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Nonaxisymmetric patterns in the linear theory of MHD Taylor-Couette instability

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Abstract. The linear stability of MHD Taylor-Couette flow of infinite vertical extension is considered for various magnetic Prandtl numbers Pm. The calculations are performed for a wide gap container with $\hat{\eta} = 0.5$ with an axial uniform magnetic field excluding counterrotating cylinders. For both hydrodynamically stable and unstable flows the magnetorotational instability produces characteristic minima of the Reynolds number for certain (low) magnetic field amplitudes. For Pm $\lesssim 1$ there is a characteristic magnetic field amplitude beyond which the instability sets in in form of nonaxisymmetric spirals with the azimuthal number $m = 1$. Obviously, the magnetic field is able to excite nonaxisymmetric configurations despite the tendency of differential rotation to favor axisymmetric magnetic fields, which is known from the dynamo theory. If Pm is too big or too small, however, the axisymmetric mode with $m = 0$ appears to be the most unstable one possessing the lowest Reynolds numbers – as it is also true for hydrodynamic Taylor-Couette flow or for very weak fields. That the most unstable mode for modest Pm proves to be nonaxisymmetric must be considered as a strong indication for the possibility of dynamo processes in connection with the magnetorotational instability.

Key words. magnetohydrodynamics – accretion, accretion disks – turbulence

1. Introduction

To discuss possible experimental realizations of the magnetorotational instability as the main transporter of angular momentum in all kinds of accretion disks, several recent studies of Taylor-Couette flow for electro-conducting fluids between rotating cylinders under the influence of an uniform axial magnetic field have been carried out (Ji et al. 2001; Rüdiger & Zhang 2001; Willis & Barenghi 2002). The numbers describing the geometry of the container and the magnetic Prandtl number of the fluid have been considered as the free parameters. For a given magnetic field amplitude (the Hartmann number) the critical angular velocity of the inner cylinder (the critical Reynolds number) is computed for the onset of an instability of the rotation law between the cylinders.

In Rüdiger & Shalybkov (2002) the instability pattern is considered as axisymmetric. The main result for a resting outer cylinder is that for high magnetic Prandtl numbers for weak magnetic fields the excitation of the instability is easier than without a magnetic field but for strong magnetic fields the excitation of the instability is more complicated. The effect, however, disappears for small magnetic Prandtl number, i.e. for lower electric conductivity of the fluid, as it may occur in protoplanetary disks.

On the other hand for a rotating outer cylinder, when no instability without a magnetic field exists, the magnetic field always produces critical Reynolds numbers which, however, change with 1/Pm. For Pm of the order of 10^{-5} the critical Reynolds number is of the order of 10^6 which is the experimental limit.

In the present paper the nonaxisymmetric perturbations are included into the discussion of the magnetorotational instability (MRI). This is of particular relevance for the question of whether the Cowling theorem for dynamo action can be fulfilled, after which a dynamo can only work with nonaxisymmetric fields. We shall find that indeed for certain parameters – despite the smoothing action of the differential rotation – nonaxisymmetric modes can be excited more easily than axisymmetric modes. This is in contrast to earlier results of Taylor-Couette flow without magnetic fields where always the axisymmetric modes possess the lowest Reynolds numbers (Roberts 1965; DiPrima 1961)[1].

Here, the dependence of a real Taylor-Couette flow on the magnetic Prandtl number and on the azimuthal "quantum number m" is investigated. The simple model of uniform density

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[1] For counterrotating cylinders, however, the preference for nonaxisymmetric modes is already known, see Krüger et al. (1966), Chen & Chang (1998).

