

# Particle acceleration in rotating and shearing jets from AGN

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**Abstract.** We model the acceleration of energetic particles due to shear and centrifugal effects in rotating astrophysical jets. The appropriate equation describing the diffusive transport of energetic particles in a collisionless, rotating background flow is derived and analytical steady state solutions are discussed. In particular, by considering velocity profiles from rigid, over flat to Keplerian rotation, the effects of centrifugal and shear acceleration of particles scattered by magnetic inhomogeneities are distinguished. In the case where shear acceleration dominates, it is confirmed that power law particle momentum solutions  $f(p) \propto p^{-(3+\alpha)}$  exist, if the mean scattering time  $\tau_c \propto p^\alpha$  is an increasing function of momentum. We show that for a more complex interplay between shear and centrifugal acceleration, the recovered power law momentum spectra might be significantly steeper but flatten with increasing azimuthal velocity due to the increasing centrifugal effects. The possible relevance of shear and centrifugal acceleration for the observed extended emission in AGN is demonstrated for the case of the jet in the quasar 3C273.

**Key words.** acceleration of particles – galaxies: active – galaxies: jets

## 1. Introduction

Astrophysical jets emerge from a variety of astrophysical sources, ranging from protostellar objects, to micro-quasars, active galactic nuclei (AGN) and probably Gamma Ray Bursts (GRBs). Currently, the observations of jets from radio-loud AGN may be counted among the most interesting ones as they constitute test laboratories for studying the spatial structure in relativistic jets. Today there is convincing evidence that the central engine in these AGN is a rotating supermassive black hole surrounded by a geometrically thin accretion disk, which gives rise to the formation of a pair of relativistic jets. The observation of superluminal motions and theoretical opacity arguments indicate that the bulk plasma in these jets moves at relativistic speeds along the jet axis. Models based on the assumption of a one-dimensional velocity structure have thus allowed useful insights into the emission properties of AGN jets. However, as real jets are generally expected to show a significant velocity shear perpendicular to their axes, such models may be adequate only for a first approximation. In particular, several independent arguments suggest that AGN jet flows might be characterized by an additional rotational

velocity component:

(1) The analysis of the jet energetics in extragalactic radio sources have revealed a remarkably universal correlation between a disk luminosity indicator and the bulk kinetic power in the jet (Rawlings & Saunders 1991; Celotti & Fabian 1993) and supported a close link between jet and disk. The successful application of models within the framework of a jet-disk symbiosis (Falcke & Biermann 1995; Falcke et al. 1995) indicates that for radio-loud objects the total jet power may approach 1/3 of the disk luminosity so that a considerable amount of accretion energy, and hence rotational energy of the disk (cf. virial theorem), is channelled into the jet leading to an efficient removal of angular momentum from the disk.

(2) Observational findings, including helical motion of knots or periodic variabilities, seem to provide additional evidence for intrinsic rotation in AGN jets (cf. Biretta 1993 for M87; Camenzind & Krockenberger 1992 for 3C273; Schramm et al. 1993 for 3C345).

(3) From a more theoretical point of view, intrinsic jet rotation is generally expected in magnetohydrodynamical (MHD) models for the formation and collimation of astrophysical jets (e.g. Begelman 1994; Sauty et al. 2002). In such models, intrinsic rotation with speeds up to a considerable fraction of the velocity of light is a natural consequence of the assumption that the flow is centrifugally accelerated from the accretion disk. It should be noted however, that the rotation profile in the jet does not

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necessarily have to be disk-like, i.e. the set of available jet rotation profiles could be much wider and might include, for example, rigid, flat and Keplerian profiles (e.g. Hanasz et al. 2000). In particular, rigid rotation inside a well-defined light cylinder might be related to foot points of the magnetic field lines concentrated near the innermost stable orbit (e.g. Camenzind 1996; Fendt 1997a), while more generally differential rotation would be intuitively expected if there is an intrinsic connection between jet motion and the rotating disk (cf. also Fendt 1997b; Lery & Frank 2000).

In this paper, we are interested in the influence of such rotation and shear profiles on the acceleration of particles in AGN jets. So far, several authors have contributed to our understanding of particle acceleration by shear. A kinetic analysis was used in the pioneering approach of Berezhko (1981, 1982) and Berezhko & Krymskii (1981). Their results showed that the particle distribution function in the steady state might follow a power law if the mean interval between two scattering events increases with momentum according to a power law. Later on, particle acceleration in the diffusion approximation at a gradual shear transition in the case of non-relativistic flow velocities was studied independently by Earl et al. (1988). They re-derived Parker's equation (i.e. the transport equation including the well-known effects of convection, diffusion and adiabatic energy changes), but also augmented it with new terms describing the viscous momentum transfer and the effects of inertial drifts. Jokipii & Morfill (1990) used a microscopic treatment to analyse the non-relativistic particle transport in a moving, scattering fluid which undergoes a step-function velocity change in the direction normal to the flow. They showed that particles may gain energy at a rate proportional to the square of the velocity change. Matching conditions in conjunction with Monte Carlo simulations for shear discontinuities were derived by Jokipii et al. (1989). The Monte Carlo analysis was extended by Ostrowski (1990, 1998), who also studied the acceleration at a sharp tangential velocity discontinuity, including relativistic flow speeds. He found that only relativistic flows can provide conditions for efficient acceleration, resulting in a very flat particle energy spectra which depends only weakly on the scattering conditions. The relevance of such a scenario for the acceleration of particles at the transition layer between AGN jets and their ambient medium was stressed in recent contributions by Ostrowski (2000) and Stawarz & Ostrowski (2002).

The work on gradual shear acceleration by Earl et al. (1988) was successfully extended to the relativistic regime by Webb (1989, 1992). Assuming the scattering to be strong enough to keep the distribution function almost isotropic in the comoving frame (so that the diffusion approximation applies), he derived the general relativistic diffusive particle transport equation for both rotating and shearing flows. Subsequently, Green's formula for the relativistic diffusive particle transport equation was developed by Webb et al. (1994). Applying their results to the cosmic ray transport in the galaxy, they found that the acceleration of cosmic rays beyond the knee by means of galactic rotation might be possible, but not to a sufficient extent. In its form presented however, it rather remains an essentially theoretical approach which could not be easily related to observations.

In a previous contribution (Rieger & Mannheim 2001b) we developed and applied a new model, that utilizes the relativistic transport theory advanced by Webb and that permits the analysis of the acceleration of energetic particles in rotating and shearing AGN jets. By studying velocity profiles known to be typical for such jets, we obtained results indicating that the resultant particle energization is in general a consequence of both centrifugal and shear effects. The formation of power law particle spectra under a wide range of conditions reveals the significant potential of shear and centrifugal acceleration as a natural explanation for the origin of the extended, continuous emission recently observed from AGN (e.g. Meisenheimer et al. 1997; Jester et al. 2001). In the present paper we aim to provide a more detailed investigation of this model, including the derivation of the relevant particle transport equation (cf. Appendix B).

The paper is organised as follows: after a short review of the underlying theoretical background in part 2 (see also Appendix A), we present several applications to relativistic jet flows with rigid, Keplerian and flat intrinsic rotation profiles in part 3, leaving the detailed derivation of the particle distribution function to Appendix C. Part 4 provides a discussion of our results. The observational relevance of the model presented is pointed out in part 5 with reference to recent observations. The paper finally closes with a short conclusion.

## 2. Model background

By starting from the relativistic Boltzmann equation and by using both the differential moment equations and a simple BKG (Bhatnagar, Gross & Krook) time relaxation for the scattering term, the relativistic particle transport equation in the diffusion approximation was derived by Webb (1989, 1992; cf. also Appendix A). In the underlying physical picture, it is assumed that scattering of high energy particles occurs by small-scale magnetic field irregularities carried in a collisionless, systematically moving background flow. In each scattering event the particle momentum is randomized in direction, but its magnitude  $p'$  is assumed to be conserved in the (local) comoving flow frame, where the electric field vanishes. Since the rest frame of the scattering centres is regarded to be essentially that of the background flow, particles neither gain energy nor momentum merely by virtue of the scattering if there is no shear or rotation present and the flow is not diverging. However, in the case of a shear in the background flow, the particle momentum relative to the flow changes for a particle travelling across the shear. As the particle momentum in the local flow frame is preserved in the subsequent scattering event, a net increase in particle momentum may occur (cf. Jokipii & Morfill 1990). Thus, if rotation and shear is present, high energy particles, which do not corotate with the flow, will sample the shear flow and may be accelerated by the centrifugal and shear effects (cf. Webb et al. 1994).

For the present application, we consider a rather idealized (hollow) cylindrical jet model where the plasma moves along the  $z$ -axis at constant (relativistic)  $v_z$ , while its velocity component in the plane perpendicular to the jet axis is purely azimuthal and characterized by the angular frequency  $\Omega$ . Using

cylindrical coordinates for the position four vector, i.e.  $x^\alpha = (ct, r, \phi, z)$ , the metric tensor becomes coordinate-dependent (i.e.  $(g_{\alpha\beta}) = \text{diag}\{-1, 1, r^2, 1\}$ ) and for the chosen holonomic basis the considered flow four velocity may be written in shortened notation as

$$u^\alpha = \gamma_f (1, 0, \Omega/c, v_z/c), \quad (1)$$

$$u_\alpha = \gamma_f (-1, 0, \Omega r^2/c, v_z/c), \quad (2)$$

where the normalization

$$\gamma_f = 1/\sqrt{1 - \Omega^2 r^2/c^2 - v_z^2/c^2} \quad (3)$$

denotes the Lorentz factor of the flow and where the angular frequency may be a function of the radial coordinate, i.e.  $\Omega = \Omega(r)$ .

As suggested by Webb et al. (1994), the resulting transport equation may be cast in a more suitable form if one replaces the comoving variable  $p'$  by the variable  $\Phi = \ln(H)$ , where  $H$  is given by

$$H = p'^0 c \exp\left(-\int^r dr' \frac{\gamma_f^2 \Omega^2 r'}{c^2}\right). \quad (4)$$

In the case of highly relativistic particles (with  $p'^0 \approx p'$ ) and using an (isotropic) diffusion coefficient of the form

$$\kappa = \kappa_0 p'^\alpha r^\beta, \quad (5)$$

we finally arrive at the steady state transport equation (cf. Appendix B) for the (isotropic) phase space distribution function  $f(r, z, p')$

$$\begin{aligned} \frac{\partial^2 f}{\partial r^2} + \left(\frac{1+\beta}{r} + [3+\alpha] \frac{\gamma_f^2 \Omega^2 r}{c^2}\right) \frac{\partial f}{\partial r} \\ + \frac{\gamma_f^4 r^2}{5 c^2} (1 - v_z^2/c^2) \left(\frac{d\Omega}{dr}\right)^2 \left([3+\alpha] \frac{\partial f}{\partial \Phi} + \frac{\partial^2 f}{\partial \Phi^2}\right) \\ - \frac{\gamma_f v_z}{\kappa} \frac{\partial f}{\partial z} + (1 + \gamma_f^2 v_z^2/c^2) \frac{\partial^2 f}{\partial z^2} = -\frac{Q}{\kappa}. \end{aligned} \quad (6)$$

In general, the solution of Eq. (6) can be quite complicated. However, as we are especially interested in the azimuthal effects of particle acceleration in a rotating flow further along the jet axis, it seems sufficient for a first approach to search for a  $z$ -independent solution of the transport equation, i.e. we may be content with an investigation of the one-dimensional Green's function, which preserves much of the physics involved and which corresponds to the assumption of a continuous injection along the jet (cf. Appendix C). In this case the corresponding source term becomes

$$Q = \frac{q_0}{p'_s} \delta(r - r_s) \delta(\Phi - \Phi_s), \quad (7)$$

describing mono-energetic injection of particles with momentum  $p' = p'_s$  from a cylindrical surface at  $r = r_s$ . Here,  $\delta$  denotes the Dirac delta distribution and the constant  $q_0$  is defined as  $q_0 = N_s/(8\pi^2 p_s'^2 r_s)$  with the total number  $N_s$  of injected particles. In order to solve the  $z$ -independent transport equation, we then apply Fourier techniques and consider the Green's solutions satisfying homogeneous, i.e. zero Dirichlet boundary conditions.

