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Chaotic orbits in a galaxy model with a massive nucleus

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Abstract. The transition from regular to chaotic motion is studied in an axially symmetric galaxy model with a disk-halo and a spherical nucleus. This model has the characteristic that the mass of the nucleus increases exponentially, because mass is transported from the disk to the nucleus while the total mass of the galaxy remains constant. Stars with values of angular momentum L_z less or equal to a critical value L_{zc}, moving near the galactic plane, are scattered to the halo when approaching the nucleus. The corresponding orbits are chaotic. A linear relationship is found to exist between the critical angular momentum and the final mass of the nucleus M_{nf}. Our results suggest that the stars in the central regions of disk galaxies with massive nuclei must be in chaotic orbits. Comparison with previous work is also made.

Key words. galaxies: kinematics and dynamics

1. Introduction

During the last three decades a large number of interesting papers have been published on the regular and chaotic behavior of orbits in galactic potentials (see e.g. Saito & Ichimura 1979; Carlberg & Innanen 1987; Caranicolas 1990a; Caranicolas & Innanen 1991; Elipe et al. 1995; Karanis & Caranicolas 2001). Usually two types of models were used by the investigators. The first type of model describes global motion in galaxies. An example is the axially symmetric mass model used by Caranicolas (1997). The second type of model describes local galactic motion (i.e. near an equilibrium point) and it is made up of perturbed harmonic oscillators (see Innanen 1985; Caranicolas & Karanis 1999, and references therein).

Numerical, analytical, or semi-analytical methods were used during these years. Many were based on the classical Poincare surface of section technique. Recent papers use new methods, based on the spectra of orbits (Karanis & Caranicolas 2002) or modern analytical calculations (Elipe & Deprit 1999; Elipe 2000).

In the present article we study the transition from regular to chaotic motion in an axially symmetric galaxy described by the disk-halo potential

$$\Phi(r, z) = -\frac{M_1}{R}, \tag{1}$$

with

$$R^2 = \left[\alpha + \sum_{i=1}^{3} \beta_i \sqrt{z^2 + h_i^2}\right]^2 + b^2 + r^2, \tag{2}$$

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where r, z are the usual cylindrical coordinates. Here M_1 is the mass, α is the scale length of the disk, h corresponds to the disk scale height while b is the core radius scale-length of the halo component. β_1, β_2, β_3 represent the fractional portions of old disk, dark matter and young disk, respectively (see Carlsberg & Innanen 1987). To this potential we add a spherically symmetric nucleus

$$\phi(r, z) = -\frac{M_2}{[r^2 + z^2 + c^2]^{1/2}}, \tag{3}$$

where M_2 is the mass and c is the scale length of the nucleus.

Caranicolas & Innanen (1991) found that low angular momentum stars, moving near the galactic plane, when passing near the nucleus, are scattered to higher scale heights displaying chaotic motion. Furthermore, they confirmed both analytically and numerically that a linear relationship exists between the critical angular momentum L_{zc} (that is the the maximum angular momentum L_z, for which stars scattered to the halo display chaotic motion, for a given value of the mass of the nucleus M_n) and M_n.

In the present work we shall study the transition from regular to chaotic motion in the above described galaxy model, when a mass transport is taking place. We assume that mass is transported from the disk to the nucleus in such a way that we have an exponential increase in the mass of the nucleus while an exponential decrease occurs in the mass of the disk. Thus we write

$$M_1 = M_{di} - m(1 - \exp kt), \tag{4}$$

$$M_2 = M_{ni} + m(1 - \exp kt), \tag{5}$$

where M_{di}, M_{ni} are the initial values of the mass of the disk and nucleus, m is the portion of the disk mass that is transferred, t is the time while $k > 0$ is a parameter. In particular we shall try to find the relation – if any – between the critical value of the angular momentun and the final mass of the nucleus M_{nf}. As final mass we consider the limit of the current mass M_2 as $t \to \infty$. The numerical experiments for the search of this relation are presented in Sect. 2. In the same section we explain the general characteristics of regular and chaotic orbits. Furthermore, we study the dependence of the Lyapunov Characteristic Number (LCN) (see for details Lichtenberg & Lieberman 1983) on the value of the parameter k. Section 3 contains a discussion and the conclusions of this work.

