Spectroscopic mode identification for the $\beta$ Cephei star EN (16) Lacertae*

C. Aerts$^1$, H. Lehmann$^2$, M. Briquet$^3$, R. Scuflaire$^3$, M. A. Dupret$^3$, J. De Ridder$^1$, and A. Thoul$^3$

1 Institut voor Sterrenkunde, Katholieke Universiteit Leuven, Celestijnenlaan 200 B, 3001 Leuven, Belgium
2 Thüringer Landessternwarte, 07778 Tautenburg, Germany
3 Institut d’Astrophysique et de Géophysique de Liège, Université de Liège, allée du Six Août 17, 4000 Liège, Belgium

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Abstract. We perform for the first time spectroscopic mode identification in the eclipsing binary $\beta$Cephei star EN (16)Lac. This mode identification is based upon a time series of 942 line profiles of the HeI $\lambda\lambda$5876 line in its spectrum. All three known frequencies $f_1$, $f_2$, $f_3$ of the star are present in the line-profile variations, but we failed to find additional modes. Using different identification methods we find conclusive evidence for the radial nature of the main mode and for the $\ell = 2, m = 0$ identification of the mode with frequency $f_2$. A unique identification of the third mode is not possible from the spectra, but we do derive that $t_1 < 3$. Fits to the amplitude and phase variability of the modes imply a rotation frequency between 0.1 and 0.4 c d$^{-1}$. The star’s rotation axis is not aligned with the orbital axis.

Key words. stars: binaries: spectroscopic – stars: variables: early-type – stars: individual: EN(16)Lac

1. Introduction

Recently, Lehmann et al. (2001, hereafter called Paper I) have made a very detailed study of the radial-velocity variations of the eclipsing and spectroscopic binary EN (16)Lacertae (hereafter shortened as EN (16)Lac; HD 216916, HR 8725, spectral type B2IV). The goal of their study was to disentangle the orbital and pulsational velocity variations of this binary such that more accurate orbital elements could be derived. In the current paper we elaborate further on the data presented in Paper I with the specific goal of analysing the line-profile variations of EN (16)Lac in full detail. With this study, we hope to settle the issue of mode identification of the pulsation modes in this well-studied $\beta$Cephei star.

EN (16)Lac was the first $\beta$Cephei star subjected to an asteroseismic analysis (Dziembowski & Jerzykiewicz 1996). The reason for choosing this star as a test case for asteroseismology is obvious: the star is a single-lined spectroscopic and an eclipsing binary with well-known orbital elements, which helps to constrain the physical parameters of the $\beta$Cephei-type primary. Moreover, the primary exhibits multiperiodic pulsations the periods of which have been studied in quite some detail in the literature and are known with a high precision. However, general agreement on the mode identification was never reached. In fact, Dziembowski & Jerzykiewicz (1996) rejected some previously suggested identifications and used their theoretical modelling as mode identification tool in order to find overall agreement between the observed variations and the excitation models. The seismic application remained limited, however, precisely due to the lack of unambiguous mode identifications. A value of 40 km s$^{-1}$ was assumed for the equatorial rotational velocity – we come back to this value later on in the paper. It is not clear to us how important this adopted value is for the results obtained by Dziembowski & Jerzykiewicz (1996).

Dziembowski & Jerzykiewicz clearly stressed the importance of updating the mode identifications for EN (16)Lac from more accurate and preferably high-resolution spectroscopic line-profile data. Presently, all attempts to identify the modes are based upon multicolour photometry and/or radial-velocity measurements. A problem for the application of such methods to EN (16)Lac is the fact that the photometric amplitudes of the third frequency vary considerably in time (see, e.g. Jerzykiewicz & Piątkowski 1999) and so the ratio of the radial-velocity and magnitude amplitude and/or the photometric amplitude ratios are uncertain diagnostics to identify this mode. This has led to different mode identifications in the past, even by one and the same author. The most recent summary of the observational attempts of mode identification, together with a proposal for an “unambiguous” identification of the pulsational degrees, was written by Chapellier et al. (1995). These authors come to the conclusion that the three main frequencies 5.9112, 5.8551 and 5.5033 c d$^{-1}$ correspond to respectively $\ell = 0, 2, 1$ modes.