### 3. Applications

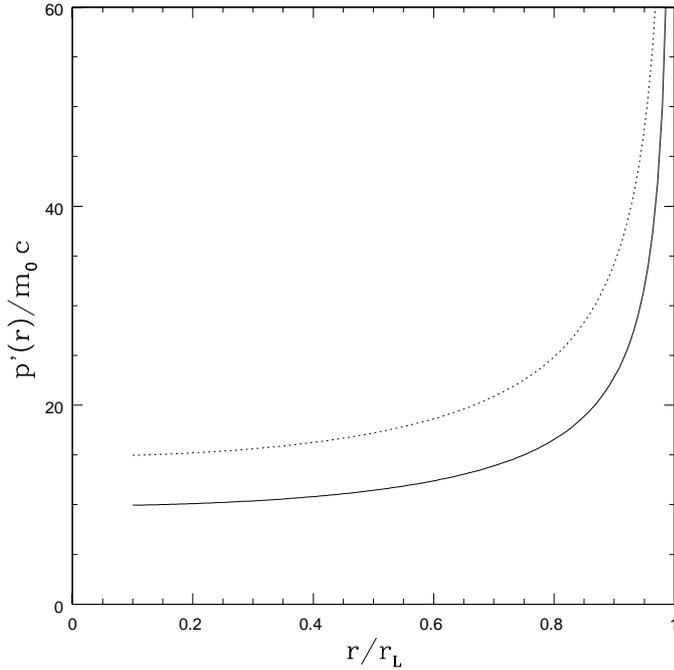
#### 3.1. Rigid rotation – no shear

In order to gain insight into the particle transport and acceleration process, let us first consider the influence of rigid rotation profiles. One may associate such rotation profiles with dynamo action in the inner accretion disk around a spinning black hole, which creates a jet magnetosphere filled with disk plasma and rotating with the angular frequency of its foot points concentrated near the innermost stable orbit (e.g. Camenzind 1996). The acceleration of particles due to rigid rotation could represent an efficient process for the production of high energy particles. This was demonstrated by Rieger & Mannheim (2000) for the centrifugal acceleration of charged test particles at the base of a rigidly rotating jet magnetosphere for conditions assumed to prevail in AGN-type objects. In such cases an upper limit for the maximal attainable Lorentz factor can be established, which is given by the breakdown of the bead-on-the-wire approximation in the vicinity of the light cylinder. There are several similarities to the application presented here: If we consider the transport of relativistic particles in a rigidly rotating background flow (i.e.  $\Omega = \Omega_0 = \text{const.}$ ) by utilising the above model, shearing is absent (i.e.  $d\Omega/dr = 0$ ) and thus particle energization only occurs as a consequence of the interaction with the accelerating background flow, i.e. due to the presence of centrifugal effects. It can then be readily shown that for the case considered here, Eq. (4) is analogous to the Hamiltonian for a bead on a rigidly rotating wire (cf. Webb et al. 1994, also Rieger & Mannheim 2001a), i.e.

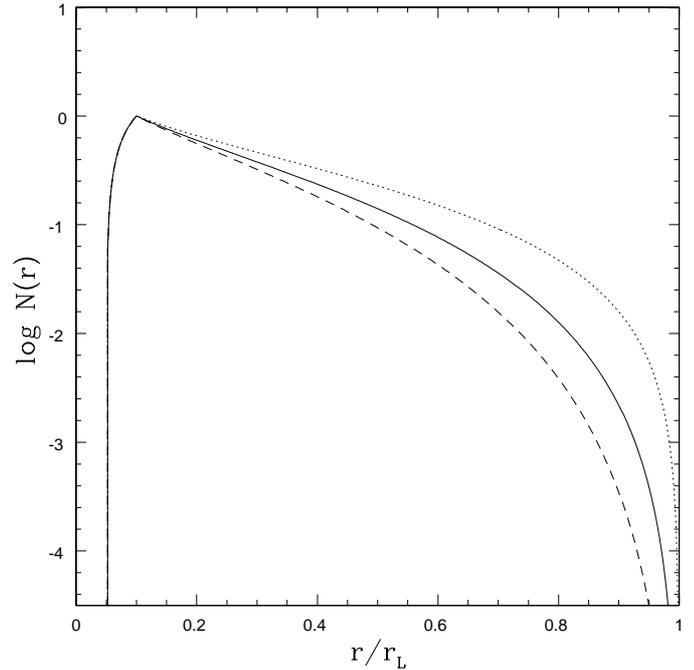
$$H = \frac{p'^0 c}{\gamma_f}, \quad (8)$$

while the transport equation becomes purely spatial. The variable  $H$  introduced above, could thus be regarded as describing the balance between the centrifugal force and the inertia of the particle in the comoving frame. By means of Noether's theorem,  $H$  could be shown to be a constant of motion, and hence Eq. (C.12) indicates that the ratio of particle to flow Lorentz factor is fixed. The comoving particle momentum  $p'$  can then be simply expressed as a function of the radial coordinate  $r$ , i.e.  $p' = p'(r)$  (cf. Fig. 1).

We may easily solve the resultant (spatial) transport equation analytically, assuming homogeneous boundary conditions at the jet inner radius  $r_{\text{in}}$  and its relevant outer radius  $r_{\text{out}}$ . In the present case, one may choose the size of the inner radius  $r_{\text{in}}$  to be of the order of the radius of the innermost stable orbit around a rotating black hole, while the outer radius  $r_{\text{out}}$  is at any rate physically constrained to be smaller than the light cylinder radius (note that for relativistic MHD winds, the Alfvén point may be close to the light cylinder). Results concerning the particle transport are presented in Figs. 1 and 2. They indicate that the energetic particles injected at  $r_s$  will gain energy by being transported in the rigidly rotating background flow, i.e. due to the impact of centrifugal acceleration the particle momentum increases with position as expected. The precise evolution in the immediate vicinity of the light cylinder should however be treated with some caution due to the limitations imposed by



**Fig. 1.** Particle momentum  $p'$  as a function of the radial coordinate in the case of rigid rotation, for particles injected at  $r_s = 0.1 r_L$  with initial Lorentz factor  $\gamma_s = 10$  (solid) and 15 (dashed). Here,  $r_L$  is defined by  $r_L = c(1 - v_s^2/c^2)^{0.5}/\Omega_0$



**Fig. 2.** Spatial distribution  $N(r)/N(r_s)$  for rigid rotation using a different energy dependence of the diffusion coefficients, i.e.  $\alpha = \beta = 0$  (solid line),  $\alpha = -2, \beta = -0.01$  (dotted line) and  $\alpha = 2, \beta = -0.01$  (dashed line). Boundary conditions  $r_{in} = 0.05 r_L, r_s = 0.1 r_L$  and  $r_{out} = 0.999 r_L$  have been used for the calculations.

the applied diffusion approach. Figure 2 reveals a decrease in efficiency if the time  $\tau_c$  between two collisions increases with momentum (in case of a very weak spatial dependence), i.e. if scattering occurs less rapidly for the higher than for the lower energy particles.

### 3.2. Keplerian rotation – shear dominance

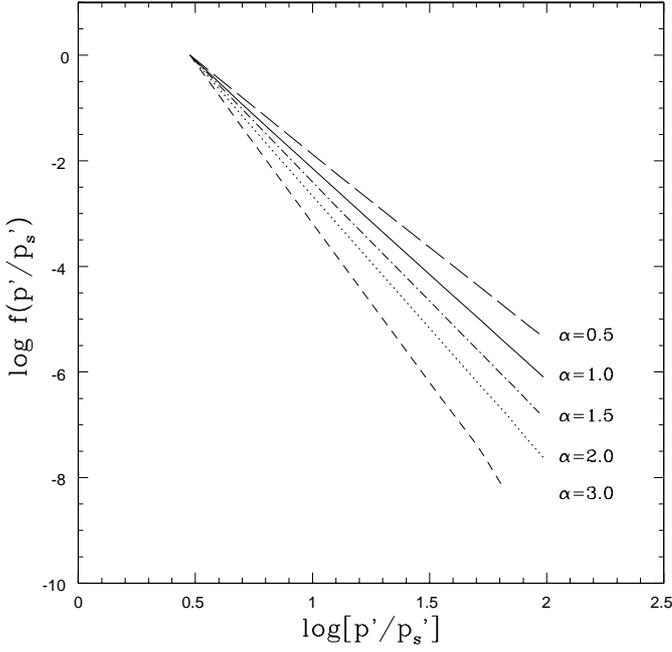
Let us now consider the acceleration of particles due to a rotating background flow with keplerian rotation profile  $\Omega(r) = k r^{-3/2}$ , where  $k = \sqrt{GM}$ . Again, such flow profiles might be associated with jets or disk winds originating from the accretion disk around the black hole and dragging the Keplerian disk rotation with them. Hence, for reasons of self-similarity, keplerian rotation may be intuitively regarded as one of the most characteristic descriptions with respect to the velocity profile of intrinsically rotating flows. For example, Lery & Frank (2000) recently investigated the structure and stability of astrophysical jets including Keplerian rotation in the outermost part of the outflow and rigid rotation close to its axis (cf. also Hanasz et al. 2000). They also studied the application to non-relativistic outflows from young stellar objects. One may thus eventually consider a simple model where particles, accelerated in a rigidly rotating flow, are subsequently injected into a Keplerian rotating flow.

Generally, if we consider Keplerian rotation, both shear and centrifugal effects are present. For non-relativistic rotation, analytical solutions of the Fourier transformed transport equation are given in the Appendix C.2. in terms of the confluent hypergeometrical functions. Such solutions should be appropriate

for the outer jet regions. As the relative strength of the contribution by shear to that of centrifugal energization evolves with  $r$ , shear effects will eventually dominate over centrifugal acceleration (see discussion). This is particularly illustrated in Fig. 3, where we have plotted the logarithm of the (normalized) particle distribution function  $f$  above  $p'/p'_s \geq 3$  for  $\beta = 0$  (i.e. allowing no spatial dependence of the diffusion coefficient) and different momentum dependence of  $\kappa$ . Most interestingly, well-developed power law momentum spectra are recovered, which suggest a spectral slope linearly related to the momentum index of the diffusion coefficient, so that we have  $f \propto p'^{-(3+\alpha)}$ . This excellently confirms the results previously derived by Berezhko & Krymskii (1981). For a collisionless plasma with a simple shear flow  $U(y) e_x$ , and by using a simple BGK term, they found the shear acceleration to give rise to a power-law momentum spectrum for the steady state comoving particle distribution  $n(r, p') \propto p'^2 f \propto p'^{-(1+\alpha)}$ , if the collision time  $\tau_c$  depends on momentum as  $\tau_c \propto p'^\alpha$  and  $\alpha > 0$ . However, if the momentum index  $\alpha$  is smaller than zero, i.e.  $\alpha < 0$ , an exponential spectrum may developed.

