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Vacuum nonlinear electrodynamics curvature of photon trajectories in pulsars and magnetars

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Abstract. Near a magnetic neutron star electromagnetic emission should undergo nonlinear electrodynamic effects in strong magnetic fields. Manifestations of this effect in detected hard emission from magnetic neutron stars are discussed on the base of nonlinear generalizations of the Maxwell equation in vacuum. The dispersion equations for electromagnetic waves propagating in the magnetic dipole field were obtained in the framework of these theories.

Key words. pulsars: general – scattering – stars: neutron – waves

1. Introduction

Studies of phenomena in the vicinity of a neutron star make it possible to obtain information on the properties of matter in states, which are unattainable in ground laboratories. The last experiments on light-to-light scattering made at Stanford (Burke et al. 1997) show that electrodynamics in vacuum is really a nonlinear theory. However, magnetic fields ($B \sim 10^6$ G) available in ground laboratories give no opportunity to test the different models of nonlinear electrodynamics of vacuum and their predictions because the typical value of magnetic field induction necessary for essential manifestation of electrodynamics nonlinearity in vacuum is about $B_q = m^2c^3/e\hbar = 4.41 \times 10^{13}$ G. Since magnetic fields of some pulsars can be characterized by such magnitudes, and for magnetars can reach much greater values ($B \sim 10^{15}$ G) (Thomson & Duncan 1995, 1996), the nonlinear effects of electrodynamics in vacuum should be most pronounced in the vicinity of such astrophysical objects. Different nonlinear electrodynamic effects in the vicinity of a strongly magnetized neutron star were previously studied in the context of quantum electrodynamics (Meszaros 1992; Gal'tsov & Nikitina 1983; Bussard et al. 1986; De Lorenci et al. 2001). However, the analysis presented above concentrated on the validity of quantum electrodynamics without comparison with possible alternative theories. Thus, we try to obtain some specific predictions by using post-Maxwellian items of different nonlinear generalizations of electrodynamics.

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2. Nonlinear models of vacuum electrodynamics

Several nonlinear generalizations of the Maxwell equation in vacuum are considered in the framework of the field theory. The most well known among them are the Born-Infeld (BI) (Born & Infeld 1934) and the Heisenberg-Euler (HE) electrodynamics (Heisenberg & Euler 1936).

Born and Infeld proceeded from the idea of a limited value of the electromagnetic field energy of a point particle. This and some other reasons led them to the following Lagrangian of the nonlinear electrodynamics in vacuum:

$$L = -\frac{1}{4\pi a^2}\Big[\sqrt{1 + a^2\left(\boldsymbol{B}^2 - \boldsymbol{E}^2\right) - a^4\left(\boldsymbol{B}\times\boldsymbol{E}\right)^2} - 1\Big], \qquad (1)$$

where $1/a = 9.18 \times 10^{15}$ G was obtained (Born & Infeld 1934) from the atomic physics constraints. In the case of weak fields Lagrangian of the BI nonlinear electrodynamics can be expanded into the small parameters $a^2\boldsymbol{E}^2 \ll 1$ and $a^2\boldsymbol{B}^2 \ll 1$:

$$L = -\frac{1}{8\pi}\left(\boldsymbol{B}^2 - \boldsymbol{E}^2\right) + \frac{a^2}{32\pi}\Big[\left(\boldsymbol{B}^2 - \boldsymbol{E}^2\right)^2 + 4(\boldsymbol{B}\times\boldsymbol{E})^2\Big].$$

The HE nonlinear electrodynamics is based on the quantum electrodynamic (QED) effect of electron-positron vacuum polarization by electromagnetic fields. If electromagnetic fields are not strong ($B \ll B_q$, $E \ll B_q$) the first two terms in the vacuum electromagnetic field nonlinear Lagrangian expansion in the small parameters $(\boldsymbol{B}^2 - \boldsymbol{E}^2)/B_q^2$ and $(\boldsymbol{B}\times\boldsymbol{E})/B_q^2$, should have the following form (Heisenberg & Euler 1936):

$$L = -\frac{1}{8\pi}\left[\boldsymbol{B}^2 - \boldsymbol{E}^2\right] + \frac{\alpha}{360\pi^2 B_q^2}\Big\{\left(\boldsymbol{B}^2 - \boldsymbol{E}^2\right)^2 + 7(\boldsymbol{B}\times\boldsymbol{E})^2\Big\},$$

where $\alpha = e^2/\hbar c \approx 1/137$ is the fine structure constant.

