The multimode pulsation of the $\delta$ Scuti star V784 Cassiopeae*

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Abstract. We present an analysis of new Johnson and Strömgren photometric and medium-resolution spectroscopic observations of the $\delta$ Scuti type variable star V784 Cassiopeae. The data were obtained in three consecutive years between 1999 and 2001. The period analysis of the light curve resulted in the detection of four frequencies ranging from 9.15 d$^{-1}$ to 15.90 d$^{-1}$, while there is a suggestion for more, unresolved frequency components, too. The mean Strömgren indices and Hipparcos parallax were combined to calculate the following physical parameters: $\langle T_{\text{eff}} \rangle = 7100 \pm 100$ K, $\log g = 3.8 \pm 0.1$, $M_{\text{bol}} = 1^{m}50 \pm 0^{m}15$. The position of the star in the HR diagram was used to derive evolutionary mass and age yielding to a consistent picture of an evolved $\delta$ Scuti star with a mixture of radial plus non-radial modes.

Key words. stars: fundamental parameters – stars: oscillations – $\delta$ Sct – stars: individual: V784 Cas

1. Introduction

$\delta$ Scuti-type variable stars are pulsating variables located in the lower part of the classical instability strip near or slightly above the main-sequence. The characteristic time-scale of the light variation is in the order of $\mathcal{O}$1, while the observed light curves are usually multiperiodic due to the simultaneously excited radial and/or non-radial modes. Reliable mode identification requires long-term and/or multi-site observing campaigns, which are well-illustrated, e.g. by recent results from the Delta Scuti Network (Breger et al. 1998; Breger 2000), STEPHI program (Alvarez et al. 1998) or the Whole Earth Telescope (Handler et al. 1997). A handbook of reviews and discussion of the astrophysical importance of these variables has been published very recently (Breger & Montgomery 2000). The most complete catalogue of $\delta$ Scuti stars has been tailored and analysed by Rodríguez et al. (2000) and Rodríguez & Breger (2001).

The light variation of the short period variable V784 Cas (=HD 13122 = BD+59$^\circ$422, $P = 0.1092$ d, $\Delta V = 0^m06$, spectral type F5II, ESA 1997) was discovered by the Hipparcos astrometric satellite. The R00 catalogue (Rodríguez et al. 2000) includes this star listing the parameters derived from the Hipparcos observations. The star lies about 1$^\circ$ NW of the open cluster Stock 2, but it is not associated with this strongly reddened cluster located at 316 pc (Krzemiński & Serkowski 1967). The Hipparcos parallax ($9.81 \pm 0.75$ mas) supports the close proximity of the star (102$^{+5}_{-7}$ pc). A few radial velocity measurements can be found in the literature, they range from $-6$ km s$^{-1}$ (De Medeiros & Mayor 1999) to +20 km s$^{-1}$ (Dufot et al. 1995). $UVBVR$ photometry was given by Fernie (1983), while the star was included in the list of bright northern stars with interesting Strömgren indices by Olsen (1980). V784 Cas was also studied in a sample of bright giant stars by Lébre & De Medeiros (1997), where no emission features, neither time variations or asymmetries of the Hr line profiles have been detected (this star was observed two times separated by ten months). The measured rotational velocity is 66 km s$^{-1}$ (De Medeiros & Mayor 1999). Neither of the studies mentioned above dealt with the time-dependent phenomena, only the scatter of the velocity measurements (4 km s$^{-1}$), as listed in De Medeiros & Mayor (1999), suggested the possible variability. Most recently, Gray et al. (2001) included the star in their large sample of late A-, F- and early G-type stars and determined its spectral type (F0-F2III). They also noted that V784 Cas is a mild $Am$ star, the lines of Sr II $\lambda 4077$ and $\lambda 4216$ are enhanced. There is no metallicity determination in the literature.

We started a long-term observational project of obtaining follow-up observations of bright, new variable stars discovered...
by the Hipparcos satellite. We have so far identified a candidate second overtone field RR Lyrae variable (Kiss et al. 1999a), a new high-amplitude δ Scuti star (Kiss et al. 1999b) and revealed the misclassification of a contact binary (Csák et al. 2000). The main aim of this paper is to present an analysis of new photometric and spectroscopic observations of V784 Cas. The paper is organised as follows: the observations are described in Sect. 2. Sect. 3 deals with the period analysis, while radial velocities are discussed in Sect. 4. Finally, the physical parameters are presented in Sect. 5.