### 3.3. Flat rotation – interplay between shear and centrifugal effects

In the case of flat rotation (i.e.  $\Omega = \Omega_0 r_0/r = v_{\phi f}/r$ ) one may easily investigate a more complex interplay between shear and centrifugal effects. For in this case, the relative strength of shear to centrifugal effects is independent of the radial coordinate and the general solution of the fourier-transformed equation could be cast in simple analytical terms allowing



**Fig. 3.** The momentum-dependence of the (normalized) distribution function  $f$  for Keplerian rotation using  $\beta = 0$ , calculated for  $\alpha = 0.5, 1.0, 1.5, 2.0, 3.0$  at fixed  $r = 40 r_{\text{ms}}$ . Boundary and injection conditions have been specified as  $r_{\text{in}} = 10 r_{\text{ms}}, r_{\text{out}} = 1000 r_{\text{ms}}, r_s = 20 r_{\text{ms}}$ , where  $r_{\text{ms}}$  is given by  $r_{\text{ms}} = GM/(c^2 - v_z^2)$ .

basic inverse Fourier integration to be done, using homogeneous boundary conditions at  $r_{\text{in}}$  and  $r_{\text{out}}$ . For the present application, we have considered typical jet flows with relativistic  $v_z/c = 0.95$  and different azimuthal velocities  $v_{\phi f}$  (i.e. for a range of  $\gamma_f \sim (3 - 4)$ ). The corresponding results are plotted in Fig. 4 using a constant diffusion coefficient (i.e.  $\alpha = \beta = 0$ ). Again, the calculated distribution functions reveal a powerlaw-type momentum dependence, where for low azimuthal velocities very steep momentum spectra are recovered, i.e. the momentum exponents are found to be near by  $-8.8$  (for  $\beta_\phi = 0.075$ ),  $-6.9$  (for  $\beta_\phi = 0.10$ ),  $-5.1$  (for  $\beta_\phi = 0.15$ ) and  $-4.4$  (for  $\beta_\phi = 0.20$ ). The observed flattening of the spectra with increasing azimuthal velocities is indicative of the increasing impact of centrifugal effects.

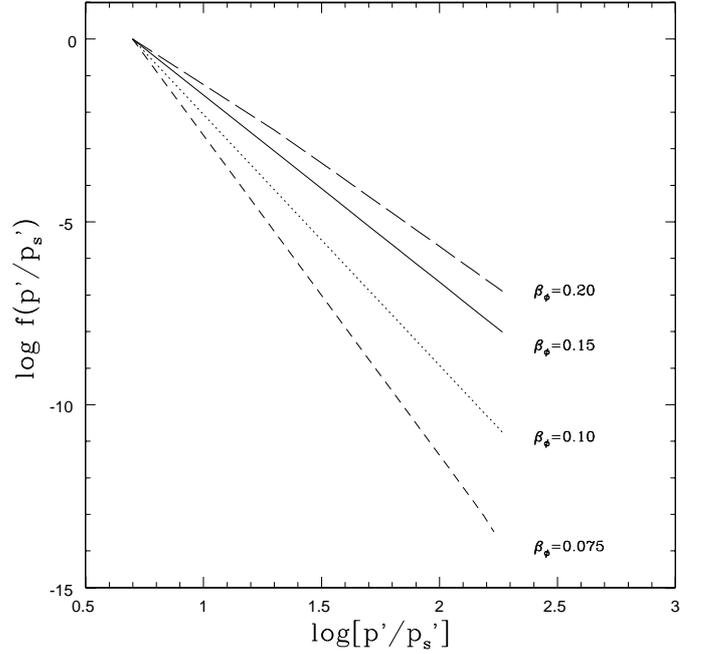
#### 4. Discussion

The applications presented so far differ in their relative contributions from shear and centrifugal effects. We may estimate the relative strength of these two more transparently by comparing their corresponding acceleration coefficients: for the shear acceleration coefficient we have

$$\langle \dot{p} \rangle_s = \frac{1}{p'^2} \frac{\partial}{\partial p'} (p'^4 \tau_c \Gamma) = (4 + \alpha) p' \frac{\lambda}{v'} \Gamma \quad (9)$$

where  $\Gamma$  denotes the viscous energization coefficient given by Eq. (B.10) and  $\tau_c \propto \tau_0(r) p'^\alpha$  with  $\tau_c = \lambda/v'$ . On the other hand, centrifugal acceleration may be described by the coefficient

$$\langle \dot{p} \rangle_c = \frac{1}{p'^2} \frac{\partial}{\partial p'} \left( \kappa (p'^0)^4 \gamma_f^4 \frac{\Omega^4 r^2}{c^4} \right), \quad (10)$$



**Fig. 4.** The momentum-dependence of the (normalized) distribution function  $f(r, p')$  for flat rotation, calculated for  $r_{\text{in}}/r_{\text{out}} = 0.02, r_s/r_{\text{out}} = 0.04$  at position  $r/r_{\text{out}} = 0.2$ . Chosen azimuthal velocities are  $v_{\phi f}/c = 0.075$  (short-dashed),  $0.10$  (dotted),  $0.15$  (solid),  $0.20$  (long-dashed).

which for highly relativistic particles ( $p' \simeq p'^0$ ) and  $\kappa = v'^2 \tau_c/3$  reduces to

$$\langle \dot{p} \rangle_c = \frac{4 + \alpha}{3} p' \frac{\lambda}{v'} \gamma_f^4 \frac{\Omega^4 r^2}{c^4}. \quad (11)$$

Hence, for the ratio  $c_s = \langle \dot{p} \rangle_c / \langle \dot{p} \rangle_s$  of centrifugal to shear acceleration we finally arrive at

$$c_s \simeq \begin{cases} 2.2 r_{\text{ms}}/r & \text{for Keplerian rotation} \\ 5 v_{\phi f}^2 / (c^2 - v_z^2) & \text{for flat rotation.} \end{cases} \quad (12)$$

For the case of Keplerian rotation and for the parameters chosen above, the relative strength becomes  $c_s \simeq 0.05$ , i.e. the shear effects essentially dominate over those of centrifugal ones. On the other hand, in the case of galactic rotation we have  $c_s \simeq (0.3 - 2)$ , which allows a complex interplay between centrifugal and shear acceleration with increasing azimuthal velocity.

Our approach utilises the relativistic diffusive particle transport theory as advanced by Webb (1989) and Webb et al. (1994) and thus assumes the diffusion approximation to be valid, i.e. the deviation of the particle distribution from isotropy to be small. In a strict sense, this requires the particle mean free path  $|c \tau_c|$  to be much smaller than both, the typical length scale for the evolution of the mean (momentum-averaged) distribution function and the typical length scale of variation for the background flow. Our conclusions are therefore of restricted applicability if highly anisotropic distributions are expected as, for example, near ultra-relativistic shocks (cf. Kirk & Schneider 1987; Kirk & Webb 1988).

In the application presented here, energy changes as a result of radiative (e.g. synchrotron) losses or second-order Fermi effects due to stochastic motions of the scattering centres have not been considered. One expects the inclusion of radiative losses to introduce an upper bound to the possible particle energy at the point where acceleration is balanced by losses, thus leading to a cut-off in the particle momentum spectrum. On the other hand, the inclusion of second-order Fermi effects would give an additional diffusion flux in momentum space. It may formally be taken into account by a more careful treatment of the scattering term in the Boltzmann equation. For non-relativistic jet flows, a numerical study of second-order Fermi acceleration has recently been given by Manolakou et al. (1999). Our present omission of second-order Fermi acceleration in the relativistic transport equation appears justifiable for the cases where the typical random velocities of the scattering centres (as measured in the comoving frame, i.e. relative to the flow) are smaller than a product of the radial particle mean free path times the rotational flow velocity gradient. However, one should note, that estimating the effects of second-order Fermi acceleration for the case of flat rotation, for example, clearly show them to be of increasing relevance for decreasing azimuthal flow velocities. A more detailed analysis will thus be given in a subsequent publication, while the purpose of the present model is confined to the analysis of steady-state solutions and the essential physical features of shear and centrifugal acceleration.

So far, shear-type acceleration processes in the context of AGN jets have been investigated by Subramanian et al. (1999) and Ostrowski (1998, 2000): By following the road suggested by Katz (1991), who considered the particle acceleration in a low density corona due to flux tubes anchored in a keplerian accretion disk, Subramanian et al. (1999) investigated the acceleration of protons driven by the shear of the underlying Keplerian accretion disk. They demonstrated that the shear acceleration may transfer the energy required for powering the jet and showed the shear to dominate over second-order Fermi acceleration. However, their model does not deal with the acceleration of particles emitting high energy radiation, but is basically confined to the bulk acceleration of the jet flow up to asymptotic Lorentz factors of  $\sim 10$ . On the other hand, Ostrowski (1998, 2000) examined the acceleration of cosmic ray particles at a sharp tangential flow discontinuity. He demonstrated that both, the acceleration to very high energies as well as the production of flat particle spectra are possible. In order for this model to apply efficiently, one requires several conditions to be satisfied, including the presence of a relativistic velocity difference and a thin (not extended) boundary, as well as a sufficient amount of turbulence on both sides of the boundary. It seems however that such conditions might be realized, for example, at the jet side boundary layer in powerful FR II radio sources.

With respect to shear acceleration, our approach can thus be regarded as complementary to the one developed by Ostrowski. While our analysis considers the influence of a gradual (azimuthal) shear profile in the jet interior, the analysis by Ostrowski deals with a sharp tangential shear profile at the jet boundary layer. Generally, one expects both processes to occur and to contribute to the production of high energy particles,

their relative contributions being dependent on the specific conditions realized in the jet interior and its boundary.