In other theoretical models of nonlinear electrodynamics the coefficients at the terms $(\boldsymbol{B}^2 - \boldsymbol{E}^2)^2$ and $(\boldsymbol{B}\times\boldsymbol{E})^2$ in the

Lagrangian expansion can be absolutely arbitrary. We will use a parameterized post-Maxwell formalism, which was elaborated by Denisov & Denisova (2001a,b). We will assume, that the main prerequisite for this formalism is that the Lagrangian of nonlinear electrodynamics in vacuum is an analytical function of invariants $J_1 = (\boldsymbol{E}^2 - \boldsymbol{B}^2)/B_q^2$ and $J_2 = (\boldsymbol{E}\times\boldsymbol{B})^2/B_q^4$, at least, near their zero values. Thus, in the case of a weak electromagnetic field ($J_1 \ll 1,\ J_2 \ll 1$) this Lagrangian can be expanded into a converging set in integer powers of these invariants:

$$L = \frac{B_q^2}{8\pi}\sum_{n=0}^{\infty}\sum_{m=0}^{\infty} L_{nm} J_1^n J_2^m. \tag{2}$$

Since, at $J_1 \to 0, J_2 \to 0$ the theory with Lagrangian (6) should be reduced to Maxwell electrodynamics, then $L_{00} = 0, L_{10} = 1$.

For such an approach, a quite definite number of post-Maxwell parameters L_{nm} will correspond to each nonlinear electrodynamics. If we limit oneselves only to a few first terms in expansion (2), then according to the parameterized post-Maxwell formalism the generalized Lagrangian of the nonlinear vacuum electrodynamics in the case of weak fields can be represented as (Denisov & Denisova 2001a,b):

$$L = \frac{1}{8\pi}\left\{\left[\boldsymbol{E}^2 - \boldsymbol{B}^2\right] + \xi\left[\eta_1\left(\boldsymbol{E}^2 - \boldsymbol{B}^2\right)^2 + 4\eta_2(\boldsymbol{B}\times\boldsymbol{E})^2\right]\right\}, \tag{3}$$

where $\xi = 1/B_q^2$, and the value of the dimension-less post-Maxwell parameters η_1 and η_2 depends on the choice of the model of nonlinear vacuum electrodynamics.

In particular, in the nonlinear HE electrodynamics parameters η_1 and η_2 have quite definite values $\eta_1 = \alpha/(45\pi) = 5.1\times10^{-5}$, $\eta_2 = 7\alpha/(180\pi) = 9.0\times10^{-5}$, while in the BI theory they can be expressed through the same unknown constant a^2: $\eta_1 = \eta_2 = a^2B_q^2/4$.

3. The effect of nonlinear-electrodynamic bending of a ray

The electromagnetic ray is exactly the agent, which passing through the neutron star magnetic field undergoes nonlinear electromagnetic influence from this field independently of the spectral range. Studying the main parameters of incoming electromagnetic emission, such as dependence of ray bending angle on impact distance, the law of emission intensity decreasing in the course of time, etc., it is possible (Denisov et al. 2001) to reveal the main dependencies of nonlinear electrodynamic interactions of electromagnetic fields. We will assume that a "weak" plane electromagnetic wave propagates through the permanent magnetic field $\boldsymbol{B}_0$ of a neutron star. Then in the geometric optics approach we can write the following relations:

$$\boldsymbol{E} = \boldsymbol{e}\ \exp[-\mathrm{i}(\omega t - \boldsymbol{k}\times\boldsymbol{r})],$$
$$\boldsymbol{B} = \boldsymbol{B}_0 + \boldsymbol{b}\ \exp[-\mathrm{i}(\omega t - \boldsymbol{k}\times\boldsymbol{r})],$$

where ω is the frequency, $\boldsymbol{k}$ is the wave vector and vectors $\boldsymbol{b}$ and $\boldsymbol{e}$ are slowly changing functions of t and $\boldsymbol{r}$, in comparison with the $\exp[-\mathrm{i}(\omega t - \boldsymbol{k}\times\boldsymbol{r})]$ function.

