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Fourier decomposition and frequency analysis of the pulsating stars with $P < 1$ d in the OGLE database

II. Multiperiodic RR Lyrae variables in the Galactic Bulge

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Abstract. We present the results of a systematic search for multiperiodic pulsators among the Galactic Bulge RR Lyrae stars of the OGLE-1 sample. We identify one "canonical" double-mode variable (RRd star) pulsating in two radial modes. In 38 stars we detect secondary periodicities very close to the primary pulsation frequency. This type of multiperiodic variables constitute ~23% of RRab and ~5% of RRc population of the Bulge. With the observed period ratios of 0.95−1.02 the secondary periods must correspond to nonradial modes of oscillation. Their beating with the primary (radial) pulsation leads to a long-term amplitude and phase modulation, known as the Blazhko effect. The Blazhko RRab variables occur more frequently in the Galactic Bulge than in the LMC. The opposite tendency is seen in case of the RRd stars. The differences of incidence rates are most likely caused by different metallicity of the two populations. We discuss pulsation properties of the OGLE-1 Blazhko stars and compare them with predictions of theoretical models. We argue, that the oblique magnetic pulsator model of Shibahashi (2000) cannot account for the observations and should be ruled out.

Key words. stars: horizontal branch – stars: oscillations – stars: variables: RR Lyr

1. Introduction

The Optical Gravitational Lensing Experiment (OGLE; Udalski et al. 1992) is devoted to search for dark matter in our galaxy through the detection of microlensing events. As a by-product of this program a large amount of photometric data has been collected for stars in the Galactic Bulge and Magellanic Clouds. In this series of papers we examine short period pulsating variables discovered in the OGLE-1 Galactic Bulge sample (Udalski et al. 1994, 1995, 1996, 1997). Paper I (Poretti 2001) was devoted to *monoperiodic* δ Scuti and RR Lyrae variables. In the current paper, second in the series, we discuss *multiperiodic* RR Lyrae stars.

The existence of multimode RR Lyrae variables has been known since the discovery of double-mode pulsations in AQ Leonis (Jerzykiewicz & Wenzel 1977). In following years about 300 similar variables have been identified in several stellar systems, including 181 stars in the Large Magellanic Cloud (see Kovács 2001 for comprehensive review). All these variables share common properties: they pulsate in *two radial modes*, namely fundamental and the first overtone, with the period ratio of P_1/P_0 = 0.742–0.747. These stars are commonly referred to as RRd variables.

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In recent years a new class of multiperiodic RR Lyrae variables has been identified (Olech et al. 1999). They are characterised by the presence of two or more very closely spaced frequencies, with period ratios in the range of 0.95−1.05. Such period ratios are incompatible with the radial mode oscillations and point strongly towards the presence of *nonradial* modes. This new form of multiperiodicity was first discovered in the RRc variables, but later was shown to occur in the RRab variables as well (Moskalik 2000). For the most complete summary of recent observations see Kovács (2002).

Massive photometry collected during microlensing surveys is particularly well suited for a systematic search for multiperiodic RR Lyrae stars. Such a search has already been performed in the Large Magellanic Cloud, using MACHO data (Alcock et al. 2000; Welch et al. 2002). In this paper we present a complete inventory of multiperiodic RR Lyrae stars in the OGLE-1 Galactic Bulge sample. Our primary goal is to establish the incidence rates of different types of multimode oscillations in the Galactic Bulge population. Comparison between the rates in the Bulge and in the LMC can shed new light on the physical conditions favouring such oscillations. We also discuss pulsation properties of the identified multiperiodic RR Lyrae stars. Preliminary results of this work have already been published by Moskalik & Poretti (2002). Here we present the complete discussion of our findings.

2. Search for multiperiodicity

The OGLE-1 database contains 215 RR Lyrae stars, 150 of them have been classified as RRab (fundamental-mode pulsators) and 65 as RRc (first overtone pulsators). The photometric data span ~900 days, with typically 130-150 I-band measurements published for each star. As a first step, the lightcurve is fitted with the Fourier sum of the form

$$m_I(t) = \langle m_I \rangle + \sum_k A_k \cos(2\pi k f_0 t + \phi_k). \quad (1)$$

The pulsation frequency $f_0 = 1/P_0$ is also adjusted in the fitting process. For several stars slow instrumental drifts are present in the data. These are modeled by a cosine term with period of 50 000 day added to Eq. (1).

In the next step, the search for additional periodicities is performed. This is done with two different, independently applied methods:

1. The fit Eq. (1) is subtracted from the data. The Fourier power spectrum of the residuals is then computed in order to reveal a secondary frequency, if present.
2. The fitting formula Eq. (1) is supplemented with an additional cosine term with frequency f_1. The data is then fitted for different trial values of f_1, keeping fixed the primary frequency f_0 and the number of its harmonics, but recalculating their amplitudes and phases for each trial. A secondary frequency, if present in the data, should reduce the dispersion of the fit in a significant way.

The two procedures give the same results for all the stars except one, strengthening our confidence in the frequency identifications. In the following, we accept as multiperiodic only those variables in which a secondary period is detected with both methods.

As a third step, the Fourier fit with two identified frequencies and *their linear combinations* is performed. To this effect, we fit the lightcurve with the following formula:

$$m_I(t) = \langle m_I \rangle + \sum_{k,n} A_{kn} \cos[2\pi(k f_0 + n f_1)t + \phi_{kn}]. \quad (2)$$

The search for additional periodicities is then repeated, again using both methods: Fourier transform of residuals (method 1) and minimization of dispersion with respect to a new trial frequency f_2 (method 2). The process is stopped when no significant new terms appear.

As an example, we show the application of the first procedure described above to the case of BW6 V20. In Fig. 1 we present the spectral window of the data. This window is typical for OGLE-1 photometry. Figure 2 shows consecutive steps of frequency analysis. The top panel displays the Fourier power spectrum of the original data. The blow-up of the neighbourhood of the dominant peak is shown in the right column. The second panel displays the power spectrum of data prewhitened by the first frequency and its harmonics. A new frequency has clearly emerged. The next prewhitening (panel three) reveals another frequency on the opposite side of the dominant peak. Finally, after prewhitening by three frequencies no more periodicities are detected (panel four). The analysis is stopped.