## 5. Observational relevance

The present results may serve as an instructive example revealing the significant potential of shear and centrifugal acceleration in the jets of AGN. Since intrinsic jet rotation is typically expected in the AGN setting, the proposed mechanism might operate in a natural manner over a wide range. This suggests that particle acceleration in a rotating and shearing background flow could be of particular relevance for an explanation of the continuous optical emission observed from several AGN jets (e.g. for 3C273; M87; PKS 0521-365; cf. Meisenheimer et al. 1997; Jester et al. 2001, 2002), as in contrast to shock acceleration, this mechanism generally would not be constrained to localized regions. Recent observations in fact indicate that the radiating particles might be widely distributed so that the optical emission from radio jets for example, is not confined to bright individual knots (cf. Meisenheimer et al. 1996, 1997). In order to explain the apparent absence of strong radiative cooling between knots, Meisenheimer et al. (1997) suggested the operation of an extended so-called “jet-like” acceleration mechanism, which was thought to be associated with velocity shear and expected to contribute in addition to standard diffusive shock acceleration. Meanwhile, the need of such an extended acceleration mechanism is strengthened by recent, high-resolution HST observations of the jet in the 3C273 (Jester et al. 2001, 2002). Classified as a blazar source, 3C273 is known as the brightest and nearest ( $z = 0.158$ ) quasar, with a jet extending up to several tens of kpc. The absence of a detectable counter-jet suggests bulk relativistic motion even up to kpc-scales, a conclusion also supported by recent models for the X-ray emission observed with Chandra (cf. Sambruna et al. 2001). The observations performed by Jester and collaborators have revealed an extremely well-collimated jet, where the optical spectral index varies only smoothly along the jet indicating only smooth variations of physical conditions. Their phenomenological analysis yields an optical spectral flux index of  $\sim -(0.65-0.8)$  and provides strong evidence for an underlying, universal powerlaw electron distribution, which is maintained almost entirely throughout the jet. If in the context of shear and centrifugal acceleration, the particle energy is locally dissipated by synchrotron radiation, the spectral emissivity  $j_\nu$  for a power-law particle number density distribution  $n \propto p'^2 f \propto p'^{-\delta}$  is given by  $j_\nu \propto \nu^{-s}$  with  $s = (\delta - 1)/2$ . Thus, for  $s = (0.65 - 0.8)$  we require  $\delta = (2.3 - 2.6)$ , which for example, may be realized for the case of a constant diffusion coefficient and high flat rotation or for the case of a simple shear flow with moderate momentum-dependent diffusion.

## 6. Conclusion

Observational and theoretical arguments suggest that astrophysical jets should exhibit intrinsic rotation of material perpendicular to the jet axis. Motivated by such arguments, we have proposed and analysed a basic model for the acceleration of energetic particles by centrifugal and viscous shear

effects which is applicable to relativistic, intrinsically rotating jet flows. The results obtained indicate that particle acceleration in rotating jets might represent an active mechanism for the production of the synchrotron-emitting high energy particles by giving rise to power law particle momentum distributions over a wide range of conditions. Due to its anticipated non-localized operation, such a mechanism is expected to be of particular relevance for the explanation of the recently observed extended emission in the jets from AGN. Moreover, as a general mechanism this process is able to provide high energy particles, which are required as seed particles for the (localised) first-order Fermi acceleration widely believed to occur in relativistic jets (e.g. Drury 1983; Kirk & Duffy 1999). Particularly, in the beginning the injection of seed particles could naturally occur at the base of the jet due to centrifugal acceleration (cf. Rieger & Mannheim 2000).

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## Appendix A: The general steady state transport in the diffusion approximation

In the case where we are concerned with the scattering of particles in relativistic bulk flows, it has been typically found useful to evaluate the scattering operator in the local Lorentz frame in which the fluid is at rest, i.e. in the so-called comoving frame  $K'$  (e.g. Webb 1985; Riffert 1986; Kirk et al. 1988)<sup>1</sup>. For in this frame  $K'$ , a simple form of the scattering operator could be applied if one assumes, as in the present approach, the rest frame of the scattering centres to be essentially that of the background flow. Quantities, which are operated upon the scattering operator, e.g. the momentum, might then conveniently be evaluated in this comoving frame, while the time and space coordinates are still measured in the laboratory frame  $K$  characterized by its metric tensor  $g_{\alpha\beta}$ . We would like to note however, that  $K'$  will in general be a non-inertial coordinate system (i.e. an accelerated frame) and therefore the related connection coefficients will not vanish. With reference to the considered particle transport, the covariant form of the Boltzmann equation is thus required, which may be achieved by replacing the ordinary (partial) space-time derivatives by their covariant derivatives.

Now, by starting from the relativistic Boltzmann equation and by using a perturbation solution of the moment equations in the diffusion approximation, i.e. by assuming the deviation of the particle distribution from isotropy in the comoving frame to be small, Webb (1989) has derived the general equation describing steady state particle transport in relativistic rotating and shearing flows. Following Eq. (4.4) of Webb (1989), the special relativistic diffusive particle transport equation for the isotropic, mean scattering frame distribution (averaged over all

momentum directions)  $f'_0(x^\alpha, p') = \langle f' \rangle$  may be written as

$$\frac{1}{p'^2} \frac{\partial}{\partial p'} \left[ -\frac{p'^3 c}{3} f'_0 \nabla_\beta u^\beta - p' (p'^0)^2 \dot{u}_\alpha q^\alpha - \Gamma p'^4 \tau_c \frac{\partial f'_0}{\partial p'} \right] + \nabla_\alpha (c u^\alpha f'_0 + q^\alpha) = 0, \quad (\text{A.1})$$

with  $x^\alpha$  the position four vector in the laboratory frame  $K$ , where the background plasma is in motion with four velocity  $u^\alpha$ , and  $p' = m' v'$  the (magnitude of the) particle momentum as measured in the local (comoving) fluid frame  $K'$ . Note, that in order to simplify matters, the subscript 0 and the prime superscript are omitted in the following characterisations of the isotropic part of the particle distribution function, i.e.  $f$  now denotes the isotropic part! The total particle energy and momentum in the frame  $K'$  may be written as

$$E' = p'^0 c \quad \text{and} \quad p' = \sqrt{(p'^0)^2 - m_0^2 c^2}, \quad (\text{A.2})$$

respectively, with  $m_0$  the rest mass of the particle and  $c$  the speed of light.

The terms in the first line of Eq. (A.1) represent particle energy changes due to adiabatic expansion or compression of the flow (i.e. the term proportional to the fluid four divergence  $\nabla_\beta u^\beta$ ), due to shear energization (i.e. the term involving  $\Gamma$ ) and due to the fact that  $K'$  is an accelerated frame (i.e. the term  $\propto \dot{u}_\alpha$ , cf. also Webb 1985). The second line gives the effects of diffusion and convection. In Eq. (A.1),  $\nabla_\alpha$  denotes the covariant derivative while  $q^\alpha$  denotes the heat flux. This heat flux contains a diffusive particle current plus a relativistic heat inertial term  $\propto \dot{u}_\beta$  and is given by

$$q^\alpha = -\kappa^{\alpha\beta} \left( \nabla_\beta f - \dot{u}_\beta \frac{(p'^0)^2}{p'} \frac{\partial f}{\partial p'} \right). \quad (\text{A.3})$$

As shown by Webb (1989), Eq. (A.1) could be regarded as the relativistic generalization of the non-relativistic particle transport equation first derived by Earl et al. (1988).

The acceleration four vector  $\dot{u}_\alpha$  of the comoving (or scattering) frame in Eqs. (A.1) and (A.3), is defined by

$$\dot{u}_\alpha = u^\beta \nabla_\beta u_\alpha. \quad (\text{A.4})$$

The fluid energization coefficient  $\Gamma$  in Eq. (A.1) represents energy changes due to viscosity. Since the acceleration of particles draws energy from the fluid flow field, one expects on the other hand the flow to be influenced by the presence of these particles. As was shown, for example, by Earl et al. (1988) and Katz (1991), the resultant dynamical effect on the flow could be modelled by means of an (induced) viscosity coefficient. If one considers the strong scattering limit, i.e. the case where  $\omega \tau_c \ll 1$ , with  $\omega$  the gyrofrequency of the particle in the scattering frame (i.e.  $\omega = q B' c / p'^0$ ),  $\tau_c = 1/\nu_c$  the mean time interval between two scattering events and  $\nu_c$  the collision frequency, this fluid energization coefficient could be written as (Webb 1989, Eq. (34))

$$\Gamma = \frac{c^2}{30} \sigma_{\alpha\beta} \sigma^{\alpha\beta}, \quad (\text{A.5})$$

<sup>1</sup> Note, that in the following, quantities which are measured relative to  $K'$  are labelled with a prime superscript.

where  $\sigma_{\alpha\beta}$  is the (covariant) fluid shear tensor given by

$$\sigma_{\alpha\beta} = \nabla_{\alpha}u_{\beta} + \nabla_{\beta}u_{\alpha} + \dot{u}_{\alpha}u_{\beta} + \dot{u}_{\beta}u_{\alpha} + \frac{2}{3}(g_{\alpha\beta} + u_{\alpha}u_{\beta})\nabla_{\delta}u^{\delta}, \quad (\text{A.6})$$

with  $g_{\alpha\beta}$  the (covariant) metric tensor. Additionally, for the strong scattering limit the spatial diffusion tensor  $\kappa^{\alpha\beta}$  reduces to a simple form given by

$$\kappa^{\alpha\beta} = \kappa(g^{\alpha\beta} + u^{\alpha}u^{\beta}), \quad \text{with } \kappa = v'^2 \tau_c/3, \quad (\text{A.7})$$

the isotropic diffusion coefficient and  $v'$  the comoving particle speed.

## Appendix B: Derivation of the steady state diffusive particle transport equation in cylindrical coordinates

Consider for the present purpose a cylindrical jet model where the plasma moves along the  $z$ -axis at constant (relativistic)  $v_z$  while the perpendicular velocity component is purely azimuthal, i.e. characterized by the angular frequency  $\Omega$ , in which case it proves useful to apply cylindrical coordinates  $x^{\alpha} = (ct, r, \phi, z)$ . One may then choose a set of (non-normalized) holonomic basis vectors  $\{e_{\alpha}, \alpha = 0, 1, 2, 3\}$  with  $e_{\alpha} = \partial\mathbf{x}/\partial x^{\alpha}$ , which determine a 1-form basis  $\{e^{\alpha}\}$ , known as its dual basis. For cylindrical coordinates  $x^{\alpha}$  the metric tensor  $g_{\alpha\beta}$  becomes coordinate-dependent, i.e. for the covariant metric tensor we have

$$(g_{\alpha\beta}) = \text{diag}\{-1, 1, r^2, 1\}, \quad (\text{B.1})$$

while for the contravariant counterpart is consequently given by  $(g^{\alpha\beta}) = \text{diag}\{-1, 1, 1/r^2, 1\}$ . In particular, all partial derivatives of the metric coefficients vanish, except for the  $g_{22}$  coefficient, and therefore all connection coefficients or Christoffel symbols of second order vanish, except for

$$\Gamma_{22}^1 = -r, \quad \Gamma_{21}^2 = \Gamma_{12}^2 = \frac{1}{r}. \quad (\text{B.2})$$

Using the chosen holonomic basis we consider a simple plasma flow where the four velocity could be written in coordinate form as  $u^{\alpha}e_{\alpha} = \gamma e_0 + (\gamma\Omega/c)e_2 + (\gamma v_z/c)e_3$  and  $u_{\alpha}e^{\alpha} = -\gamma e^0 + (\gamma r^2\Omega/c)e^2 + (\gamma v_z/c)e^3$ , respectively, or in shortened notation as

$$u^{\alpha} = \gamma(1, 0, \Omega/c, v_z/c), \quad (\text{B.3})$$

$$u_{\alpha} = \gamma(-1, 0, \Omega r^2/c, v_z/c), \quad (\text{B.4})$$

where the normalization

$$\gamma = \frac{1}{\sqrt{1 - \Omega^2 r^2/c^2 - v_z^2/c^2}} \quad (\text{B.5})$$

denotes the Lorentz factor of the flow and where the angular frequency may be selected to be a function of the radial coordinate, i.e.  $\Omega = \Omega(r)$ .

