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On the precession of the isolated pulsar PSR B1828-11

A time-varying magnetic field

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Abstract. Analysis of both pulse timing and pulse shape variations of the isolated pulsar PSR B1828-11 shows highly correlated and strong Fourier power at periods ≃1000, 500, 250, and 167 d (Stairs et al. 2000). The only description based on a free precession of the star's rigid crust coupled to the magnetic dipole torque, explains the 500 d-component, as the fundamental Fourier frequency, with its harmonic 250 d-component (Link & Epstein 2001). In this paper, we study a time-varying magnetic field model and show that *if* the dipole moment vector rotates with a period *nearly equal* to the longest (assumed fundamental) observed period (≃1000 d) relative to the star's body axes, the resulting magnetic torque may produce the whole Fourier spectrum consistently. We also find the second and fourth harmonics at periods ≃500 and 250 d are dominant for small wobble angle ≃3° and large field's inclination angle ≥89°. We note that, although our model simply explains the observed pulse timing and pulse shape variations of PSR B1828-11, it also evokes some serious problems with our currently understanding from the physics of the neutron star structure.

Key words. pulsar: individual: PSR B1828-11 – stars: neutron – stars: magnetic fields

1. Introduction

The monitoring of long-term and periodic variations both in pulse shape and slow-down rate of the isolated pulsar PSR B1828-11 shows strong Fourier power at periods of ≃1000, 500, 250, and 167 d, with the strongest one at period ≃500 d (Stairs et al. 2000). The close relationship between the periodic changes in the beam shape and the spin-down rate of the pulsar suggests the possibility of precession of the spin axis in a rotating body. The precession of the spin axis would provide cyclic changes in the inclination angle χ between the spin and magnetic symmetry axes. The result will be periodic variations both in the observed pulse-profile and spin-down rate of the pulsar.

Recently Jones & Andersson (2001) and Link & Epstein (2001) studied a freely precessing neutron star due to a small deformation of the star from spherical symmetry coupled to a torque such as magnetic dipole moment, gravitational radiation, etc., and explained some part of the observed data. Because of the strong periodicity at period ≃504 d seen in the data, Jones & Andersson (2001) reasonably suggested that the actual free precession period is $P_{\rm pre} = 1009$ d. A coupling between the magnetic dipole moment and star's spin axis can provide a strong modulation at period $P_{\rm pre}/2 \simeq 504$ d, when the magnetic dipole is nearly orthogonal to the star's deformation axis. But their model could not explain the strong Fourier component corresponding to a period of ≃250 d (see Stairs et al. 2000). The latter component has a significant contribution in the observed variations of period residual Δp, its derivative $\Delta\dot{p}$, and pulse shape. For this reason, Link & Epstein (2001) assumed that the strongest Fourier component (≃500 d) represent the actual free precession period. They found that for a small deformation parameter of $\epsilon = (I_3 - I_1)/I_1 \simeq 9 \times 10^{-9}$, a free precession of the angular momentum axis around the symmetry axis of the crust could provide a period at $P_{\rm pre} \simeq 511$ d. Here $I_1 = I_2 < I_3$ are the principle moment inertia of the star. Further, they showed that a coupling of nearly orthogonal (fixed to the body of the star) magnetic dipole moment to the spin axis would provide the observed harmonic at period ≃250 d. Their model has good agreement with observations in the pulse period, but as they mentioned, it failed to explain the Fourier component at period ≃1000 d seen in the data (as well as 167 d).

The existence of precession in a neutron star is in strong conflict with the superfluid models for the neutron star interior structure. These models have successfully explained the glitch phenomena (with both pre- and post-glitch behavior) in most neutron stars in which the pinned vortices to the star crust become partially unpinned during a glitch (Alpar et al. 1984). As shown by Shaham (1977) and Sedrakian et al. (1999), the precession should be damped out by the pinned (even imperfect) vortices on a time scale of few precession periods. For example, PSR B1828-11 with typical degree of vortex pinning,

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$I_{\rm pinned}/I_{\rm star} \sim 1.4\%$ (indicated by pulsar glitches in stars that frequently glitch), would precess for $\ll$40 s, far shorter than the observed periods (Link & Epstein 2001). Here $I_{\rm star}$ is the total moment inertia of the star, while $I_{\rm pinned}$ is the portion of star's fluid moment inertia that is pinned to the crust.

