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The magnetic geometry of magnetic-dominated thin accretion disks

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Abstract. The presence of an imposed external magnetic field drastically influences the structure of thin accretion disks. The magnetic field energy is assumed to be in balance with the thermal energy of the accretion flow, and a diffusion approximation simulates the (vertical) energy transport. Our main result is that inside the corotation radius the resulting radial inclination i of the magnetic field lines from the rotation axis easily exceeds the critical value 30° (required to launch cold jets) even for magnetic Prandtl numbers of order unity. The self-consistent consideration of both magnetic field and accretion flow demonstrates only a weak dependence of the inclination angle on the magnetic Prandtl number. The surface values of the toroidal magnetic fields necessary to induce considerably high values for the radial inclination are much smaller than expected. As the innermost part of the disk produces the largest $B^{\rm tor}$, the largest radial inclination can also be expected there. The idea is therefore supported that the cold jets are basically launched in the central disk area.

Key words. accretion, accretion disks – magnetic fields – MHD

1. Introduction

The structure of magnetic accretion disks is the problem of major importance for understanding the origin of astrophysical jets. If the inclination angle, i, of the magnetic field lines from the vertical exceeds 30°, the plasma can be accelerated when spiraling along the field line (Blandford & Payne 1982; Lynden-Bell 1996; Campbell 1997; Krasnopolsky et al. 1999). Accretion disk-driven magnetocentrifugal winds have been widely used to model the astrophysical jets (e.g. Ustyugova et al. 1999; Ouyed & Pudritz 1999; Krasnopolsky et al. 1999).

The present study is motivated by a series of papers dealing with the interaction of accretion disks with external magnetic fields (Livio & Pringle 1992; Lubow et al. 1994; Bardou & Heyvaerts 1996; Reyes-Ruiz & Stepinski 1996; Ogilvie 1997; Ogilvie & Livio 1998, 2001; Campbell 1998; Brandenburg & Campbell 1998). To find the structure of accretion disks threaded by a inclined magnetic field is a complex problem. In the presence of a vertical magnetic field, the vertical gradient of angular velocity generates the toroidal field due to the stretching effect. This field generally influences the vertical and radial structure of a disk and alter the angular momentum transport. If the magnetic field is sufficiently strong it even will influence the rotation law (Ogilvie 1997; Ogilvie & Livio 1998).

The standard accretion disk theory yields the accretion rate, $\dot{M}$, for any possible column density, Σ, with given viscosity-alpha $\alpha_{\rm SS}$ and opacity law. The same is done here for an accretion disk threaded by a magnetic field with a given vertical component $B^{\rm vert}$ and a given toroidal component

Such configuration can only exist for a certain accretion rate and with a certain distribution of a radial magnetic field component B_R. The value of the latter taken at the surface defines the radial inclination angle i known from the jet theory. Along this way the well-known dragging problem – to find the radial inclination i – has been unified with the theory of the vertical structure of accretion disks and can be solved.

After Campbell (1992) the shear between the rigidly rotating halo and the Kepler disk induces a toroidal magnetic field at the surface with the amplitude

$$B^{\rm tor} = -\gamma \frac{R}{H} \frac{\rm Pm}{\alpha_{\rm SS}} \frac{\Omega_{\rm Kep} - \Omega_*}{\Omega_{\rm Kep}} B^{\rm vert}, \quad (1)$$

Ω_* is the stellar rotation rate, R and H are the local radius and height of the disk. The magnetic field threading the accretion disk is thus always inclined in the *azimuthal direction* and we shall show that – if the accretion disk is magnetic dominated – it is thus always inclined in the *radial direction*. The toroidal surface field changes its sign at the corotation radius where $\Omega_{\rm Kep}$ equals the stellar rotation rate Ω_*. The magnetic torque results as negative inside the corotation radius and positive outside the corotation radius. For a disk embedded in vacuum, of course, the γ in (1) vanishes. We consider the γ as representing the unknown halo conductivity. We shall show that we only need very small γ in order to produce inclination angles i of the interesting value of 30° and more – also independent of the magnetic

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Prandtl number Pm. We also can take from (1) and from the simulations by Elstner & Rüdiger (2000) that the ratio

$$\beta = \frac{B^{\rm tor}}{B^{\rm vert}} \qquad (2)$$

– which is negative (positive) inside (outside) the corotation radius grows (by absolute value) inwards.