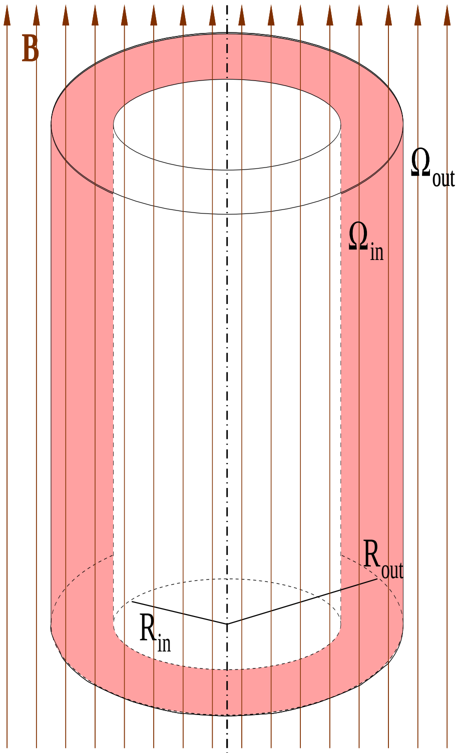

Fig. 1. Cylinder geometry of the Taylor-Couette flow.

fluid contained between two vertically-infinite rotating cylinders is used with a constant magnetic field parallel to the rotation axis. The unperturbed state is a stationary circular flow with the rotation law

$$\Omega(R) = a + b/R^2, \tag{1}$$

where a and b are two constants related to the angular velocities $\Omega_{\rm in}$ and $\Omega_{\rm out}$ with which the inner and the outer cylinders are rotating. If $R_{\rm in}$ and $R_{\rm out}$ ($R_{\rm out} > R_{\rm in}$) are the radii of the two cylinders then

$$a = \frac{\hat\mu - \hat\eta^2}{1 - \hat\eta^2}\,\Omega_{\rm in}, \qquad b = \frac{1 - \hat\mu}{1 - \hat\eta^2}\,\Omega_{\rm in}\,R_{\rm in}^2, \tag{2}$$

with

$$\hat\mu = \Omega_{\rm out}/\Omega_{\rm in} \qquad \text{and} \qquad \hat\eta = R_{\rm in}/R_{\rm out}. \tag{3}$$

After the Rayleigh stability criterion, $\mathrm{d}(R^2\Omega)^2/\mathrm{d}R > 0$, rotation laws are hydrodynamically stable for $\hat\mu > \hat\eta^2$. Taylor-Couette flows with resting outer cylinders ($\hat\mu = 0$) are thus never stable. Here in order to isolate the MRI, we are also interested in flows with rotating outer cylinders so that the hydrodynamical stability criterion, $\hat\mu > \hat\eta^2$, is fulfilled. Our standard examples are formed with $\hat\eta = 0.5$, $\hat\mu = 0$ and $\hat\mu = 0.33$, resp. The first example ($\hat\mu = 0$) is hydrodynamically unstable and the second one ($\hat\mu = 0.33$) is hydrodynamically stable. Due to our astrophysical motivation we are here only interested in flow patterns of containers with positive $\hat\mu$.

2. Basic equations

The MHD equations which have to be solved are

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u}\nabla)\boldsymbol{u} = -\frac{1}{\rho}\nabla p + \nu\Delta\boldsymbol{u} + \boldsymbol{J}\times\boldsymbol{B} \tag{4}$$

and

$$\frac{\partial \boldsymbol{B}}{\partial t} = \mathrm{rot}(\boldsymbol{u}\times\boldsymbol{B}) + \eta\Delta\boldsymbol{B} \tag{5}$$

with $\mathrm{div}\,\boldsymbol{u} = \mathrm{div}\,\boldsymbol{B} = 0$. They are considered in a cylindrical geometry with R, ϕ, and z as the cylindrical coordinates. A viscous electrically-conducting incompressible fluid between two rotating infinite cylinders in the presence of a uniform magnetic field parallel to the rotation axis leads to the basic solution $U_R = U_z = B_R = B_\phi = 0$, $B_z = B_0 = \text{const.}$, and $U_\phi = aR + b/R$, with $\boldsymbol{U}$ as the flow and $\boldsymbol{B}$ as the magnetic field. We are interested in the stability of this solution. The perturbed state of the flow may be described by u'_R, $U_\phi + u'_\phi$, u'_z, B'_R, B'_ϕ, $B_0 + B'_z$, p', with p' as the pressure perturbation.

