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An extension of Herschel's method for dense and extensive catalogues

Application to the determination of solar motion

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Abstract. We present a way of handling and interpreting stellar proper motions, in which the idea of substituting them by great circles, proposed by Herschel in 1783, allows us to take the entire celestial sphere as a field of application, making possible the study of dense and extensive stellar zones. A suitable mathematical processing of data together with Herschel's method make up a good tool in order to search for and detect associations, clusters, systematic patterns of motion and to go deeper in the local galactic kinematic. In this paper, we explain our vision of Herschel's method, illustrating it with some practical examples. The particular case of solar motion is developed in a specific and complete way, using the whole Hipparcos catalogue, Herschel's method and our own mathematical tool, originally designed to determine systematic trends in plates or between catalogues. The solar apex and velocity obtained here are expressed as a continuous function, depending on the distribution of the stellar spectral types.

Key words. astrometry – galaxies: stars clusters – methods: data analysis – stars: kinematics

1. Introduction

The proper motion is defined by the projection of the real motion of stars over the celestial sphere. It is calculated from observations but, in general, the range of collected data epochs are insignificant in comparison with the time scale of the proper motion. So, we can say that the proper motion is simply an approximate representation of the linear spatial motion. This led to the idea of representing it as a motion over a great circle of the celestial sphere. This is not a new idea since in 1783 Herschel used it to determine the solar motion by selecting 12 stars in the solar vicinity and interpreting the intersection of their great circles as the solar apex (Trumpler & Weaver 1953).

More recent examples includes Schwan (1991), who selected the stars of the open stellar cluster Hyades and calculated their apex based on the great circles and their intersections. Agekyan & Popovich (1993) who employed the poles of the great circles to detect the solar motion using the Luyten Catalogue (1976) stars with $\mu \geq 0.2''$, even though Jaschek & Valbousquet in 1992 had found that the poles of 5800 stars from the Bright Star Catalogue (Hoffleit & Jaschek 1982) were rather randomly distributed, he argued that solar motion was not clearly perceptible. Other applications of the great circles include the study of the Coma Berenices open cluster by Abad (1996).

We think that Herschel's method deserves major attention since it has been exclusively used to verify facts with preselected stars. The advantage of the method consists of the possibility to visualize the proper motion over the entire sphere in different forms: first, graphically, using the path of the stars; second, using the intersection of those paths and viewing them in density diagrams, and third, more complete, using the poles of paths, taking advantage of the bi-univoque correspondence for each star between position and motion, and the pole. We consider that this pole is extremely important in our procedures because, being only one point, it represents a position plus a motion, simplifying the work with dense and extensive stellar catalogues. In the pole the motion direction is represented without considering, temporally, the proper motion modulus. It is thus excluded from this work.

Statistical or numerical tools and Herschel's method complement each other. The order of application will be given by

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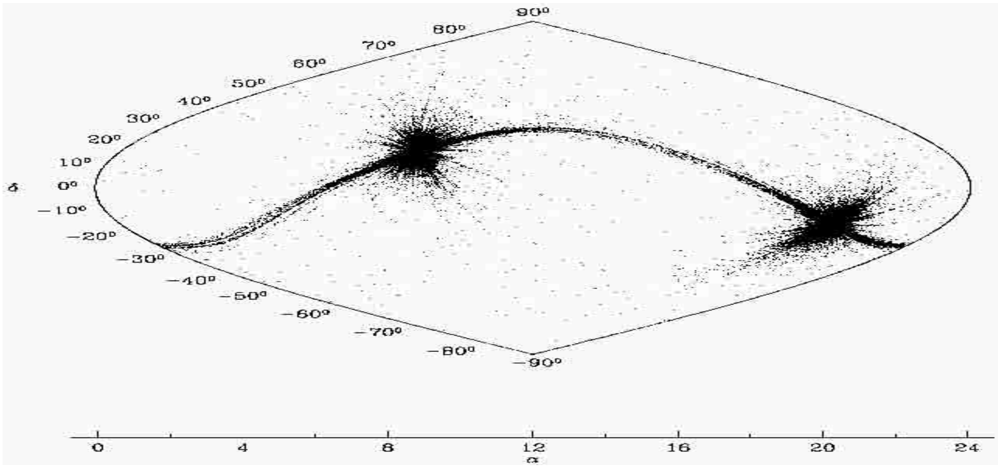