2. Regular and chaotic motion

In this paper we use a system of galactic units where the unit of length is 1 kpc, the unit of time is 0.977748×10^8 yr and the unit of mass is 2.325×10^7 $M_\odot$. The velocity unit is 10 km s^{-1} while G is equal to unity. In the above units we use the following values of the parameters: $\alpha = 3$ kpc, $b = 8$ kpc, $M_{di} = 12\,000$, $(\beta_1, \beta_2, \beta_3) = (0.4, 0.5, 0.1)$, and $(h_1, h_2, h_3) = (0.325, 0.090, 0.125)$ kpc. We take $M_{ni} \leq 100$ while m, c and k are treated as parameters.

The Hamiltonian describing the motion in the meridian $r-z$ plane of the galaxy is

$$H = \frac{1}{2}\left(p_r^2 + p_z^2\right) + \Phi_{\rm eff}(r, z), \tag{6}$$

where p_r, p_z are the momenta, per unit mass, conjugate to r and z, respectively, while

$$\Phi_{\rm eff} = \frac{L_z^2}{2r^2} + \Phi_{\rm tot}(r, z), \tag{7}$$

is the effective potential, with

$$\Phi_{\rm tot}(r, z) = \Phi(r, z) + \phi(r, z). \tag{8}$$

Our study is based on the numerical integration of the equations of motion, which are

$$\ddot{r} = -\frac{\partial \Phi_{\rm eff}}{\partial r}, \; \ddot{z} = -\frac{\partial \Phi_{\rm eff}}{\partial z}, \tag{9}$$

where the dot indicates derivative with respect to the time.

Orbits were started at $r = 8.5$, $z = 0$ with radial and vertical velocities smaller than 30 km s^{-1}. Because it is not possible to use the Poincare surface of section in the corresponding time dependent Hamiltonian we decided, in order to distinguish between regular and chaotic motion, to calculate the maximal Lyapunov Characteristic Number (LCN).

Our numerical calculations suggest that a linear relationship exists between L_{zc} and M_{nf}. This can be easily seen looking at the diagram shown in Fig. 1. Each line divides the $[L_{zc} - M_{nf}]$ plane in two parts. Orbits with values of the physical parameters L_{zc} and M_{nf} on the left hand side part of the diagram, including the line, are chaotic while orbits on the right hand side part are regular. Note that, for very small values of the angular momentum, the slope of the line is different. In order to obtain this part of the line we have used small values of M_{ni} and m. In all other cases we used $M_{ni} > 100$. One observes that this diagram is very similar to the diagram given in Fig. 1 in Caranicolas & Innanen (1991) hereafter CI.