Generally, for a contravariant four vector  $A^{\alpha}$  the covariant derivative is given by

$$A^{\alpha}{}_{\parallel\beta} = \frac{\partial A^{\alpha}}{\partial x^{\beta}} + \Gamma_{\beta\mu}^{\alpha} A^{\mu}, \quad (\text{B.6})$$

while for the covariant derivative of a covariant four vector  $A_{\alpha}$  one has

$$A_{\alpha\parallel\beta} = \frac{\partial A_{\alpha}}{\partial x^{\beta}} - \Gamma_{\alpha\beta}^{\mu} A_{\mu}. \quad (\text{B.7})$$

Hence, for the assumed four velocity Eq. (B.3), the fluid four divergence becomes zero, i.e.  $\nabla_{\beta}u^{\beta} = 0$ , while the fluid four acceleration Eq. (A.4) reduces to

$$\dot{u}_{\alpha}e^{\alpha} = -u^2(\Gamma_{21}^2 u_2)e^1 = -(\gamma^2 \Omega^2 r/c^2)e^1. \quad (\text{B.8})$$

For the components of the shear tensor Eq. (A.6) we may then derive the following relations

$$\begin{aligned} \sigma_{01} &= \sigma_{10} = -(\gamma^3 r^2/c^2)\Omega \frac{d\Omega}{dr} \\ \sigma_{00} &= \sigma_{11} = \sigma_{22} = \sigma_{33} = \sigma_{02} = \sigma_{20} = \sigma_{23} = \sigma_{32} = 0 \\ \sigma_{12} &= \sigma_{21} = (\gamma^3 r^2/c) \frac{d\Omega}{dr} (1 - v_z^2/c^2) \\ \sigma_{13} &= \sigma_{31} = (\gamma^3 r^2 v_z/c^3)\Omega \frac{d\Omega}{dr}. \end{aligned} \quad (\text{B.9})$$

The viscous energization coefficient Eq. (A.5) then becomes

$$\Gamma = \frac{1}{15} \gamma^4 r^2 \left( \frac{d\Omega}{dr} \right)^2 (1 - v_z^2/c^2), \quad (\text{B.10})$$

noting that  $\sigma^{01} = -\sigma_{01}$ ,  $\sigma^{12} = \sigma_{12}/r^2$  and  $\sigma^{13} = \sigma_{13}$ .

As the fluid four divergence vanishes, the first term in brackets of Eq. (A.1) becomes zero, while for the second term in brackets one finds

$$\dot{u}_{\alpha}q^{\alpha} = \dot{u}_1 q^1, \quad (\text{B.11})$$

with the radial particle current  $q^1 \equiv q^r$  (cf. Eq. (A.3))

$$q^1 \equiv q^r = -\kappa \left( \frac{\partial f}{\partial r} + \frac{\gamma^2 \Omega^2 r (p^0)^2}{c^2} \frac{\partial f}{p'} \frac{\partial f}{\partial p'} \right). \quad (\text{B.12})$$

In the steady-state the fifth term in brackets of Eq. (A.1) becomes

$$\nabla_{\alpha}q^{\alpha} = \frac{1}{r} \frac{\partial}{\partial r}(r q^r) - \kappa(1 + \gamma^2 v_z^2/c^2) \frac{\partial^2 f}{\partial z^2}, \quad (\text{B.13})$$

noting that the third component of the heat flux is given by

$$q^3 = -\kappa(1 + \gamma^2 v_z^2/c^2) \frac{\partial f}{\partial z}. \quad (\text{B.14})$$

Finally, for the fourth term in brackets of Eq. (A.1) we have

$$u^{\alpha} \nabla_{\alpha} f = (\gamma v_z/c) \frac{\partial f}{\partial z}. \quad (\text{B.15})$$

Now, by collecting together all the relevant terms of Eq. (A.1) and by introducing a source term  $Q$  (which may depend on  $r$ ,  $z$  and  $p'$ ), we finally arrive at the relevant relativistic steady-state transport equation in cylindrical coordinates being appropriate

for the considered rotating and shearing jet flows:

$$\begin{aligned} & \frac{1}{p'^2} \frac{\partial}{\partial p'} \left[ p' (p'^0)^2 \frac{\gamma^2 \Omega^2 r}{c^2} q' \right. \\ & \quad \left. - \frac{\gamma^4 r^2}{15} (1 - v_z^2/c^2) \left( \frac{d\Omega}{dr} \right)^2 p'^4 \tau_c \frac{\partial f}{\partial p'} \right] \\ & + \gamma v_z \frac{\partial f}{\partial z} - \kappa (1 + \gamma^2 v_z^2/c^2) \frac{\partial^2 f}{\partial z^2} + \frac{1}{r} \frac{\partial}{\partial r} (r q') = Q. \end{aligned} \quad (\text{B.16})$$

For purely azimuthal, special relativistic flows with  $v_z = 0$ , the transport Eq. (B.16) reduces to Eq. (5.2) derived in WJM 94.

As suggested by WJM 94, the derived transport equation may be cast in a more suitable form by introducing the variable

$$\Phi = \ln(H) \quad (\text{B.17})$$

replacing the comoving particle momentum variable  $p'$ . Following WJM 94, we may define  $H$  such that  $(\partial f/\partial r)_H = -q'/\kappa$ , with  $q'$  given by Eq. (B.12), the index  $H$  denoting a derivative at constant  $H$ , i.e.

$$H = p'^0 c \exp\left(-\int^r dr' \frac{\gamma^2 \Omega^2 r'}{c^2}\right). \quad (\text{B.18})$$

With respect to a physical interpretation of  $H$  we would like to note, that in the case of rigid rotation (i.e.  $\Omega = \text{const.}$ )  $H$  could be related to the Hamiltonian for a bead on a rigidly rotating wire (cf. WJM 94; see also Eq. (C.12)).

Hence, by writing  $f(r, z, p') \rightarrow f(r, z, \Phi)$ , the relevant derivatives transform like

$$\begin{aligned} \left( \frac{\partial f(r, z, \Phi)}{\partial r} \right)_\Phi &= \frac{\partial f(r, z, p')}{\partial r} + \left( \frac{\partial p'}{\partial r} \right)_\Phi \frac{\partial f(r, z, p')}{\partial p'} \\ &= -\frac{q'}{\kappa}, \end{aligned} \quad (\text{B.19})$$

using  $\partial p'/\partial p'^0 = p'^0/p'$  and noting that  $p'^0 c = \exp[\int dr' \gamma^2 \Omega^2 r'/c^2] \exp \Phi$ . As usual, the index  $\Phi$  in Eq. (B.19) denotes a derivative at constant  $\Phi$ . Similarly, for the momentum-derivatives we have

$$\frac{\partial f(r, z, p')}{\partial p'} = \frac{\partial \Phi}{\partial p'} \frac{\partial f(r, z, \Phi)}{\partial \Phi} = \frac{p'}{(p'^0)^2} \frac{\partial f(r, z, \Phi)}{\Phi} \quad (\text{B.20})$$

and consequently

$$\begin{aligned} \frac{\partial^2 f(r, z, p')}{\partial p'^2} &= \frac{(p'^0)^2 - 2 p'^2}{(p'^0)^4} \frac{\partial f(r, z, \Phi)}{\partial \Phi} \\ &+ \frac{p'^2}{(p'^0)^4} \frac{\partial^2 f(r, z, \Phi)}{\partial \Phi^2}. \end{aligned} \quad (\text{B.21})$$

Now, using Eq. (B.19) and collecting all terms together in the transport Eq. (B.16) which depend on  $\partial f/\partial r$  and additionally recalling that  $\kappa = (v'^2 \tau_c)/3$  we may arrive at

$$\begin{aligned} & \left[ \frac{1}{r} + \frac{\partial \kappa/\partial r}{\kappa} \right] \left( \frac{\partial f}{\partial r} \right)_\Phi + \frac{1}{p'^2} \frac{\partial}{\partial p'} \left[ p' (p'^0)^2 \frac{\gamma^2 \Omega^2 r}{c^2} \right] \left( \frac{\partial f}{\partial r} \right)_\Phi \\ &= \frac{1}{r} (1 + \beta) \left( \frac{\partial f}{\partial r} \right)_\Phi + [3 + \alpha] \left( \frac{p'^0}{p'} \right)^2 \frac{\gamma^2 \Omega^2 r}{c^2} \left( \frac{\partial f}{\partial r} \right)_\Phi \end{aligned} \quad (\text{B.22})$$

where the position and momentum dependence of the collision time  $\tau_c$  (and thus of the diffusion coefficient) has been caught into the definition of the variables  $\alpha$  and  $\beta$ , i.e.  $\alpha$  and  $\beta$  are given by

$$\alpha = \frac{\partial \ln \tau_c}{\partial \ln p'} \quad \text{and} \quad \beta = \frac{\partial \ln \tau_c}{\partial \ln r}, \quad (\text{B.23})$$

respectively. Equation (B.22) has been obtained noting that  $v'^2 = c^2 p'^2/(p'^0)^2$  and  $[\partial \tau_c/\partial p']/\tau_c = \alpha/p'$  and using

$$\frac{\partial \kappa}{\partial p'} = \frac{1}{3} \left( \frac{\partial v'^2}{\partial p'} \tau_c + v'^2 \frac{\partial \tau_c}{\partial p'} \right). \quad (\text{B.24})$$

In a similar manner, the terms depending on  $\partial f/\partial p'$  in Eq. (B.16) may be rewritten using Eq. (B.20) as

$$\begin{aligned} & \frac{1}{p'^2} \left[ \frac{\partial}{\partial p'} \left( \frac{\gamma^4 r^2}{15} (1 - v_z^2/c^2) \left( \frac{d\Omega}{dr} \right)^2 p'^4 \tau_c \right) \right] \frac{\partial f(r, z, p')}{\partial p'} \\ &= \kappa \frac{\gamma^4 r^2}{5 c^2} (1 - v_z^2/c^2) [4 + \alpha] \left( \frac{d\Omega}{dr} \right)^2 \frac{\partial f(r, z, \Phi)}{\partial \Phi}. \end{aligned} \quad (\text{B.25})$$

Likewise, for the terms in Eq. (B.16) which depend on  $\partial^2 f/\partial p'^2$  one finds

$$\begin{aligned} & \frac{1}{p'^2} \left( \frac{\gamma^4 r^2}{15 c^2} (1 - \frac{v_z^2}{c^2}) \left( \frac{d\Omega}{dr} \right)^2 p'^4 \tau_c \right) \frac{\partial^2 f(r, z, p')}{\partial p'^2} \\ &= \kappa \frac{\gamma^4 r^2}{5 c^2} (1 - \frac{v_z^2}{c^2}) \left( \frac{d\Omega}{dr} \right)^2 \left( \left[ 1 - 2 \frac{p'^2}{(p'^0)^2} \right] \frac{\partial f(r, z, \Phi)}{\partial \Phi} \right. \\ & \quad \left. + \frac{p'^2}{(p'^0)^2} \frac{\partial^2 f(r, z, \Phi)}{\partial \Phi^2} \right) \end{aligned} \quad (\text{B.26})$$

where Eq. (B.21) has been used.