2. Observations

2.1. Photometry

Photoelectric Johnson photometry was carried out on 6 nights in September-December, 1999 using the 0.4-m Cassegrain telescope of Szeged Observatory (Hungary). The detector was a single-channel SSP-5A photoelectric photometer. We made differential photometry with respect to HD 14172 (V = 6.96, B − V = +0.22, U − B = +0.20, Krzeminski & Serkowski 1967), which is the bright member of the visual double star BDS 1193. The check star was HD 14173 (V = 7.20, B − V = +1.00, U − B = +0.66, Krzeminski & Serkowski 1967), the faint member of the system, located at ¬1′ from the comparison star. Since the diaphragm of the photometer is 30", we could well measure the individual stars. We note, that U-band observations were carried out only on the last two nights of the observing run.

The Strömgren u'by' photometric observations were acquired on 4 nights in August and September, 2000 and 12 nights in August and September, 2001 using the 0.9-m telescope of the Sierra Nevada Observatory (Spain) equipped with a four-channel spectrograph photometer. The differential photometric data were obtained using the same comparison and check stars (HD 14172: b − y = +0.154, m1 = +0.111, c1 = +1.153; HD 14173: b − y = +0.642, m1 = +0.257, c1 = +0.455, Olsen 1993). The brightness and colour differences of the comparison stars have been found to be constant at ±0.01 (suggested by the rms of the data). The overall accuracy of the standard transformations is estimated to be about ±0.01 for the V (both from the Johnson and Strömgren data), B − V and b − y data, ±0.015 for U − B and m1 and ±0.02 for the c1 data. Because of the same comparison, the standardized V light curves from the Johnson and Strömgren measurements are well comparable (the observed light range – 6.60–6.70 – was the same for both datasets). The journal of observations is presented in Table 1.

We have obtained 3125 individual V points, (512 from UBV and 2613 from u'by'). 512 B − V points, 132 U − B points and 2613 Strömgren indices1. The whole dataset has been phased with the Hipparcos ephemeris (P = 0.1092130, \(E = 2448500.0700, \)ESA 1997) and the resulting phase diagrams are plotted in Fig. 1. The observed behaviour of the colour variations is typical in pulsating stars thus excluding the possibility of other type of variation (e.g. eclipsing or ellipsoidal). However, the light curve showed such cycle-to-cycle changes that the assumption of the monoperiodic nature had to be rejected.

2.2. Spectroscopy

The spectroscopic observations were carried out at the David Dunlap Observatory with the Cassegrain spectrograph attached

Table 1. The journal of observations.

<table>
<thead>
<tr>
<th>JD</th>
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<th>hours</th>
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<th>type</th>
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<td>spectr.</td>
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</tr>
<tr>
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<td>u'by</td>
<td>5.4</td>
</tr>
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<td>2451 785</td>
<td>u'by</td>
<td>1.0</td>
<td>2452 160</td>
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<td>5.4</td>
<td>2452 200</td>
<td>spectr.</td>
<td>5.3</td>
</tr>
</tbody>
</table>

Fig. 1. The light and colour curves phased with the Hipparcos ephemeris. Note the ~−0.1 shift of the maximum indicating the outdated light elements.

1 Individual data are available in electronic form at the CDS via anonymous ftp to cdsarc.u-strasbg.fr (139.79.128.5) or via http://cdsweb.u-strasbg.fr/cgi-bin/qcat?J/A+A/394/97
to the 74" telescope on one night in October, 1999 and two nights in October, 2001. The detector and the spectrograph setup were the same as used by Vinkó et al. (1998). The resolving power ($\lambda/\Delta\lambda$) was 11,000 and the signal-to-noise ratio reached 70–100. The spectra in 1999 were centered on 6635 Å and the wavelength span was 200 Å in both cases. The data were reduced with standard tasks in IRAF\(^2\), including bias removal, flat-fielding, cosmic ray elimination, aperture extraction (with the task doslitt) and wavelength calibration. For the latter, two FeAr spectral lamp exposures were used, which were obtained before and after every five stellar exposures. Because of the short period of V784 Cas the observing sequence of FeAr-var-var-var-var-var-FeAr was chosen. Careful linear interpolation between the two comparison spectra was applied in order to take into account the sub-pixel shifts of the five stellar spectra caused by the tracking of the telescope. We chose an exposure time of 6 min, which corresponds to ~0.04 phase of the dominant period. The spectra were normalized to the continuum by fitting a cubic spline, omitting the region of H$\gamma$.

