

Radiation pattern of the isolated pulsar PSR B1828-11

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Abstract. Based on the free precession model of the isolated pulsar PSR B1828-11, Link & Epstein (2001) showed that the observed pulse durations require the radio beam to have a non-standard shape: the beam duration is larger for beam sweeps farthest from the dipole axis. In their analysis they assumed that the actual precession period is ≈ 500 d. Recent theoretical studies suggested that the actual precession period might be ≈ 1000 d as seen in observations (Rezania 2003; Wasserman 2002). In this paper, in a good agreement with the observed data (Stairs et al. 2000), we model the changes of the pulse shape in a precession cycle with period ≈ 1000 d and find that the variation of the pulse duration follows from a *standard* beam pattern in each cycle.

Key words. pulsar: individual: PSR B1828-11 – stars: neutron

1. Introduction

Analysis of the long-term observation of the spin behavior of the isolated pulsar PSR B1828-11 reveals periodic variations both in the pulse shape and the slow-down rate of the pulsar and shows strong Fourier power at periods of ≈ 1000 , 500, 250, and 167 d (Stairs et al. 2000). Close correlations of the periodic changes in the pulse shape and duration with variations in the spin-down rate of the pulsar strongly suggest that the star's spin axis is freely precessing around the star's symmetry axis.

Recently Link & Epstein (2001) studied the behavior of the observed pulse duration of PSR B1828-11 based on the free precession model. Since both 500 d and 250 d Fourier components have dominant contributions in the observed variations of period residual Δp , its derivative $\Delta \dot{p}$, and pulse shape of the beam, they suggested that the actual free precession period of the star is close to the strongest Fourier component ≈ 500 d. Then a coupling of nearly orthogonal magnetic dipole moment to the star's spin axis would provide the observed harmonic at period 250 d. On this basis, they modeled the pulse duration variations as a function of beam's sweep angle $\Delta\Theta$ (see below for definition), and found that the beam pattern of radiations *must* be non-standard: the beam duration is larger for beam sweeps farthest from the dipole axis. As a result, both upper and lower parts of the emission region are wider than its middle, see Fig. 2.

As they mentioned this is *required* by the precession interpretation of PSR B1828-11.

Recent theoretical investigations on the free precessing motion of PSR B1828-11 provide new explanations for the reported data with the fundamental period ~ 1000 d, which was originally suggested by Jones & Andersson (2001). Rezania (2003) considered the case in which the magnetic field of the star varies with time while the star is precessing. Then he found a condition under which a coupling of the star's crust with the time-varying magnetic radiation torque would produce the whole observed Fourier spectrum consistently. In a good agreement with the data he found that the fundamental precession period would be ≈ 1000 d. Alternatively, Wasserman (2002) showed that in general an oblique rotator must precess. By analysis of the mechanical energy of the system, he found that the minimum energy state for such star is a state where the star precesses. For strong magnetic stresses in the star's type II superconductor core he estimated the precession period as $P_{\text{pre}} \approx 2460 \text{ d}/(\beta \cos \chi B_{12} H_{15})$, where $B_{12} = B/10^{12}$ G is the star's magnetic field strength, $H_{15} = H/10^{15}$ G corresponds to the first critical field strength in a type II superconductor ($H_{\text{cr}} \sim 10^{15}$ G), $\beta \sim 1$ and χ is the inclination angle of magnetic symmetry axis. For the case PSR B1828-11 with $B \sim 5 \times 10^{12}$ G, the precession period will be $P_{\text{pre}} \sim 1000$ d, if the inclination angle is $\chi \approx 60^\circ$.