Here only the linear stability problem is considered. By analyzing the disturbances into normal modes the solutions of the linearized magnetohydrodynamical equations are of the form

$$\begin{aligned} u'_R &= u_R(R)\mathrm{e}^{\mathrm{i}(m\phi+kz-\omega t)}, & B'_R &= B_R(R)\mathrm{e}^{\mathrm{i}(m\phi+kz-\omega t)},\\ u'_\phi &= u_\phi(R)\mathrm{e}^{\mathrm{i}(m\phi+kz-\omega t)}, & B'_\phi &= B_\phi(R)\mathrm{e}^{\mathrm{i}(m\phi+kz-\omega t)},\\ u'_z &= u_z(R)\mathrm{e}^{\mathrm{i}(m\phi+kz-\omega t)}, & B'_z &= B_z(R)\mathrm{e}^{\mathrm{i}(m\phi+kz-\omega t)}. \end{aligned} \tag{6}$$

Only marginal stability will be considered where the imaginary part of ω vanishes. Let $D = R_{\rm out} - R_{\rm in}$ be the gap between the cylinders. We use

$$H = \sqrt{R_{\rm in}\,D} \tag{7}$$

as unit of length, the η/H as unit of perturbed velocity and B_0 as unit of perturbed magnetic field with the magnetic Prandtl number

$$\mathrm{Pm} = \frac{\nu}{\eta}, \tag{8}$$

ν is the kinematic viscosity, η is the magnetic diffusivity. Note H^{-1} as the unit of wave numbers and ν/H^2 as the unit of frequencies. After elimination of both pressure fluctuations and the fluctuations of the vertical magnetic field, B'_z, the linearized equations are

$$\frac{\partial u_R}{\partial R} + \frac{u_R}{R} + \frac{\mathrm{i}m}{R}u_\phi + \mathrm{i}ku_z = 0, \tag{9}$$

$$\begin{aligned} &\frac{\partial^2 u_\phi}{\partial R^2} + \frac{1}{R}\frac{\partial u_\phi}{\partial R} - \frac{u_\phi}{R^2} - \left(\frac{m^2}{R^2} + k^2\right)u_\phi\\ &-\mathrm{i}\left(m\mathrm{Re}\frac{\Omega}{\Omega_{\rm in}} - \omega\right)u_\phi + \frac{2\mathrm{i}m}{R^2}u_R - \mathrm{Re}\frac{1}{R}\frac{\partial}{\partial R}\left(R^2\frac{\Omega}{\Omega_{\rm in}}\right)u_R\\ &-\frac{m}{k}\left[\frac{1}{R}\frac{\partial^2 u_z}{\partial R^2} + \frac{1}{R^2}\frac{\partial u_z}{\partial R} - \left(\frac{m^2}{R^2} + k^2\right)\frac{u_z}{R}\right.\\ &\left.-\mathrm{i}\left(m\mathrm{Re}\frac{\Omega}{\Omega_{\rm in}} - \omega\right)\frac{u_z}{R}\right] + \frac{m}{k}\mathrm{Ha}^2\left[\frac{1}{R}\frac{\partial B_R}{\partial R} + \frac{B_R}{R^2}\right]\\ &+\frac{\mathrm{i}}{k}\mathrm{Ha}^2\left(\frac{m^2}{R^2} + k^2\right)B_\phi = 0, \end{aligned} \tag{10}$$