It is well-known by now that line-profile variations offer the possibility to discover new pulsation modes, which are hardly or not visible in photometry. Recent examples of this phenomenon for $\beta$Cephei stars can be found in Aerts et al. (1998, $\beta$Crucis), Telting & Schrijvers (1998, $\omega$ Sco), and so the ratio of the radial-velocity and magnitude amplitude and/or the photometric amplitude ratios are uncertain diagnostics to identify this mode. This has led to different mode identifications in the past, even by one and the same author. The most recent summary of the observational attempts of mode identification, together with a proposal for an “unambiguous” identification of the pulsational degrees, was written by Chapellier et al. (1995). These authors come to the conclusion that the three main frequencies 5.9112, 5.8551 and 5.5033 c d$^{-1}$ correspond to respectively $\ell = 0, 2, 1$ modes.

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Send offprint requests to: C. Aerts,
e-mail: conny@ster.kuleuven.ac.be

* Based on observations gathered with the coude spectrograph attached to the 2.0 m reflector telescope at Tautenburg Observatory.
Ausseloos et al. (2002, βCentauri) and Schrijvers & Telting (2002, νCen). For the former example, it was shown convincingly that the modes detected only in the line-profile variations so far are also markedly present in space photometry gathered by the WIRE satellite (Cuyvers et al. 2002). These examples show that an independent search for new modes is warranted in all line-profile studies.

In this paper, we perform for the first time mode identification on the basis of the line-profile variations of EN (16) Lac. We will subsequently use the results of the current paper in a future study which is devoted to seismic modelling (Aerts et al., in preparation). The current paper is organised as follows. We give a brief description of the data in Sect. 2. A frequency analysis and the calculation of several line-profile diagnostics are presented in Sect. 3 while Sect. 4 is devoted to the spectroscopic mode identification. Finally, we discuss our findings and outline our future follow-up study in Sect. 5.

2. Observations

The data we explored for this paper consists of 942 high-resolution échelle spectra recorded with the coudé spectrograph attached to the 2.0 m reflector telescope at Tautenburg Observatory. For a full description of these data we refer to Paper I. We have distilled the spectra taken at Tautenburg as they have by far the highest signal-to-noise ratio. The total time base of this subset is 474 days, which is considerably shorter than the full dataset presented in Paper I. The considered spectra were taken during 16 runs, of which most consisted of only one night during which the star was intensively monitored. The temporal resolution of the spectra is better than 2% for all the pulsation modes mentioned in this paper. The wavelength ranges from 4780 Å–7080 Å.

We entirely focus the line-profile study for mode identification on the deepest, least blended line in this wavelength range. This is the He Iλλ6678.151 Å line. We have shifted the spectra by subtracting the orbital velocity according to the orbital elements listed as “Solution III” in Table 10 of Paper I.

In order to give the reader a feeling of the line-profile variability of EN (16) Lac and of the quality of our data we show in Fig. 1, 30 arbitrarily chosen profiles of the selected He line. One can see clear line-profile variability characterised by global asymmetries. Such variability is typical for low-degree (ℓ ≤ 4) (non-)radial pulsation modes. The line extends over a total range of some 100 km s⁻¹, while the FWHM of the line is about 40 km s⁻¹. The latter is therefore an upper limit of the rotational broadening.

We have determined the equivalent width of the He line and find that this quantity remains constant to within a few percent. Moreover, the small variations that occur do not behave periodically.