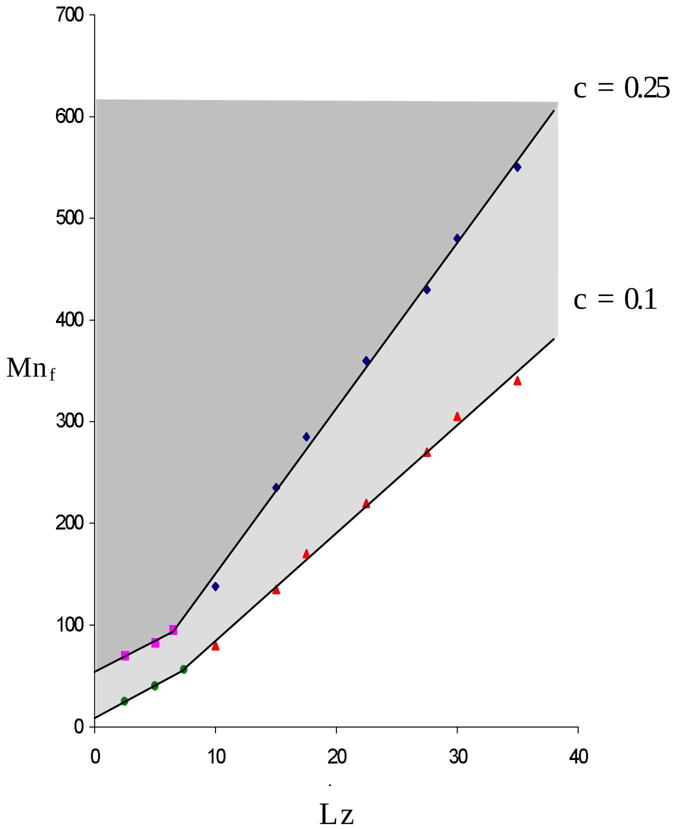

Fig. 1. Relation between L_z and M_{nf}. Orbits in the shaded region are chaotic. Units for L_z are 10 (km s^{-1}) kpc while the unit for the mass is given in text.

Figure 2 shows a chaotic orbit when $L_z = 22.5$, $M_{nf} = 400$, $c = 0.25$, $k = 0.004$.The corresponding LCN is shown in Fig. 3. The orbit was calculated for 1000 time units. Observing the orbit, one can see that it remains for a time period near the galactic plane before it was scattered to higher scale heights. This time period is 600–700 time units. This happens because, for a given value of the angular momentum, there is a critical value of the mass of the nucleus which is needed to scatter the star to the halo. This time period is necessary for the nucleus to gain the mass through the transportation mechanism described by Eqs. (4) and (5).

Useful information can be obtained by observing Fig. 3. We see that at least a few thousand time units are needed in order to obtain an estimation of the LCN. Furthermore, one can say that we are dealing with a case of fast chaos because we have a LCN which is relatively large (see Caranicolas 1990b and references therein).

Figure 4 shows a regular orbit when $L_z = 22.5$, $M_{nf} = 100$, $c = 0.25$, $k = 0.004$. The corresponding LCN is shown in Fig. 5. Note that the LCN goes to zero, which means that the orbit remains always in the galactic plane .

A large number of orbits were computed using different starting distances from the centre of galaxy while keeping the corresponding radial and vertical velocities smaller than 30 km s^{-1}. Orbits starting at the same distance r were considered to belong to the same group. All numerical outcomes suggest that, for each group of orbits, a linear relationship exists

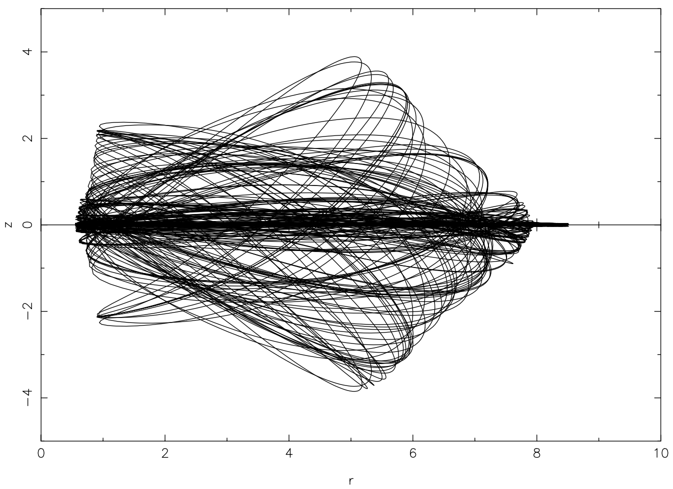

Fig. 2. A chaotic orbit when $L_z = 22.5$, $M_{\rm nf} = 400$, $c = 0.25$, $k = 0.004$.

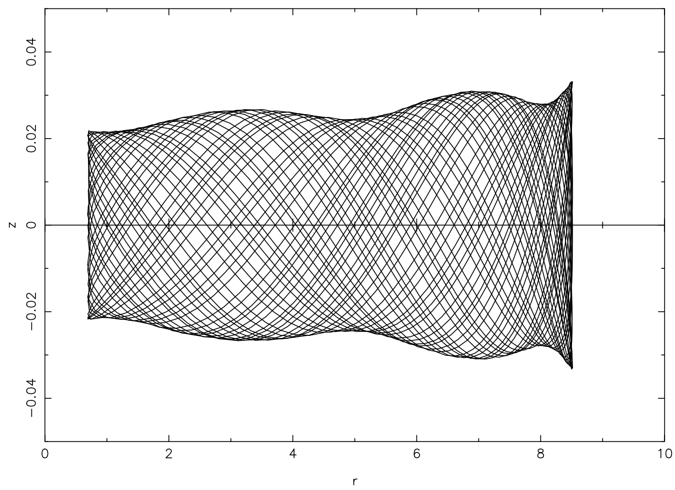

Fig. 4. A regular orbit when $L_z = 22.5$, $M_{\rm nf} = 100$, $c = 0.25$, $k = 0.004$. Note that the orbit stays near the galactic plane.