$$
\begin{aligned}
&\frac{\partial^3 u_z}{\partial R^3} + \frac{1}{R}\frac{\partial^2 u_z}{\partial R^2} - \frac{1}{R^2}\frac{\partial u_z}{\partial R} - \left(\frac{m^2}{R^2} + k^2\right)\frac{\partial u_z}{\partial R} \\
&+ \frac{2m^2}{R^3}u_z - \mathrm{i}\left(m\mathrm{Re}\frac{\Omega}{\Omega_{\rm in}} - \omega\right)\frac{\partial u_z}{\partial R} - \mathrm{i}m\mathrm{Re}\frac{\partial}{\partial R}\left(\frac{\Omega}{\Omega_{\rm in}}\right)u_z \\
&-\mathrm{Ha}^2\left[\frac{\partial^2 B_R}{\partial R^2} + \frac{1}{R}\frac{\partial B_R}{\partial R} - \frac{B_R}{R^2} - k^2 B_R\right. \\
&\left. + \frac{\mathrm{i}m}{R}\frac{\partial B_\phi}{\partial R} - \frac{\mathrm{i}m}{R^2}B_\phi\right] - \mathrm{i}k\left[\frac{\partial^2 u_R}{\partial R^2} + \frac{1}{R}\frac{\partial u_R}{\partial R} - \frac{u_R}{R^2}\right. \\
&\left. - \left(k^2 + \frac{m^2}{R^2}\right)u_R\right] - k\left(m\mathrm{Re}\frac{\Omega}{\Omega_{\rm in}} - \omega\right)u_R \\
&-2\frac{km}{R^2}u_\phi - 2\mathrm{i}k\mathrm{Re}\frac{\Omega}{\Omega_{\rm in}}u_\phi = 0, \qquad (11)
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial^2 B_R}{\partial R^2} + \frac{1}{R}\frac{\partial B_R}{\partial R} - \frac{B_R}{R^2} - \left(\frac{m^2}{R^2} + k^2\right)B_R \\
&-\frac{2\mathrm{i}m}{R^2}B_\phi - \mathrm{i}\mathrm{Pm}\left(m\mathrm{Re}\frac{\Omega}{\Omega_{\rm in}} - \omega\right)B_R + \mathrm{i}ku_R = 0, \qquad (12)
\end{aligned}
$$

$$
\begin{aligned}
&\frac{\partial^2 B_\phi}{\partial R^2} + \frac{1}{R}\frac{\partial B_\phi}{\partial R} - \frac{B_\phi}{R^2} - \left(\frac{m^2}{R^2} + k^2\right)B_\phi + \frac{2\mathrm{i}m}{R^2}B_R \\
&-\mathrm{i}\mathrm{Pm}\left(m\mathrm{Re}\frac{\Omega}{\Omega_{\rm in}} - \omega\right)B_\phi + \mathrm{i}ku_\phi + \mathrm{Pm}\ \mathrm{Re}\ R\frac{\partial \Omega/\Omega_{\rm in}}{\partial R}B_R = 0. \qquad (13)
\end{aligned}
$$

The Reynolds number Re and the Hartmann number Ha are defined as

$$\mathrm{Re} = \frac{\Omega_{\rm in} H^2}{\nu}, \qquad \mathrm{Ha} = \frac{B_0 H}{\sqrt{\mu_0 \rho \nu \eta}}. \qquad (14)$$

For a given Hartmann number and Prandtl number in the present paper we shall derive in a linear theory the critical Reynolds number of the rotation of the inner cylinder for various mode numbers m.

3. Boundary conditions, numerics

An appropriate set of ten boundary conditions is needed to solve the system (9)–(13). Always no-slip conditions for the velocity on the walls are used, i.e. $u_R = u_\phi = \mathrm{d}u_R/\mathrm{d}R = 0$. The boundary conditions depend on the electrical properties of the walls. The tangential currents and the radial component of the magnetic field vanish on conducting walls hence $\mathrm{d}B_\phi/\mathrm{d}R + B_\phi/R = B_R = 0$. These boundary conditions may hold both for $R = R_{\rm in}$ and for $R = R_{\rm out}$.