3. Line-profile diagnostics

3.1. Frequency search

A search for the best multiple frequency solution based on the known three main frequencies was already presented in Paper I – see Tables 8 and 9 for the outcome. That frequency search was done using different weighting schemes on the radial velocities which were derived from several spectral lines in the spectrum and for spectra of different quality. Overall agreement was found in Paper I with the three well established frequencies f₁, f₂, f₃ and modulation time scales of respectively 76.7 years and 344 and 674 days were proposed. In the following of the paper we adopt the values f₁ = 5.9113, f₂ = 5.8529, f₃ = 5.5026 d⁻¹ as derived in Paper I for the frequencies.

As the three main modes are clearly present in the radial-velocity data treated in Paper I, we proceeded here as follows. We determined the radial velocities of the selected He line by calculating its first moment (for a definition of the line moments – see Aerts et al. 1992). We imposed a new fit for the multiple periodic solution given in Table 8 of Paper I, considering only the distilled data set described in Sect. 2, and subtracted it, i.e. we prewhitened not only with the three main periods, but also with the modulation time scales. We used P₁, P₁’, P₂, P₂’, P₃, P₃’ (notation of Paper I) for the prewhitening. These notations Pᵢ and Pᵢ’ correspond to the observed frequency splitting which gives rise to the time scales of amplitude modulation for the frequency fᵢ. Subsequently we searched for new frequencies in the residuals by means of the different methods. None of the test frequencies reached a significant amplitude. In particular, we also mention not to have found evidence of the frequency 7.194 c d⁻¹ suggested by Jerzykiewicz (1993) from photoelectric observations.

3.2. Amplitude and phase behaviour across the line profile

Schrijvers et al. (1997) have provided for the first time a systematic study of the amplitude and phase variability across the line profile and showed that these quantities are powerful
diagnostics to analyse non-radial pulsations in stars. We show the amplitude and phase behaviour according to their formalism for all three modes of EN (16) Lac in Fig. 2. We only plot the phase behaviour for the frequencies themselves, and not for their first harmonic, as the latter is not sufficiently well determined and leads to scattered phase curves. It can be seen from Fig. 2 that all three modes exhibit clear variability throughout the whole profile. For all modes the amplitude drops considerably in the line center and the phase curves are very smooth, showing a well-determined signature. The phase difference across the profile amounts to 1.0–1.1 in units of \( \pi \) radians for jumps of 2\( \pi \). From left to right we show the plots for \( f_1 \), \( f_2 \), \( f_3 \).

### 3.3. Moment variations

The moment method (Aerts et al. 1992; Aerts 1996) is a very suitable identification method for stars with low-degree modes (\( \ell \leq 4 \)) and a relatively low rotation rate. Both these conditions apply to EN (16) Lac, as the profiles shown in Fig. 1 are only mildly broadened and do not show any moving subfeatures. We hence determine the moments of the He I line of EN (16) Lac as they have a high diagnostic value. In the versions of the moment method for multiperiodic pulsations in the slow-rotation approximation, the three lowest-order moments of the line profiles are considered (see e.g. Aerts 1996; Briquet & Aerts 2003). The observed moments are shown as dots in Figs. 3–5 for six arbitrarily selected nights. The amplitudes of the triperiodic fit to the first moment are listed in Table 1.

Mathias et al. (1994) have provided theoretical expressions for the moments in the general case of a multiperiodic pulsation with \( n \) frequencies. We have determined a fit to the first moment based upon these expressions for \( f_1, f_2, f_3 \). This fit is shown as full line in Fig. 3. However, as already mentioned above, it was shown very clearly in Paper I that amplitude modulation occurs in EN (16) Lac, with modulation time scales of 77 years, 344 days and 674 days (see Table 8 of Paper I). The former time scale is irrelevant for our subset of spectra, but the latter two may be present in the moment variations. In order to check this we have determined the fit to the first moment for the values of \( f_1, f_2, f_3 \) and including the two short modulation time scales. The resulting fit is represented as a dotted line in Fig. 3. It is clear from this comparison that the short modulation time scales are indeed present, although they by no means dominate the time behaviour of the first moment.