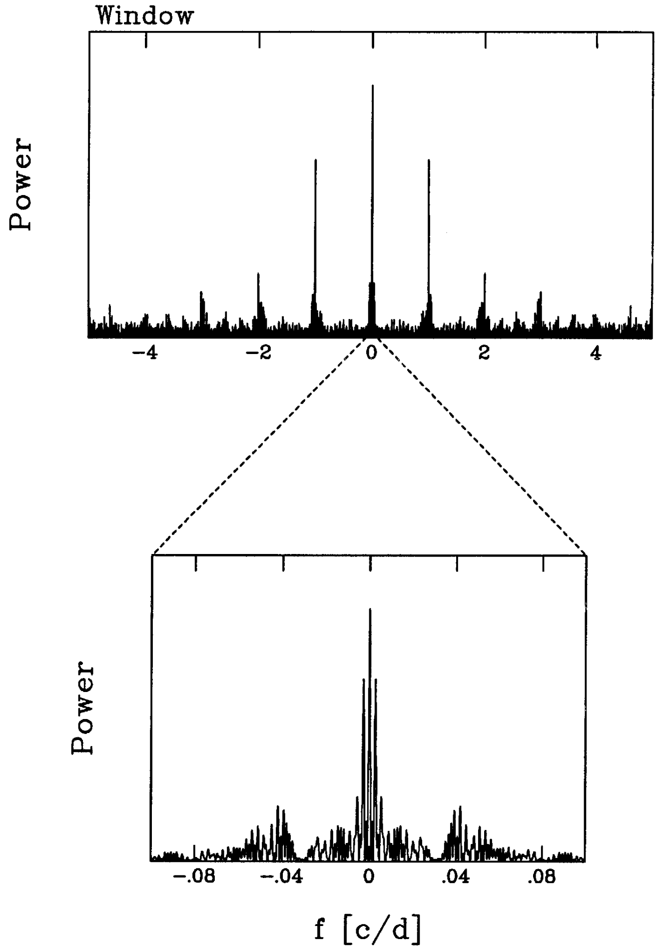

Fig. 1. Spectral window for BW6 V20 dataset.

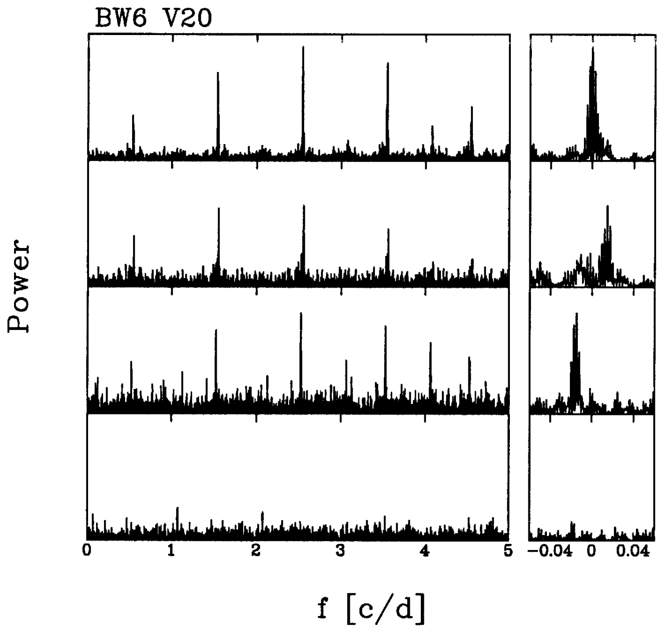

Fig. 2. Frequency analysis of BW6 V20. Subsequent panels show the power spectrum of the original data and the results of consecutive prewhitening steps. The right column of the plot displays fine structure around the main peak. The spectra are normalised separately in each panel.

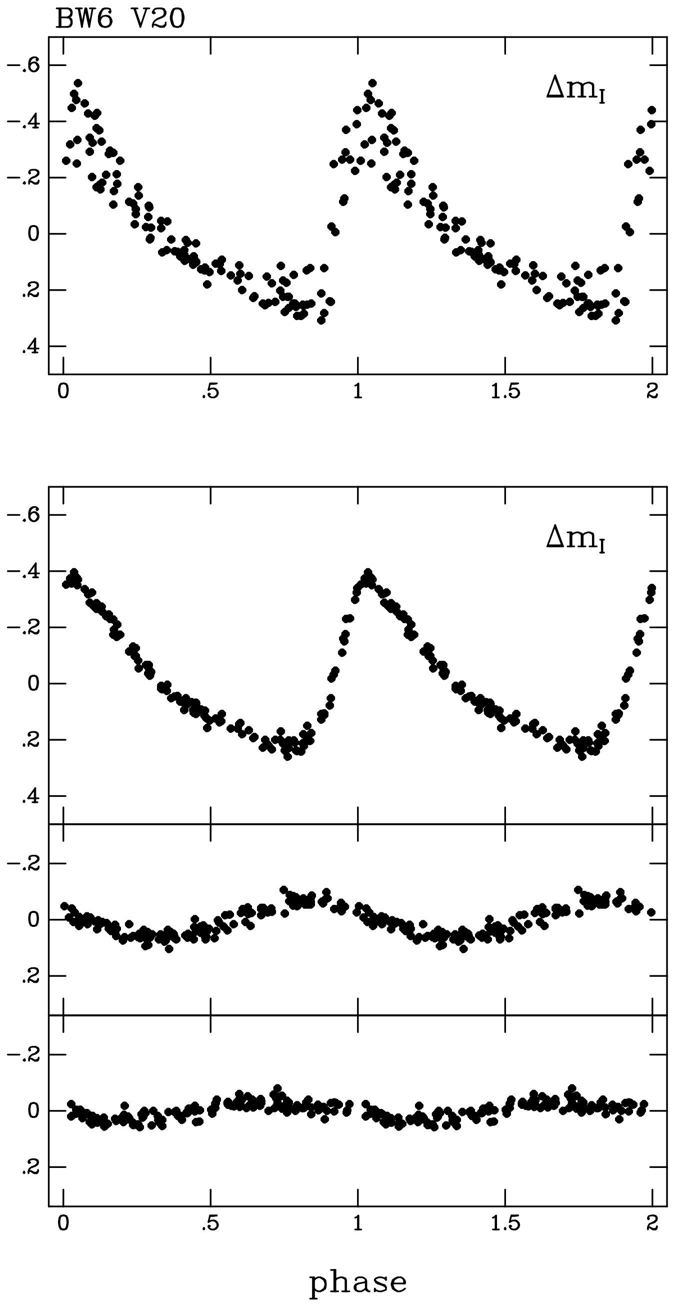

Fig. 3. Phased lightcurve (I-magnitude) of BW6 V20. *Top*: original data phased with the primary period. *Bottom*: lightcurves of the three detected frequencies, f_0 = 2.539947 c/d, f_1 = 2.554618 c/d and f_2 = 2.525276 c/d, respectively.

In Fig. 3 we display the phased lightcurve of BW6 V20, first for the original data (top panel), then separately for the three detected periods (three bottom panels).

3. Results

Most of the RR Lyrae stars in the OGLE-1 sample are strictly periodic. Those stars are discussed in Paper I. Departures from a monoperiodic behaviour have been detected in 49 RR Lyrae variables. They can be divided in several distinctive types.

Table 1. RR-ν1 variables of OGLE-1 sample.