The homogeneous set of Eqs. (9)–(13) with the boundary conditions for conducting walls determine the eigenvalue problem of the form $\mathcal{L}(k, m, \mathrm{Re}, \mathrm{Ha}, \mathcal{R}(\omega)) = 0$ for given Pm. The real part of ω, $\mathcal{R}(\omega)$, describes a drift of the pattern along the azimuth which only exists for nonaxisymmetric flows. $\mathcal{L}$ is a complex quantity, both its real part and its imaginary part must vanish for the critical Reynolds number (Fig. 2). The latter is minimized by choice of the wave number k. $\mathcal{R}(\omega)$ is the second quantity which is fixed by the eigen equation.

The system is approximated by finite differences with typically 81 gridpoints. The resulting determinant, $\mathcal{L}$, takes the value zero if and only if the values Re are the eigenvalues.

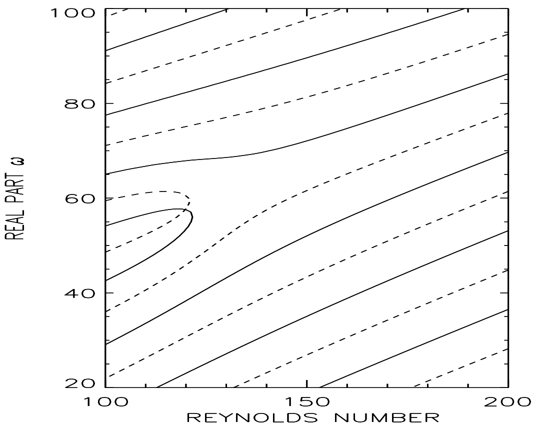

Fig. 2. The zero-lines of the real (solid) and the imaginary (dashed) part of the system determinant $\mathcal{L}$. At the crossing point both parts of the determinant simultaneously vanish fixing the Reynolds number and the drift frequency $\mathcal{R}(\omega)$. Pm and k are prescribed.

We can also stress that the results are numerically robust as an increase of the number of gridpoints does not change the results remarkably. For a fixed Hartmann number, a fixed Prandtl number and a given vertical wave number k we find the eigenvalues of the equation system. They are always minimal for a certain wave number which by itself defines the marginally unstable mode. The corresponding eigenvalue is the desired Reynolds number.

4. Results for conducting walls

Only such a container is considered in the present paper with one and the same gap geometry, i.e. $\hat{\eta} = 0.5$. Then the flow between the cylinders is hydrodynamically unstable between $\hat{\mu} = 0$ and $\hat{\mu} = 0.25$. We shall work with both a hydrodynamically unstable container with $\hat{\mu} = 0$ and also with the hydrodynamically stable container with $\hat{\mu} = 0.33$. If such a container is filled with liquid sodium the relation between Hartmann number and magnetic field is simply

$$\mathrm{Ha} \simeq 0.45\ \frac{B_0}{\mathrm{Gauss}}\ \frac{R_{\rm out}}{10\ \mathrm{cm}}. \qquad (15)$$

For containers with about 20 cm outer radius, therefore, a magnetic field of 1 Gauss corresponds to a Hartmann number of almost unity.

4.1. Resting outer cylinder (steep rotation law)

We start with the results for containers with resting outer cylinders (Fig. 3). Provided a critical rotation rate of the inner cylinder is exceeded they are hydrodynamically unstable. Of course, for $\mathrm{Ha} = m = 0$ the known critical Reynolds number $\mathrm{Re} = 68$ is reproduced. For $m > 0$ the critical Reynolds numbers exceed the value for $m = 0$. Without a magnetic field the instability yields rolls. The critical Reynolds number for $m = 1$ is 75 and for $m = 2$ it is 127.