2. Basic equations

An axisymmetric disk in a steady-state regime is considered; R, ϕ, and z are the cylindrical coordinates. The resulting kinetic and magnetic equations are given in Shalybkov & Rüdiger (2000) as the Eqs. (1)–(13) there. Here we have only to add the energy equation. All generated energy is assumed to be transfered from the disk by radiation neglecting convection, and the disk is optically thick. Then the energy equation is

$$\frac{1}{R}\frac{\partial}{\partial R}(RF_R) + \frac{\partial F_z}{\partial z} = \rho\nu_{\rm T}\left[2\left(\frac{\partial u_R}{\partial R}\right)^2 + 2\left(\frac{u_R}{R}\right)^2 \right.$$
$$+2\left(\frac{\partial u_z}{\partial z}\right)^2 + \left(R\frac{\partial}{\partial R}\left(\frac{u_\phi}{R}\right)\right)^2$$
$$+\left(\frac{\partial u_\phi}{\partial z}\right)^2 + \left(\frac{\partial u_R}{\partial z} + \frac{\partial u_z}{\partial R}\right)^2$$
$$\left.-\frac{2}{3}\left(\frac{1}{R}\frac{\partial}{\partial R}(Ru_R) + \frac{\partial u_z}{\partial z}\right)^2\right] + \frac{\eta_{\rm T}}{\mu_0}\left[\left(\frac{\partial B_\phi}{\partial z}\right)^2 \right.$$
$$\left.+\left(\frac{\partial B_R}{\partial z} - \frac{\partial B_z}{\partial r}\right)^2 + \left(\frac{1}{R}\frac{\partial}{\partial R}(RB_\phi)\right)^2\right], \qquad (3)$$

where the energy flux is $\boldsymbol{F} = -(16\sigma T^3/3\kappa\rho)\ \nabla T$ with σ the Stefan-Boltzmann constant, T the temperature and κ mean opacity.

The equations must be supplemented by relations specifying the gravitational potential, the equation of state, the opacity, the viscosity, and the magnetic diffusivity. We neglect the self-gravitation of the disk so that simply $\psi = -{\rm G}M_*/(R^2+z^2)^{1/2}$, with M_* as the mass of the central object. We adopt the ideal-gas equation $P = \mathcal{R}\rho T/\mu$ ($\mathcal{R}$ molecular gas constant, μ mean molecular mass) and the opacity as $\kappa = k_0\rho T^{-3.5}$ with constant k_0. A Shakura-Sunyaev parameterization is used for the turbulent viscosity, $\nu_{\rm T} = \alpha_{\rm SS}P/(\rho\Omega)$, where $\alpha_{\rm SS}$ is a constant. For the magnetic diffusivity we assume that the magnetic Prandtl number ${\rm Pm} = \nu_{\rm T}/\eta_{\rm T}$ is constant. The last relation implies that the magnetic diffusivity runs with the temperature and therefore, after our boundary conditions, that it vanishes at the disk surface. Such a behavior is fully consistent with the assumption of a perfect-conducting plasma in the halo. We shall here only consider models with $\alpha = 0.1$.