Besides the telluric features on the blue side of the H$\alpha$ line, we could detect a few weak and broad metallic lines. The H$\alpha$ profile remained symmetric during the observations excluding the presence of high-amplitude non-radial oscillations. A sample spectrum taken in 1999 is shown in Fig. 2. We have collected 98 individual spectra for V784 Cas.

### 3. Period analysis

The period analysis was performed by means of standard Fourier-analysis with subsequent prewhitening steps. For this we have used Period98 of Sperl (1998) which also includes multifrequency least squares fitting of the parameters.

First we decided to analyse the merged transformed data. As has been mentioned above, the use of the same comparison star suggested a common analysis of the standard V-band data. The variable has very similar $B - V$ and $b - y$ indices as the comparison, thus the colour-dependent terms in the standard transformation equations are only small corrections. Either the mean brightness (6\%65) or the extrema of the light curve are considered, the standard V data originated from Johnson-BV and Strömgren-\textit{by} measurements agree very well (the difference does not exceed 0\%05). However, some spurious low-frequency components yielded by the period analysis enforced us to reject this homogeneity assumption. During the prewhitening steps two low-frequency components (at 0.0007 d\(^{-1}\) and 0.5 d\(^{-1}\)) appeared suggesting: 1. a possible shift by a few millimagnitudes of the mean brightness from Johnson and Strömgren photometry and 2. possible slow and low-amplitude variation of the comparison star mimicking changes of the mean brightness on a daily basis. A close look at the comparison \textit{minus} check magnitudes revealed indeed some slight changes of the daily averages with no systematic short-term variations. Therefore, we have adjusted the individual light curves (21 together) by subtracting nightly mean values (the differences are of order of a few millimagnitudes). Although the low-frequency components have been removed, we have to admit that the mean values on some nights are fairly uncertain because of shorter time-spans than mean periods. That is the reason why the most critical subset obtained on JD 2451 785 was excluded from the frequency analysis.

The calculated amplitude spectra are presented in Fig. 3, where we show the individual frequency spectra after consecutive prewhitennings. In order to illustrate the difficulty of the whole analysis we also present close-up views of the main peaks. Besides the strong 1/day alias structure the 1/year is also strong. A certain amount of ambiguity cannot be excluded due to the complex spectral window and finite spectral resolution.

The primary peak at $f_1 = 9.15650$ d\(^{-1}\) is in very good agreement with the Hipparcos result (9.15642 d\(^{-1}\)). In every step of the prewhitening procedure we allowed all of the parameters to vary to get the “best” Fourier-fit of the light curve. In order to check the reality of the components, we have determined the signal to noise ratios ($S/N$) following the suggestions of Breger et al. (1993). The calculated $S/N$ for the sixth components is 4.1 so it satisfies the proposed criterion of Breger et al. ($S/N(\text{real}) > 4$) for accepting ambiguous peaks in the frequency spectrum. Therefore, the final set consists of six frequencies ranging from 9.15 c/d to 15.9 c/d and it is summarized in Table 2.

### Table 2. The result of the period analysis. $f_4$ and $f_6$ seems to be caused by an unresolved frequency pair, thus discarded from further analysis.