Fig. 1. Paths intersections of all stars in a 2° × 2° area from Wang et al. Catalogue, containing the Praesepe open cluster. A preferred direction is clear.

the original data and the goals pursued. In the specific work about solar motion presented here, the numerical tool applied to the original data is followed by Herschel's method applied to the results already obtained, while in most of the examples in this paper, the order of application is the opposite, beginning with Herschel's method on the catalogue data. Other applications of this method will be published soon.

2. The Convergent Point Method (CPM)

The motion of a star can be associated with a great circle of the celestial sphere or *path*, defined by the direction of the proper motion of the star. Two different paths will intersect at two opposite points in the sky. Vector cross and dot products of stellar positions make the mathematical description of paths and intersection points simple.

Let $\boldsymbol{V} = (x, y, z)$ be a unit vector representing the position of a star whose coordinates on the sphere are (α, δ), and $\boldsymbol{V}_t = (x_t, y_t, z_t)$ another unit vector representing the position t years later, which is obtained from the star proper motion. The path, seen as the intersection of a plane with the celestial sphere, can be described by the normal vector to this plane, henceforth the *pole*. It is defined by the cross product $\boldsymbol{W} = \boldsymbol{V} \times \boldsymbol{V}_t$. The intersection between two planes or paths is defined by the cross product of their respective poles, $\boldsymbol{Z} = \boldsymbol{W}_1 \times \boldsymbol{W}_2$, where the orientation of this vector depends on the order the product is made.

Whenever a set of stars exists that have a parallel principal component in their spatial motion, their paths should intersect at two opposite points on the celestial sphere. If the spatial motion of the stars is due exclusively to a tangential velocity, the intersecting points will be at an angular distance of 90 degrees with respect to the geometrical center of the set. On the other hand, if there is only radial velocity, one of these points will coincide with the geometrical center. If the motion is a combination of both velocities, the two convergent points could be anywhere between these two extreme cases.

2.1. Paths and their intersections

A stellar association implies parallelism and therefore the existence of an apex on the sphere; the paths of the stars in an association will be heading toward a convergent point and the distribution of intersection points will be more dense near that direction (and near its opposite). On the other hand, if the intersection between paths is spread on the celestial sphere in a random way, we can assume that there is no parallelism in the spatial motion of the set of stars considered.

Thus, when we select an extended area of the sky, the use of the intersection points between paths of the stars, whether they are associated or not, could indicate the existence of preferential motion directions over the celestial sphere and therefore, the possibility of the existence of real stellar associations.

Even if this part is not different to Herschel's method, please note the following details applied to extended fields.

First, if we have a field with a lot of stars, the representation on the celestial sphere of the intersections of theirs paths can highlight preferred directions of motion of some star groups. It is a graphical effect but enough to begin the isolation of the stars sharing such a preferred direction. Figure 1 is an example of that, showing a complete 2° × 2° area from the Wang et al. (1995) catalogue, containing the Praesepe open cluster.

Second, if the motion convergence point or points are not clear, a study of the density distribution of the number of intersections is necessary for search members of moving groups. In Fig. 2, we show the density distribution of Hipparcos catalogue stars (Perryman & ESA 1997) in a 10° × 10° area centered at $(\alpha, \delta) = (55^\circ 5, 24^\circ 1)$, including the Hyades and Pleiades clusters. In the upper panel, intersections between paths are shown, and in the lower one, we show the respective density diagram, on a smaller zone including the selected area. This area with its opposite (not shown) will have the largest number of intersections, however, other concentrations exist in the preferred directions detected in the upper panel of this figure. When the density of intersections is too large, we obtain the same results as for a random sample of stars. The application of statistical tests on the density diagram is useful in the search for moving groups.