With magnetic fields ($\mathrm{Ha} > 0$) the magnetic Prandtl number becomes relevant. Results for $\mathrm{Pm} = 10, 1, 0.1$ and 0.01 are presented in Fig. 3. For $\mathrm{Pm} \geq 1$ the electrical conductivity is so high that the magnetorotational instability

(Balbus & Hawley 1991; Brandenburg et al. 1995; Ziegler & Rüdiger 2000) for Ha $\simeq$ 5 produces a characteristic minimum of the critical Reynolds number but for stronger magnetic fields the suppressing action of the magnetic field starts to dominate. In contrast to the expectations, however, for the magnetic Prandtl numbers which are not too high and not too low the mode with $m = 1$ becomes more and more dominant. This is a new and interesting result: The linear instability of the Taylor-Couette flow without magnetic field is formed by axisymmetric rolls but the magnetic field favors the excitation of bisymmetric spirals. For Ha $> 10 \ldots 40$ the instability sets in in the form of a drifting pattern with maximum and minimum separated by 180°. However, as can be seen in Fig. 3 (last plot) for small magnetic Prandtl number (here Pm = 0.01) the axisymmetric pattern with $m = 0$ again starts to dominate with the lowest critical Reynolds number.

The modes with $m = 2$, which we have also considered, never have the lowest Reynolds numbers, they are not important for the discussion of the pattern of the instability. What we have found is that in contrast to the hydrodynamic case (Pm = 0) there are experimental combinations where the nonaxisymmetric mode with $m = 1$ has a lower Reynolds number than the axisymmetric mode with $m = 0$. This is one of the most surprising structure-forming consequences of the inclusion of magnetic fields to the Taylor-Couette flow experiment found first in astrophysical simulations.

4.2. Rotating outer cylinder (flat rotation law)

If the outer cylinder rotates with an angular velocity with $\hat{\mu} \geq \hat{\eta}^2$ then the linear instability without magnetic field disappears and the critical Reynolds number for Ha = 0 moves to infinity. However, for finite Hartmann number (again of order 10) the instability survives practically for the same Reynolds numbers. The consequence is the occurrence of typical minima in the stability diagram (Fig. 4 for $\hat{\mu} = 0.33$).

The minima also occur for the nonaxisymmetric solutions with $m = 1$. For very high electrical conductivity (Pm = 10) there seems to be no intersection between both the bifurcation profiles. The ring-like structure with $m = 0$ always possesses the lowest critical Reynolds number.

This is not true, however, for smaller magnetic Prandtl numbers, i.e. for lower electrical conductivity. For Pm $\lesssim$ 1 we always find intersections between the lines for $m = 0$ and $m = 1$. Again there is a critical Hartmann number at which the ring geometry ($m = 0$) of the excited flow and field pattern changes to a nonaxisymmetric geometry with $m = 1$[2].

Hence, also in experiments with a rotating outer cylinder the magnetic field is able to produce nonaxisymmetric structures. After the Cowling theorem which requires the existence of nonaxisymmetric magnetic modes for the existence of a dynamo, a selfexcited dynamo might thus exist, but only for certain magnetic Prandtl numbers, i.e. for Pm $\lesssim$ 1. The magnetic Prandtl number for experiments with liquid metals like sodium or gallium with Pm of the order of $10^{-(5\ldots6)}$ are still smaller than the considered values. The magnetic Prandtl number of stellar plasma (also in accretion disks) is of the order of 10^{-2}. For the central regions of galaxies values much larger than unity are reported (Kulsrud & Anderson 1992).

[2] For differentially rotating spheres we find a similar behavior but only for stress-free boundary conditions (Kitchatinov & Rüdiger 1997).