The free precession description provides an effective decoupling between the internal superfluid and the crust. Recently, Link & Cutler (2002) studied the problem more carefully by considering dynamics of the pinned vortices in a free precessing star under both Magnus, $f_{\rm m}$, and hydrodynamics (due to the precession), $f_{\rm p}$, forces. They found that the precessional (free) motion itself prevents the vortex pinning process and keeps the vortices unpinned in the crust of PSR B1828-11 while precessing, for a force density (per unit length) $f_{\rm p} \sim 10^{16}$ dyn cm^{-1}. As a result, they found that partially pinned-vortex configuration cannot be static.

The effective core-crust decoupling causes the core and the crust to rotate at different rates (Sedrakian et al. 1999)[1]. The latter would increase core magnetic flux-tube displacement relative to the crust, and then sustain the magnetic stresses on the solid crust in forcing it to break (in platelets) and move as the star rotate (Ruderman 1991a,b). The stresses are strong enough to move the crustal lattice by continual cracking, buckling, or plastic flow to relative stresses beyond the lattice yield strength. As a result, because of the very high electrical conductivity of the crust ($\sigma \sim 10^{26}$ s^{-1}), the foot points of external magnetic field lines move with the conducting plates in which the field is entangled. Furthermore, since the core magnetic flux tubes are frozen into the core's fluid, the precessing crust drags them and then increases core flux-tube displacements. Therefore, during precession of the crust such plate motion is unavoidable. In addition, as shown by Malkus (1963, 1968) the precessional motion of the star exerts torques to the core and/or crust resulting from shearing flow at the thin core-crust boundary region. These torques, so-called precessional torque, are able to sustain a turbulent hydromagnetic flow in the boundary region, and then increase local magnetic field strength. This would excite convective fluid motions in the core-crust boundary, increase magnetic stresses on the crust and cause it to break down in platelets.

In this paper, motivated by the above conjecture, we suggest that the magnetic field may vary somewhat with time, relative to the body axes of the star. The question that arises now is whether the whole observed Fourier spectrum of PSR B1828-11, can be consistently generated by a *time-varying* magnetic field during the course of free precession of the star. In other words, under what conditions will the observed cyclical changes in the timing data be produced by precession of the star's crust coupled to the magnetic dipole torque of a time-varying magnetic field. In Sect. 2 we address this question in detail. Following Link & Epstein (2001) we assume that the star precesses freely around the spin axis, but with period $P_{\rm pre} \simeq 1000$ d. Then we show that the magnetic torque exerted by a dipole moment may produce the other observed harmonics as seen in data, *if* the magnetic dipole vector rotates with a period close to $P_{\rm pre}$ relative to the star's body axes. Section 3 is devoted to further discussion.

[1] Actually Sedrakian et al. (1999) have shown that even for partially pinned vortices the core and the crust would rotate at different angular velocities.

2. The precession

Consider a rigid, biaxial rotating star with the principle axes $\boldsymbol{e}_1, \boldsymbol{e}_2, \boldsymbol{e}_3$ and corresponding principle moment of inertia $I_1 = I_2 \neq I_3$. The star's angular momentum $\boldsymbol{L}$ is misaligned to the symmetry axis $\boldsymbol{e}_3$ by a wobble angle θ, i.e. $\boldsymbol{L}\cdot\boldsymbol{e}_3 = L\cos\theta$. In general, we assume that the stellar magnetic field is dipolar and changes with time as

$$\boldsymbol{m} = m_0 \sin\chi f_1(t)\boldsymbol{e}_1 + m_0 \sin\chi f_2(t)\boldsymbol{e}_2 + m_0 \cos\chi f_3(t)\boldsymbol{e}_3, \qquad (1)$$

where m_0 is the average value of $|\boldsymbol{m}|$ over a period $2\pi/\omega_p$ and χ is the angle between $\boldsymbol{m}$ and $\boldsymbol{e}_3$. The functions $f_1(t), f_2(t)$ and $f_3(t)$ are arbitrary functions in time and will be determined later by using the data. The equations of motion in the corotating frame are