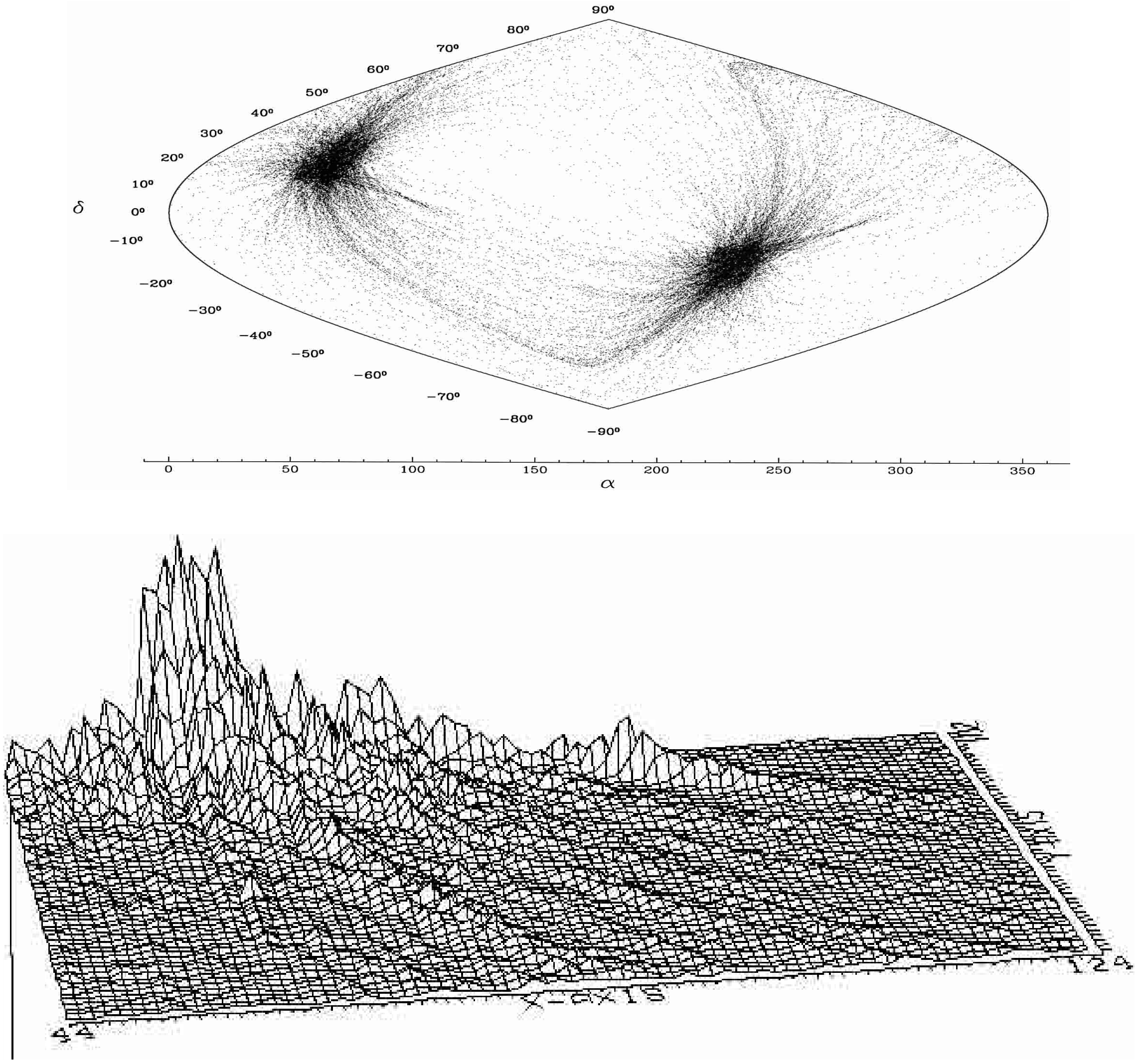

Fig. 2. Upper panel: Paths intersections of all Hipparcos stars in a 10° × 10° area, containing the Pleiades and Hyades open clusters. At least two preferred directions stand out. Both α and δ are expressed in degrees. Lower panel: Density diagram on a smaller zone including the same area. Some directions and a concentration on one of them, are visible in this figure.