Following Regev (1983) and Kluzniak & Kita (2000) we scale all quantities by their correspondent characteristic values. This makes the equations dimensionless and allows to compare the relative significances of each term. The radial distances are scaled by some characteristic radius, $\tilde{R}$, and vertical distances by a typical vertical height of the disk, $\tilde{H}$. We represent the angular velocity in units of the Keplerian velocity at the characteristic radius, $\tilde{\Omega}^2 = {\rm G}M_*/\tilde{R}^3$. With three characteristic quantities all others can be defined. Note, that the typical magnetic field energy is defined as in balance with the thermal energy of the accretion flow. Geometrically thin disks are considered and all variables are expanded by powers of H/R. At the leading order, the angular velocity is the Keplerian one ($\Omega_0 = R^{-1.5}$)[1], the vertical component of the magnetic field does not depend on the vertical coordinate ($B_{z0} = B_{z0}(R) = B^{\rm vert}$) and the vertical velocity is absent ($u_{z0} = 0$). At the first order the system takes the form[2]

$$u_{R0}B^{\rm vert} + \eta_{\rm T0}\frac{\partial B_{R0}}{\partial z} = 0, \qquad (4)$$

$$RB^{\rm vert}\frac{\partial\Omega_1}{\partial z} + RB_{R0}\frac{\partial\Omega_0}{\partial R} + \frac{\partial}{\partial z}\left(\eta_{\rm T0}\frac{\partial B_{\phi 0}}{\partial z}\right) = 0, \qquad (5)$$

$$-2\rho_0\Omega_0\Omega_1 R = B^{\rm vert}\frac{\partial B_{R0}}{\partial z} + \frac{\partial}{\partial z}\left(\rho_0\nu_{\rm T0}\frac{\partial u_{R0}}{\partial z}\right), \qquad (6)$$

$$\rho_0\frac{u_{R0}}{R}\frac{\partial}{\partial R}(R^2\Omega_0) = B^{\rm vert}\frac{\partial B_{\phi 0}}{\partial z} + \frac{\partial}{\partial z}\left(\rho_0\nu_{\rm T0}R\frac{\partial\Omega_1}{\partial z}\right), \qquad (7)$$

$$\frac{\partial P_0}{\partial z} + \rho_0\frac{z}{R^3} + B_{\phi 0}\frac{\partial B_{\phi 0}}{\partial z} + B_{R0}\frac{\partial B_{R0}}{\partial z} = 0, \qquad (8)$$

$$\frac{\partial F_0}{\partial z} = \rho_0\nu_{\rm T0}\left[\left(R\frac{\partial\Omega_0}{\partial R}\right)^2 + \left(R\frac{\partial\Omega_1}{\partial z}\right)^2 + \left(\frac{\partial u_{R0}}{\partial z}\right)^2\right]$$
$$+\eta_{\rm T0}\left[\left(\frac{\partial B_{\phi 0}}{\partial z}\right)^2 + \left(\frac{\partial B_{R0}}{\partial z}\right)^2\right] \qquad (9)$$

with $F_0 = -\rho_0^{-2}T_0^{-0.5}\partial T/\partial z$. Note that the system depends on R only as a parameter and that all energy is assumed to be transported in the vertical direction.

Solving the equations for given R and M_*, the vertical structure for given $\alpha_{\rm SS}$, Pm and $B^{\rm vert}$ is found if 10 boundary conditions are known.

For disks symmetric with respect to the midplane ($z = 0$) the physical quantities such as Ω, u_R, T are even functions of z while B_R, B_ϕ, F are odd functions of z. Such symmetries provide the boundary conditions at the midplane ($z = 0$) as $B_R = B_\phi = \partial u_R/\partial z = \partial u_\phi/\partial z = F = 0$. At the disk surface, $z = 1$, any external pressure is not allowed. The simplest boundary condition for the temperature, $T(1) = 0$, is used. It reflects the fact that for optically thick disks the surface temperature must be much smaller than the temperature at the disk midplane.