In this paper we study the pulse shape of PSR B1828-11 by assuming that the actual precession period of the star is ≈ 1000 d rather than ≈ 500 d. We note that Link & Epstein's calculations give the right behavior of the pulse shape variations *provided* the actual precession period is close to ≈ 500 d *only*. For the case $P_{\text{pre}} \approx 1000$ d, their analysis gives an incorrect prediction for the observed shape variations. Here, we generalize link & Epstein's analysis and show that, for $P_{\text{pre}} \approx 1000$ d,

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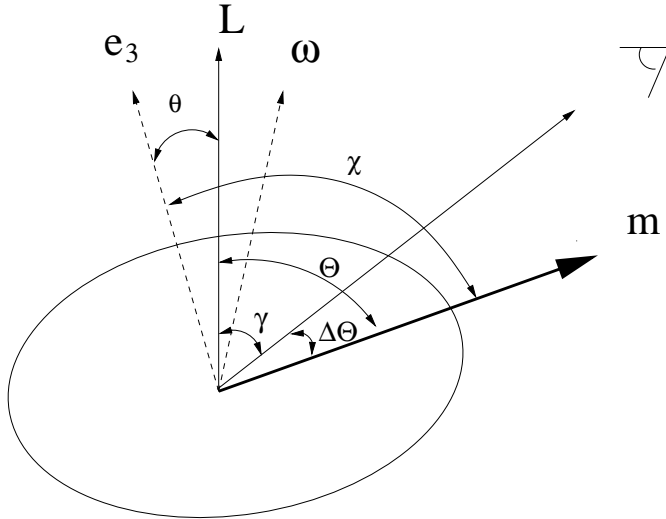


Fig. 1. Observing geometry. The angles defined at the instant the dipole moment \mathbf{m} is in the plane containing the angular momentum \mathbf{L} and the observer.

to get the correct behavior for the shape variations (compared with data) the radio beam of PSR B1828-11 must have the *standard* pattern.

2. The model

Let x , y , and z represent an inertial coordinate system \mathcal{S} , and an observer is in the $x-z$ plane with $x > 0$ and $z > 0$. Now consider a rigid, biaxial rotating star with angular velocity ω , the principal axes $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$, and corresponding principal moment of inertia $I_1 = I_2 \neq I_3$. The star's angular momentum \mathbf{L} , along the z -axis in \mathcal{S} , is misaligned to the star's symmetry axis \mathbf{e}_3 by a wobble angle θ , i.e. $\mathbf{L} \cdot \mathbf{e}_3 = L \cos \theta$, see Fig. 1. To study the pulse shape variations we look at the variation of the polar angle of the beam with respect to \mathbf{L} (fixed in \mathcal{S}) that is equal to the polar angle of the magnetic dipole moment \mathbf{m} ¹. The azimuthal and polar angles of the magnetic dipole moment \mathbf{m} , Φ and Θ in the inertial frame \mathcal{S} are given by $\tan \Phi = m_y/m_x$ and $\cos \Theta = m_z/m = \sin \theta \sin \psi \sin \chi + \cos \theta \cos \chi$, where θ is the wobble angle, $\psi = \tan^{-1} \omega_1/\omega_2 = \pi/2 - \omega_p t - \beta$ ($\omega_i = \mathbf{e}_i \cdot \omega$) and χ is the inclination of magnetic field symmetry axis from the star's symmetry axis \mathbf{e}_3 , i.e. $\mathbf{m} \cdot \mathbf{e}_3 = m \cos \chi$. Here $\omega_p = 2\pi/P_{\text{pre}}$ is free precession frequency and β is a constant phase. Expanding $\cos \Theta$ to the first order in θ one finds $\Theta = \chi - \theta \cos(\omega_p t + \beta)$. The latter shows the beam polar angle Θ changes sinusoidally about χ , as the star precesses.

Now let $\gamma \equiv \xi + \chi$ be the polar angle of the observer in \mathcal{S} (the constant angle between the observer and the angular momentum vector), see Fig. 1. Following Link & Epstein (2001)

¹ Note that though the pulsar beam's direction is not necessarily in the same direction as the dipole moment \mathbf{m} , for simplicity Link & Epstein (2001) defined a pulse as occurring when the azimuthal angle Φ of the magnetic dipole equals to the azimuth of the observer. As a result, $\dot{\Phi}$ is the observed pulse frequency. To compare our concluding result with one obtained by Link & Epstein (2001), we use the same definitions.