OGLE No.	P_0 [d]	Δf [c/d]	A_0 [mag]	A_1 [mag]
		RR1-ν1		
BWC V47	0.256926	–0.02439	0.067	0.050
MM5A V20[a]	0.391192	0.04546	0.126	0.009
		RR0-ν1		
MM5A V46[a]	0.370187	0.02530	0.202	0.041
BW3 V17	0.403331	0.12005	0.258	0.013
BW11 V10	0.450658	0.01864	0.252	0.049
BW3 V13	0.457517	0.03255	0.230	0.042
BWC V41	0.462144	0.01767	0.221	0.041
BW11 V44	0.482493	0.03329	0.204	0.028
BW5 V34	0.490265	–0.00873	0.223	0.028
BW2 V23	0.492764	0.01198	0.244	0.044
BW7 V15[b]	0.497071	0.00941	0.226	0.035
BW7 V8	0.507103	–0.03312	0.244	0.024
BW7 V33	0.511226	–0.00689	0.261	0.030
MM5A V9	0.516204	0.03580	0.251	0.016
BW10 V21	0.521996	0.01740	0.136	0.042
MM5B V4	0.524886	0.02237	0.184	0.024
BW10 V20	0.527123	–0.02450	0.244	0.023
BW10 V44	0.527263	–0.01206	0.191	0.036
BW8 V15	0.577234	0.02543	0.167	0.028
BW5 V36[c]	0.594501	0.01445	0.139	0.015
BW2 V24	0.597426	0.01641	0.118	0.018
BWC V61	0.615946	0.02145	0.096	0.022
BW10 V41	0.626882	0.00855	0.215	0.049
BW1 V34	0.632719	0.01434	0.154	0.031
BWC V51	0.649494	0.01618	0.071	0.031
BW6 V17	0.651601	0.00545	0.146	0.022
BW10 V40	0.682741	0.01393	0.117	0.021

[a] Amplitude and/or phase of primary peak variable.
[b] Second harmonic of primary frequency variable.
[c] Amplitude and/or phase of secondary peak variable.

3.1. RRd stars: Variables with radial fundamental and first overtone modes

This type of oscillations has been detected in one object of the sample: BW7 V30. The star pulsates with $P_0 = 0\overset{d}{.}486889 \pm 0\overset{d}{.}000010$ and $P_1 = 0\overset{d}{.}36202176 \pm 0\overset{d}{.}00000083$. The resultant period ratio of P_1/P_0 = 0.743541 ± 0.000016 is typical for the RRd variables. The first overtone strongly dominates over the fundamental mode, with A_0/A_1 = 0.142. Two harmonics of the first overtone are detected, but no harmonics of the fundamental and no frequency combinations can be identified. This is perhaps not surprising, considering the extremely low amplitude of the fundamental mode (A_0 = 0.018 mag).

3.2. RR-ν1 stars: Blazhko variables with two closely spaced frequencies

In 38 RR Lyrae variables secondary peaks close to the dominant pulsation frequency are detected. They are well-resolved within our dataset and are not due to a secular period

Table 2. Amplitudes of Fourier components for selected RR-ν1 variables of OGLE-1 sample.

Frequency	BWC V47	MM5A V46	BW11 V10	BWC V41	BW3 V13	BW5 V34	MM5A V9
Δf	0.005	—	0.014	—	—	—	—
f_0	0.067	0.202	0.252	0.221	0.230	0.223	0.251
$f_0 + \Delta f$	0.050	0.041	0.049	0.041	0.042	0.028	0.016
$2f_0$	0.008	0.068	0.115	0.087	0.103	0.107	0.123
$2f_0 + \Delta f$	—	0.028	0.040	0.032	0.027	0.025	0.010
$3f_0$		0.021	0.072	0.044	0.062	0.072	0.080
$3f_0 + \Delta f$		0.011	0.033	0.022	0.022	0.024	—
$4f_0$			0.043	0.028	0.031	0.038	0.052
$4f_0 + \Delta f$			0.028	—	—	—	—
$5f_0$			0.027			0.021	0.031
$5f_0 + \Delta f$			0.024			—	—
$6f_0$			0.018			0.016	0.017
$6f_0 + \Delta f$			0.018			—	—
$7f_0$							0.010
$7f_0 + \Delta f$							—

	BW10 V21	BW10 V20	BW10 V44	BWC V61	BW10 V41	BWC V51	BW6 V17
Δf	—	—	—	—	—	—	—
f_0	0.136	0.244	0.191	0.096	0.215	0.071	0.146
$f_0 + \Delta f$	0.042	0.023	0.036	0.022	0.049	0.031	0.022
$2f_0$	0.066	0.118	0.083	0.034	0.100	0.019	0.068
$2f_0 + \Delta f$	0.029	0.022	0.024	0.014	0.032	0.009	0.010
$3f_0$	0.033	0.062	0.052	0.015	0.052	0.006	0.041
$3f_0 + \Delta f$	0.023	0.024	0.008	0.016	0.029	—	0.017
$4f_0$	0.013	0.036	0.028		0.035		0.019
$4f_0 + \Delta f$	—	0.016	0.021		—		0.010
$5f_0$		0.017					0.013
$5f_0 + \Delta f$		0.015					—
$6f_0$							0.007
$6f_0 + \Delta f$							—

variability. Their beating with the primary (radial) pulsation results in an apparent long-term amplitude and phase modulation, a phenomenon referred to as Blazhko effect.

In most cases (27 stars) only one secondary peak is present, forming a close *doublet* with the primary peak. Such a frequency pattern occurs in 2 RRc and in 25 RRab stars. Following Alcock et al. (2000), we denote these variables as RR1-ν1 and RR0-ν1, respectively. Their main properties (period P_0 and amplitude A_0 of the primary peak, frequency separation $\Delta f = f_1 - f_0$ and amplitude of the secondary peak A_1) are listed in Table 1. In most stars the frequency separation is between 0.008 c/d and 0.036 c/d, corresponding to a beat (Blazhko) period of 28−125 days. The amplitude of the secondary peak is always very small, never exceeding 0.05 mag. Contrary to the case of the LMC stars (Alcock et al. 2000; Welch et al. 2002), combination frequencies $kf_0 + f_1 = (k + 1)f_0 + \Delta f$ can be detected in about half of the RR-ν1 stars of the OGLE-1 sample. The amplitudes of all components identified in these variables are given in Table 2. The amplitudes and frequencies listed in Tables 1 and 2 are determined through a least–squares fit of Eq. (2) to the data.

3.3. RR-BL stars: Blazhko variables with three symmetrically spaced close frequencies

In several RR Lyrae variables *two* secondary frequencies, located on opposite sides of the primary peak, are present. Together with the primary frequency they form a close *equally spaced triplet*. Such a frequency pattern is found in 1 RRc and in 9 RRab stars. Following Alcock et al. (2000), we denote these variables as RR1-BL and RR0-BL, respectively. Their properties are listed in Table 3. The amplitudes of secondary peaks are somewhat larger than in the RR-ν1 stars, reaching up to 0.1 mag. The frequency separations Δf are similar to those observed in RR-ν1 stars. With the exception of one star, the triplets are *exactly equidistant*. This has been verified by fitting the data with Eq. (2), with all 3 frequencies of a triplet treated as independent parameters. The resulting separations $-\Delta f_- = -(f_- - f_0)$ and $\Delta f_+ = f_+ - f_0$ differ by less than 0.00005 c/d, which within our dataset is indistinguishable from zero. Thus, in the final solution $\Delta f_- = -\Delta f_+$ is assumed. The one deviating case is BW9 V24, where $\Delta f_- + \Delta f_+ = 0.00054$ c/d. We note, that in this particular star the triplet is barely resolved.