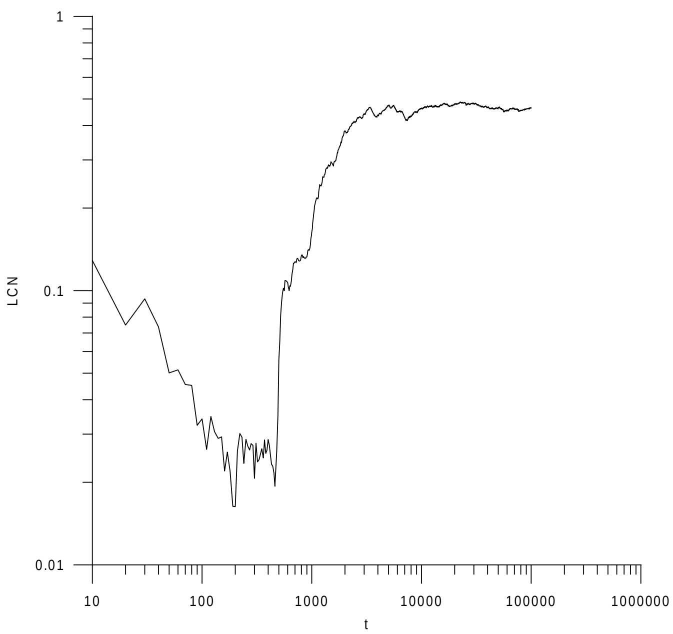

Fig. 3. LCN for the chaotic orbits of Fig. 2.

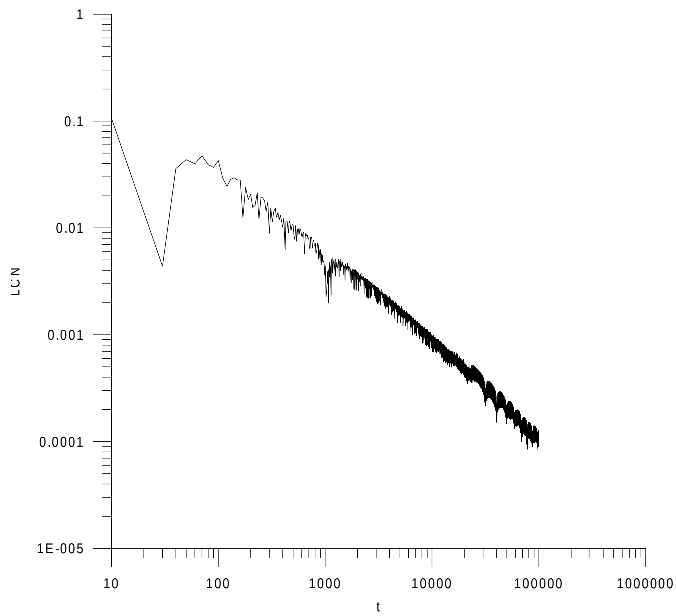

Fig. 5. LCN for the regular orbits of Fig. 4.

between the critical value of the angular momentum and the final mass of nucleus.

Figure 6 shows the behaviour of the LCN of a chaotic orbit for two different values of k. The values of the parameters are $L_z = 22.5$, $M_{\rm ni} = 100$, $m = 300$, $c = 0.25$. The dotted line stands for $k = 0.02$ while the solid line $k = 0.001$. One can see that, for several hundreds of time units, the LCN with the smaller value of k is two orders of magnitude smaller than that belonging to $k = 0.02$. Furthermore, for about two thousand time units the dotted LCN belonging to $k = 0.02$ shows significant changes. The two lines seem to converge after a few hundreds of thousand time units.