$$\boldsymbol{I}\cdot\frac{\mathrm{d}\boldsymbol{\omega}}{\mathrm{d}t} + \boldsymbol{\omega}\times\boldsymbol{L} = \frac{2\omega^2}{3c^3}(\boldsymbol{\omega}\times\boldsymbol{m})\times\boldsymbol{m} - \frac{1}{5Rc^2}(\boldsymbol{\omega}\cdot\boldsymbol{m})(\boldsymbol{\omega}\times\boldsymbol{m}), \quad (2)$$

where R is the average radius of the star. The first term of the magnetic torque, $\boldsymbol{T}_{\rm ff}$, is due to the far-field radiation and has components both parallel and perpendicular to the spin axis. It is responsible for spinning down the star. The second term, $\boldsymbol{T}_{\rm nf}$, represents the near-field radiation torque and is exactly perpendicular to the spin axis. It has no contribution to the energy/angular momentum transfer from the star. This torque does affect the wobble angle and spin rate of a freely precessing star. Following Link & Epstein (2001), for small wobble angle $\theta \simeq 3°$, suggested by the observed pulse shape variations of PSR B1828-11 over one precession period, and for small oblateness $\epsilon \simeq 10^{-8}$, we have $(\omega_0\tau_{\rm ff})^{-1} \ll (\omega_0\tau_{\rm nf})^{-1} < \epsilon\theta \ll \theta \ll 1$. Here ω_0 is the angular frequency of the star, $\tau_{\rm nf}$ ($\sim 10^4$ yr), and $\tau_{\rm ff}$ ($\sim 10^8$ yr) are the corresponding near- and far-field radiation torque time scales, respectively. So up to the first order of θ, the magnetic torques in the RHS of Eq. (2) can be neglected. Therefore in this order, the angular velocity vector $\boldsymbol{\omega}$ precesses freely around the star's symmetry axis as

$$\boldsymbol{\omega}(t) \simeq \theta\omega_0\cos(\omega_{\rm p}t + \beta_{\rm p})\boldsymbol{e}_1 + \theta\omega_0\sin(\omega_{\rm p}t + \beta_{\rm p})\boldsymbol{e}_2 + \omega_0\boldsymbol{e}_3, \quad (3)$$

with the precession frequency $\omega_{\rm p} = \epsilon\omega_0$, and a constant phase $\beta_{\rm p}$. For the case of PSR B1828-11, observations suggest that $\epsilon \simeq 4.7\times10^{-9}$ (Stairs et al. 2000). Then $\omega_{\rm p} \simeq 7.29\times10^{-8}$ Hz or equivalently $P_{\rm pre} = 2\pi/\omega_{\rm p} \simeq 997$ d.

2.1. Timing

The observed timing behavior can be understood by considering the contribution of other torque's components in the variations of the spin rate. By multiplying $\boldsymbol{\omega}$ in Eqs. (2) we have

$$\frac{\mathrm{d}\omega^2}{\mathrm{d}t} = \frac{2}{I_1}\left(\boldsymbol{\omega}\cdot\boldsymbol{T} - \epsilon\frac{I_1}{I_3}\omega_3 T_3\right). \qquad (4)$$

Equation (4) shows the torque-induced variations in the spin rate of star. From Eq. (2) it is clear that the $\boldsymbol{\omega}\cdot\boldsymbol{T}$ term does not depend on the near-field torque. It contributes to the spin rate change only through the negligible final term. Using Eqs. (1) and (2), by calculating $\boldsymbol{\omega} \cdot \boldsymbol{T}_{\rm ff}$ and subtracting Eq. (4) from the secular spin down of the star (in the absence of precession), $-(\omega \sin^2\chi/\tau_{\rm ff})(I_3/I_1)$, one can find the spin rate due to the far-field torque as (dropping constant terms)

$$\begin{aligned}\frac{\Delta\dot{\omega}}{\omega_0} \simeq \frac{1}{\tau_{\rm ff}}\frac{I_3}{I_1}\Big[& \cos^2\chi f_3^2(t) + \theta\sin 2\chi \\ & \times\Big(\cos(\omega_{\rm p}t+\beta_{\rm p})f_1(t)f_3(t) + \sin(\omega_{\rm p}t+\beta_{\rm p})f_2(t)f_3(t)\Big) \\ & -\frac{\theta^2}{2}\sin^2\chi\Big(\sin^2(\omega_{\rm p}t+\beta_{\rm p})f_1^2(t) + \cos^2(\omega_{\rm p}t+\beta_{\rm p})f_2^2(t) \\ & -2\sin(\omega_{\rm p}t+\beta_{\rm p})\cos(\omega_{\rm p}t+\beta_{\rm p})f_1(t)f_2(t)\Big)\Big]. \qquad (5)\end{aligned}$$