2.2. Pole of the paths

When a stellar association exists, the poles of its members are located on a great circle whose poles are precisely the apex and antapex of the association, and these two points are calculated by the cross product of all of the poles of the members between themselves.

As is clearly shown in Fig. 3, the pole of the paths offers more graphic possibilities than the vector point diagram μ_δ vs. μ_α to detects particular trends in the star motion, specially in dense areas. In the left panel the vector point diagram of all O and B stars in the Hipparcos catalogue is shown, while in the right panel, their respective poles are illustrated. Several concentrations located on great circles easily stand out in the second figure, while in the first one, the existence of any moving group is not evident. In fact, all the mathematical methods (generally statistic) applied to a vector point diagram to detect moving groups can be applied here to obtain good results in cases where the vector point diagram is limited because of the density of stars and data precision.

Other way to mathematically process the data to improve the results of Herschel's method is to use numerical tools. In the next part we apply one of these techniques to study and determine the solar motion.

3. The Solar motion

3.1. Stellar proper motion: The sum of several movements

In the proper motion μ, measured in arc seconds per year, we observe the tangential component V_t of the vector V of spatial motion of the star, scaled by the distance d in pc to the star, by the equation $V_t = 4.738\mu\ d$, where V_t is given in km s^{-1}. This μ contains not only the intrinsic movement of the star but also the reflex of the reference frame motion under which it is observed. Of these contributions, the most important will be the differential galactic rotation and the reflex of solar motion.

In Mignard (2000), the systematic velocity field relative to the Sun due to the galactic rotation is modeled without any

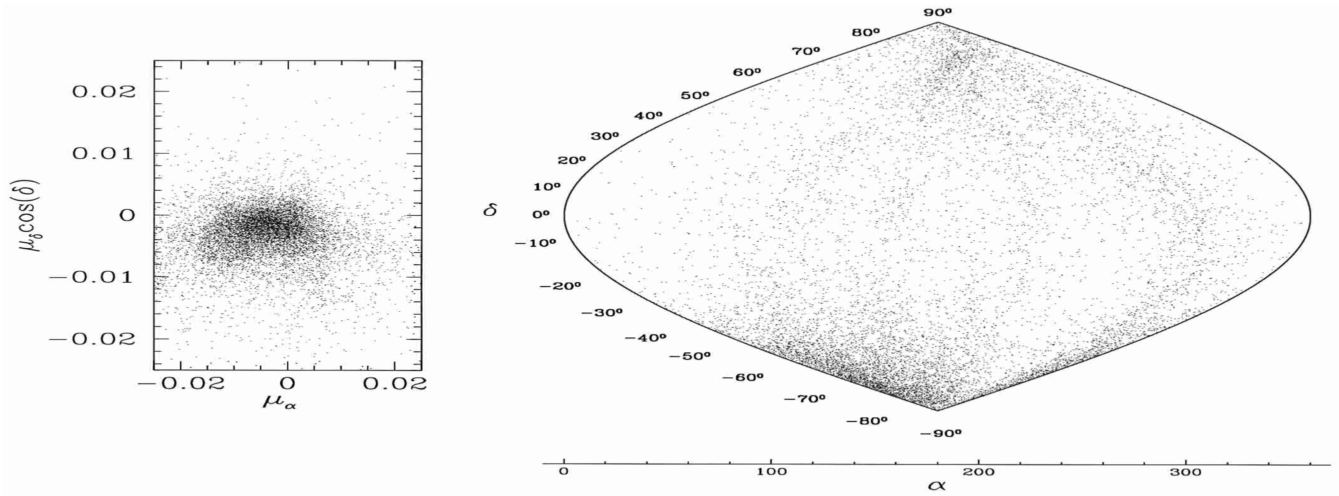

Fig. 3. Left panel: Vector-Point Diagram of all O and B stars in Hipparcos catalogue. Both $\mu_\alpha \cos(\delta)$ and μ_δ are expressed in arcseconds. Right panel: Poles of these stars. It is clear that they arranged on great circles. Both α and δ are expressed in degrees.