Table 3. RR-BL variables of OGLE-1 sample.

OGLE No.	P_0 [d]	Δf_- Δf_+ [c/d]	A_0 [mag]	A_- A_+ [mag]
		RR1-BL		
BW8 V34	0.320365	0.01544	0.127	0.030 0.016
		RR0-BL		
BW6 V20	0.393709	0.01467	0.244	0.031 0.065
BW9 V34	0.449297	0.00896	0.254	0.042 0.063
BWC V15	0.458717	0.00376	0.228	0.072 0.056
BW9 V24	0.476339	–0.00185 0.00239	0.206	0.021 0.103
BW6 V7	0.525015	0.03363	0.257	0.035 0.042
BW1 V18	0.529560	0.01876	0.250	0.032 0.034
BWC V33	0.550304	0.00865	0.175	0.020 0.034
BW6 V29	0.562906	0.02283	0.164	0.019 0.022
BW10 V66	0.581234	0.01410	0.156	0.030 0.053

Table 4. Amplitudes of Fourier components for RR-BL variables of OGLE-1 sample.

Frequency	BW8 V34	BW6 V20	BW9 V34	BWC V15	BW9 V24	BW6 V7	BW1 V18	BWC V33	BW10 V66
Δf	—	—	—	—	—	—	0.020	—	—
$f_0 - \Delta f$	0.030	0.031	0.042	0.072	0.020	0.035	0.032	0.020	0.030
f_0	0.127	0.244	0.254	0.228	0.206	0.257	0.250	0.175	0.156
$f_0 + \Delta f$	0.016	0.065	0.063	0.056	0.103	0.042	0.034	0.034	0.053
$2f_0 - \Delta f$	0.008	0.028	0.039	0.059	—	0.034	0.028	0.019	0.027
$2f_0$	0.014	0.104	0.103	0.081	0.068	0.124	0.127	0.087	0.062
$2f_0 + \Delta f$	—	0.042	0.051	0.044	0.066	0.038	0.020	0.033	0.035
$3f_0 - \Delta f$		0.021	0.030	0.035	—	0.029	0.019	—	0.016
$3f_0$		0.040	0.052	0.036	0.018	0.065	0.080	0.051	0.036
$3f_0 + \Delta f$		0.030	0.050	0.024	0.051	0.038	—	0.030	0.028
$4f_0 - \Delta f$		0.012	0.022	0.016	—	0.024	0.018	—	—
$4f_0$		0.022	0.020	0.022	—	0.038	0.049	0.033	0.017
$4f_0 + \Delta f$		0.029	0.027	0.014	0.026	0.030	—	0.023	0.019
$5f_0 - \Delta f$		—		0.011		0.015	—		—
$5f_0$		0.013		0.017		0.018	0.020		—
$5f_0 + \Delta f$		0.015		0.013		0.022	—		0.014

Since the A_- component in BW9 V24 is very weak, the insufficient resolution can have a significant effect on its frequency determination. The deviation from equidistant spacing seen in BW9 V24 needs to be confirmed with a longer dataset.

In all but one RR-BL stars we detect combination frequencies $kf_0 \pm \Delta f$. The amplitudes of all identified components are given in Table 4. The combination peaks, together with harmonics of f_0, form a sequence of equidistant triplets. This structure can be followed up to the third or fourth harmonic. We note in passing, that the observed frequency pattern is the same as found in several well-studied field RR Lyrae Blazhko variables (Borkowski 1980; Smith et al. 1994, 1999; Kovács 1995; Nagy 1998; Szeidl & Kolláth 2000).

3.4. RR-ν2 stars: Blazhko variables with three nonequidistant close frequencies

Yet another pulsation patter is found in BWC V106. In this star, two secondary frequencies are detected, but both located on *the same side* of the primary peak. Together with the primary frequency they form a close triplet, which is *neither equally spaced, nor centered on the primary*. Following

Alcock et al. (2000) we classify BWC V106 as an RR0-ν2-type variable. This type of pulsation pattern is extremely rare, only 3 similar stars have been identified among 1327 first overtone RR Lyrae pulsators in the LMC (Alcock et al. 2000). The properties of BWC V106 are listed in Table 5. No combination peaks are detected in this star.

3.5. Miscellaneous RR Lyrae variables

In 10 RR Lyrae stars we find after prewhitening a significant residual power at a frequency almost identical to that of the (just removed) primary component. Such behaviour is a signature of a slow phase and/or amplitude variability, not resolved within the length of available data (~900 days for OGLE-1).

For a closer examination of the long-term variations in these objects we have performed Fourier fits of their lightcurves *separately for each observing season*. In 6 stars, all pulsating in the first overtone, we find that the pulsation phase varies from year to year, but the amplitude of the pulsation remains constant. These stars are listed in Table 6. Clearly, they undergo a secular period change. Assuming that the period varies linearly in time, we can estimate for each object the *slowest* $\mathrm{d}P/\mathrm{d}t$ consistent with the data. We show them in the last two columns of Table 6. We stress that the numbers derived here are based on three observing seasons only and therefore should be treated as preliminary. Nevertheless, we can conclude that in all 6 cases the pulsation period changes on a timescale of no more than $\sim 10^4$ yr, which is much shorter than the timescale predicted by the theory of stellar evolution.

In the remaining 4 stars (1 RRc and 3 RRab-type) *both* amplitude and phase vary from year to year. We list these objects in Table 7, where in the last two columns we give the values of the Fourier amplitude A_1 and phase ϕ_1 (see Eq. (1)) for each observing season. Seasonal lightcurves of the stars are displayed in Fig. 4. The amplitude variability is hardly noticeable in the RRc star BW11 V34, but in the RRab stars it is rather obvious. In the extreme case of BW5 V13 the amplitude increases by a factor of five within one year. The change of the pulsation amplitude is accompanied by a change of phase and change of shape of the lightcurve. This last effect is most noticeable in BW4 V5 and BW6 V18. All 4 stars are most likely Blazhko variables with modulation period (beat period between the modes) longer than the span of available data. However, with the data in hand we are not able to make any firm statement about the nature of the observed long term behaviour. In the following discussion we will treat all 4 variables as "nonclassified".

Table 5. RR0-ν2 variable of OGLE-1 sample.

OGLE No.	P_0 [d]	Δf [c/d]	A_0 [mag]	A_1 [mag]
BWC V106	0.464967	0.00708	0.188	0.020
		0.02675		0.020

Table 6. Variable period RRc stars of OGLE-1 sample.