<table>
<thead>
<tr>
<th>No.</th>
<th>freq.</th>
<th>ampl.</th>
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<tr>
<td>$f_1$</td>
<td>9.1565</td>
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<td>27.6</td>
</tr>
<tr>
<td>$f_2$</td>
<td>9.4649</td>
<td>8.4</td>
<td>9.5</td>
</tr>
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<td>$f_3$</td>
<td>15.4036</td>
<td>5.5</td>
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<td>($f_4$)</td>
<td>9.8800</td>
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<td>$f_5$</td>
<td>15.9013</td>
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<td>5.3</td>
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<td>($f_6$)</td>
<td>9.7777</td>
<td>3.3</td>
<td>4.1</td>
</tr>
</tbody>
</table>

\(^2\) IRAF is distributed by the National Optical Astronomy Observatories, which are operated by the Association of Universities for Research in Astronomy, Inc., under cooperative agreement with the National Science Foundation.
The period analysis has been checked by making use of the Hipparcos Epoch Photometry data consisting of 117 points. They were analysed separately and the calculated frequency spectrum (middle panel in Fig. 4) yields essentially to the same dominant period, that has been used to phase the data (top panel in Fig. 4). After prewhitening with this frequency, no further periodicity can be inferred from these data (bottom panel in Fig. 4). A further consistency check is a comparison of Hipparcos and our dominant frequencies. It is shown in Fig. 5, where a close-up to the main peaks is presented. As Hipparcos has good spectral window (in sense of having only weak and asymmetric sidelobes), the very good agreement of the main peaks supports our frequency analysis (see Jerzykiewicz & Pamyatnykh 2000 for a recent discussion on the use of Hipparcos photometric data).

Since the bulk of the data was obtained in 2001, a separate analysis of the 2001 data was carried out to yield insights into the reality of the six-frequency solution. Furthermore, the noise in the Stömgren \(v\) data is significantly reduced compared to transformed Johnson \(V\) magnitudes (i.e. the full amplitude is larger and the photometric accuracy is better through the \(v\) filter). The prewhitening and simultaneous nonlinear parameter fitting resulted in largely similar frequencies at several 1/year aliases of the finally adopted set (for instance, even the dominant component at 9.15 d\(^{-1}\) is approximately a 1/year alias of the true frequency). That is why we favoured frequencies resulted from the whole two-years long data.

Taking the six frequencies in Table 2, strong interaction between \(f_4\) and \(f_6\) is obvious. A visual inspection of the corresponding close-up views in Fig. 3 reveals that the peaks are broader than expected from the length of the data (the horizontal axes in the right column have the same frequency scales). It is suggested that some very close components may exist which are beyond our spectral resolution (approximately 1.5/\(\Delta T\),
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Fig. 4. The phase diagram of the Hipparcos data taken from the Hipparcos Epoch Photometry database calculated with a period of 0.1092130 d (top panel) and the frequency spectrum (middle panel). No secondary period can be inferred from these data (bottom panel).

Loumos & Deeming (1978). For our data $\Delta T = 729$ d, which corresponds to a $\Delta f = 0.002$ d$^{-1}$. Most recently, Breger & Bischof (2002) studied close frequency pairs ($\Delta f < 0.06$ d$^{-1}$) in δ Scuti stars. A detailed discussion of the behaviour of BI CMi led these authors to conclude that very close frequency pairs do indeed exist and their presence should be taken into account when planning photometric observations with the best available spectral resolution. We conclude that similar close frequencies might also exist in V784 Cas making difficult to interpret the presently available data. As a result, we kept only four frequencies for further analysis. The observed individual $V$ light curves are compared with the four-component harmonic fit in Fig. 5. We turn back to the adopted frequencies in Sect. 5.

4. Radial velocity variations

Radial velocity variation of V784 Cas was determined by measuring Doppler-shifts of the Hα line. This is not an ideal choice because the line forming region of the Hα line extends to a much wider region than the photosphere, e.g. it may have strong chromospheric component in the line core (Lêbre & De Medeiros 1997 and references therein). However, the observed spectral region does not contain other strong lines, the detected metallic lines are too weak for radial velocity determination. And, as it will be shown below, they are substantially asymmetric suggesting the presence of non-radial oscillation.