assumption of the nature of this rotation. The galactic rotation is represented as a continuous smooth flow, characterized by the generalized Oort constants, obtained in her paper, which depend exclusively on galactic longitude and latitude, and not on the distance.

A more important systematic motion shared by all the stars around us is the solar motion. Solar motion introduces a systematic effect on the observed proper motion of the stars: the addition of a component directed to the solar antapex, whose modulus depends on d as well as their angular distance λ to the solar apex, by the equation

$$\mu_\odot = \frac{V_\odot}{4.738\ d} \sin\lambda \quad (1)$$

where $V_\odot$ is the Sun velocity in km s^{-1} with respect to the Local Standard of Rest. In general, any parallel space motion shared by a set of stars follows this equation. All stars in a gravitational linked association share a common spatial motion $\boldsymbol{V}$, which is observed as a vector of proper motion directed toward the apex of the association, with a modulus depending on the angular distance λ from the star to this apex, following a sinusoidal shape given by

$$V_t = 4.738\ \mu\ d = V \sin(\lambda). \quad (2)$$

This equation says that V_t has a minimum in the apex and the antapex, and a maximum on all points at 90° to these last two points, that is, on the great circle whose poles are the apex and antapex. Thus, it is logical to think that the solar motion could be easily detected using the CPM on all stars. Some authors, such Agekyan & Popovich (1993) have tried this method, but Jaschek & Valbousquet (1992) disagreed because of the poor results obtained.

According to (1), all the stars with the same d and λ have exactly the same solar motion component. If we look for a systematic pattern produced by solar motion on stars in a given region, we have to take into account the distances to each one of them, because it is a factor that is hidden in the calculation. So, a re-scaling of proper motions to a fixed distance will result in that all the stars have the same solar motion component. Only after this, can we try to detect solar motion as a systematic pattern shared by all stars in the sky. This is done through a fitting function defined in a numerical way. Obviously, before the re-scaling it is necessary to correct the proper motion by the galactic rotation, which does not depend on the distance.

Finally, a star can be a member of an open cluster, stellar association or even be flowing in a stream, so that its proper motion contains information on the kinematics, too.

3.2. The fitting function

To know the systematic behavior of the proper motion in a certain position (l_0, b_0) over the sky, the fitting function developed by Stock & Abad (1988) is applied to each of the proper motion components in galactic coordinates, μ_l and μ_b, selecting all the stars located inside an encircled area of fixed radius r centered on that point. This radius must be large enough to contain a significant number of stars and to avoid local patterns. Each datum is weighted by a number from 0 to 1, in such a manner that it is zero when a point is located at the edge of the encircled area and reaches a maximum of one when located at the center. To guarantee the continuity and derivability of the fitting until order $n - 1$, the weight has an exponent n.

The fit at one point will be totally independent of other points far away. The result obtained for each fitted point is a vector of proper motion, representative of the proper motions of all the stars used in the calculation. This fact allows us to detect the existence of a systematic pattern over all the sky when independent data show a common behavior.

4. Results obtained using Hipparcos

The use of CPM on the proper motions obtained from the fitting function applied in a total or partial way on large samples of stars can detect the existence of common trends in the proper motions, matching a common parallel space movement shared by all stars. This is the procedure we use to detect the solar motion.