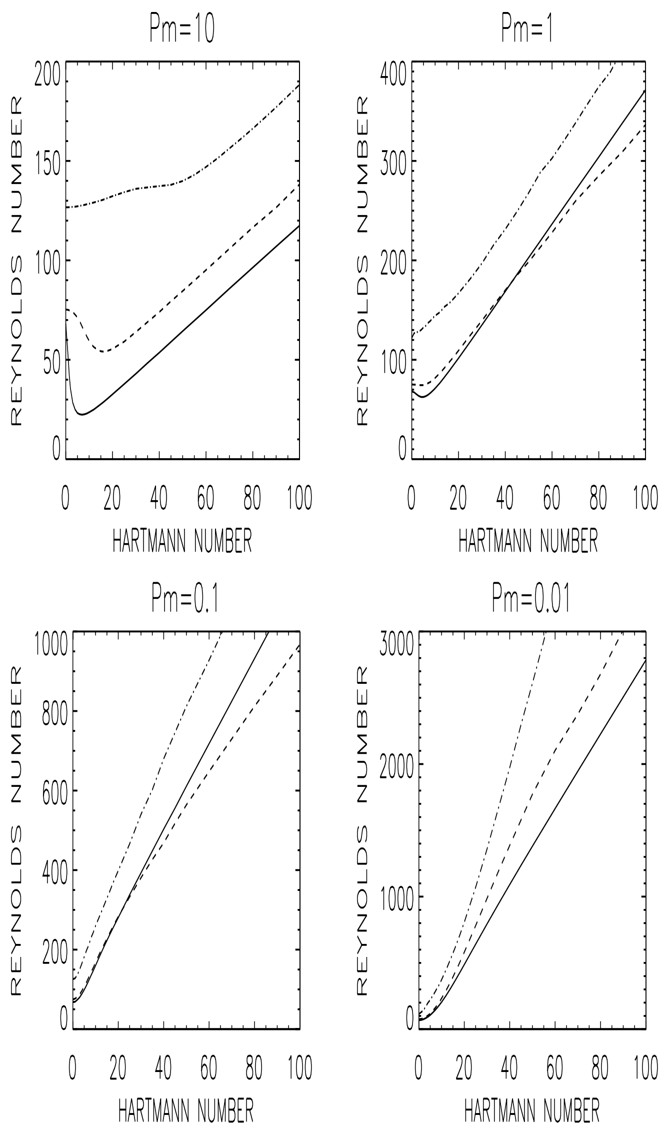

Fig. 3. Resting outer cylinder: Stability lines for axisymmetric ($m = 0$, solid lines) and nonaxisymmetric instability modes with $m = 1$ (dashed) and $m = 2$ (dashed-dotted). Results are given for Pm = 10, 1, 0.1 and 0.01. Note that for Pm = 1 and for Pm = 0.1 for certain magnetic fields the nonaxisymmetric mode with $m = 1$ possesses the lowest Reynolds number.

5. Wave number and drift frequencies

The wave numbers have been discussed in detail for the axisymmetric modes in an earlier paper (Rüdiger & Shalybkov 2002). Generally, in the cells for the nonaxisymmetric modes become more and more elongated in the vertical direction. Here we only add remarks about the drift velocity $\mathcal{R}(\omega)$ which