OGLE No.	P_0 [d]	A_0 [mag]	$\dot{P}$ [s/s]	$P/\dot{P}$ [yr]
BWC V37	0.380122	0.140	1.7×10^{-7}	6.1×10^{3}
BW2 V10	0.507782	0.118	3.1×10^{-7}	4.5×10^{3}
BW7 V51	0.272148	0.154	2.9×10^{-8}	2.5×10^{4}
BW9 V38	0.305731	0.117	-4.0×10^{-8}	-2.1×10^{4}
BW11 V55	0.286179	0.123	-5.1×10^{-8}	-1.5×10^{4}
MM5A V24	0.379396	0.093	-5.4×10^{-8}	-1.9×10^{4}

Table 7. Suspected Blazhko variables of OGLE-1 sample.

OGLE No.	P_0 [d]	A_1 [mag]	ϕ_1 [rad]
		RRc	
BW11 V34	0.341674	0.139 ± 0.002	0.00 ± 0.02
		0.122 ± 0.002	0.14 ± 0.02
		0.119 ± 0.003	0.03 ± 0.02
		RRab	
BW4 V5	0.474673	0.167 ± 0.013	0.00 ± 0.05
		0.223 ± 0.004	0.13 ± 0.02
		0.182 ± 0.003	–0.04 ± 0.02
BW5 V13	0.494952	0.065 ± 0.006	0.00 ± 0.10
		0.326 ± 0.006	–1.42 ± 0.04
		0.286 ± 0.005	–1.01 ± 0.02
BW6 V18	0.541402	0.290 ± 0.014	0.00 ± 0.03
		0.199 ± 0.004	–0.22 ± 0.02
		0.205 ± 0.005	–0.24 ± 0.03

4. Discussion

4.1. Incidence rates

Tables 8 and 9 present the inventory of different types of RR Lyrae variables in the OGLE-1 Galactic Bulge sample, separately for the overtone and for the fundamental mode pulsators. The canonical double mode variables (RRd stars) are somewhat arbitrarily included into the table for overtone variables. This is justified by the strong dominance of the overtone mode in these stars – as a result they are usually disguised, until full frequency analysis, as RRc stars. For each type of variability we list the number of stars found (third column) and the estimated incidence rate in the population (fourth column). The standard deviations of the incidence rates are calculated assuming Poisson distribution. For the purpose of comparison, we also show the corresponding rates for the LMC sample (Alcock et al. 2000; Welch et al. 2002), these are listed in the last column of the tables. The LMC rates have not been published for all the subclasses; the missing entries in the tables are raplaced by question marks.
Several important conclusions can be drawn from data of Tables 8 and 9:

1. Blazhko variables (RR-ν1, RR-ν2, RR-BL) occur more frequently among fundamental-mode pulsators (RRab) than among overtone pulsators (RRc). The property is common

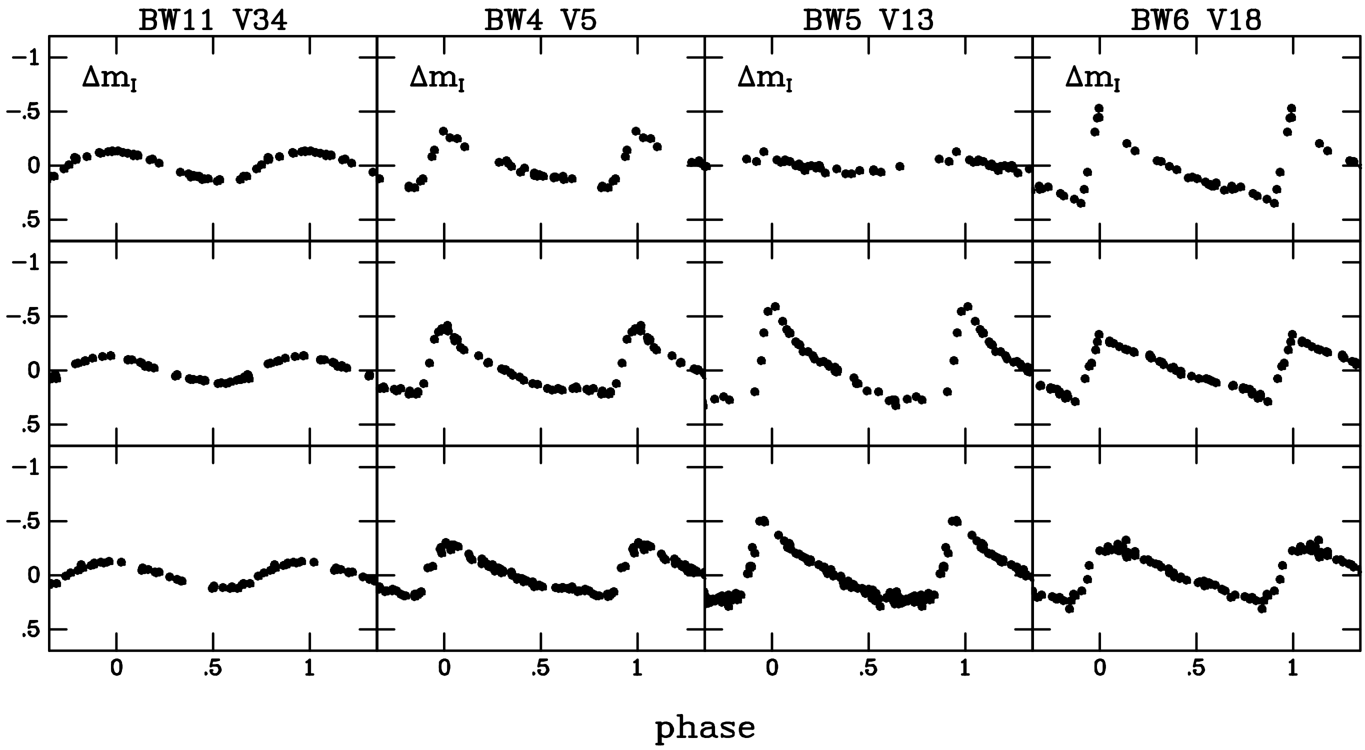

Fig. 4. Seasonal lightcurves (*I*-magnitude) of suspected Blazhko variables. Panels from top to bottom correspond to observing seasons of 1992, 1993 and 1994, respectively.

Table 8. Variable types in OGLE-1 Galactic Bulge sample: first overtone RR Lyrae stars (RRc stars).

Type	Description	Number	%	LMC %
RRc	single-mode	54	83.1 ± 11.3	69.0 ± 1.3
RRd	F/1H double mode	1	1.5 ± 1.5	13.6 ± 0.9
RR1-ν1	two close components	2	3.1 ± 2.2	1.8 ± 0.4
RR1-BL	symmetric triplet	1	1.5 ± 1.5	2.1 ± 0.4
RRc-PC	period change	6	9.2 ± 3.8	10.6 ± 0.9
RRc-NC	nonclassified	1	1.5 ± 1.5	0.5 ± 0.2

Table 9. Variable types in OGLE-1 Galactic Bulge sample: fundamental Mode RR Lyrae stars (RRab stars).