3. Discussion

Astronomers know that the infall of matter onto compact objects, such as massive nuclei or black holes, may lead to the emission of electromagnetic radiation and possibly to cosmic ray production. In the case of black holes the accretion process involves speeds that approach the speed of light. In that case a relativistic formulation of hydrodynamics must be used.

In the present paper we have studied the transition from regular to chaotic motion in a galactic potential with an exponential mass transport from the disk to the nuclear region. We have adopted this simple model without going into details for two basic reasons. The first reason is that as it was mentioned above, it would be very difficult to make a model to explain such a mechanism, and the second reason is that we are not interested in the physical structure of the galaxy but only in its dynamical behaviour. It is very well known that such mechanisms are met in the central regions of active galaxies and are responsible for the enormous luminosity of quasars (see Collin & Zahn 1999 and references therein).

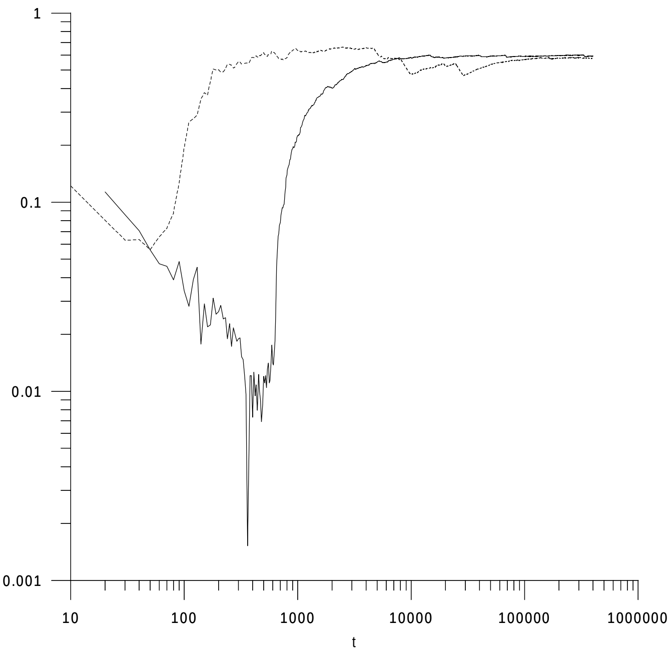

Fig. 6. LCNs for two chaotic orbits when $L_z = 22.5$, $M_{\rm nf} = 400$, $c = 0.25$. Solid line $k = 0.001$, dotted line $k = 0.02$. Details are given in text.

Our numerical calculations show that a linear relationship exists between the critical value of the angular momentum and the final mass of the nucleus. Note that the slope of the line increases as the value of c increases. This fact indicates that a nucleus of the same mass but with greater concentration (smaller value of c) is able to scatter stars with greater values of the angular momentum. Therefore one can say that the present results are similar to the results obtained in CI.

In addition to the observed similarities we have also observed some differences in the behaviour of orbits in this model and the model used by CI. The most important difference was that in CI the star moved towards the halo after several encounters with the nucleus. Here the star can stay for long periods (this depends on the value of the parameter k, that is on how fast mass is transported from the disk onto the nucleus) before it goes to the halo. Therefore, one must be careful because, for small time periods, a chaotic orbit may look regular. It seems that, in this case, one is not able to use methods for fast detection of chaos especially when small values of k are used (see Karanis & Caranicolas 2002 and references therein).

Going a step further into the physical interpretation of the outcomes of this work we can say the following: There is no doubt that a large number of galaxies show significant activity in their nuclear region. We adopt a simple model to express this activity which is described by a mass transport from the disk to the nuclear region. What we did in this note is to investigate how the dynamical properties of the galaxy, that is, the behavior of orbits, have been affected by the mass transfer phenomenon and to establish a relationship between two important physical quantities: the final mass of the nucleus and the star's angular momentum. The physical conclusion is that in active galaxies a number of low angular momentum stars are in chaotic orbits. Furthermore, some orbits that look regular must be chaotic when the transportation of mass is slow.

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