Under this approach the dispersion equation can be obtained from Lagrangian (3) directly. To ensure the necessary accuracy of calculations, we will add to Lagrangian (3) the terms of higher approximations and, hence, write it with surplus accuracy:

$$L = \frac{1}{8\pi}\Big\{\left[\boldsymbol{E}^2 - \boldsymbol{B}^2\right] + \xi\left[\eta_1\left(\boldsymbol{E}^2 - \boldsymbol{B}^2\right)^2 + 4\eta_2(\boldsymbol{B}\times\boldsymbol{E})^2\right]$$
$$+\xi^2\left[\eta_3\left(\boldsymbol{E}^2 - \boldsymbol{B}^2\right)^3 + \eta_4\left(\boldsymbol{E}^2 - \boldsymbol{B}^2\right)(\boldsymbol{B}\times\boldsymbol{E})^2\right]\Big\}.$$

This relation is used to obtain the dispersion equations:

$$\omega_{1,2}(\boldsymbol{k}) = ck\left\{1 - \frac{2\eta_{1,2}\xi}{k^2}[\boldsymbol{k}\times\boldsymbol{B}_0]^2 + O\left(\xi^2\boldsymbol{B}_0^4\right)\right\}, \tag{4}$$

where ω_1 and ω_2 are the frequences of two main polarization modes of electromagnetic wave. It is necessary to note, that the same dispersion equations can also be obtained from a simpler Lagrangian (3).

As it was shown previously (Denisov 2000), the exact dispersion equation for electromagnetic wave propagating in the magnetic field $\boldsymbol{B}_0$ in the BI theory (1) has the form

$$\left(1 + a^2\boldsymbol{B}_0^2\right)\frac{\omega^2}{c^2} - \boldsymbol{k}^2 - a^2(\boldsymbol{k}\ \boldsymbol{B}_0)^2 = 0, \tag{5}$$

independently of its polarization and for any $a^2\boldsymbol{B}_0^2$ value.

The solution of the nonlinear electrodynamics equations for electromagnetic waves propagating in the magnetic field shows that at $\eta_1 \neq \eta_2$ the waves of both types with the dispersion Eqs. (4) are polarized linearly in mutually normal planes and propagate with different group velocities. This property of electromagnetic waves is well known as birefringence.

At $\eta_1 = \eta_2$ both types of electromagnetic waves will coincide to the accuracy of terms proportional to ξ^2. As a result, electromagnetic waves of the same type with arbitrary polarization will propagate in each direction.

Let us find now the eikonal equation for an electromagnetic wave propagating in the dipole magnetic field of a neutron star under the laws of nonlinear vacuum electrodynamics. For this purpose we will raise relations (4) to the second power. Retaining terms linear in ξ and taking into account that $\omega = \partial S/\partial t,\ \boldsymbol{k} = \nabla S$, we obtain:

$$\frac{1}{c^2}\left(\frac{\partial S}{\partial t}\right)^2 - \left[1 - 4\eta_1\xi\boldsymbol{B}_0^2\right]\left(\nabla S\right)^2 - 4\eta_1\xi\left(\boldsymbol{B}_0\times\nabla S\right)^2 = 0, \tag{6}$$
$$\frac{1}{c^2}\left(\frac{\partial S}{\partial t}\right)^2 - \left[1 - 4\eta_2\xi\boldsymbol{B}_0^2\right]\left(\nabla S\right)^2 - 4\eta_2\xi\left(\boldsymbol{B}_0\times\nabla S\right)^2 = 0.$$

In the BI theory, as it follows from relation (5), the eikonal equation valid for any $a^2\boldsymbol{B}_0^2$, values has the form

$$\frac{1}{c^2}\left(\frac{\partial S}{\partial t}\right)^2\left[1 + a^2\boldsymbol{B}_0^2\right] - (\nabla S)^2 - a^2(\nabla S\,\boldsymbol{B}_0)^2 = 0. \tag{7}$$

The solution of these equations in the general case is not known. Thus, further on we will consider solution of the Eqs. (6) only for rays lying in the dipole magnetic field equator plane.