In the version of the moment method by Aerts (1996), which, for a multiperiodic pulsation, is based upon the theory by Mathias et al. (1994), it is of utmost importance to determine the theoretical fits to the moments in the best possible way, as the mode identification is entirely based on the amplitudes of these fits to the moments. This poses a problem in the case of EN (16) Lac. Indeed, our data only span 474 days and they are very poorly sampled. Because of this it is not possible to determine a fit to the moments that includes the periods \( P_2^3 \) and \( P_3^5 \) that correspond to the modulation time scales, as this would lead to unreliable amplitudes. The reason is that an increase of 2 frequencies in the first moment implies the addition of 18 frequencies in the second moment and of 38 in the third moment (see Mathias et al. 1994). Our data are insufficient in amount and in time spread to fit such a large number of free amplitudes.

The only strategy to follow in such a case is to consider only \( f_1, f_2, f_3 \) and all the relevant coupling frequencies given in Mathias et al. (1994) in the fit to the first three moments. Such a fit to the second and third moment is shown as full line in Figs. 4 and 5. While such a fit is quite satisfactory for the third moment (explaining 92% of the observed variance), it is far from perfect for the second moment (for which “only” 60% of the variance is explained). For this reason we have used the new version of the moment method recently proposed by Briquet & Aerts (2003). In this new numerical version of the mode identification method one identifies multiple modes simultaneously and one considers the moment values calculated at each time of observation instead of the amplitudes of the fits to the moments. We thus use only the values shown as dots in Figs. 3–5, and not the amplitudes of the fits shown as full lines. As shown by Briquet & Aerts (2003), this increases considerably the feasibility and the accuracy of the mode identification for multiperiodic stars compared to the versions presented in Mathias et al. (1994) and in Aerts (1996). We refer to Briquet & Aerts (2003) for a detailed description.

### Table 1. Amplitudes of the first velocity moment resulting from a fit to the data for \( f_1, f_2, f_3 \). The \( K \)-values for these frequencies are also listed.

<table>
<thead>
<tr>
<th>Frequency (c d(^{-1}))</th>
<th>Amplitude (km s(^{-1}))</th>
<th>( K )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_1 = 5.9113 )</td>
<td>2.57 ± 0.07</td>
<td>0.082</td>
</tr>
<tr>
<td>( f_2 = 5.8529 )</td>
<td>2.71 ± 0.07</td>
<td>0.084</td>
</tr>
<tr>
<td>( f_3 = 5.5026 )</td>
<td>1.10 ± 0.06</td>
<td>0.095</td>
</tr>
</tbody>
</table>

**Fig. 2.** Amplitude and phase behaviour across the He line profile for the 3 frequencies \( f_1, f_2, f_3 \). Upper panel: average line profile; middle panel: amplitude variability in continuum units for the frequency; bottom panel: phase behaviour of the frequency in units of \( \pi \) radians corrected for jumps of 2\( \pi \). From left to right we shown the plots for \( f_1 \to f_3 \).
4. Mode identification

Several ways of identifying modes from line-profile variations have been proposed in the literature. A recent overview including applications can be found in Aerts & Eyer (2000) and for more detailed descriptions of each of the methods we refer to the references in that review paper. Basically, two main complementary quantitative and objective methods are available. We apply them both to our data of EN (16) Lac.

4.1. Identification based on the phase variation

The mode identification method published by Telting & Schrijvers (1997) is based on the amplitude and phase behaviour across the profile. The authors have performed extensive simulation studies and propose a linear relation between the degree $\ell$ of the mode and the phase difference across the profile for the frequency, and between the azimuthal number $m$ and the phase difference for the first harmonic of the frequency. The phase differences across the profile for $f_1$, $f_2$, $f_3$ amount to respectively $1.06\pi$, $1.10\pi$ and $1.20\pi$ (see Fig. 2). Applying their formula to these three phase differences leads to $\ell = 1 \pm 1$, i.e. $\ell < 3$ for all three modes.