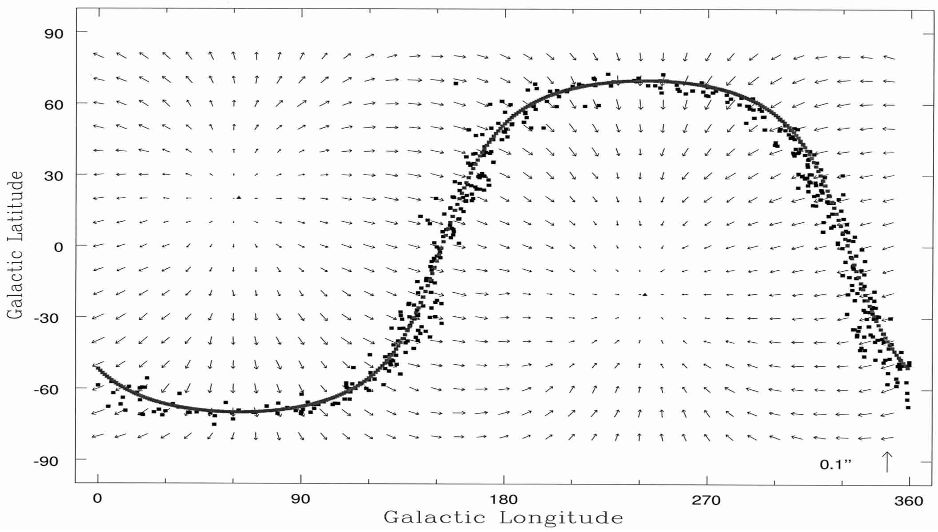

Fig. 4. The Solar motion using all Hipparcos stars. Arrows represent the common behavior, as obtained from the fitting function, of all stellar proper motions inside a 20° diameter encircled area centered on its application point. Points are their respective poles and the continuous line is a great circle that fit them. From here we obtain a solar ápex $(l, b) = (61.39, 20.42) \pm (3.13, 2.62)$.

No star from the Hipparcos Catalogue has been excluded a priori from the calculus, although we do this search by taking preferentially all the stars in Hipparcos catalogue having $3\sigma_\pi < \pi$ (π means parallax), corrected for galactic rotation (according to Mignard 2000) and re-scaling proper motions to a distance of 100 pc by the equation $\mu_{\rm esc} = \frac{d}{100}\mu$ (we will see later that the value of the distance for which proper motions are re-scaled does not affect the results). We apply the fitting function to get the systematic pattern in the stellar proper motions and then use CPM to test if they correspond to a parallel spatial motion.

To have a graphical idea of the fitting function, Fig. 4 shows the results obtained with Hipparcos catalogue. In this figure, a set of vectors uniformly distributed on the celestial sphere are shown. Each is the result of the application of the fitting function to those Hipparcos stars contained in a 20° diameter area centered on the vector application point. It is clear that a common behavior exists in the stellar proper motions, but it is the CPM which definitively confirms that it matches a parallel motion. Each vector can be considered as a proper motion μ applied to its origin and so, its path can be traced and its poles can be calculated. These poles are represented in the figure by dots to which a great circle can be fitted. The poles of this great circle meet the path intersection points.

Vector moduli can be used to determine the spatial velocity. It is easily seen from Fig. 4 that these moduli follow a pattern: they are almost zero near the apex and antapex, and have a maximun length at the great circle whose poles are these two point. As expressed in (2), any parallel space movement is characterized by a V_t following a sinusoidal shape as a function of λ; Fig. 5 reveals such a feature. The reliability of the results obtained are assured by the precision of the Hipparcos data.

This process reveals the existence of a parallel space motion shared by all the Hipparcos stars, directed to the point $(l, b) = (61.39, 20.42) \pm (3.13, 2.62)$ and with a spatial velocity $V = 21.96 \pm 3.72$ km s^{-1}. The only known motion that can produce an effect like this is solar motion.