Using Fourier expansion, we expand the functions $f_1(t)$, $f_2(t)$, and $f_3(t)$ as follow

$$\begin{aligned} f_1(t) &= \sum_{n=0} a_n\cos(n\omega_{\rm d} + n\beta_{\rm d}), \\ f_2(t) &= \sum_{n=0} b_n\sin(n\omega_{\rm d} + n\beta_{\rm d}), \\ f_3(t) &= \sum_{n=0} c_n\cos(n\omega_{\rm d} + n\beta_{\rm d}), \qquad (5a)\end{aligned}$$

where $\omega_{\rm d}$ is frequency of the magnetic field's variation and $\beta_{\rm d}$ is constant. The coefficients a_n, b_n, and c_n will be determined by fitting the data. Equation (5) shows that the spin rate variations depend on both the precession frequency and the variation frequency of the dipole field. For the case $a_0 = c_0 = 1$ and $a_n = b_{n-1} = c_n = 0$ for $n \geq 1$, Eq. (5) reduces to one obtained by Link & Epstein (2001) (except by a factor I_3/I_1).

To obtain the reported spectrum of PSR B1828-11, it is enough to consider $n = 0$ and 1 terms in Eq. (5) only. The $n \geq 2$ terms will produce the higher harmonics which will be discussed later. Simply a correct behavior of the spin rate can be found by setting $c_1 = 0$, $c_0 = 1$, and $b_1 = -a_1$. Therefore Eq. (5) reduces to (dropping constant terms)

$$\begin{aligned}\frac{\Delta\dot{\omega}}{\omega_0} \simeq \frac{\theta}{\tau_{\rm ff}}\frac{I_3}{I_1}\Big[& \sin 2\chi\Big(a_0\cos(\omega_{\rm p}t+\beta_{\rm p}) \\ & +a_1\cos[(\omega_{\rm p}+\omega_{\rm d})t+\beta_{\rm p}+\beta_{\rm d}]\Big) \\ & -\frac{\theta}{2}\sin^2\chi\Big(2a_0a_1\cos(\omega_{\rm d}t+\beta_{\rm d}) - a_0^2\cos(2\omega_{\rm p}t+2\beta_{\rm d}) \\ & -2a_0a_1\cos[(2\omega_{\rm p}+\omega_{\rm d})t+2\beta_{\rm p}+\beta_{\rm d}] \\ & -a_1^2\cos[2(\omega_{\rm p}+\omega_{\rm d})t+2(\beta_{\rm p}+\beta_{\rm d})]\Big)\Big]. \qquad (6)\end{aligned}$$

As one expected, the expression for observable variations in period derivative, $\Delta\dot{p}$, will be modified as well. The star's residual in $\dot{p}$ is owing to both torque variation, Eq. (5), and the geometrical effect. The later is due to the orientation of the star's angular velocity vector $\boldsymbol{\omega}$ relative to the observer. As expected, the torque effects dominate the geometrical effects by a factor $(P_{\rm pre}^2/\pi P_0\tau_{\rm ff})(I_3/I_1)\sin^2\chi \simeq 100$–$1000$ for the precession period $\simeq 1000$ d, and so we neglected it here. Therefore

$$\begin{aligned}\Delta\dot{p} &\simeq -\frac{P_0^2}{2\pi}\Delta\dot{\omega} \\ &\simeq -\frac{P_0}{T}\theta\Big[\cot\chi\Big(a_0\cos(\omega_{\rm p}t+\beta_{\rm p}) \\ &\quad +a_1\cos[(\omega_{\rm p}+\omega_{\rm d})t+\beta_{\rm p}+\beta_{\rm d}]\Big) \\ &\quad -\frac{\theta}{4}\Big(2a_0a_1\cos(\omega_{\rm d}t+\beta_{\rm d}) - a_0^2\cos(2\omega_{\rm p}t+2\beta_{\rm d}) \\ &\quad -2a_0a_1\cos[(2\omega_{\rm p}+\omega_{\rm d})t+2\beta_{\rm p}+\beta_{\rm d}] \\ &\quad -a_1^2\cos[2(\omega_{\rm p}+\omega_{\rm d})t+2(\beta_{\rm p}+\beta_{\rm d})]\Big)\Big], \qquad (7)\end{aligned}$$