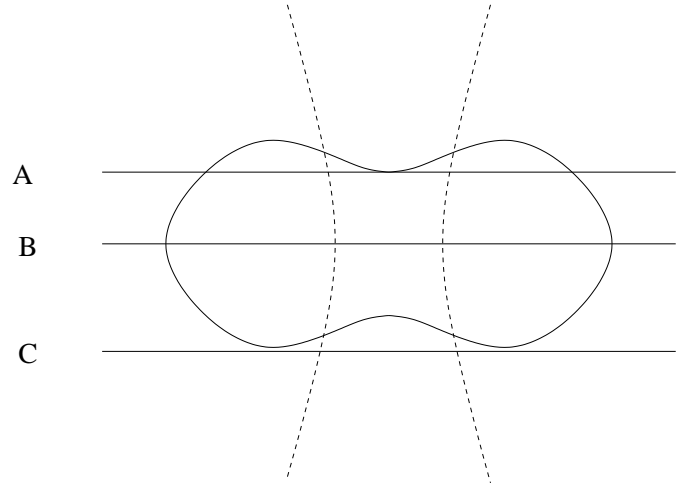


Fig. 2. The beam pattern for the radio emission from PSR B1828-11. The dashed parabolas represent the beam pattern proposed by Link & Epstein (2001) with $P_{\text{pre}} \approx 500$ d, while the solid closed curve correspond to the 1000 d precession period. The radiation pattern is very close to multipole radiation pattern with $(\ell, m) = (1, \pm 1)$. The lines B, A, C represent the viewing angle at which the beam is the narrowest ($S \approx 1$), wider ($S \approx 0.4$), and widest ($S \approx 0$), respectively, for Link & Epstein's model, while correspond to the widest ($S \approx 0$), narrower ($S \approx 0.4$), and narrowest ($S \approx 1$), respectively, for the model discussed here.

we define the sweep angle $\Delta\Theta$ as the difference in polar angle of observer, γ , and the dipole, Θ at the time of the pulse:

$$\Delta\Theta \equiv \gamma - \Theta = \xi + \theta \cos(\omega_p t + \beta) + O(\theta^2). \quad (1)$$

Here ξ is a free parameter which will be fixed later by fitting the data. Link & Epstein assumed that the pulse duration w is a function of $\Delta\Theta$ only, and has an extremum at $\Delta\Theta = 0$. Then they expanded the pulse duration w in terms of $\Delta\Theta$ as $w = w_0 + w_2(\Delta\Theta)^2$. As it is clear, the latter expression would provide both P_{pre} and $P_{\text{pre}}/2$ components (due to the $\cos(\omega_p t + \beta)$ and $\cos^2(\omega_p t + \beta)$ terms) in the pulse duration. As a result, it gives the correct behavior of the observed pulse duration for $P_{\text{pre}} \approx 500$ d. But with $P_{\text{pre}} \approx 1000$ d, the 250 d Fourier component is missing. To get the correct behavior for the pulse shape variations during a 1000 d precession cycle, we take a more general expansion rather than Link & Epstein (2001) as

$$w = w_0 + w_2(\Delta\Theta)^2 + w_4(\Delta\Theta)^4. \quad (2)$$

It is interesting to note that Eq. (2) provides the contribution of P_{pre} , $P_{\text{pre}}/2$, $P_{\text{pre}}/3$, and $P_{\text{pre}}/4$ terms in the pulse duration. So with $P_{\text{pre}} \approx 1000$ d, one would expect to observe 500 d, 333 d, and 250 d Fourier components in the pulse shape parameter. As reported by Stairs et al. (2000), the 1000 d and 333 d components though small (in comparing with 500 d and 250 d components) they have *non-zero* amplitude in the observed pulse shape parameter. These terms are missing in the Link & Epstein's model.