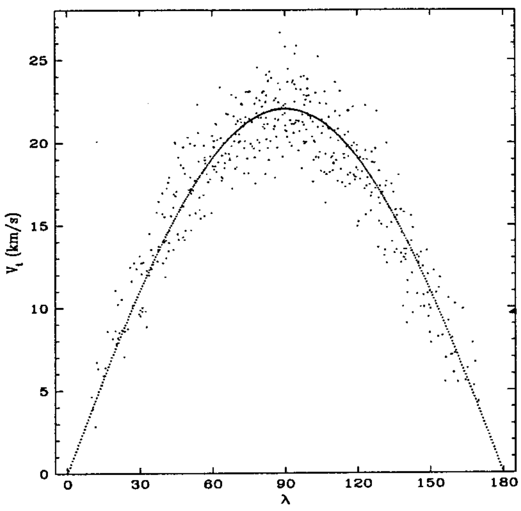

Fig. 5. V_t vs. λ based on the vectors moduli from Fig. 4. Solar velocity obtained is $V_\odot = 21.96 \pm 3.72$ km s^{-1}.

This procedure has been applied to samples of stars according to the spectral type and subtype, distance, absolute magnitude and luminosity class. Results are shown in Table 1, where we highlight the systematic drift of the solar apex and its velocity, according to the spectral type. It has been pointed out that stellar kinematics can vary systematically with stellar type, in the sense that groups of stars that are on average younger have smaller velocity dispersions and larger mean galactic rotation velocities than older stellar groups (Mihalas & Binney 1981).

In order to further study this situation, we apply the fitting function on several subsamples centered on a certain spectral type and subtype, taking into account only those stars no farther away than four subtypes to each side, and giving more importance to those stars whose spectral subtype is nearer to the central spectral subtype, using an additional weight to that related to the distance (as mentioned in Sect. 3.2).

Table 1. Results of solar motion. $l_\odot, b_\odot, \sigma_{l\odot}$ and $\sigma_{b\odot}$ are expressed in galactic coordinates degrees. $V_\odot$ and $\sigma_{V_\odot}$ are in km s^{-1}. d is distance to stars in pc. z is vertical distance to the galactic plane, where $z < 0$ means under that plane. r is distance to the galactic center, Sun is located at R_0 = 8.0 Kpc. M is absolute magnitude.

Criterion of selection	$l_\odot$	$b_\odot$	$V_\odot$	$\sigma_{l_\odot}$	$\sigma_{b_\odot}$	$\sigma_{V_\odot}$	**Number of stars**
(μ_l, μ_b) re-scaled to 100 pc	61.39	20.42	21.96	3.13	2.62	3.72	74316
(μ_l, μ_b) re-scaled to 250 pc	61.91	20.03	22.38	3.26	2.61	3.89	74316
(μ_l, μ_b) re-scaled to 500 pc	61.91	20.03	22.38	3.25	2.61	3.89	74316
(μ_l, μ_b) re-scaled to 750 pc	61.91	20.04	22.38	3.26	2.61	3.89	74316
Spectral Type O and B	48.01	19.96	19.51	7.24	4.11	5.35	4236
Spectral Type A	42.06	24.22	15.68	5.00	3.91	2.79	12464
Spectral Type F	55.52	20.87	18.61	4.50	3.66	2.98	19894
Spectral Type G	64.93	17.59	26.07	5.07	3.78	3.69	16498
Spectral Type K	67.19	17.69	24.82	4.30	3.31	3.29	17558
Spectral Type M	65.22	14.86	27.10	10.08	6.96	8.68	2103
000 pc < d < 100 pc	62.54	16.98	25.56	4.02	3.06	3.04	22407
100 pc < d < 200 pc	60.17	20.53	20.14	3.88	2.89	1.98	29752
300 pc < d < 300 pc	59.94	21.18	19.57	5.03	3.85	3.34	16116
300 pc < d	56.47	20.09	24.06	8.58	5.48	7.46	5922
–200 pc < z < –100	64.61	24.18	19.93	4.22	3.09	4.32	8509
–100 pc < z < 000	63.60	19.32	22.68	3.80	2.15	3.79	27825
000 pc < z < 100	58.58	19.89	23.96	3.52	2.06	3.74	26449
100 pc < z < 200	58.03	21.27	20.82	4.24	3.44	4.21	7977
7800 pc < r < 7900	59.28	19.72	21.17	4.43	3.30	3.51	8958
7900 pc < r < 8000	61.93	17.39	23.15	2.78	2.82	2.27	27477
8000 pc < r < 8100	61.36	19.53	24.15	3.00	2.86	3.97	25443
8100 pc < r < 8200	59.87	23.47	19.98	4.75	3.66	2.79	8947
M < 0.00	56.02	20.84	21.71	8.25	5.50	6.26	6410
0.00 < M < 1.00	59.92	21.87	21.16	5.65	4.17	4.45	11846
1.00 < M < 2.00	57.23	20.69	18.89	5.11	3.54	2.75	14899
2.00 < M < 3.00	56.72	21.38	18.58	4.85	3.21	2.88	13804
3.00 < M < 4.00	62.00	19.04	20.04	5.16	3.99	3.32	12590
4.00 < M < 5.00	66.78	15.45	27.31	6.53	4.83	5.60	7224
M < 5.00	63.77	14.93	33.12	5.72	4.29	5.71	7424
Main sequence stars	58.96	19.93	21.87	3.31	2.43	2.14	52819
Off Main sequence stars	65.79	19.36	21.68	4.29	3.37	2.78	21378
All Hipparcos	61.82	20.44	24.28	3.27	2.81	1.78	118218