It should be emphasized that the relation of Telting & Schrijvers is particularly suited to identify high-degree ($\ell \geq 4$) modes in rapid rotators – a situation not encountered for EN (16) Lac. Hence it is not surprising that its predictive power for the nature of the modes of EN (16) Lac is limited. The phase behaviour for the harmonic of the frequencies is too noisy to be of any use as an estimate for $m$. On the other hand, the amplitude and phase behaviour shown in Fig. 2 is very useful to check the validity of particular solutions for the mode identification, by constructing theoretical profiles and comparing their amplitude and phase variation with the observed ones (for such a detailed modelling application in a $\beta$Cephei star with similar line-profile variations as EN (16) Lac, see Telting et al. 1997). We will perform such detailed modelling of the amplitude and phase behaviour after having obtained estimates for the wavenumbers by means of another method, i.e. after having eliminated all unrealistic combinations of the wavenumbers and having selected the most likely ones.

4.2. Identification based on the moment variations

The moment method has been successfully applied to several $\beta$Cephei stars already (see e.g. Aerts et al. 1994). Despite the fact that the even moments are always more noisy than the odd ones, the second moment is a very good diagnostic for evaluating the nature of the azimuthal number $m$. Indeed, comparing the importance of the frequencies $f_i$ and $2f_i$ allows one to discriminate between $m = 0$ and $m \neq 0$ (Aerts et al. 1992). In the former case, the double sine is dominant while this is not the case for non-axisymmetric modes. This simple rule already allows us to conclude from the observed variations of the second moment that the modes with frequencies $f_1$ and $f_2$ in EN (16) Lac are axisymmetric: $m_1 = m_2 = 0$. Moreover, in
the case of EN(16)Lac, it is highly unlikely that the modes have \( \ell > 2 \) as we would see more complex line-profile variability in that case.

In order to calculate the discriminants for the three modes, whatever the version of the moment method used, we need to determine the ratio of the horizontal to the vertical velocity amplitude for each of the modes. These so-called \( K \)-values are to a good approximation given by \( GMf/\ell j^2 R^3 \) for the three modes with frequency \( f \). Accurate estimates for the mass and radius of EN(16)Lac are available (Pigulski & Jerzykiewicz 1988) and amount to respectively 9.7 \( M_\odot \) and 6.3 \( R_\odot \). The three \( K \)-values are listed in Table 1. Besides these three known numbers we also need to give the linear limb-darkening coefficient, which, for the effective temperature of EN(16)Lac (21 530 K; Pigulski & Jerzykiewicz 1988) and the wavelength 6678 Å, amounts to 0.21 (Wade & Rucinski 1985).

The overall broadening is about 40 km s\(^{-1} \) (see Fig. 1). One could try to take into account that EN(16)Lac is a young (age \( \sim 1.3 \times 10^7 \) years) eclipsing binary of which we know the orbital inclination: \( \iota_{\text{orb}} = 83^\circ \) (Pigulski & Jerzykiewicz 1988). Assuming a synchronised star, and aligned rotation and orbital axes, leads to \( v \sin i = 26.2 \) km s\(^{-1} \). This is not realistic as the overall pulsational broadening is less than 10 km s\(^{-1} \) (see Table 1 and Fig. 3). It is therefore clear that the star cannot be synchronised yet and/or has a rotational inclination very different from the orbital inclination. We therefore did not restrict the inclination angle nor \( v \sin i \).