Now, collecting all relevant terms together the steady state transport Eq. (B.16) may finally be rewritten as

$$\begin{aligned} & \frac{\partial^2 f}{\partial r^2} + \left( \frac{1 + \beta}{r} + [3 + \alpha] \frac{\gamma^2 \Omega^2 r}{c^2} \left( \frac{p'^0}{p'} \right)^2 \right) \frac{\partial f}{\partial r} \\ & + \frac{\gamma^4 r^2}{5 c^2} (1 - v_z^2/c^2) \left( \frac{d\Omega}{dr} \right)^2 \left( \left[ 5 + \alpha - 2 \left( \frac{p'}{p'^0} \right)^2 \right] \frac{\partial f}{\partial \Phi} \right. \\ & \quad \left. + \left( \frac{p'}{p'^0} \right)^2 \frac{\partial^2 f}{\partial \Phi^2} \right) \\ & - \frac{\gamma v_z}{\kappa} \frac{\partial f}{\partial z} + (1 + \gamma^2 v_z^2/c^2) \frac{\partial^2 f}{\partial z^2} = -\frac{Q}{\kappa}, \end{aligned} \quad (\text{B.27})$$

with  $f \equiv f(r, z, \Phi)$  and where the  $r$ -derivatives should be understood as derivatives keeping  $\Phi$  constant, i.e. we may treat  $r, z, \Phi$  as independent variables. Again, for  $v_z = 0$  the partial differential equation (B.27) reduces to the steady state transport equation given in WJM 94 (e.g. see their Eq. (5.5)).

For the present application where we are interested in highly relativistic particles with  $p'^0 \simeq p'$ , the steady-state

transport Eq. (B.27) simplifies to

$$\begin{aligned} \frac{\partial^2 f}{\partial r^2} + \left( \frac{1+\beta}{r} + [3+\alpha] \frac{\gamma^2 \Omega^2 r}{c^2} \right) \frac{\partial f}{\partial r} \\ + \frac{\gamma^4 r^2}{5c^2} (1 - v_z^2/c^2) \left( \frac{d\Omega}{dr} \right)^2 \left( [3+\alpha] \frac{\partial f}{\partial \Phi} + \frac{\partial^2 f}{\partial \Phi^2} \right) \\ - \frac{\gamma v_z}{\kappa} \frac{\partial f}{\partial z} + (1 + \gamma^2 v_z^2/c^2) \frac{\partial^2 f}{\partial z^2} = -\frac{Q}{\kappa}. \end{aligned} \quad (\text{B.28})$$

### Appendix C: Green's solutions for the steady state diffusive particle transport equation

For general applications one may search for (two-dimensional) Green's solutions of the steady state transport equation (see also WJM 94), i.e. solutions of Eq. (B.28) with source term

$$Q = q_0 \delta(r - r_s) \delta(p' - p'_s) \delta(z - z_s), \quad (\text{C.1})$$

or equivalently (i.e. utilizing the properties of the delta function) with source term

$$Q = \frac{q_0}{p'_s} \delta(r - r_s) \delta(\Phi - \Phi_s) \delta(z - z_s), \quad (\text{C.2})$$

describing monoenergetic injection of particles with momentum  $p' = p'_s$  at position  $r = r_s$ ,  $z = z_s$ . For consistency, the constant  $q_0$  in Eqs. (C.1) and (C.2) has to be defined such, that the relevant expression satisfies the requirement that  $\delta(\mathbf{r} - \mathbf{r}_s) \delta(\mathbf{p}' - \mathbf{p}'_s)$  vanishes unless  $r = r_s$ ,  $\phi = \phi_s$ ,  $p' = p'_s$  and integrate to unity (or  $N_s$  if  $N_s$  particles are injected) over all space and momentum directions. Using cylindrical coordinates this requires

$$q_0 = \frac{N_s}{8\pi^2 p_s'^2 r_s}. \quad (\text{C.3})$$

Solutions of the transport Eq. (B.28) may then be found by applying Fourier techniques, i.e. by using the double Fourier transform defined by

$$F(r; \mu, \nu) = \int_{-\infty}^{\infty} dz \int_{-\infty}^{\infty} d\Phi \exp[i(\nu \Phi + \mu z)] f(r, z, \Phi), \quad (\text{C.4})$$

where the inverse Fourier transform is given by

$$f(r, z, \Phi) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} d\mu \int_{-\infty}^{\infty} d\nu \exp[-i(\nu \Phi + \mu z)] \times F(r; \mu, \nu). \quad (\text{C.5})$$

Denoting the Fourier transform of the source term  $-Q/\kappa$  by  $\tilde{Q}$ , we have

$$\begin{aligned} \tilde{Q} &= - \int_{-\infty}^{\infty} dz \int_{-\infty}^{\infty} d\Phi \exp[i(\nu \Phi + \mu z)] \frac{q_0}{\kappa p'_s} \\ &\quad \times \delta(r - r_s) \delta(\Phi - \Phi_s) \delta(z - z_s) \\ &= -\frac{q_0}{\kappa_s p'_s} \exp[i(\nu \Phi_s + \mu z_s)] \delta(r - r_s). \end{aligned} \quad (\text{C.6})$$

Taking the Fourier transform of the transport Eq. (B.28) one arrives at

$$\begin{aligned} \frac{\partial^2 F}{\partial r^2} + \left( \frac{1+\beta}{r} + [3+\alpha] \frac{\gamma^2 \Omega^2 r}{c^2} \right) \frac{\partial F}{\partial r} \\ - \left[ \frac{\gamma^4 r^2}{5c^2} (1 - \frac{v_z^2}{c^2}) \left( \frac{d\Omega}{dr} \right)^2 ([3+\alpha] i\nu + \nu^2) \right] F \\ - \left[ \frac{\gamma v_z}{\kappa} i\mu + (1 + \gamma^2 \frac{v_z^2}{c^2}) \mu^2 \right] F = \tilde{Q}. \end{aligned} \quad (\text{C.7})$$

Let  $F_G$  be the (two-dimensional) Green's solution of this equation satisfying homogeneous, i.e. zero Dirichlet boundary conditions, then Fourier inversion [i.e. Eq. (C.5)] yields the required Green's function solution  $f_G(r, z, p'; r_s, z_s, p'_s)$ . Generally, the considered Fourier techniques yield the Green's function for infinite domains. The proper Green's function for bounded domains (i.e. for finite  $z$ ) may be obtained, for example, by the method of images (e.g. Morse & Feshbach 1953, pp. 812-816) in the case where the boundaries are restricted to straight lines in two dimensions or planes in three dimensions. The steady state version of the Green's formula (cf. WJM 94, Eqs. (3.24) and (7.12)) may then be applied in order to arrive at the general solution of the considered transport equation.

Yet, even for the simplification  $v_z = 0$  and  $\gamma = \text{const.}$  (i.e. using a simple galactic rotation law  $\Omega \propto 1/r$ ), the Green's function solution for the steady state transport Eq. (B.28) is not straightforward to evaluate (see WJM 94). However, as we are primarily interested in an analysis of the azimuthal effects of particle acceleration in rotating jet flows, we may be content with a  $z$ -independent solution of the transport equation, i.e. with an investigation of the so-called one-dimensional Green's function which preserves much of the physics involved (cf. Morse & Feshbach 1953, pp. 842-847). This (one-dimensional) Green's function may be found by integrating the two-dimensional ring source  $Q$  (which depends on the space coordinates  $r_s, z_s$ ) from  $z_s = -\infty$  to  $z_s = \infty$ ; i.e. the one-dimensional Green's function represents the Green's solution for a steady state monoenergetic injection of particles from a infinite cylindrical surface parallel to the jet axis at radius  $r_s$ . Hence, by utilizing the Fourier integral theorem it could be directly shown, that the one-dimensional Green's function  $f_{G,1D}$  corresponds to a  $z$ -independent solution of the transport Eq. (B.28) with source term  $Q(r, p') = q_0 \delta(r - r_s) \delta(p' - p'_s)$ , i.e. the (one-dimensional) Green's function we are seeking for in the following analysis, represents the solution of the modified transport equation

$$\begin{aligned} \frac{\partial^2 f}{\partial r^2} + \left( \frac{1+\beta}{r} + [3+\alpha] \frac{\gamma^2 \Omega^2 r}{c^2} \right) \frac{\partial f}{\partial r} \\ + \frac{\gamma^4 r^2}{5c^2} (1 - v_z^2/c^2) \left( \frac{d\Omega}{dr} \right)^2 \left( [3+\alpha] \frac{\partial f}{\partial \Phi} + \frac{\partial^2 f}{\partial \Phi^2} \right) \\ = -\frac{q_0}{\kappa p'_s} \delta(r - r_s) \delta(\Phi - \Phi_s). \end{aligned} \quad (\text{C.8})$$

### C.1. Rigid rotation profiles

In the case of solid body (uniform) rotation shearing in the background flow is absent since the fluid moves without internal distortions. For  $\Omega = \Omega_0 = \text{const.}$ , Eq. (C.8) reduces to the purely spatial transport equation

$$\frac{\partial^2 f}{\partial r^2} + \left[ \frac{1+\beta}{r} + (3+\alpha) \frac{\tilde{\Omega}_0^2 r/c^2}{(1-\tilde{\Omega}_0^2 r^2/c^2)} \right] \frac{\partial f}{\partial r} = Q_0, \quad (\text{C.9})$$

where the constant  $\tilde{\Omega}_0$  is defined by

$$\tilde{\Omega}_0 = \frac{\Omega_0}{\sqrt{1-v_z^2/c^2}}, \quad (\text{C.10})$$

while  $Q_0 = -q_0 \delta(r-r_s) \delta(\Phi - \Phi_s) / (\kappa_s p'_s)$ , with  $q_0$  given by Eq. (C.3), and where the diffusion coefficient is assumed to be of the form

$$\kappa = \kappa_0 \left( \frac{p'}{p'_0} \right)^\alpha \left( \frac{r}{r_0} \right)^\beta. \quad (\text{C.11})$$

with  $\kappa_0$ ,  $p'_0$  and  $\alpha, \beta$  as constants, cf. Eqs. (A.7) and (B.23).

For rigid rotation Eq. (B.18) yields

$$H(r, p') = p'^0 c (1 - \Omega_0^2 r^2/c^2 - v_z^2/c^2)^{1/2} = \frac{p'^0 c}{\gamma}, \quad (\text{C.12})$$

where  $\gamma$  denotes the Lorentz factor of the flow. As  $H = H_s(r_s, p'_s)$  is a constant of motion (cf. Noether's theorem, see also WJM 94), the particle momentum  $p'$  in the comoving frame could be simply expressed as a function of the radial coordinate

$$p'(r) = m_0 c \sqrt{\frac{H_s^2}{m_0^2 c^4 (1 - \Omega_0^2 r^2/c^2 - v_z^2/c^2)} - 1}, \quad (\text{C.13})$$

with  $m_0$  the rest mass of the particle.