We additionally fix the toroidal and radial magnetic field components at the surface. Hence, the boundary conditions at $z = 1$ are $P = T = u_R = 0$, $B_\phi = B^{\rm tor}$, $B_R = B_{R\rm s}$ where $B^{\rm tor}$ and $B_{R\rm s}$ are constants. These constants are not independent and they are connected with the accretion rate[3]. As usual in the theory of the vertical structure of accretion disks, the accretion rate (or the column density Σ) remains the only free parameter.

[1] The same symbols for the physical and for the normalized quantities are used from hereon.

[2] Leading order terms are described by "0" and first order terms by "1".

[3] Easy to see by integrating Eqs. (4) and (7) over z after multiplication with the density.

Table 1. Azimuthal inclination β and resulting radial inclination i ($\alpha_{\rm SS} = 0.1$, Pm = 1, $B^{\rm vert} = 1$).

β	−0.0004	−0.0026	−0.0082	−0.019	−0.035	−0.059
i	14°	27°	37°	45°	51°	56°

The inclination angle, i, of the magnetic field lines to the rotation axis is $\tan i = B_{R{\rm s}}/B^{\rm vert}$. Its determination for prescribed accretion rate is the central point of the often formulated dragging problem in the theory of the vertical structure of accretion disks in external magnetic fields.

Below we shall use the parameter values $R = 10^{10}$ cm, $M_* = 1\ M_\odot$, $\mu = 0.6$, $k_0 = 6.6 \times 10^{22}$ cgs. The Kepler velocity is $\Omega_{\rm Kep} = 1.2 \times 10^{-2}$ s^{-1}.

3. The numerical results

The differential equations are solved by a relaxation method. The dependent variables are P, T, F, u_R, u_ϕ, B_R, and B_ϕ and the parameters are $\alpha_{\rm SS}$, Pm, $B^{\rm vert}$ and $B^{\rm tor}$ or $\dot{M}$.

The vertical disk structure for a fixed set of parameters is illustrated in Fig. 1. Almost the entire variation of the disk variables happens for $0 < z < 0.5$ hence always we have "atmospheres" for $z > 0.5$. The atmosphere becomes thinner for increasing values of the vertical field. The flow is subsonic (Ma < 1) and the applied surface toroidal magnetic field is much smaller in comparison with $B^{\rm vert}$ for all calculated cases ($|\beta| \leq 0.1$). We find that the density and the temperature weakly depend on the turbulence parameters $\alpha_{\rm SS}$ and Pm. The accretion flow at the disk midplane can be comparable with the sound velocity. However, due to the weak dependence of the temperature on $\alpha_{\rm SS}$, the Mach number behaves almost linear with $\alpha_{\rm SS}$ and therefore Ma $\ll 1$ for $\alpha_{\rm SS} \ll 1$.

The accretion rate for magnetic disks drastically increases in comparison with nonmagnetic disks (see Fig. 2). Nevertheless, the accretion rate for magnetic disks can be even smaller than for nonmagnetic disks for large Pm and small i (the last case is the same as the case with large $B^{\rm vert}$ when $\hat{\Sigma}$ fixed).

The surface density for magnetic disks is usually smaller than for nonmagnetic disks for the same accretion rate. The accretion flow of the magnetic disk exceeds the accretion flow of the nonmagnetic disk. The surface density does not depend on $\alpha_{\rm SS}$ because the toroidal magnetic field is always small and we can neglect its influence on the pressure.

We find that for rather low β the inclination angle i can be larger than the critical value of 30° (Blandford & Payne 1982) required for a cold jet launching even for Pm of order unity (Table 1). Moreover, for given surface density the inclination angle hardly depends on Pm (Fig. 3) – in accordance with the results for the polytropic models considered by Shalybkov & Rüdiger (2000). This means that the radial velocity is proportionate to 1/Pm in this case. Nevertheless, the inclination angle depends more strongly on Pm for given accretion rate. The inclination angle increases for decreasing $\alpha_{\rm SS}$ for fixed accretion rate and decreases for fixed surface density.