The moment method in the formulation by Aerts (1996) is more limited in predictive power as the number of modes increases, as outlined explicitly by Briquet & Aerts (2003). For reasons mentioned in the previous section, we report here only the results based upon the new version of the moment method by Briquet & Aerts (2003). The outcome of their mode identification is repeated here in Table 2 in order to clarify the discussion below. In this table, \( A_p \) is proportional to the amplitude of the radial part of the pulsation velocity and is fixed by requesting that the observed amplitude of the first moment is compatible with the computed one (which contains the factor \( A_p \)). It is expressed in km s\(^{-1} \). Additionally, \( i \) is the inclination angle; \( v \sin i \) is the projected rotational velocity, expressed in km s\(^{-1} \) and \( \sigma \) is the intrinsic line-profile width, also expressed in km s\(^{-1} \). For the three-dimensional grid of continuous parameters we adopted: \( v \sin i \in [1, 40] \) km s\(^{-1} \) in steps of 1 km s\(^{-1} \), \( i \in [0^\circ, 90^\circ] \) in steps of 5\(^\circ \) and \( \sigma \in [1, 20] \) km s\(^{-1} \) in steps of 1 km s\(^{-1} \) while all testcases with \( \ell \leq 3 \) were calculated. The lower \( \Sigma \), the more likely the solution is. In determining Table 2, we have eliminated solutions for which the modes with \( f_1 \) and \( f_2 \) have the same \( (\ell, m) \), as the frequency values of these modes are too close to each other for them to differ only in radial order.

From Table 2 we conclude the following:
- the main mode is axisymmetric, with \( \ell = 0 \) or 1;
- the second mode is an \( \ell = 2, m = 0 \) mode;
- the nature of the third mode is unclear, but it is either an \( \ell = 1 \) or 2 mode.

We point out that the estimation of the continuous parameters is difficult from the moment variations, as outlined by de Ridder et al. (2002). As a consequence, we are unable to pinpoint the values for \( (v \sin i, i, \sigma) \) with a precision of, let’s say, several km s\(^{-1} \). In view of the effective temperature of the star, the \( \sigma \)-value of 19 km s\(^{-1} \) is quite high but we did not want to be too restrictive in the grid of parameters.

We can eliminate further the solutions labeled as combinations numbers 2, 9 and 10 in Table 2 on astrophysical grounds, as a radial mode and an \( \ell = 1 \) do not have so close frequencies in \( \beta \) Cephei models. This, together with the fact that \( \ell < 3 \) for all modes, encourages us to try and find the most likely identification of the three modes by cross-validation, i.e. by combining the diagnostics of both identification methods.

### 4.3. Towards a unique spectroscopic identification

We have checked the possible combinations of \( (\ell_p, m) \) according to the discriminant values listed in Table 2, keeping in mind that the continuous parameters \( (v \sin i, i, \sigma) \) are not accurately determined by the discriminant (de Ridder et al. 2002). We have generated theoretical profiles for all acceptable combinations of wavenumbers listed in Table 2. We subsequently determined the amplitude and phase variability across the profile and compared the outcome with those of the observed spectra shown in Fig. 2. We have next adapted \( (v \sin i, i, \sigma) \) until the best agreement with the observed curves of Fig. 2 is found for each of the acceptable combinations for the wavenumbers.

The results of this iterative process are the following:
- all solutions explain well the phase behaviour across the profile;
- all solutions with \( \ell_1 = 1, \ell_2 = 2, m_1 = m_2 = 0 \) do not give satisfactory fits to the amplitude variability of the two dominant modes and can hence be excluded (see full line in Fig. 6 for a representative case);
- we are unable to discriminate between the \( \ell = 1, 2 \) solutions for the third mode;
- for all cases the best agreement is found for \( v \sin i \) between 20 and 32 km s\(^{-1} \), for \( i \) between 15\(^\circ\) and 35\(^\circ\) and for \( \sigma \) between 8 and 13 km s\(^{-1} \).

Some of the best trial cases are shown in Fig. 6. The fact that the different line styles are hardly distinguishable proves that several solutions are equivalent. We stress that also the combination number 5 is very much alike combination number 3, and so is acceptable. In addition, also the combinations with \( \ell_1 = 0, \ell_2 = 2, m_1 = m_2 = 0 \) and \( \ell_3 = 1, m_3 = \pm 1 \) are acceptable, for certain combinations of the continuous parameters. These additional cases are not shown in Fig. 6 for clarity.