Let us denote the plane normal to the magnetic dipole momentum vector $\boldsymbol{m}$, as plane XOY. In this case only one component of the vector $\boldsymbol{m} = |\boldsymbol{m}|\boldsymbol{e}_z$ will be nonzero and vector $\boldsymbol{B}_0$ in this plane can be represented as: $\boldsymbol{B}_0 = -|\boldsymbol{m}|\boldsymbol{e}_z/r^3$.

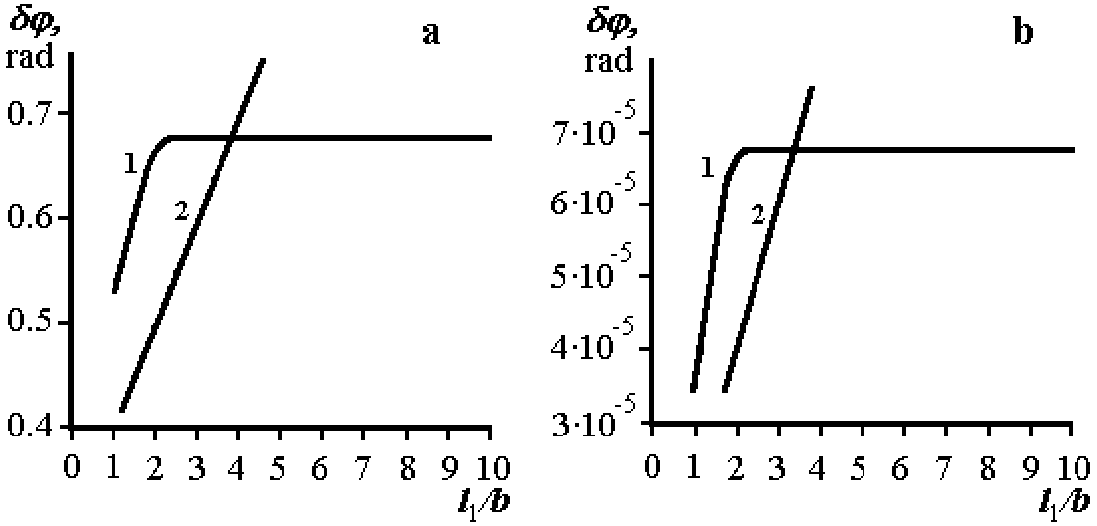

Fig. 1. The BI dependence $\delta\varphi$ versus l_1/b for $\boldsymbol{B}^2(b)/B_q^2 = 10^2$ (curve 1, panel **a**)), $\boldsymbol{B}^2(b)/B_q^2 = 1$ (curve 1, panel **b**)). The $\delta\varphi$ versus l_1/b from gravitational bending are plotted for $r_g/l_1 = 0.1$ (dependence 2 on the panel **a**)), $r_g/l_1 = 10^{-5}$ (dependence 2 on the panel **b**)).