where $T = (\tau_{\rm ff}/2\sin^2\chi)(I_1/I_3) \simeq t_{\rm age}$ is approximately equal to the characteristic spin-down age and P_0 is the spin period of star. Equation (7) gives the period derivative residual due to far-field torque variations. Now let us set $\omega_{\rm p}+\omega_{\rm d} \simeq 2\pi/500\,{\rm d}^{-1}$ (equivalently $1/P_{\rm pre}+1/P_{\rm d} \simeq 1/500\,{\rm d}^{-1}$) in Eq. (7). By assuming the fundumental precession frequency is $\omega_{\rm p} \simeq 2\pi/500\,{\rm d}^{-1}$, i.e. $\omega_d \simeq 0$, *only* the 500 d and 250 d components will survive in Eq. (7), as obtained by Link & Epstein (2001), and all other harmonics are forbidden. Although, as shown by observations, both 500 d and 250 d Fourier components are dominant and have comparable amplitudes, the other Fourier components, especially the 1000 d-component, have nonzero amplitudes (Stairs et al. 2000). To get the 1000 d-component we choose $\omega_{\rm p} \simeq 2\pi/1000\,{\rm d}^{-1}$ corresponding to the fundamental Fourier frequency seen in the data, so we have $\omega_{\rm d} \simeq \omega_{\rm p} \simeq 2\pi/1000\,{\rm d}^{-1}$. As is clear from Eq. (7), the fundamental period $\simeq$1000 d and its first three harmonics, $\simeq$500, 333, and 250 d are present in period derivative residual variations. We note that the latter assumption requires the magnetic field of the star changes in time with period $P_{\rm d} \simeq 1000$ d close to the precession period $P_{\rm pre}$. We will get back to this point later. Therefore, Eq. (7) reduces to (for $\beta_{\rm p} = 0$ and $\beta_{\rm d} = 0$)

$$\begin{aligned}\Delta\dot{p} \simeq -\frac{P_0}{T}\theta\Big[& \cot\chi\Big(a_0\cos(2\pi t/1000) + a_1\cos(2\pi t/500)\Big) \\ & +\frac{\theta}{4}\Big(2a_0a_1\cos(6\pi t/1000) + a_1^2\cos(2\pi t/250)\Big)\Big], \qquad (8)\end{aligned}$$

where t measured in days. Here we ignore the 1000 d and 500 d contributions to the θ^2 order. For $|a_0| \ll |a_1|$, the 250 d-component will be comparable to the 500 d-component if we have $(a_1\theta/4)\tan\chi > 1$, or $\tan\chi > 4/(a_1\theta)$. For a small θ ($a_1 \geq 1$), one finds that the magnetic dipole moment must be nearly orthogonal to the symmetry axis $\boldsymbol{e}_3$. Hence for $\chi > 89°$ and $\omega_{\rm p}+\omega_{\rm d} \simeq 2\pi/500\,{\rm d}^{-1}$, the most dominant terms are the second and fourth harmonics, 500 d and 250 d, in good agreement with the observed data. Since the proposed inclination angle between the star's spin axis and the magnetic field's symmetry axis is nearly a right angle, $\chi > 89°$, one may consider the rotation of dipole vector as a magnetic poles reversal, see Sect. 3 for more discussion.

In Fig. 1, by using Eq. (8) we fit the observed Δp and $\Delta\dot{p}$ data (Stairs et al. 2000) with a precession period of $P_{\rm pre} \simeq 1015$ d, a wobble angle $\theta = 3.^{\circ}2$, and the inclination angle $\chi = 89°$ between the magnetic dipole and star's symmetry axis.