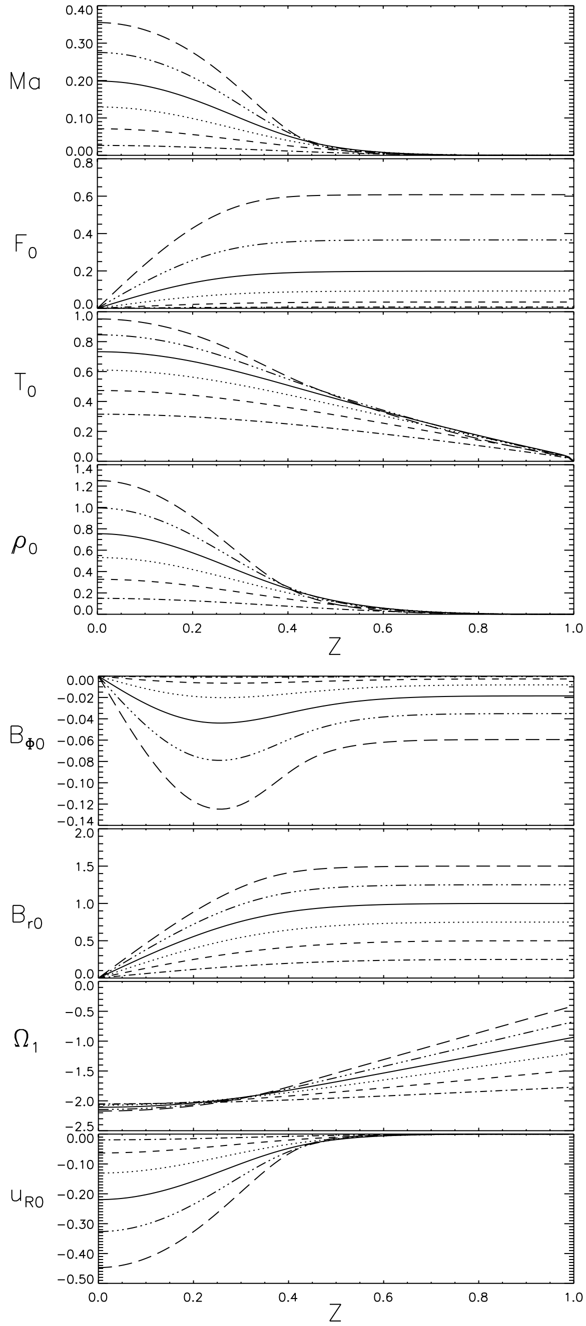

Fig. 1. The vertical disk structure for $\alpha_{\rm SS} = 0.1$, Pm = 1 and $B^{\rm vert} = 1$. The β is -4×10^{-4} (dot-dashed), -2.6×10^{-3} (short-dashed), -8.2×10^{-3} (dotted), -1.9×10^{-2} (solid), -3.5×10^{-2} (dot-dot-dot-dashed), -5.1×10^{-2} (long-dashed).

As already stressed, the accretion rate is connected with β. So, for given $B^{\rm vert}$ there is a direct relation between the β and the resulting inclination angle i, Table 1 presents the numbers. Our model yields high accretion rates and high inclination angles already for rather small β. The larger the β the higher the i. According to (1), the β increases inwards. So we can conclude that the Blandford-Payne condition is fulfilled most easily for the inner part of the accretion disk.

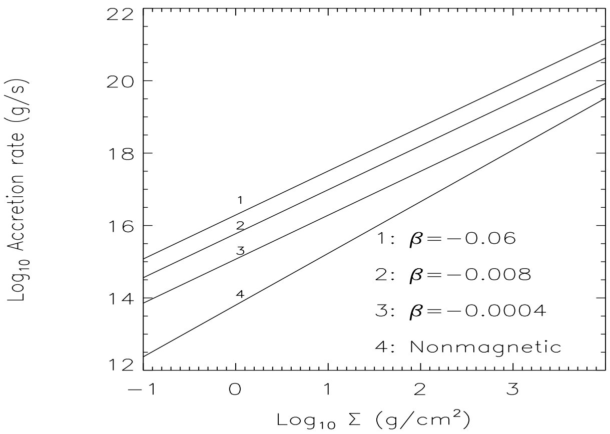

Fig. 2. The accretion rate as a function of the surface density Σ for α_{SS} = 0.1, Pm = 1, B^{vert} = 1.