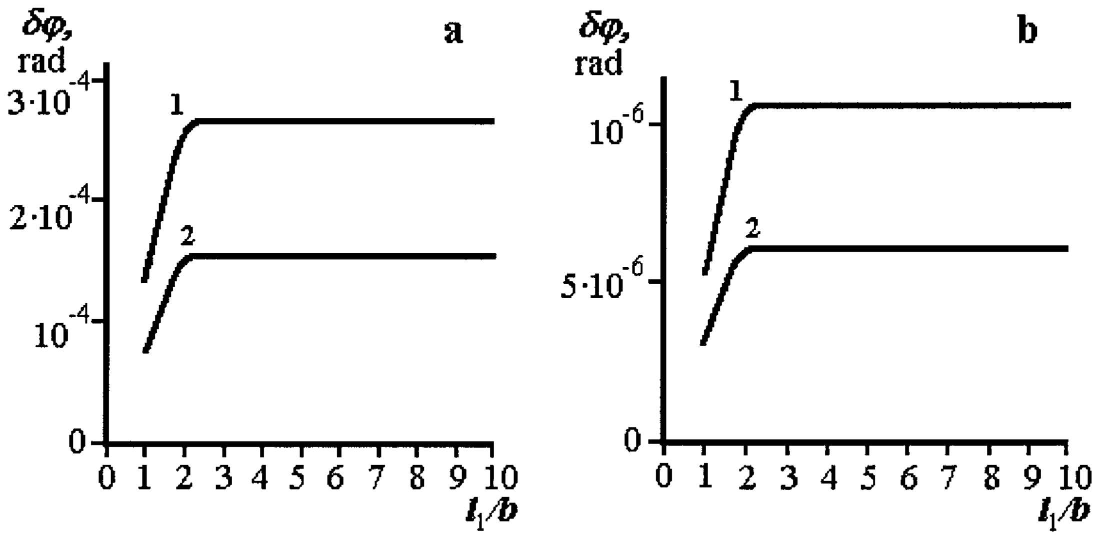

Fig. 2. The HE $\delta\varphi_1^*$ (curves 1), $\delta\varphi_2^*$ (curves 2) versus l_1/b for $\boldsymbol{B}^2(b)/B_q^2 = 0.25$ (panel **a**)), $\boldsymbol{B}^2(b)/B_q^2 = 10^{-2}$ (panel **b**)).

Hence the first of the eikonal Eq. (6) for electromagnetic wave polarized in the XOY plane, which lie in the same plane, will be:

$$\frac{1}{c^2}\left(\frac{\partial S_1}{\partial t}\right)^2 - \left[1 - \frac{4\eta_1\xi\boldsymbol{m}^2}{r^6}\right]\left[\left(\frac{\partial S_1}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial S_1}{\partial \varphi}\right)^2\right] = 0. \qquad (8)$$

A similar equation with the replacement of the η_1 parameter by the η_2 parameter and S_1 by S_2, can be written for a ray of electromagnetic wave polarized along the z axis.

As it is accepted in theoretical mechanics (Landau & Lifshitz 1984), we will find the partial solution of Eq. (8) using the variables separation method. As a result, we obtain:

$$S_1 = -\mathcal{E}_0 t + \alpha\varphi + \int^r \mathrm{d}r \sqrt{\frac{\mathcal{E}_0^2}{c^2}\left[1 + \frac{4\eta_1\xi\boldsymbol{m}^2}{r^6}\right] - \frac{\alpha^2}{r^2}}, \qquad (9)$$

where $\mathcal{E}_0$, α are the constants of integration and all calculations were made with accuracy, linear in the small value $\eta_1\xi\boldsymbol{m}^2/r^6$.

It should be noted, that in the magnetic equator plane the expression (9) is also the solution of BI exact eikonal Eq. (7), if we take into account, that in this theory $\eta_1 = \eta_2 = a^2 B_q^2/4$.

Using relation (9) we can determine the kinematic and dynamic parameters describing photon propagation in the dipole magnetic field.

Let us consider the case, when the gamma ray source is located at a limited distance l_1 from a neutron star or even in its nearest vicinity. Let us denote the distance from neutron star to the detector as l_2. Then the distance l_1 is much smaller than l_2 and comparable with the neutron star radius R. Hence, the dependence of impact distance on time for a circular orbit in the first approximation can be represented as: $b(t) = b_0 + R_1 \cos\Omega t$, where R_1 is the orbit radius, Ω is the orbital frequency.

If we are considering the propagation of a X-ray or gamma ray photon from a source located near a galactic neutron star, it is convenient to direct the X and Y axes in such a way, that a ray from the source travels along the X axis with the impact distance b, the center of the dipole magnetic field is placed in the center of the coordinate system and the spacecraft with the detectors is located at the distance l_2 from the center of the coordinate system near the point $x \approx l_2$.