Since the Hα line is symmetric due to its saturation, the radial velocities were determined by fitting a parabola to the lowest points of the line profile. The barycentric corrections were calculated with the IRAF task $rvcorr$. The observed velocities are presented in Table 3. Their estimated accuracy is about ±1 km s$^{-1}$, which is based on our earlier experiences when using the same equipment for studying other bright variables observed at similar $S/N$ ratios (Kiss et al. 1999a,b). As an independent check, we have also determined line bisector velocities at various levels (see Kiss & Vinkó 2000 for an application of this technique in Cepheid variables). The mean difference of the resulting data is $\approx 0.5$ km s$^{-1}$ with a standard deviation of 1.0 km s$^{-1}$ (even the most deviant points did not differ more than 2 km s$^{-1}$ from the line-core velocities).

The light and radial velocity variations have been compared using the light curve fit consisting of four frequencies. The comparison is shown in Fig. 7. It is intended to illustrate the overall characteristics of the correlation between the light and radial velocity variations and the ability of the four-frequency fit to predict light variation both as interpolation (JD 2 451 480) and extrapolation (JD 2 452 191 and 2 452 200). We have estimated the value of $2K/\Delta V$ by taking the full amplitudes of the radial velocity curves and the calculated light curves. The result is $130 \pm 30$ km s$^{-1}$ mag$^{-1}$, being somewhat larger then the mean value of 93 km s$^{-1}$ mag$^{-1}$ found by Breger (1979). This ratio depends on the non-adiabatic behaviour of the modes and it is different depending on $n$ and $l$ values. That is why we do not find exact fit with the photometric solution. A much longer spectroscopic data series is required to draw firm conclusions on the non-radial nature of oscillation.

By a close visual inspection of the individual spectra, we found the metallic lines to show significant and highly variable asymmetries (see Fig. 8). The most straightforward explanation is the line profile distortion caused by non-radial pulsation. Line profile variations among the multiply periodic δ Scuti stars
Fig. 6. The observed individual $V$ light curves (gray dots) with the four-component harmonic fit.
are found frequently and their analysis is a common method of mode identification (see, e.g., Schrijvers et al. 1997; Telting & Schrijvers 1997 for theory and Mantegazza et al. 2000 for a recent application). Unfortunately, neither the resolution, nor the relatively high noise level allow us to use such spectroscopic methods, but the brightness of V784 Cas makes the star a good target object for further spectroscopic investigations.

As can be seen in Fig. 8, the spectral lines of V784 Cas are significantly broader than those of HD 187691 (for which SIMBAD lists \( \text{v} \sin \text{i} = 3 \text{ km s}^{-1} \)). Following the work of Solano & Fernley (1997), we have estimated the rotational velocity of the star from the blend-free line Fe I 6677.99 Å. The resulting \( \text{v} \sin \text{i} = 55 \pm 10 \text{ km s}^{-1} \) is in agreement with the published value \( 66 \pm 10 \text{ km s}^{-1} \) of De Meireiros & Mayor (1999). The most important point is that rapid rotation affecting the photometric parameters (Pérez Hernández et al. 1999) can be excluded.

Finally, we have to discuss the possible binarity, which may also produce asymmetric line profiles when the partially resolved components are of similar brightness. In our case the radial velocity measurements in the literature show quite high scatter that may be associated with long-term orbital motion in an unresolved binary system. We have searched the SIMBAD database for published radial velocities and found the following data: i) Evans (1967) lists \( \text{v}_{\text{rad}} = +15 \text{ km s}^{-1} \); ii) the catalog of Fehrenbach et al. (1996) gives \( \text{v}_{\text{rad}} = +20 \text{ km s}^{-1} \); iii) the latter value has been adopted in Duflot et al. (1995); iv) the first high-precision data were published by De Meireiros & Mayor (1999) giving \( \text{v}_{\text{rad}} = -6.09 \text{ km s}^{-1} \) (and it was used in Lèbre & De Meireiros 1997). Our mean value is \( \langle \text{v}_{\text{rad}} \rangle = -6.3 \text{ km s}^{-1} \) supporting the latest available data, which were taken in the early 90’s. Although we do not know the uncertainty of the early data, the \( \pm 25 \text{ km s}^{-1} \) difference found seems to be too high to just ignore it. Therefore, subsequent spectroscopy is highly desirable, either the mode identification, or the possible binarity is concerned. We note, that in light of the results presented in the next section, we favour the non-radial pulsation and associate the large velocity difference to the effects of metallic line profile distortions. Furthermore, the lack of any variable asymmetry in the H\( \alpha \) profile makes unlikely the presence of a secondary of similar brightness as the primary one producing strong metallic lines and having practically no contribution to the H\( \alpha \) line. We did not find any change in the systematic velocity between 1999 and 2001, that is why we consider non-radial pulsation to be more likely cause of these asymmetric instead of a peculiar companion.