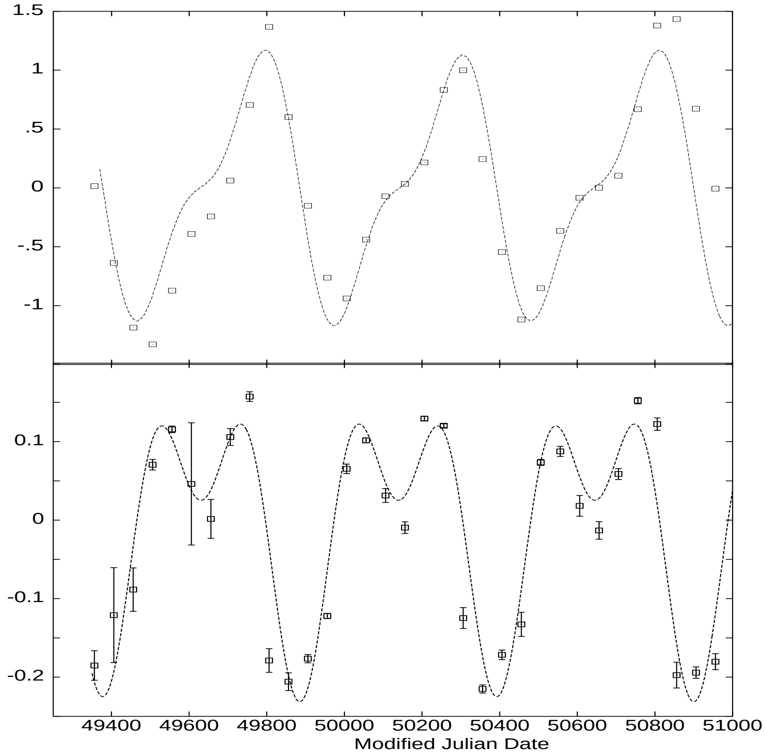

Fig. 1. Timing data for PSR B1828-11 (from Stairs et al. 2000). The top panel shows period residual Δp (in ns) relative to the star's secular spin-down. The bottom panel gives the time derivative $\Delta\dot{p}$ (in units of 10^{-15}). The solid curves are the fit explained in the text.

The Fourier expansion coefficients are $a_0 = .01$ and $a_1 = 1$. We note that our fit is indistinguishable from the one obtained by Link & Epstein (2001).

It is interesting to note that Eq. (8) includes 1000, 500, 333, and 250 d Fourier components. Further, by considering $n \geq 2$ terms in Eq. (5), one can find the higher harmonics in $\Delta\dot{p}$. These terms were missing in the Link & Epstein's model. In Table 1 we compare the time-varying magnetic field model with one suggested by Link & Epstein (2001), and the observations made by Stairs et al. (2000).

3. Discussion

In this paper, motivated by the effective core-crust decoupling during the precessional motion (Link & Cutler 2002), we considered the case of time-varying magnetic field of the star. Then we endeavored to find out under what condition a *time-varying* magnetic field is able to provide a consistent explanation for the reported PSR B1828-11 timing analysis by Stairs et al. (2000). We studied the free precession of spin axis of PSR B1828-11 under the magnetic radiation torque caused by an inclined time-varying magnetic dipole moment vector $\boldsymbol{m}(t)$, with *constant* inclination angle χ. In general, we assumed that the dipole field vector changes with time relative to the star's body axes. Then we expanded its components in terms of the Fourier expansion as $m_1 = \boldsymbol{m}\cdot\boldsymbol{e}_1 = m_0 \sin\chi \sum_{n=0} a_n \cos(n\omega_d + n\beta_d)$, $m_2 = \boldsymbol{m}\cdot\boldsymbol{e}_2 = m_0 \sin\chi \sum_{n=0} b_n \sin(n\omega_d + n\beta_d)$, and $m_3 = \boldsymbol{m}\cdot\boldsymbol{e}_3 = m_0 \sin\chi \sum_{n=0} c_n \cos(n\omega_d + n\beta_d)$, where a_n, b_n, and c_n were determined by fitting the data. Finally, we showed that if $P_d/P_{pre} \simeq (P_{pre}/508-1)^{-1}$, one may consistently explain the whole observed spectrum of the Fourier power analysis of PSR B1828-11. We find the same fit as obtained by Link & Epstein (2001) to the data with a wobble angle $\theta = 3.^\circ2$, the inclination angle $\chi = 89^\circ$ between the magnetic dipole and star's symmetry axis, but with a precession period of 1015 d, see Fig. 1. Note that the chosen Fourier expansion coefficients are $a_0 = .01$, $-b_1 = a_1 = 1 = c_0$, and $a_n = b_0 = b_n = c_{n-1} = 0$ for $n \geq 2$.