Basically, the solution space of the homogeneous part of Eq. (C.9) could be described by a set of two independent solutions, e.g. by the functions  $y_1(r)$  and  $y_2(r)$  with Wronskian  $W(r) \equiv W(y_1(r), y_2(r)) = y_1 dy_2/dr - y_2 dy_1/dr$ , where for an appropriate choice  $y_2(r) \equiv 1$ . The relevant analytical expressions might be directly written down for some special cases of interest:

a.) for a constant diffusion coefficient, i.e.  $\alpha = \beta = 0$ , two independent solutions are given by  $y_2(r) \equiv 1$  and

$$y_1(r) = \sqrt{1 - \tilde{\Omega}_0^2 r^2/c^2} \left( \frac{4}{3} - \frac{\tilde{\Omega}_0^2 r^2}{3c^2} \right) - \ln \frac{c(1 + \sqrt{1 - \tilde{\Omega}_0^2 r^2/c^2})}{\tilde{\Omega}_0 r} \quad (\text{C.14})$$

where for the Wronskian one simply finds

$$W(r) = -r^{-1} \left( 1 - \frac{\tilde{\Omega}_0^2 r^2}{c^2} \right)^{3/2}; \quad (\text{C.15})$$

b.) in the case, where  $\beta$  is negative, the solution  $y_1(r)$  could be expressed in terms of the incomplete Beta function (cf.

Abramowitz & Stegun 1965, p. 263), i.e. one finally may arrive at the system  $y_2(r) \equiv 1$  and

$$y_1(r) = \frac{1}{2} \left( \frac{\tilde{\Omega}_0}{c} \right)^\beta \times B \left( \frac{\tilde{\Omega}_0^2 r^2}{c^2}, -\frac{\beta}{2}, \frac{5+\alpha}{2} \right) \quad \text{for } \alpha > -5, \text{ and } \beta < 0. \quad (\text{C.16})$$

Note, that now the solutions  $y_1, y_2$  have been defined such that the appropriate Wronskian reduces to Eq. (C.15) for  $\alpha = \beta = 0$ , i.e. we have

$$W(r) = -r^{-(1+\beta)} \left( 1 - \frac{\tilde{\Omega}_0^2 r^2}{c^2} \right)^{(3+\alpha)/2}. \quad (\text{C.17})$$

The general (one-dimensional Green's) solution of the inhomogeneous differential equation Eq. (C.9) with monoenergetic source term  $Q_0$  defined above could then be written as (e.g. Morse & Feshbach 1953, p. 530)

$$f(r, p') = y_1(r) \left[ k_1 - \int^r \frac{Q_0 y_2(r)}{W(r)} dr \right] + y_2(r) \left[ k_2 + \int^r \frac{Q_0 y_1(r)}{W(r)} dr \right], \quad (\text{C.18})$$

where  $k_1, k_2$  are integration constants specified by the boundary conditions.

In the disk-jet scenario the accretion disk is usually assumed to supply the mass for injection into the jet, thus for simplicity one may consider a rather hollow jet structure (cf. Marcowith et al. 1995; Fendt 1997a; Subramanian et al. 1999) where the plasma motion in the azimuthal direction is restricted to a region  $r_{\text{in}} \leq r \leq r_{\text{out}} < r_L$  where  $r_{\text{in}}$  denotes the jet inner radius,  $r_{\text{out}}$  the relevant outer radius and  $r_L$  the light cylinder radius. Particles are supposed to be injected at position  $r_s$  with initial momentum  $p'_s$ , where  $r_{\text{in}} < r_s < r_{\text{out}}$ . By choosing homogeneous boundary conditions  $f(r = r_{\text{in}}) = 0$  and  $f(r = r_{\text{out}}) = 0$ , the integration constants in Eq. (C.18) are determined by

$$k_1 = -k_2 \frac{1}{y_1(r_{\text{in}})} \quad \text{and} \quad k_2 = \frac{\tilde{q} [y_1(r_{\text{out}}) - y_1(r_s)]}{1 - y_1(r_{\text{out}})/y_1(r_{\text{in}})}, \quad (\text{C.19})$$

where

$$\tilde{q} = -\frac{q_0}{\kappa_s p'_s W(r_s)} \delta(\Phi - \Phi_s), \quad (\text{C.20})$$

with  $q_0$  given by Eq. (C.3), i.e.  $q_0 = N_s / (8\pi p'_s r_s)$ .

Therefore the (one-dimensional) Green's solution may be written as

$$f(r, p'; r_s, p'_s) = y_1(r) [k_1 \theta(r - r_s) + k_1 \theta(r_s - r) - \tilde{q} \theta(r - r_s)] + [k_2 \theta(r - r_s) + k_2 \theta(r_s - r) + \tilde{q} y_1(r_s) \theta(r - r_s)], \quad (\text{C.21})$$

where  $\theta(x)$  denotes the Heaviside step function. The delta function in Eqs. (C.20) and (C.21) indicates that the particle momentum in the comoving frame is directly related to the relevant radial position by Eq. (C.13). In order to gain insight into

the efficiency of the acceleration process one may introduce a spatial weighting function  $N(r)$  defined by

$$f(r, p'; r_s, p'_s) = N(r) \delta(\Phi - \Phi_s) = N(r) H_s \delta(H - H_s). \quad (\text{C.22})$$

### C.2. Keplerian rotation profiles

In the case of Keplerian rotating background flow with  $\Omega(r) = k r^{-3/2}$ ,  $k = \sqrt{GM}$  generally both, shear and centrifugal acceleration, will occur. By applying a simple Fourier transformation (cf. Eqs. (C.4) and (C.5)), i.e.

$$F = \int_{-\infty}^{\infty} d\Phi \exp[i\nu\Phi] f, \quad (\text{C.23})$$

where the inverse Fourier transform is given by

$$f = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu \exp[-i\nu\Phi] F, \quad (\text{C.24})$$

the transport equation (C.8) could be written as

$$\begin{aligned} \frac{\partial^2 F}{\partial r^2} + \frac{1}{r} \left( a_1 + a_2 \frac{\tilde{\gamma}(r)^2}{r} \right) \frac{\partial F}{\partial r} \\ - \frac{\tilde{\gamma}(r)^4}{r^3} (i\nu a_3 + a_4 \nu^2) F = \tilde{Q}_0, \end{aligned} \quad (\text{C.25})$$

where  $\tilde{\gamma}(r)$  is defined by  $\tilde{\gamma}(r) = \gamma(r) \sqrt{1 - v_z^2/c^2}$ , with  $\gamma$  the Lorentz factor of the flow, and where  $\tilde{Q}_0$  denotes the Fourier transform of the source term  $Q_0 = -q_0 \delta(r - r_s) \delta(\Phi - \Phi_s) / (\kappa_s p'_s)$ , i.e.

$$\tilde{Q}_0 = -\frac{q_0 \exp[i\nu\Phi_s]}{\kappa_s p'_s} \delta(r - r_s). \quad (\text{C.26})$$

The abbreviations  $a_1, a_2, a_3, a_4$  in Eq. (C.25) are defined by

$$a_1 = (1 + \beta), \quad (\text{C.27})$$

$$a_2 = (3 + \alpha) \frac{GM}{(c^2 - v_z^2)}, \quad (\text{C.28})$$

$$a_3 = \frac{9}{20} (3 + \alpha) \frac{GM}{(c^2 - v_z^2)}, \quad (\text{C.29})$$

$$a_4 = \frac{9}{20} \frac{GM}{(c^2 - v_z^2)}. \quad (\text{C.30})$$

An analytical evaluation of Eq. (C.25) is rather complicated. However, a simple set of solutions may be found in the case of  $r$  being large such that the rotational velocity becomes non-relativistic and the approximation  $\tilde{\gamma}(r) = (1 - GM/[(c^2 - v_z^2)r])^{-1/2} \simeq 1$  holds. For, the Fourier transformed transport equation then simplifies to

$$\frac{\partial^2 F}{\partial r^2} + \frac{1}{r} \left( a_1 + \frac{a_2}{r} \right) \frac{\partial F}{\partial r} - \frac{1}{r^3} (i\nu a_3 + a_4 \nu^2) F = \tilde{Q}_0, \quad (\text{C.31})$$

which, using the substitution  $y = a_2/r$ ,  $a_2 \neq 0$  (i.e.  $\alpha \neq -3$ ), leads to

$$y \frac{\partial^2 F}{\partial y^2} + (2 - a_1 - y) \frac{\partial F}{\partial y} - \frac{i\nu a_3 + a_4 \nu^2}{a_2} F = 0, \quad (\text{C.32})$$

for the homogeneous part of Eq. (C.31). Equation (C.32) is known in the literature as Kummer's equation (e.g. Abramowitz & Stegun 1965, p. 504). For the general case where  $a_2 \neq 0$  and  $(2 - a_1) \neq -n$ ,  $n \in N_0$ , the complete solution of this equation, i.e. of the homogeneous part of Eq. (C.31), may be written as

$$F_H(r, \nu) = c_1 f_1(r, \nu) + c_2 f_2(r, \nu) \quad (\text{C.33})$$

where the functions  $f_1, f_2$  are given by

$$f_1(r, \nu) = M\left(\frac{i\nu a_3 + a_4 \nu^2}{a_2}, 2 - a_1, \frac{a_2}{r}\right) \quad (\text{C.34})$$

$$f_2(r, \nu) = U\left(\frac{i\nu a_3 + a_4 \nu^2}{a_2}, 2 - a_1, \frac{a_2}{r}\right). \quad (\text{C.35})$$

Here,  $M(a, b, y)$  and  $U(a, b, y)$  denote the confluent hypergeometric functions (cf. Abramowitz & Stegun 1965, pp. 504f; Buchholz 1953, pp. 1-9), with  $M(a, b, y)$  being characterized by the series representation

$$\begin{aligned} M(a, b, y) = 1 + \frac{a}{1!b} y + \frac{a(a+1)}{2!b(b+1)} y^2 \\ + \frac{a(a+1)(a+2)}{3!b(b+1)(b+2)} y^3 + \dots \end{aligned} \quad (\text{C.36})$$

while  $U(a, b, y)$  is given by the series

$$\begin{aligned} U(a, b, y) = \frac{\pi}{\sin \pi b} \left( \frac{M(a, b, y)}{\Gamma(1+a-b)\Gamma(b)} \right. \\ \left. - y^{1-b} \frac{M(1+a-b, 2-b, y)}{\Gamma(a)\Gamma(2-b)} \right), \end{aligned} \quad (\text{C.37})$$

with  $\Gamma(x)$  the Gamma function.

$M(a, b, z)$  has a simple pole at  $b = -n$  for  $a \neq -m$  or for  $a = -m$  if  $m > n$ , and is undefined for  $b = -n$ ,  $a = -m$  and  $m \leq n$ .  $U(a, b, z)$ , however, is defined even for  $b \rightarrow \pm n$ .