Since the value $\xi \boldsymbol{m}^2/r^6$ is small, in order to find the bending angles $\delta\varphi^*_{1,2}$ we can use the algorithm (Darwin 1961), well established for calculations of angles of light gravitational bending. For the case, when $b < l_1 \ll l_2$, and $l_2 \gg b$ we obtained the following relations for the bending angles:

$$\begin{aligned}
\delta\varphi_1^* = {} & \arctan\left[\frac{bQ_1}{\sqrt{Q_1^2 l_1^2 - 1}}\right] + \frac{15\pi\eta_1\xi\boldsymbol{m}^2}{4b^6} \qquad (10)\\
& - \left[1 + \frac{15\eta_1\xi\boldsymbol{m}^2}{4b^6}\right]\arcsin\left(\frac{1}{Q_1 l_1}\right)\\
& - \frac{\eta_1\xi\boldsymbol{m}^2}{16b^6}\left\{\sin\left[4\arcsin\left(\frac{1}{Q_1 l_1}\right)\right]\right.\\
& \left. - 16\sin\left[2\arcsin\left(\frac{1}{Q_1 l_1}\right)\right]\right\},\\
\delta\varphi_2^* = {} & \arctan\left[\frac{bQ_2}{\sqrt{Q_2^2 l_1^2 - 1}}\right] + \frac{15\pi\eta_2\xi\boldsymbol{m}^2}{4b^6}\\
& - \left[1 + \frac{15\eta_2\xi\boldsymbol{m}^2}{4b^6}\right]\arcsin\left(\frac{1}{Q_2 l_1}\right)\\
& - \frac{\eta_2\xi\boldsymbol{m}^2}{16b^6}\left\{\sin\left[4\arcsin\left(\frac{1}{Q_2 l_1}\right)\right]\right.\\
& \left. - 16\sin\left[2\arcsin\left(\frac{1}{Q_2 l_1}\right)\right]\right\}, \qquad (11)
\end{aligned}$$

where

$$Q_1 = \frac{1}{b}\left[1 + \frac{2\eta_1\xi\boldsymbol{m}^2}{b^6}\right], \quad Q_2 = \frac{1}{b}\left[1 + \frac{2\eta_2\xi\boldsymbol{m}^2}{b^6}\right].$$

The plus sign in this relation shows, that the magnetic dipole field in the magnetic equator plane effects the electromagnetic waves as a convex lens. To illustrate dependeces (10) in BI and HE theories we plot $\delta\varphi_1^*$ and $\delta\varphi_2^*$ versus b/l_1 for different values $\boldsymbol{B}^2(b)/B_q{}^2$ (Figs. 1, 2). Here $\boldsymbol{B}^2(b)$ is the square of magnetic field at the distance b from a neutron star.

Thus, nonlinear models of vacuum electrodynamics with $\eta_1 \neq \eta_2$, predict different angles of ray bending for electromagnetic waves with different polarization.

It should be noted, that besides the nonlinear electrodynamic bending (10) of electromagnetic rays will also undergo the well known gravitational bending. However, because of the different bending angle dependence on the impact distance b ($1/b$ and $1/b^2$ in the case of gravitational bending (Epstein & Shapiro 1980; Meszaros & Riffert 1988; Riffert & Meszaros 1988) and $1/b^6$ in the case of nonlinear electrodynamic bending) mathematical processing allows to resolve each of these parts from the observational data if the time dependence of impact distance b is harmonical $b(t) = b_0 + R_1 \cos \Omega t$. For comparison, we present in Fig. 1 $\delta\varphi$ versus l_1/b from gravitational bending ($\delta\varphi = 2r_g/b = (2r_g/l_1)(l_1/b)$ for different values of the attitude r_g/l_1.

4. Conclusion

Although the observation of the manifestations of the nonlinear electrodynamics effects in astrophysical objects requires special conditions, in principle, they can be observed. The main astrophysical objects, where the nonlinear electrodynamic effects can be revealed more clearly, are certain kinds of gamma-ray pulsars and magnetars. These effects can be manifested as some peculiarities in the form of their hard emission pulsation.

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