### 5. Physical parameters

The most important physical parameters of V784 Cas were determined using the mean Strömgren colours and the accurate Hipparcos parallax. Unfortunately, there is no \( \beta \) measurement for this star in the literature, therefore, we could not follow the “standard” procedures of Strömgren photometric calibrations (e.g. Zhou et al. 2001a,b). The parallax is \( 9.81 \pm 0.75 \text{ mas} \) (ESA 1997) that corresponds to a distance of \( 102^{+5}_{-2} \text{ pc} \). Consequently, the reddening can be neglected even in this low galactic latitude region \( (b = -1.39^\circ) \). The distance and \( \text{V}-\text{brightness} \) result in \( \text{M}_\text{V} = 1^{o}61 \pm 0^{o}15 \). The spectral type (or, equivalently, the mean \( B - V = 0^{o}33 \)) implies a bolometric correction of \( BC = -0^{o}11 \) (Carroll & Ostlie 1996). Thus, the bolometric absolute magnitude is \( \text{M}_\text{bol} = 1^{o}50 \pm 0^{o}15 \) \((L/L_\odot =20)\), which is in accordance with the expected absolute magnitude of an evolved early F-type star. The atmospheric parameters were estimated from the mean Johnson and Strömgren colours \((\langle b - y \rangle = 0^{o}20, \langle m_1 \rangle = 0^{o}17, \langle c_1 \rangle = 0^{o}74)\).
We can draw some constraints on the stellar mass and age using the evolutionary tracks from Claret (1995) for solar abundances. They are plotted in Fig. 9 with the corresponding position of V784 Cas in the log $T_{\text{eff}}$ - log $L$ plane. Two models with log $M/M_\odot$ = 0.25–0.30 are closest to the stellar error box and they result in an evolutionary mass of 1.89 ± 0.11 $M_\odot$. The corresponding ages range from 1.03 to 1.65 Gyr.

The given parameters permit calculation of the pulsational constants of the determined frequencies. The equation

$$\log Q = -6.456 + \log P + 0.5 \log g + 0.1 M_{\text{bol}} + \log T_{\text{eff}}$$

was used in terms of four observables (Breger et al. 1993). For the adopted set of four frequencies we have calculated the pulsational constants listed in Table 4 (assuming 20% uncertainty). We have also determined amplitude and phase relations for the given frequencies utilizing Strömgren data obtained in 2001. By fixing the frequencies but allowing their amplitudes and phases to vary for different wavebands, we could draw some contraints on the pulsation modes (see a description of the discrimination procedure in Garrido et al. 1990). For the given parameters, it seems to be clearly established the radial character of $f_1$ and the non-radial character of $f_3$ (possibly $l = 2$). We can say nothing definitive for $f_2$ and $f_5$ because errors are higher than values, but the small values seem to indicate $l = 1$ modes. We note, that when the rotation is high enough the mode identification becomes unclear (see Daszyńska-Daszkiewicz et al. 2002).

The parameters and pulsation pattern outlined above suggest V784 Cas to be an evolved $\delta$ Scuti-type variable star with a mixture of radial plus non-radial modes. The star is presently about 1.4 mag brighter than main sequence stars of the same spectral type (e.g. Carroll & Ostlie 1996) and its position above the main-sequence does not contradict the luminosity class III determined by Gray et al. (2001). The star is located on the HR diagram about halfway between the theoretical Blue Edge for radial overtones and the empirical Red Edge (see Fig. 1 in Breger & Pamyatnykh 1998). Furthermore, comparing V784 Cas with the evolved Am stars in Fig. 5 of Rodríguez & Breger (2001), its position is also in agreement with the weak Am nature suggested spectroscopically by Gray et al. (2001). The temperature, surface gravity, mass and luminosity give a consistent picture compared with the standard evolutionary models used by Breger & Pamyatnykh (1998). The determined physical parameters place V784 Cas in that region where no fast evolutionary period changes are expected. Therefore, the slight phase shift of $\approx$0.1 between the Hipparcos data and our observations (see Fig. 1) may indicate either more and yet undetected pulsational frequencies or non-evolutionary period change due to, for instance, light-time effect in a binary system (Kiss & Szatmáry 1995).