Without loss of generality we assume that $w_4 = 1$. Following Stairs et al. (2000), we define the shape parameter as

$S = \frac{A_N}{A_N + A_W}$ where A_N and A_W are the fitted heights of the narrower and wider standard profiles respectively, so that $S \approx 0$ for the wide pulses and $S \approx 1$ for narrow ones. As a result, one can relate the shape parameter S to the pulse duration w of the observed beam by

$$w = \max(w)(1 - S) + \min(w)S, \quad (3)$$

where $\max(w)$ and $\min(w)$ are the maximum and minimum values of the beam duration in the precession cycle. Combining Eqs. (2) and (3), one finds for $w_2 > 0$

$$S = 1 - \frac{w_2 + (\Delta\Theta)^2}{w_2 + (|\xi| + \theta)^2} \left(\frac{\Delta\Theta}{|\xi| + \theta} \right)^2, \quad (4)$$

while for $w_2 < 0$ we have

$$S/S_0 = \begin{cases} \frac{|w_2| - (\Delta\Theta)^2}{|w_2| - (|\xi| + \theta)^2} \left(\frac{\Delta\Theta}{|\xi| + \theta} \right)^2 & \text{if } |w_2| > (|\xi| + \theta)^2, \\ 1 - \frac{|w_2| - (\Delta\Theta)^2}{|w_2| - (|\xi| + \theta)^2} \left(\frac{\Delta\Theta}{|\xi| + \theta} \right)^2 & \text{if } |w_2| < (|\xi| + \theta)^2, \end{cases} \quad (5)$$

where S_0 is a normalization factor. It is clear that the shape parameter depends on ω_p , β , θ , ξ , and both the sign and magnitude of w_2 . With the given values of $\theta \approx 3^\circ$ and $P_{\text{pre}} \approx 1000$ d the shape parameter is determined by ξ , w_2 , and $\text{sign}(w_2)$ completely. In a good agreement with data, we find that w_2 must be *negative* with magnitude larger but close to $|w_2| \geq (|\xi| + \theta)^2$. Inserting the latter results in Eq. (2), one can easily show that the pulse duration w is bigger when the beam sweeps closer to the dipole axis ($\Delta\Theta = 0$) rather than when it sweeps farther ($\Delta\Theta \neq 0$). This means the beam pattern is *standard*: the beam duration is smaller for beam sweeps farthest from the dipole axis. Our result is completely in contrast with the result of Link & Epstein (2001) who found that a non-standard beam pattern (the beam duration is larger for beam sweeps farthest from the dipole axis) is *required* to explain the data. In Fig. 2 we sample the pulse profile for different observer's viewing angles (the solid closed curve) and compare with the one (the dashed parabola) obtained by Link & Epstein (2001). We find the radiation pattern of PSR B1828-11 is very close to multipole field radiation pattern with $(\ell, m) = (1, \pm 1)$ where ℓ and m are orbital and azimuthal numbers. As the star precesses, the observer sees different sweeps corresponded to different viewing angles. Line B corresponds to the viewing angle (closest to the dipole axis) for which pulse is the widest, $S \approx 0$ (narrowest, $S \approx 1$) provided the star precesses with precession period $P_{\text{pre}} \approx 1000$ d ($P_{\text{pre}} \approx 500$ d). Accordingly, the pulse profile is narrower, $S \approx 0.4$ (wider, $S \approx 0.4$) for line A and narrowest, $S \approx 1$ (widest, $S \approx 0$) for line C.

By choosing an appropriate values for $|w_2|$ and ξ one can easily fit the data. We found an acceptable fit on the pulse shape data of PSR B1828-11 by choosing $|w_2| \approx 0.0034$ and $\xi = -0:01$ for the given values of $\theta = 3:2$ and $\omega_p = 2\pi/1016$ d⁻¹, see Fig. 3. Here the value of $(|\xi| + \theta)^2 \approx 0.0031$. We note that our fit curve which is calculated for free precession period $P_{\text{pre}} = 1016$ d, is indistinguishable from the one that was calculated by Link & Epstein (2001) with free precession period $P_{\text{pre}} = 511$ d.