Type	Description	Number	%	LMC %
RRab	single-mode	112	74.7 ± 7.1	?
RR0-ν1	two close components	25	16.7 ± 3.3	6.5 ± 0.7
RR0-ν2	three close components	1	0.7 ± 0.7	?
RR0-BL	symmetric triplet	9	6.0 ± 2.0	3.7 ± 0.5
RRab-PC	period change	–	——	?
RRab-NC	nonclassified	3	2.0 ± 1.2	?

for both Galactic Bulge and LMC populations, being somewhat stronger in the Bulge.

2. For the RRc stars, the fraction of Blazhko variables is (within statistical error) the same in the Galactic Bulge and in the LMC. In the case of the RRab stars, however, the fraction is *twice higher in the Bulge* (23.3 ± 3.9% vs. 10.2 ± 0.8%).

This difference *does not* result from the lower noise level of the Galactic Bulge data. The weakest secondary peaks detected in the LMC stars have amplitudes of ~0.020 mag. Adopting the same threshold for our sample, we are still left with 31 RRab Blazhko stars, which is 20.7 ± 3.7% of the RRab population. We conclude, that the higher incidence rate of RRab Blazhko stars in the Galactic Bulge, as compared to the LMC, is real. It is tempting to speculate that this effect is caused by a difference in metallicity of the two populations.

3. The RRd variables are *significantly less common* in the Galactic Bulge than in the LMC. Assuming the same incidence rate as in the LMC, we would expect to find at least 7 variables of this type among 65 RRc stars of the Bulge (3σ lower limit). Instead, we find only one. Again, a different metallicity of the two populations is the likely culprit.
4. A secular period change is detected in about 10% of all Galactic Bulge RRc stars. This is the same fraction as for the RRc stars in the LMC. Interestingly enough, no RRab stars in the Bulge show this type of behaviour. We remind here, that the observed period variability is too fast to be of evolutionary nature. Its origin is currently unknown.

4.2. Pulsation properties of RRab Blazhko stars

With the sample of 35 fundamental mode Blazhko stars, we are in a position to discuss the group properties of this type of variables. In Fig. 5 we display the Period–Amplitude diagram for the Galactic Bulge RRab stars. In the upper panel we plot the peak-to-peak amplitude, as given by OGLE-1 catalog. In the lower panel we plot the Fourier amplitude of the radial mode, as determined by the Fourier fit Eq. (1) or Eq. (2). Monoperiodic variables are marked by asterisks, the Blazhko variables by open circles (RR0-ν1 and RR0-ν2) and by filled circles (RR0-BL). It is immediately apparent that the presence of secondary frequencies neither depends on nor affects the amplitude of the primary (radial) pulsation. It does not depend on the pulsation period, either. The stars with close frequency doublets (RR0-ν1 variables) are detected with roughly the same probability at all periods represented in the sample. The occurrence of RR0-BL variables, however, seems to be limited to $P < 0\overset{\rm d}{.}6$. Nevertheless, the overall fraction of Blazhko variables is essentially period independent. This conclusion is also supported by the histogram shown in Fig. 6. The fraction of Blazhko variables in each period bin is within 1σ consistent with the average value of 23.3%.

In Fig. 7 we show the frequency separation $\Delta f = f_1 - f_0$ for RRab Blazhko variables of our sample. Δf displays no trend with the radial mode period P_0. For 80% of RR0-ν1 stars frequency separation is positive, corresponding to the secondary frequency being *higher* than the primary one. An identical distribution of Δf has also been found for RRab stars in the LMC (Welch et al. 2002). The negative values of Δf occur only in the narrow range of periods between $0\overset{\rm d}{.}49$ and $0\overset{\rm d}{.}53$. Interestingly, this particular period range is avoided by RR0-BL stars. The values of Δf for triplets (RR0-BL stars) are on average slightly smaller than for the doublets (RR0-ν1 stars). In 8 out of 9 RR0-BL stars the higher amplitude secondary peak has also higher frequency. Again, very similar statistics have been found for the RRab stars in the LMC.

While the Δf distribution for the RRab stars is very similar in both galaxies, it is distinctively different from that for the RRc stars of the LMC (Alcock et al. 2000). In the latter case, the frequency separation is negative in more than half of the

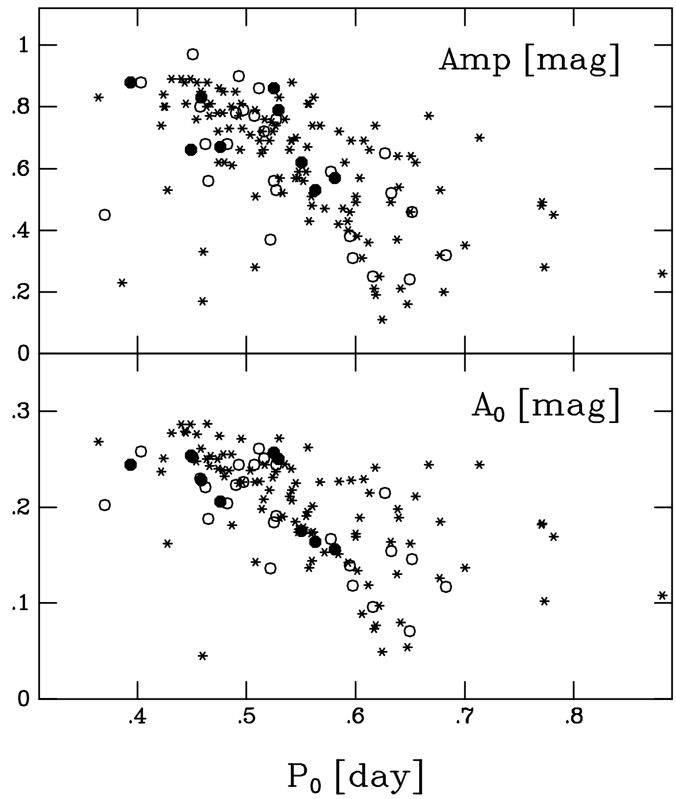

Fig. 5. Period-amplitude diagram for OGLE-1 RRab stars. *Top*: peak-to-peak amplitudes from OGLE-1 catalog, *bottom*: Fourier amplitudes of the radial mode. Monoperiodic variables marked by asterisks, Blazhko variables by open circles (RR0-ν1 and RR0-ν2) and filled circles (RR0-BL).