The acceptable intervals for \( v \sin i \) and \( \sigma \) lead to an allowed range for the equatorial rotation velocity of \( \Omega R \in [35, 135] \) km s\(^{-1} \). For the radius estimate of 6.3 \( R_\odot \) we therefore find that the rotational frequency must lie in the interval 0.11 – 0.42 c d\(^{-1} \), while the orbital frequency amounts to 0.08 c d\(^{-1} \). As \( f_2 - f_1 = 0.35 \) c d\(^{-1} \), the \( \ell_3 = 2 \) solutions with \( m_3 = \pm 1, \pm 2 \) are indeed possible, as are the \( \ell_3 = 1, m_3 = \pm 1 \) ones.

In summary, we provide in Table 3 the solutions that lead to amplitude and phase variabilities which are compatible with the observed ones. The uncertainty of \( v \sin i \) and \( \sigma \) for each of the combinations from the fitting is estimated to be \( \sim 2 \) km s\(^{-1} \).
Table 2. The ten best solutions of the mode identification for the three modes of EN (16) Lac as found by Briquet & Aerts (2003). For the meaning of the symbols, see text.

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<th>combination number</th>
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</table>

Fig. 6. Observed (bullets) and theoretical amplitude and phase behaviour across the He line profile for \(f_1, f_2, f_3\) (from left to right). Upper panel: amplitude variability in continuum units; bottom panel: phase behaviour in units of \(\pi\) radians corrected for jumps of \(2\pi\). The plotted combinations are: full line: number 4, dashed line: number 7, dashed-dot line: number 8, dotted line: number 1, dashed-dot-dot-dot line: number 3, as indicated in Table 2, but for optimized values of the continuous parameters.

Although the mode identification for the third mode is not unique, we are able to conclude from our line-profile analysis that the rotation axis cannot be aligned with the orbital axis.

5. Discussion

The ultimate goal of performing (spectroscopic) mode identification is to use the values of \((\ell, m)\), together with the frequencies, for an asteroseismic analysis of the star. Indeed, knowing which modes are excited in the star allows one to fine-tune its physical parameters, such as the mass, the luminosity, the effective temperature, the metallicity and the age. To this end, one needs to check for which values of these parameters the frequencies exactly coincide with the observed ones, taking into account the effects of rotational splitting. From the set of models that survive this test one subsequently selects only those that fulfill the mode identification. Our current paper offers a fruitfull starting point for such basic seismic modelling of EN (16) Lac, which is the topic of our subsequent study (Aerts et al., in preparation).

Ideally, one would hope to refine the modelling to a much more sophisticated level, as in helioseismology. Indeed, the modes detected and identified in the Sun are so numerous that frequency inversion, and hence derivation of the internal behaviour of the physical quantities, is possible. We are yet still far away from this level of precision for massive oscillators with \(\kappa\)-driven modes, the main reasons being the limited number of detected modes and their uncertain mode identification.
Table 3. The solutions listed here are those that were found as most promising from the moment method while they also lead to acceptable amplitude and phase variabilities. The meaning of the symbols is the same as in Table 2.

| \(l_1, m_1\) | (0, 0) | (0, 0) | (0, 0) | (0, 0) | (0, 0) | (0, 0) | (0, 0) |
| \(l_2, m_2\) | (2, 0) | (2, 0) | (2, 0) | (2, 0) | (2, 0) | (2, 0) | (2, 0) |
| \(l_3, m_3\) | (1, 0) | (2, –2) | (2, 2) | (1, 1) | (1, –1) | (2, 1) | (2, –1) |
| \(i\) | 15 | 15 | 15 | 34 | 15 | 35 | 35 |
| \(v \sin i\) | 20 | 20 | 20 | 32 | 30 | 32 | 32 |
| \(\sigma\) | 8 | 8 | 8 | 13 | 12 | 13 | 13 |

The example of \(\beta\) Cru mentioned in the introduction gives us good hope, however, that future space missions devoted to asteroseismology will imply a major step forward in seismic studies of massive stars.

References