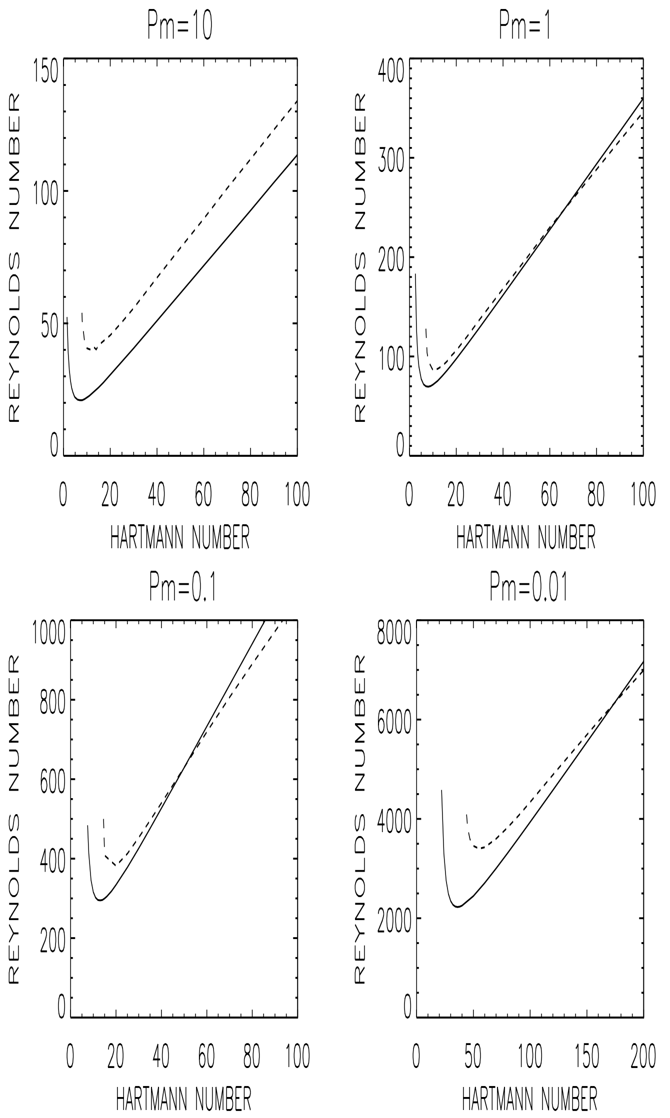

Fig. 4. The same as in Fig. 3 but for rotating outer cylinder ($\hat{\mu} = 0.33$); there is no hydrodynamical instability in the linear theory.

always is positive, i.e. the pattern drifts are in the direction of the rotation (eastward). It is

$$\dot{\phi} = \frac{\mathcal{R}(\omega)\Omega_{\rm in}}{m{\rm Re}}, \tag{16}$$

so that for $m = 1$ the drift period in units of the rotation period changes as ${\rm Re}/\mathcal{R}(\omega)$. A typical value for this ratio is 2. In Fig. 5 the expression (16) is given normalized to the rotation rate $\Omega_{\rm in}$ for the solutions with $m = 1$ and for the two low magnetic Prandtl numbers that we have considered.

6. Conclusions

We have shown that a Taylor-Couette flow which is stable in the hydrodynamic regime ($\hat{\mu} \geq \hat{\eta}^2$) is destabilized by a weak axial

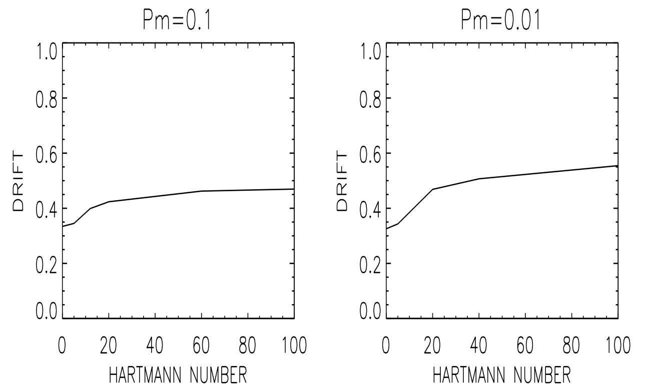

Fig. 5. The drift frequencies (16) normalized with the rotation rate $\Omega_{\rm in}$ for the solutions with $m = 1$ for resting outer cylinders. The drift of the spirals is always positive, i.e. in the direction of the rotation with about 50% of the rotation of the inner cylinder.

magnetic field. Below a critical Hartmann number of order 100 the instability sets in in the form of axisymmetric rolls while above this value the instability forms both nonaxisymmetric field and flow modes. This phenomenon exists despite the observation (e.g. in dynamo theory) that differential rotation is known to suppress the formation of nonaxisymmetric magnetic fields.

On the other hand, after the Cowling theorem of dynamo theory a magnetic field can only be maintained if it is nonaxisymmetric. Considering a variety of typical magnetic Prandtl numbers we find that for our container with conducting cylinders the dominance of the nonaxisymmetric modes only occurs for not too high and not too low magnetic Prandtl numbers. Obviously, the dissipation processes are more important for nonaxisymmetric modes rather than axisymmetric modes. Hence the dissipation allows nonaxisymmetric modes only to be preferred if both the dissipation values have nearly the same order of magnitude.

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