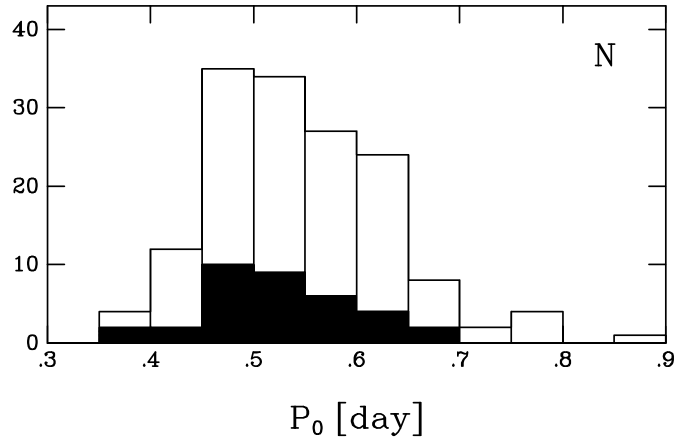

Fig. 6. Period distribution of OGLE-1 RRab stars. Black area corresponds to Blazhko variables.

doublets (RR1-ν1 stars) and the higher amplitude secondary peak in the triplets (RR1-BL stars) can occur on either side of the primary peak, with about equal probability. The values of $|\Delta f|$ in the LMC RRc stars are also noticably *larger* than in the RRab stars. Unfortunately, the number of RRc Blazhko variables in the Galactic Bulge is too small for a meaningfull discussion. Nevertheless, even for this very limited sample we find the same result: the average Δf is larger than for the RRab stars. Apparently, whatever mechanism is responsible for the

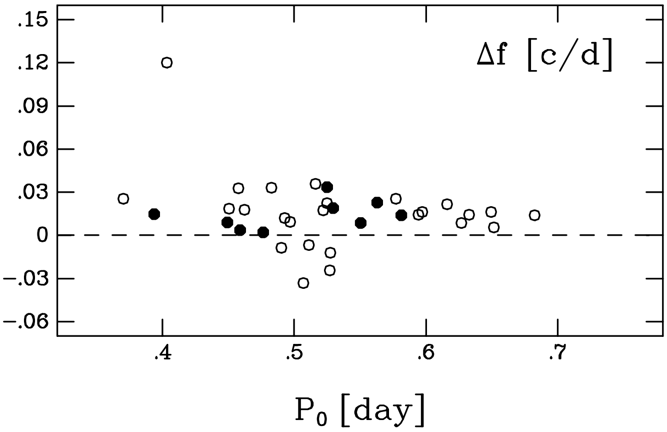

Fig. 7. Frequency difference, Δf, for OGLE-1 RRab Blazhko variables. Same symbols as in Fig. 5.

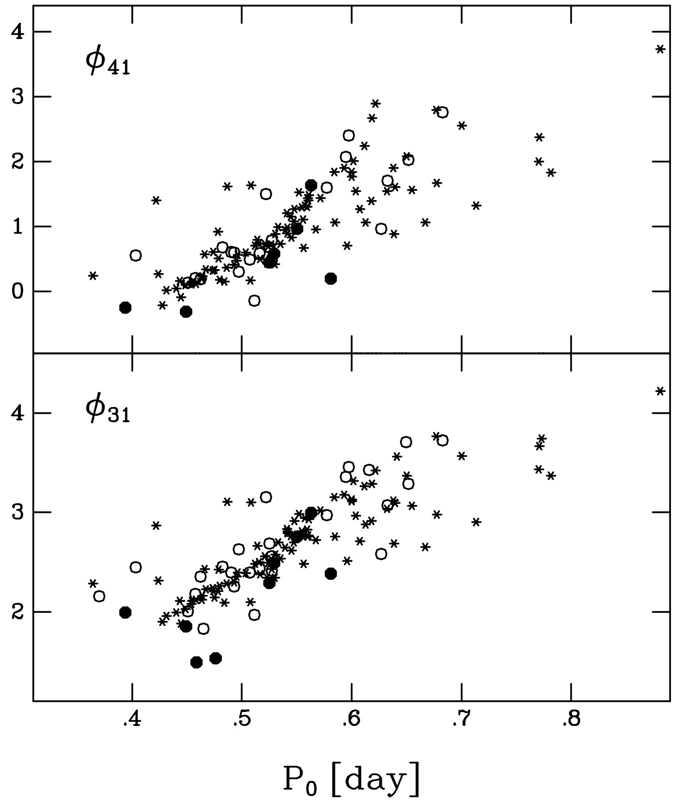

Fig. 8. Fourier phases ϕ_{31} and ϕ_{41} for OGLE-1 RRab stars. Same symbols as in Fig. 5.

excitation of the secondary frequencies, it works differently in the overtone and in the fundamental mode RR Lyrae pulsators.

4.3. Blazhko variables and metallicity

In Fig. 8 we display the Fourier phases ϕ_{31} and ϕ_{41} for all the Galactic Bulge RRab stars. This plot for the monoperiodic pulsators has already been presented in Paper I (their Fig. 12), here we have added the multiperiodic (Blazhko) stars. In the case of multiperiodic stars, the Fourier phases are derived for the primary (radial) component of the pulsation. Specifically, they are constructed with the phases of the kf_0 components of the Fourier fit Eq. (2). We stress that this is a different procedure from that used by Jurcsik et al. (2002), who have evaluated the Fourier parameters of the snapshot lightcurves at various phases of the beat (Blazhko) period. As a result, their parameters are time dependent. The phases derived in the current paper do not vary in time. They describe the shape of the primary (radial) pulsation *after secondary frequencies have been filtered out*.

In the case of monoperiodic stars the Fourier phases ϕ_{31} and ϕ_{41} follow a single progression up to $P \sim 0\overset{d}{.}55$. At longer periods the progression splits into three separate tails. As has been shown in Paper I, these tails are formed by stars of different metallicity, with the upper tail corresponding to the highest and the lower tail to the lowest value of [Fe/H].

Figure 8 shows that the Blazhko variables of our sample choose preferentially the *upper tail* of the progression. This is best visible in the plot of ϕ_{31}. Out of 11 Blazhko stars with $P > 0\overset{d}{.}55$, 7 fall on the upper tail, while only 2 belong to the central and to the lower tails, each. In the ϕ_{41} plot, these variables are split between the tails *in the same way*. The distribution of Fourier phases implies that the incidence rate of Blazhko variables increases with metallicity. We note, that the same dependence has already been suggested by comparison between the Galactic Bulge and the LMC (see Sect. 4.1).

5. Conclusions

We have conducted a systematic search for multiperiodic pulsators in the OGLE-1 sample of Galactic Bulge RR Lyrae stars. Multiperiodicity has been established for 39 variables. In addition, we have identified 6 RRc stars with changing pulsation period and 4 stars exhibiting long-term change of pulsation amplitude.