6. Conclusions

In this paper, we presented an analysis of photometric and spectroscopic observations of the recently discovered $\delta$ Scuti variable V784 Cas. The U/BV photometry was carried out at Szeged Observatory (Hungary), while the simultaneous uvy data were
Fig. 8. Asymmetric metallic line profiles of V784 Cas. The bottom spectrum is shown for comparison. The labels of the variable star spectra mean HelJD\textsubscript{obs} − 2451480.

Fig. 9. Evolutionary tracks (Claret 1995) and location of V784 Cas in the HR diagram (thick box).

1. Multicolor data consisting of more than 3000 individual points were analyzed with the standard Fourier-analysis. The multiperiodic nature of the star is revealed unambiguously. Besides the dominant period listed also in the Hipparcos catalog, we could detect three more frequencies in the 9.46–15.9 d\textsuperscript{−1} range. There is a suggestion for more, unresolved frequency components.

2. We have obtained almost 100 radial velocity measurements using the H\textalpha line. The measured radial velocity curves also show the multiperiodic nature and a close correlation with the four-component light curve fit. Spectra obtained in 1999 covered a few weak metallic lines and the varying asymmetric line profiles suggest the presence of non-radial pulsation, too.

3. Physical parameters of the star are determined from the mean Strömgren indices and synthetic colour grids. The resulting parameters give a consistent picture of an evolved δ Scuti star. Evolutionary mass (1.89 ± 0.11 M\odot) and age (1.3 ± 0.3 Gyr) is derived.

4. Possible mode identification was discussed based on the Strömgren photometric behaviour (amplitude and phase relations). We identify \( f_1 \) with the radial fundamental mode, while the remaining frequencies correspond to low-order (\( l = 1 \) or 2) non-radial modes, although some ambiguity may arise from the moderate rotation of the star.

Further observations (photometric, as well as spectroscopic) of this variable star are expected to extend the data baseline yielding to a better resolution of the pulsational pattern, mode identification and detection of time-dependent phenomena (e.g. amplitude and/or frequency modulation).

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Table 4. Pulsational constants and amplitude and phase relations for the adopted set of frequencies. Note, that ordinal numbers are the same as in Table 2.

<table>
<thead>
<tr>
<th>i</th>
<th>$f_i$ (c/d)</th>
<th>$P_i$ (d)</th>
<th>$Q_i$</th>
<th>$v/\gamma $</th>
<th>$\phi_v - \phi_y$</th>
<th>$b/\gamma $</th>
<th>$\phi_b - \phi_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9.1565</td>
<td>0.10921</td>
<td>0.030 ± 0.006</td>
<td>1.61 ± 0.03</td>
<td>5/6 ± 0.5</td>
<td>1.29 ± 0.02</td>
<td>3/2 ± 0.5</td>
</tr>
<tr>
<td>2</td>
<td>9.4649</td>
<td>0.10565</td>
<td>0.029 ± 0.006</td>
<td>1.58 ± 0.07</td>
<td>−1/1 ± 1/4</td>
<td>1.27 ± 0.06</td>
<td>−1/0 ± 1/4</td>
</tr>
<tr>
<td>3</td>
<td>15.4036</td>
<td>0.06492</td>
<td>0.018 ± 0.003</td>
<td>1.46 ± 0.09</td>
<td>1/5 ± 2/0</td>
<td>1.20 ± 0.08</td>
<td>1/2 ± 2/0</td>
</tr>
<tr>
<td>5</td>
<td>15.9013</td>
<td>0.06289</td>
<td>0.017 ± 0.003</td>
<td>1.55 ± 0.13</td>
<td>−8/2 ± 3/1</td>
<td>1.25 ± 0.13</td>
<td>−5/0 ± 3/1</td>
</tr>
</tbody>
</table>

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