For the relevant Wronskian one has (e.g. Abramowitz & Stegun 1965, p. 505)

$$W(y) \equiv W(M(a, b, y), U(a, b, y)) = -\frac{\Gamma(b) y^{-b} e^y}{\Gamma(a)}. \quad (\text{C.38})$$

Thus, for the Wronskian  $W(r) \equiv W(f_1, f_2)$  we find

$$W(r, \nu) = \frac{a_2}{r^2} \Gamma(2 - a_1) \left(\frac{a_2}{r}\right)^{-(2-a_1)} \frac{e^{a_2/r}}{\Gamma(a(\nu))}, \quad (\text{C.39})$$

where

$$a(\nu) = \frac{i\nu a_3 + a_4 \nu^2}{a_2}. \quad (\text{C.40})$$

For the general solution of the inhomogeneous fourier equation (C.25) one finds (cf. Eq. (C.18))

$$\begin{aligned} F(r, \nu) = f_1(r, \nu) [k_1 \theta(r - r_s) + k_1 \theta(r_s - r) \\ - \tilde{q}(\nu) f_2(r_s, \nu) \theta(r - r_s)] \\ + f_2(r, \nu) [k_2 \theta(r - r_s) + k_2 \theta(r_s - r) \\ + \tilde{q}(\nu) f_1(r_s, \nu) \theta(r - r_s)], \end{aligned} \quad (\text{C.41})$$

where  $\tilde{q}(\nu)$  has been defined by

$$\tilde{q}(\nu) = -q_0 \frac{\exp[i\nu\Phi_s]}{\kappa_s p'_s W(r_s, \nu)}, \quad (\text{C.42})$$

with  $q_0$  given by Eq. (C.3) and where  $k_1, k_2$  are integration constants specified by the boundary conditions. For homogeneous Dirichlet conditions at the boundaries  $r_{\text{in}}$  and  $r_{\text{out}}$ , i.e.  $F(r_{\text{in}}, \nu) = F(r_{\text{out}}, \nu) = 0$  with  $r_{\text{in}} < r < r_{\text{out}}$  and  $r_{\text{in}} < r_s < r_{\text{out}}$  the integration constants are fixed (for each  $\nu$ ) and given by

$$k_2(\nu) = -\tilde{q}(\nu) f_1(r_{\text{in}}, \nu) \times \frac{f_2(r_{\text{out}}, \nu) f_1(r_s, \nu) - f_1(r_{\text{out}}, \nu) f_2(r_s, \nu)}{f_2(r_{\text{out}}, \nu) f_1(r_{\text{in}}, \nu) - f_1(r_{\text{out}}, \nu) f_2(r_{\text{in}}, \nu)}, \quad (\text{C.43})$$

$$k_1(\nu) = \frac{f_2(r_{\text{in}}, \nu)}{f_1(r_{\text{in}}, \nu)} k_2. \quad (\text{C.44})$$

By Fourier inversion, we now may obtain the required (one-dimensional) Green's function

$$f(r, \Phi; r_s, \Phi_s) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu \exp[-i\nu\Phi] F(r, \nu), \quad (\text{C.45})$$

with  $F(r, \nu)$  given by Eq. (C.41). As may be obvious from the foregoing investigation this Fourier inversion is not easy to evaluate, not even using numerical methods. However, in order to cope with the integration, we may consider the following substitution ( $\alpha \neq -3$ )

$$\omega = \frac{3}{\sqrt{20(3+\alpha)}} \left( \nu + \frac{3+\alpha}{2} i \right), \quad (\text{C.46})$$

for which one finds (cf. Eq. (C.40) and Eqs. (C.27)–(C.30))

$$a(\nu) = \omega^2 + \sigma^2, \quad (\text{C.47})$$

where  $\sigma$  is defined by  $\sigma = 3\sqrt{3+\alpha}/\sqrt{80}$ .

Performing the substitution in Eq. (C.45), one arrives at an integral with the path of integration  $A$  now in the complex plane, i.e. parallel to the real axis at a distance  $3(3+\alpha)i/[2\sqrt{20(3+\alpha)}]$ , extending from  $-R$  to  $R$  with  $R \rightarrow \infty$ . We may close the path by choosing a rectangular contour  $C$  which consists of the stated parallel  $A$ , the real axis, and the outer lines  $B_1, B_2$  parallel to the imaginary axis at  $R$  and  $-R$ , respectively. For the relevant ranges of  $\alpha$  ( $\alpha \neq -3; 2 - a_1 \neq -n$ ) of interest, the integrand has no poles within the region bounded by  $C$  and thus, by virtue of Cauchy's integral theorem, the value of the contour integral around  $C$  sums to zero. For  $R \rightarrow \infty$  the integrals over  $B_1$  and over  $B_2$  vanish as might be verified by using asymptotic expansion formulas for the integrand. Noting that by means of Euler's equation  $e^{ix} = \cos x + i \sin x$  we have

$$\int_{-\infty}^{\infty} e^{-ix} f(x^2) dx = 2 \int_0^{\infty} f(x^2) \cos x dx, \quad (\text{C.48})$$

and collecting all relevant expressions together, one finally may arrive at the integral ( $2 - a_1 \neq -n; \alpha \neq -3$ )

$$f(r, p'; r_s, p'_s) = g(r_s, p'_s, \alpha, \beta) \exp\left[-\frac{3+\alpha}{2}(\Phi - \Phi_s)\right] \times \int_0^{\infty} d\omega \cos\left(\frac{\sqrt{20(3+\alpha)}}{3} \omega (\Phi - \Phi_s)\right) \times \Gamma(\omega^2 + \sigma^2) \times \left[ f_1(r, \omega^2) f_2(r_{\text{out}}, \omega^2) \frac{h(r_{\text{in}}, r_s; \omega^2)}{h_N(r_{\text{in}}, r_{\text{out}}; \omega^2)} + f_2(r, \omega^2) f_1(r_{\text{out}}, \omega^2) \frac{h(r_s, r_{\text{in}}; \omega^2)}{h_N(r_{\text{in}}, r_{\text{out}}; \omega^2)} \right] \quad \text{for } r > r_s \quad (\text{C.49})$$

$$= g(r_s, p'_s, \alpha, \beta) \exp\left[-\frac{3+\alpha}{2}(\Phi - \Phi_s)\right] \times \int_0^{\infty} d\omega \cos\left(\frac{\sqrt{20(3+\alpha)}}{3} \omega (\Phi - \Phi_s)\right) \times \Gamma(\omega^2 + \sigma^2) \times \left[ f_1(r, \omega^2) f_2(r_{\text{in}}, \omega^2) \frac{h(r_{\text{out}}, r_s; \omega^2)}{h_N(r_{\text{in}}, r_{\text{out}}; \omega^2)} + f_2(r, \omega^2) f_1(r_{\text{in}}, \omega^2) \frac{h(r_s, r_{\text{out}}; \omega^2)}{h_N(r_{\text{in}}, r_{\text{out}}; \omega^2)} \right] \quad \text{for } r < r_s \quad (\text{C.50})$$

where we have introduced the following abbreviations

$$g(r_s, p'_s, \alpha, \beta) = \frac{2q_0 r_s \sqrt{5(3+\alpha)}}{3\pi \kappa_s p'_s \Gamma(2-a_1)} \left(\frac{a_2}{r_s}\right)^{1-a_1} e^{-a_2/r_s}, \quad (\text{C.51})$$

$$f_1(r, \omega^2) = M(\omega^2 + \sigma^2, 2 - a_1, a_2/r), \quad (\text{C.52})$$

$$f_2(r, \omega^2) = U(\omega^2 + \sigma^2, 2 - a_1, a_2/r), \quad (\text{C.53})$$

$$h(r_{\text{in}}, r_s; \omega^2) = f_1(r_{\text{in}}, \omega^2) f_2(r_s, \omega^2) - f_2(r_{\text{in}}, \omega^2) f_1(r_s, \omega^2), \quad (\text{C.54})$$

$$h_N(r_{\text{in}}, r_{\text{out}}; \omega^2) = f_2(r_{\text{out}}, \omega^2) f_1(r_{\text{in}}, \omega^2) - f_1(r_{\text{out}}, \omega^2) f_2(r_{\text{in}}, \omega^2), \quad (\text{C.55})$$

with  $q_0$  given by Eq. (C.3). Equation (C.49) may be evaluated using numerical methods (cf. Wolfram 1996).  $f_1(r, \omega^2)$  and  $f_2(r, \omega^2)$  are bounded for  $\omega = 0$  while the integrand behaves well enough when  $\omega \rightarrow \infty$  to allow for a numerical evaluation.

### C.3. Flat rotation profiles

In the case of flat (galactic-type) rotation with  $\Omega(r) = \Omega_0 r_0/r$ , the co-operation of centrifugal and shear effects could be analysed more directly. Applying Fourier transformation (cf. Eq. (C.23)) the transport Eq. (C.8) simplifies to

$$\frac{\partial^2 F}{\partial r^2} + \frac{a}{r} \frac{\partial F}{\partial r} + \frac{b(\nu)}{r^2} F = \tilde{Q}, \quad (\text{C.56})$$

where  $a$  and  $b(\nu)$  are defined by

$$a = (1 + \beta) + (3 + \alpha) \gamma^2 \eta^2, \quad (\text{C.57})$$

$$b(\nu) = \left[ (3 + \alpha) i \nu + \nu^2 \right] \frac{\gamma^4 \eta^2}{5} (1 - v_z^2/c^2) \quad (\text{C.58})$$

with  $\gamma = 1/\sqrt{1 - \eta^2 - v_z^2/c^2}$  the Lorentz factor of the flow and  $\eta \equiv v_\theta/c = \Omega_0 r_0/c$  its rotational velocity, and where  $\tilde{Q}_0$  is

given by Eq. (C.26). Solutions for the homogeneous part of Eq. (C.56) could then be directly written down, i.e. one has

$$F_1(r, \nu) = r^{\frac{1}{2}(1-a-\sqrt{(a-1)^2+4b(\nu)})}, \quad (\text{C.59})$$

$$F_1(r, \nu) = r^{\frac{1}{2}(1-a+\sqrt{(a-1)^2+4b(\nu)})}, \quad (\text{C.60})$$

with Wronskian  $W(r, \nu)$  fixed by

$$W(r, \nu) = \frac{\sqrt{(a-1)^2+4b(\nu)}}{r^a}. \quad (\text{C.61})$$

The general solution of the inhomogeneous fourier Eq. (C.56) in the case of homogeneous Dirichlet conditions at the boundaries  $r_{\text{in}}$  and  $r_{\text{out}}$ , may then be determined following the steps given in the previous subsection, e.g. see Eqs. (C.41)–(C.45). In order to calculate the inverse Fourier transform, it proves useful to apply the following substitutions

$$\omega = \nu + \frac{1}{2}(3+\alpha)i \quad (\text{C.62})$$

$$\sigma = \frac{3+\alpha}{2} \sqrt{1 + \frac{5}{\gamma^2 \eta^2 (1 - v_z^2/c^2)} \left( \frac{\beta}{3+\alpha} + \gamma^2 \eta^2 \right)^2}, \quad (\text{C.63})$$

for which one has

$$\frac{1}{2} \sqrt{(a-1)^2+4b(\nu)} = \frac{\gamma^2 \eta}{\sqrt{5}} \sqrt{1 - v_z^2/c^2} \sqrt{\omega^2 + \sigma^2}. \quad (\text{C.64})$$

The complete solution may then be found by proceeding as presented in the previous subsection.

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