Among 39 multiperiodic variables only one is a "canonical" double-mode pulsator (RRd stars), with two radial modes – fundamental and first overtone – excited. In the remaining 38 variables a different type of multiperiodicity is found – additional peaks are present very close to the primary pulsation frequency. Their beating with the primary (radial) pulsation results in an apparent long term amplitude and phase modulation. This phenomenon is referred to as Blazhko effect. The majority of the Blazhko variables come in one of two flavours: we detect either a single secondary peak, forming a *doublet* with the primary frequency (RR-ν1 stars), or a pair of secondary peaks, which together with the primary frequency form an *equidistant triplet* centered on the primary peak (RR-BL stars). While the frequency triplet can result from periodic amplitude and/or phase modulation of a purely radial pulsation, such a process cannot produce a doublet. The observed period ratios of $P_1/P_0 = 0.95{-}1.02$ are not compatible with the excitation of two radial modes. This implies that a secondary component of the doublet must correspond to a *nonradial mode of oscillation*.

The Blazhko variables occur among both RRc (overtone) and RRab (fundamental-mode) stars, but the incidence rate is not the same. In the case of the Galactic Bulge it is 5% for the RRc stars and 23% for the RRab stars.

The incidence rates of multiperiodic RR Lyrae variables differ between the Galactic Bulge and the Large Magellanic Cloud. Specifically, the double-mode (RRd) stars occur much more frequently in the LMC, while the opposite is true for the RRab Blazhko stars. We believe that the effect is caused by a different metallicity of the two populations. Such an interpretation is supported by the Fourier analysis of the Bulge RRab stars. The hypothesis of metallicity influence can be further tested when statistics of multiperiodic variables in the SMC are established.

The occurrence of Blazhko variability among RRab stars correlates neither with amplitude nor with period of the primary pulsation. In the vast majority of doublets (RR0-ν1 stars) the secondary frequency is higher than the primary one. The same is also true for the higher of the two secondary peaks in the triplets (RR0-BL stars). This is a different distribution than is observed in the case of the RRc stars (Alcock et al. 2000). The values of the frequency separation Δf are also different, being systematically smaller than in the RRc stars. Clearly, the detailed picture of the Blazhko variability *depends on the primary mode of pulsations*.

The observational properties discussed in the current paper put constraints on any proposed explanation of the Blazhko effect. Two models are most popular nowadays: the oblique magnetic pulsator model (Shibahashi 1995, 2000) and the 1:1 resonance model (Nowakowski & Dziembowski 2001). The results presented here seem to rule out the first of the two. The oblique pulsator model predicts splitting the pulsation frequency of a mode into an equally spaced *quintuplet*. We never detect such a structure, despite specifically looking for it. We see only triplets or doublets. The model cannot explain a striking difference of the Blazhko incidence rate between RRc and RRab stars. If the modulation is indeed due to presence of the magnetic field, then it should occur equally likely independently of the pulsation mode. Finally, as the pulsation amplitude is supposed to vary with the rotation period of the stars, it is hard to understand why Blazhko periods of RRc stars are systematically shorter than those of the RRab stars.

The resonant model avoids all of the above difficulties. It generates equally split triplets, not quintuplets. It can explain naturally the asymmetry of amplitudes in the triplet. The model predicts a higher incidence rate of Blazhko variables among RRab than among RRc stars (Dziembowski & Cassisi 1999). This is in qualitative (but not quantitative!) agreement with the observations. The modulation period is not directly related to the rotation period of the stars, but is determined by the interplay between the frequency spacing and rotational splitting of the nonradial modes involved in the interaction. As such, it can be different for the overtone and for the fundamental-mode pulsators.

The outstanding difficulty of the resonant model is explaining the existence of frequency doublets. In some cases, the observed doublet can actually be a triplet, with one sidepeak too weak to be detected. For several stars, however, such an explanation would require an extreme amplitude asymmetry in order to be valid. For example, we find 3σ upper limits of $A_+/A_- < 0.08$ for BWC V47 (RRc star) and $A_-/A_+ < 0.19$ for BWC V51 (RRab star). It is not clear at present if so strongly asymmetric triplets (or pure doublets) can be reproduced by the resonant mode coupling theory.

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References

Alcock, C., Allsman, R., Alves, D. R., et al. 2000, ApJ, 542, 257
Borkowski, K. J., 1980, Acta Astron., 30, 393
Dziembowski, W. A., & Cassisi, S. 1999, Acta Astron., 49, 371
Jerzykiewicz, M., & Wenzel, W. 1977, Acta Astron., 27, 35
Jurcsik, J., Benkó. J. M., & Szeidl, B. 2002, A&A, 390, 133
Kovács, G. 1995, A&A, 295, 693
Kovács, G. 2001, in Stellar Pulsation - Nonlinear Studies, ed. M. Takeuti, & D. D. Sasselov, Astrophysics and Space Science Library, 257, 61
Kovács, G. 2002, in Radial and Nonradial Pulsations as Probes of Stellar Physics, ed. C. Aerts, & T. R. Bedding, ASP Conf. Ser., 259, 396
Moskalik, P. 2000, in The Impact of Large-Scale Surveys on Pulsating Star Research, ed. L. Szabados, & W. Kurtz, ASP Conf. Ser., 203, 315
Moskalik, P., & Poretti, E. 2002, in Radial and Nonradial Pulsations as Probes of Stellar Physics, ed. C. Aerts, & T. R. Bedding, ASP Conf. Ser., 259, 392
Nagy, A. 1998, A&A, 339, 440
Nowakowski, R. M., & Dziembowski, W. A. 2001, Acta Astron., 51, 5
Olech, A., Kałużny, J., Thompson, B., et al. 1999, AJ, 118, 442
Poretti, E. 2001, A&A, 371, 986 (Paper I)
Shibahashi, H. 1995, in GONG'94: Helio- and Asteroseismology from the Earth and Space, ed. R. K. Ulrich, E. J. Rhodes Jr., & W. Däppen, ASP Conf. Ser., 76, 618
Shibahashi, H. 2000, in The Impact of Large-Scale Surveys on Pulsating Star Research, ed. L. Szabados, & W. Kurtz, ASP Conf. Ser., 203, 299
Smith, H. A., Matthews, J. M., Lee, K. M., et al. 1994, AJ, 107, 679
Smith, H. A., Barnett, M., Silbermann, N. A., & Gay, P. 1999, AJ, 118, 572
Szeidl, B., & Kolláth, Z. 2000, in The Impact of Large-Scale Surveys on Pulsating Star Research, ed. L. Szabados, & W. Kurtz, ASP Conf. Ser., 203, 281
Udalski, A., Szymański, M., Kałużny, J., Kubiak, M., & Mateo, M. 1992, Acta Astron., 42, 253
Udalski, A., Kubiak, M., Szymański, M., et al. 1994, Acta Astron., 44, 317
Udalski, A., Olech, A., Szymański, M., et al. 1995, Acta Astron., 45, 433
Udalski, A., Olech, A., Szymański, M., et al. 1996, Acta Astron., 46, 51
Udalski, A., Olech, A., Szymański, M., et al. 1997, Acta Astron., 47, 1
Welch, D. L., Kovács, G., Cook, K. H., et al. 2002, in Radial and Nonradial Pulsations as Probes of Stellar Physics, ed. C. Aerts, T. R. Bedding, & J. Christensen-Dalsgaard, ASP Conf. Ser., 259, 412