The time-varying magnetic field model can also explain the observed timing data for PSR B1642-03. The analysis of timing data of PSR B1642-03, collected over a span of 30 years, exhibit strong Fourier power at periods ≃5000, 2500, and 1250 days (Shabanova et al. 2001). The suggested wobble and magnetic field inclination angles are $\theta \simeq 0.^\circ8$ and $\chi \simeq 60^\circ$, respectively. Similar to PSR B1828-11, the spectra of PSR B1642-03 show wide spectral features at periods ≃2500 d and 1250 d. The pulse shape variations were not detected, probably, due to their small amplitudes. Furthermore, it is interesting to note that both PSR B1828-11 and PSR B1642-03 exhibit features around the sixth harmonic, $6\omega_p$, i.e. 167 d and 667 d, respectively. As seen from Eq. (5), by including the $n = 2$ term in the expansion, one can reasonably get these harmonics in $\Delta\dot{p}$. These terms were missing in previous studies.

The necessary equality relation between P_d and P_{pre} requires the magnetic poles at the surface of the star move with relative velocity $v_{rev} \sim 2\pi R/P_{pre} \sim 7\times10^{-2}$ cm s^{-1} respect to the body axes. On the other hand, the magnetic poles reverse every $P_{pre}/2 \sim 500$ d. Because of the very high electrical conductivity of the solid crust ($\sigma \sim 10^{26}$ s^{-1}), this result would

Table 1. In this table we compare the time-varying magnetic field model for PSR B1828-11 with one suggested by Link & Epstein (2001) and the observed data reported by Stairs et al. (2000). P_0, $P_{\rm pre}$, $\epsilon = P_0/P_{\rm pre}$, θ, and χ are the star's period, precession period, star's oblateness, wobble angle, and field's inclination angle, respectively.

Data/Model	$P_{\rm pre}$ (d)	ϵ	θ	χ
Data	$\simeq$1000, 500, 250, 167	4.7×10^{-9}	$\simeq3°$	>89°
Time-varying mag. model	$\simeq1000^a$, all harmonics	4.7×10^{-9}	$\simeq3°$	>89°
Link & Epstein's model	$\simeq500^a$, 250^b	9×10^{-9}	$\simeq3°$	>89°

[a]Fundamental.
[b]Harmonic.

be hardly acceptable[2]. Variation of the magnetic dipole vector with time in a neutron star (with solid crust and no active convective zone) may be understood through the so-called *neutron star crustal tectonics* scenario which has been proposed originally to explain magnetic dipole evolution and resulting observable features in millisecond pulsars, low-mass X-ray binaries and radio pulsars (Ruderman 1991a,b). The solid crustal lattice of neutron star is subject to various strong stresses. The pinned superfluid neutron star vortex lines exert a strong force on the crustal lattice of nuclei which pin them (Anderson & Itoh 1975; Ruderman 1976; Alpar et al. 1984). Further, the evolving core magnetic flux tubes which pass through the crust, pull it strongly at the base of the crust Srinivasan et al. (1990). In the rapidly rotating weakly magnetized neutron stars such as millisecond pulsars and low-mass X-ray binaries, the lattice stresses from pinned vortices are dominant, while in the older pulsars such as radio pulsars with strong magnetic fields, the magnetic stresses from core flux-tube displacement may become important. For PSR B1828-11 and PSR B1642-03 with magnetic field strength $B \sim 10^{12}$ G and effectively unpinned superfluid vortices (Link & Cutler 2002), the latter case is more appropriate. The quantized magnetic flux tubes in a core's type II superconducting proton sea terminate at the base of the crust. These flux tubes move in response to changes in the positions of neutron star's core superfluid. If the crust were to remain immobile the shear stress, $S(B)$, on the base of the highly conducting crust (from core magnetic flux tube motion) could grow to reach

$$S(B) \sim \frac{BB_{\rm c}}{8\pi} \sim \left(\frac{B}{3\times10^{12}\ {\rm G}}\right)\times10^{26}\ \ {\rm dyn\ cm^{-2}}. \qquad (9)$$