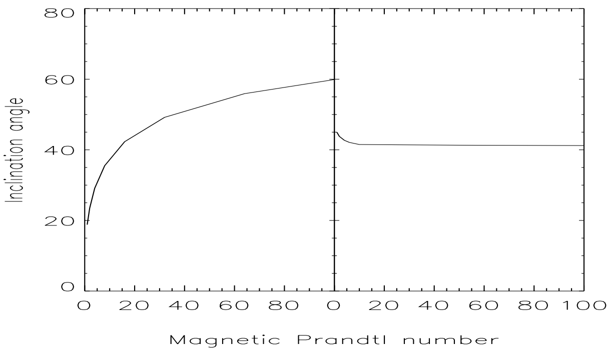

Fig. 3. The inclination angle i of magnetic field lines to the rotation axis for B^{vert} = 1. Left: toroidal field fixed ($\beta = -9 \times 10^{-4}$), Right: surface density fixed $\Sigma = 1.4$ g/cm^2.

4. Discussion and conclusion

The vertical structure of accretion disks with an imposed vertical magnetic field has been considered. The magnetic field energy is supposed to be in balance with the thermal energy of the accretion flow. The angular momentum transport is fully provided by the magnetic field and the Reynolds stress does not play any important role. Our model does not include the detailed computation of the the angular velocity shear at the disk surface. The existence of a boundary layer is assumed with the transition from almost-Keplerian rotation to the angular velocity of the central object. The structure of this boundary layer is not considered here.

The turbulent viscosity might only be important for the energy balance of the disk but even here the calculations demonstrate the dominance of the Joule heating. The magnetic field is, therefore, the essential feature of the model and the equations do not change to standard α-disk equations in the low field limit. The existence of magnetic-induced accretion is also confirmed by the results of Stehle & Spruit (2001). Our results actually demonstrate the close relation between magnetic field dragging and the vertical structure of thin accretion disks. The interaction of the magnetic field with the disk can drastically change the disk structure as well as the configuration of the magnetic field. The angular velocity differs from the Keplerian one. This difference is, however, relatively small because the magnetic energy is small compared with the gravitational energy in our model. The resulting small difference between the angular velocity and its Keplerian profile is due to the small values of the toroidal magnetic surface fields which we need, or v.v. The radial velocity, however, can increase drastically becoming comparable to the sound speed for some models. This can lead to a strong amplification of the accretion rate for a given column density (Fig. 2).

The most surprising results of our calculations concern the radial inclination i of the magnetic field lines to the rotation axis. We found that i) already rather small toroidal field components B^{tor} can produce inclinations exceeding the critical value of 30° and ii) this effect is almost independent of the magnetic Prandtl number for given surface density.

In previous studies (Lubow et al. 1994; Reyes-Ruiz & Stepinski 1996) large radial inclinations could only be obtained for Pm ≥ 100. After our results it also holds for Pm = 1. The difference is due to the fact that the former studies neglected the magnetic field influence on disk structure and used radial velocity from standard accretion disk theory. Note that also the results of Ogilvie & Livio (2001) demonstrate the possibility of large inclination angles for small β and Pm ∼ 1.

For a given accretion rate, $\dot{M}$, larger β lead to larger i and to smaller Σ (Fig. 2). As after (1) the larger (negative) β exist in the innermost accretion disk region, we have there the smaller column density and the stronger radial inclination of the field lines. The jet launching should thus be concentrated to the inner region of an accretion disk.

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