We have to ask whether situations exist where the upwards-directed turbulence buoyancy dominates the downwards-directed turbulence diamagnetism.

From Fig. 10 it is obvious that at the bottom of a convection zone, where $d\eta_T/dz > 0$, the diamagnetic advection is always downwards. This, of course, is also true for the overshoot region below the convection zone. The magnetic quenching of this effect, however, is very strong.

In the bulk of the convection zone the turbulence intensity is nearly uniform so that the downward advection should be smaller than in the overshoot regime. On the other hand, if the magnetic field is strong, the downward advection in the convection zone should be changed to an magnetic-dominated upward advection. The reason is that the density stratification $d\log \rho/dz$ is much stronger than in the overshoot region so that the turbulent buoyancy can dominate the turbulent diamagnetism if the magnetic field is strong enough. One finds this result in the behavior of the solid line $\Psi(0, \hat{B})$ in Fig. 11. For magnetic fields smaller than $\hat{B} \simeq 2$ the function $\Psi(0, \hat{B})$ is negative representing an upwards directed (buoyancy) effect. It does not appear in the overshoot region as the quantity $|\rho^{-1} d\log \rho/dz|$ is much smaller there than in the convection zone.

In summary, mean field transport in the stable layer should be downwards and dominated by turbulent diamagnetism ($\mathbf{U}^{\text{dia}}$) whereas in the convection zone for (not too) strong magnetic fields the γ -transport is upwards, dominated by the turbulent buoyancy ($\mathbf{U}^{\text{dens}}$).

A full analysis of Eq. (23) is a rather difficult task involving a consistent calculation of all coefficients. Here, a much simpler approach is used to get at least an order-of-magnitude estimate for γ_z . It is restricted to the non-rotating

¹ Here and in the following discussion the magnetic field strength is measured in units of the equipartition field of the turbulence, i.e. $\hat{B} = B/B_{\text{eq}}$.

Fig. 10. Magnetic (solid) and rotational (dashed) quenching of the diamagnetic effect given in Eq. (25) after Kichatinov (1991) and Kichatinov & Rüdiger (1992). Note that the realizations $\Phi(\Omega^*, 0)$ and $\Phi(0, \hat{B})$ are always positive.

Fig. 11. Magnetic (solid) and rotational (dashed) quenching of the turbulent buoyancy effect (Ψ) after Kichatinov (1991) and Kichatinov & Rüdiger (1992). Note that $\Psi(0, \hat{B})$ changes its sign for not too strong magnetic fields.

models M01, M02, M03, M04 assuming that $\alpha^{\text{symm}} \rightarrow 0$ as $Ta \rightarrow 0$. We will further simplify the procedure by taking averages over time and volume. Volume-averaging implies that the diffusion term in (23) vanishes. We distinguish between the stable layer and unstable layer by defining $\langle \cdot \rangle_s$ respective $\langle \cdot \rangle_c$ to be the corresponding volume average over that part of computational domain. With these presumptions and the fact that $\langle B_z \rangle_{s,c} = 0$ (which follows from the boundary conditions) and $\langle B_x \rangle_{s,c} \ll \langle B_y \rangle_{s,c}$ one obtains

$$\gamma_{z,s} \approx -\frac{\langle E_x \rangle_s}{\langle B_y \rangle_s}, \quad (27)$$

$$\gamma_{z,c} \approx -\frac{\langle E_x \rangle_c}{\langle B_y \rangle_c}. \quad (28)$$

The results are summarized in Table 2. For comparison Table 2 also lists the corresponding values of averaged vertical flows, $\langle u_z \rangle_s$ and $\langle u_z \rangle_c$. The z -dependent quantity $\langle u_z \rangle(z)$ for the considered subset of models is displayed in Fig. 9. In the stable layer turbulent pumping is directed downwards (negative $\gamma_{z,s}$) and clearly dominates mean flow transport i.e. $|\gamma_{z,s}| > |\langle u_z \rangle_s|$. This relation holds independent of β . There is, however, evidence for magnetic quenching reducing the magnitude of $\gamma_{z,s}$ from $|\gamma_{z,s}| \approx 7 \times 10^{-3}$ for $\beta = 5000$ to $|\gamma_{z,s}| \approx 10^{-3}$ for $\beta = 5$.