Here $B_{\rm c} \geq 10^{15}$ G is the average magnetic field in each core flux-tube, and B is the average magnetic field through the crust. The maximum stress that crust could bear before breaking depends on the lattice shear modulus and is calculated by Ruderman (1991a,b) as $S_{\rm max} \leq 10^{26}$ dyn cm^{-2} (for most stars $S_{\rm max} \sim 10^{23}$–10^{24} dyn cm^{-2}). If $S(B) > S_{\rm max}$, neutron stars with strongly magnetized cores would break their crust continually as they rotate. For PSR B1828-11 with $B \sim 5\times10^{12}$ G, we have $S(B) > S_{\rm max}$. Therefore, one would expect a continuous crust breaking and crustal plate motion in this pulsar[3]. In addition, the precessing crust drags the core magnetic flux tubes which are frozen into the core's fluid. This would increase core flux-tube displacements and then *increase* the characteristic velocity of conducting platelets. A typical characteristic velocity of a flux-tube array in the stellar core layer just below the core-crust interface is given by Ruderman et al. (1998) as $v_{\rm c} \sim (\omega_0/10\ {\rm Hz})(10^{12}\ {\rm G}/B)^{-1}\times10^{-7}$ cm s^{-1}. For PSR B1828-11, $v_{\rm c} \sim 3\times10^{-8}$ cm s^{-1} which is much smaller than the proposed relative velocity for magnetic poles by our calculations, $v_{\rm rev} \sim 2\pi R/P_{\rm pre} \sim 7\times10^{-2}$ cm s^{-1}. We note that in calculation of $v_{\rm c}$ the effect of precessional motion of the crust was not considered. By including the precessional effects, one may expect the value of flux-tube velocity $v_{\rm c}$ to increase significantly.

It is worth noting that the required torque variation may as well be that due to the internal torque, by the partially pinned vortices during the precessional motion of the crust. The internal torques would arise from different coupled components of the star which move with different velocities, e.g. the mutual friction torque which arise from different velocities of vortex lines and superfluid. These torques are able to sustain hydromagnetic shear flows and turbulences in the core-crust boundary, excite the fluid convection motions, and cause magnetic field variations (Malkus 1963, 1968). Further, they affect the motion of the neutron star crust, for example, by tilting away its angular velocity vector from alignment with star principle axis (Sedrakian et al. 1999). To find a clear picture of dynamics of magnetic field in a precessing neutron star, one has to consider the effect of the internal torques. This is currently under investigation (Rezania 2003).

Finally, in this paper we showed that a time-varying magnetic field model is able to explain consistently the timing analysis of both PSR B1828-11 and PSR B1642-03, if the field's symmetry axis rotates with a rate nearly equal to their

[2] Though such a short magnetic cycle has not been observed in neutron stars yet, the early observations of A-type stars (α-variables), with kilogauss magnetic field strength, showed large amplitude, nearly symmetric magnetic reversals in periods ranging from 4 to 9 days, close to the periods of the stars (Babcock 1958). Several recent observations from the young rapidly rotating stars confirmed the existence of the solar-type magnetic cycle with $P_0/P_{\rm cyc} \simeq 10^{-4}$ (Brandenburg et al. 1998, Kitchatinov et al. 2000). Of course these stars presumably have active convection zone, for the case PSR B1828-11 with proposed period for the magnetic cycle, we have $P_0/P_{\rm cyc} \simeq 10^{-9}$, which is smaller by 5 orders of magnitude relative to one obtained for the young rotating stars. This may agree with the fact that in neutron stars the convective fluid motions are hardly excited.

[3] It is interesting to note that according to the neutron star crustal tectonics scenario the magnetic fields in spinning down neutron stars move to achieve a right angle configuration relative to the star's spin axis (Ruderman 1991a,b). This is in agreement with our analysis for PSR B1828-11 as we found $\chi \geq 89°$.

precession rates, relative to the star's body axes. Unfortunately, at this stage, the large speed of the magnetic poles at the surface of the star required by this model is difficult to accept. Further studies on the evolution and dynamics of magnetic fields in precessing stars (especially the plate tectonics model) seem necessary. These will be left for future investigations.

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