The results obtained show that the shift follows a continuous behavior. In this way it is possible, as seen in Fig. 6, to get a continuous and differentiable function relating the solar apex and velocity to the spectral type and subtype. The types and subtypes are represented by numbers from 30 to 79, following a sequence, in which the stars from 30 to 39 have spectral type from A0 to A9 respectively (subtype is indicated by the second digit), and so on, until stars from 70 to 79, belonging to spectral types from M0 to M9, respectively.

For every example in this work, the apex, velocity and errors are the mean and standard deviation of simple distributions. The apex comes from a distribution of poles, which is obtained by the cross product of all possible combinations of two positional vectors of the poles of the pattern, as long as they are not too close to each other. The spatial velocity comes from a distribution of all individual spatial velocities, which is obtained applying (2) to each vector of the pattern. After an initial calculation, some poles and vectors of the pattern must be excluded from the process. Usually they are associated with vectors near the apex or antapex (too small module or $\sin(\lambda)$).

Once the solar apex and velocity are obtained, the proper motion of each star is corrected, and re-applying the procedure to the new proper motions, no systematic trend appears.

5. Conclusions

We consider that the great circles developed by Herschel more than 200 years ago have new perspectives for use in studies

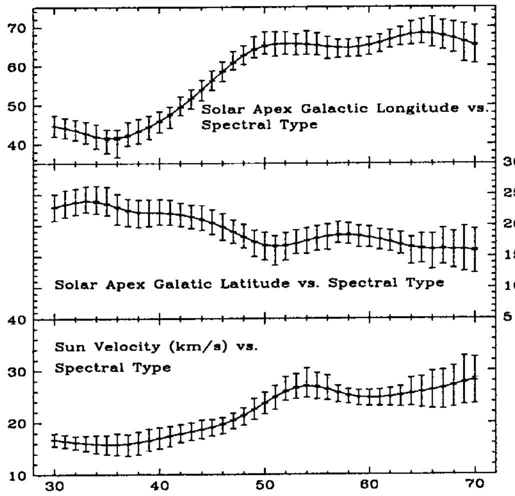

Fig. 6. Solar apex and velocity variation, according to the spectral type sampled. This last is represented by numbers, 30 to 39 have spectral type from A0 to A9 (subtype is indicated by the second digit), and so on, until stars from 70 to 79, belonging to spectral types from M0 to M9, respectively. Error bars are included.

of local galactic kinematics, through the application of several mathematical methods, not only statistical but also numerical, which increase the capability of the method to be applied on extensive or dense stellar catalogues. This application Herschel's method and the future more dense, deeper and more precise catalogues (such as Gaia, ISTM), can provide a new point of view that improves the knowledge of the kinematics of the galaxy.

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