In the unstable layer the behavior is much more complex. For $\beta = 5000$, 500 turbulent pumping turns out to be negative so that the transport of the mean field is downwards. For $\beta = 50, 5$, on the other hand, $\gamma_{z,c} > 0$ so that for stronger magnetic fields the transport is exclusively upwards². As the diamagnetic effect does not change its sign for increasing magnetic field, the behavior of γ in the unstable zone cannot be described by the diamagnetic effect alone. As described above there must be another pumping effect, the equation $\gamma = U^{\text{dia}}$ is not sufficient but must be replaced by $\gamma = U^{\text{dia}} + U^{\text{dens}}$.

The method we applied to estimate the rate of turbulent pumping is rather robust but might be too simple for this problem. In general, one could expect that quantities depend on depth and are not (piecewise) constant. Therefore, we suggest accepting the presented results as a first orientation until a more sophisticated analysis of the data is performed.

4. Conclusion

Magnetoconvection has been simulated for a convectively unstable domain rotating with various rotation rates and threaded by a given large-scale toroidal magnetic field of variable amplitude. The computational domain also contains a convectively stable layer below the convection zone in which the penetration of the convection flow is measured. We are in particular interested in knowing the depth of the penetration zone, which after the results of helioseismology only covers 10% of the pressure scale height at the bottom of the convection zone (Christensen-Dalsgaard et al. 1995). Our main result is that the depth of the overshoot region *strongly* depends on the rotation and the magnetic field. We find a very strong reduction of the penetration depth by both rotation and magnetism. In our model the (dimensionless) pressure scale height at the stable/unstable interface is $H_p = p/|dp/dz| = 0.55$. From the simulations we find the extreme values $\Delta = 0.8H_p$ for $(Ta, \beta) = (0, 0)$ and $\Delta = 0.09H_p$ for $(Ta, \beta) = (6 \times 10^5, 5)$. The latter is in agreement with observations suggesting a magnetic field of strength $\beta \approx 5$ to be present in the tachocline.

The next surprise concerns the turbulent advection of the magnetic field. Here the turbulent electromotive force $\langle \mathbf{u}' \times \mathbf{B}' \rangle$ has been computed but only for $\Omega = 0$ in order to suppress the α -effect. As expected, for weak magnetic fields the advection flow γ is negative, i.e. the advection is downwards, which can be explained by the positive gradient of the turbulent intensity (“turbulent diamagnetism”). The latter effect is, of course, smaller in the unstable layer than in the overshoot

² Note that $\langle u_z \rangle_c$ is comparable to or even exceeds $|\gamma_{z,c}|$ in contrast to the situation in the stable layer.

layer where $\langle u'^2 \rangle \rightarrow 0$ by definition. In the convectively unstable layer, therefore, for a sufficiently strong magnetic field the “turbulent buoyancy” due to the nonlinear interaction of magnetic field and density stratification may dominate, producing upwards advection. This is a striking phenomenon, which did not appear in previous numerical simulations but may be considered as confirming a SOCA-result by Kichatinov & Rüdiger (1992) (where, however, for very strong magnetic fields both a magnetic quenching and a change of the sign have been predicted).

We also attacked the diffusion problem for passive tracers like lithium. There is no doubt that after some (eddy-diffusion) time any passive tracer will reach the bottom of the overshoot region. The question is whether it can be transported deeper than the bottom of the overshoot region as suggested by Vincent et al. (1996).

In order to explain the lithium decay observed at the surface of solar-type stars along their evolution one needs an effective diffusivity exceeding the molecular one by two orders of magnitude in a stable layer beneath the overshoot region. We have shown that due to the influence of rotation on an originally horizontal turbulence field even a rather weak turbulence can explain the observations (Rüdiger & Pipin 2001). The simulations in the present paper indeed demonstrate that any chemical concentrated in the convection zone is transported deeply through the bottom of the overshoot region, i.e. $\Delta_m > \Delta$. Whether this highly interesting effect is based on the action of sporadic fast-moving plumes or by another sort of instability excited by overshooting eddies remains an open question in the present paper.

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