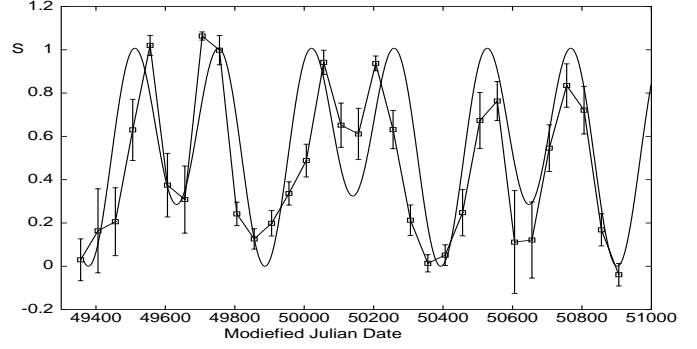


Fig. 3. The pulse shape parameter data S for PSR B1828-11 (from Stairs et al. 2000). The shape parameter is defined as $S = \frac{A_N}{A_N + A_W}$ where A_N and A_W are the fitted heights of the narrower and wider standard profiles respectively, so that $S \approx 1$ for the narrowest pulses and $S \approx 0$ for wider ones. The data points are obtained by averaging S over multiple bins, and the solid curve is the fit calculated with $P_{\text{pre}} = 1016$ d, see text.

3. Summary

Previous study on the pulse shape variations of the isolated pulsar PSR 1828-11 by Link & Epstein (2001) is based on the free precessing star with precession period ~ 500 d. Recent theoretical studies (Rezania 2003; Wasserman 2002) on the free precession of PSR B1828-11 suggested that the actual precession period might be ≈ 1000 d as seen in observations. In this paper, by assuming that the actual free precession period of the isolated pulsar PSR B1828-11 is $P_{\text{pre}} \approx 1000$ d we studied the star's pulse shape variations during a precession cycle. To get the correct behavior in the pulse shape variations in comparing with the observed data, we expanded the pulse duration w to forth order of $\Delta\Theta$ (the difference between polar angles of the observer and dipole axis at the time of the pulse). The forth order term is not considered by Link & Epstein (2001). We found that the pulse duration w is larger when the beam sweeps closer to the dipole axis ($\Delta\Theta = 0$) rather than when it sweeps farther to the dipole axis ($\Delta\Theta \neq 0$), see Fig. 2. This means that the variation of the pulse duration follows from a *standard* beam pattern in each cycle. Our result is in contrast with the result of Link & Epstein (2001) who found that a non-standard beam pattern is required to explain the data.

Further, our model provides the contribution other Fourier harmonics, 1000 d and 333 d, in the pulse shape parameter as seen in data. Although these terms have smaller (but non-zero) amplitudes rather than 500 d and 250 d Fourier components, they are forbidden in the Link & Epstein's model. In addition, by taking into account a term $(\Delta\Theta)^6$ in Eq. (2), one can explain the 167 d Fourier component seen in data of PSR B1828-11 (Stairs et al. 2000). The latter which corresponds to $P_{\text{pre}}/6$ Fourier component for $P_{\text{pre}} \approx 1000$ d is also forbidden for $P_{\text{pre}} \approx 500$ d free precession model.

Finally, we note that new observations of the pulse shape variations of PSR B1828-11 would be able to determine its radiation pattern. As predicted by Link & Epstein (2001), in a 511 d precession cycle the observer would see both the broad upper and lower parts of the radio beam. This would

be a critical observational evidence for the proposed theoretical models, since by assuming $P_{\text{pre}} \approx 1000$ d as the fundamental precession period, the middle of the beam will be broader.

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