

Astrometry of five major Uranian satellites in 1995–1997[★]

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Abstract. At the oppositions of 1995–1997, a total of 122 CCD frames were taken on the 1.56 m astrometric telescope at the Sheshan station, yielding 864 positions of the major Uranian satellites. The calibration of the images was carried out using a least-squares iterative program by fitting to the well-known orbits of the brighter moons of Uranus, based on the modern theory GUST86 and an ephemeris produced by numerical integration. The residuals lay between 0′.03 and 0′.05 for each of the inter-satellite positions, except for the innermost and faintest satellite Miranda, whose residuals exceeded 0′.08 due to the proximity of Uranus. No significant systematic errors were found when using satellites themselves for determining calibration parameters. The largest residual in the comparison between GUST86 and the numerical integration was about 0′.01.

Key words. planets and satellites: general

1. Introduction

Since 1994 several CCD observing campaigns of major planetary satellites have been carried out at the Sheshan station near Shanghai using their 1.56 m astrometric reflector. Some of these results have been already published in Qiao et al. (1999, hereafter referred to as Paper I). During this period, observing campaigns of the Saturnian satellites were paralleled by an observing campaign of the five Uranian major satellites with aid of the same instrumental set-up.

In this paper, we publish all measured positions of the five major satellites of Uranus obtained during the observing campaigns in 1995, 1996 and 1997. The procedures of observing, measuring and calibration remain essentially the same as described in Paper I, and are briefly presented in Sects. 2 and 3. Analytical and numerical techniques were used in generating the ephemerides of satellites, which were applied both in calibration reduction and in analyzing the fit to the observations. The analytical theory used for fitting to observations is GUST86 (Laskar 1986; Laskar & Jacobson 1987). The results of this fitting are described in Sect. 4.

We also performed a least squares fitting of a numerical integration to GUST86. The whole numerical procedure as well as the results of the numerical integration fitting to

observations and to analytical theory are presented in Sect. 5. Finally, in Sects. 6 and 7 we study the systematic errors and draw conclusions on the accuracy and dynamical consistency of the theory GUST86.

2. The CCD observation and measurement

All of the observations were obtained at the Sheshan station near Shanghai (E 121°18417, N 31°09611), where a 1.56 m reflector with a 1024 × 1024 pixels CCD chip is installed for astrometric purposes. The calculation and analysis made in our previous work show that the good performance of the telescope together with the better seeing at the station ensure us to obtain clear CCD images showing the major satellites on each good night. We were allocated 4–6 days of telescope time in each of the years 1995–1997. During the three-year observing run, 122 CCD images in all were obtained. A summary of the number of images of each satellite in each year is presented in Table 1.

The specifications of the reflector and the CCD chip used were the same as in Paper I. We used an I-type filter with a central wavelength of 900 nm and full width at half maximum (FWHM) of 150 nm. All positional measuring of the satellites in each CCD frame was done manually with IRAF software.

3. Observation reduction

In Paper I, we noted that the CCD device was remounted on the reflector at the Sheshan station each night, thus we had

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[★] The data are available in electronic form at the CDS via anonymous ftp to cdsarc.u-strasbg.fr (130.79.128.5) or via <http://cdsweb.u-strasbg.fr/cgi-bin/qcat?J/A+A/391/775>

Table 1. Number of images of each satellite.

Satellite	1995	1996	1997
Ariel	46	57	11
Umbriel	46	48	19
Titania	46	57	19
Oberon	46	57	19
Miranda	27	47	9
TOTAL	211	266	77

to run a separate calibration determination for each night. In another paper (Shen et al. 2001, hereafter called Paper II), a detailed description and analysis of the “brighter moon calibration” method used by us was given. This method has been widely and efficiently used in calibration reduction of satellites (see, e.g. Harper et al. 1997), although at present it still is imperfect. We have successfully applied the method to the calibration of observations of Saturnian satellites over 1994–1996. The same procedure was used in the present calibration reduction, but here the satellite ephemerides were produced from GUST86 and a numerical integration. Only four better-known satellites Ariel, Umbriel, Titania and Oberon were used as calibration satellites. Because these satellites have better ephemerides and clearer images, the calibrations were affected by smaller systematic errors. The images of Miranda are poorer due to its proximity to Uranus, hence it was excluded from the calibration.

The calibration parameters reduced from the use of the two ephemerides are presented in Table 2. Different values were obtained from analytical theory and from numerical integration. The differences prove that the use of “brighter moon calibration” method unavoidably introduces errors into the derived satellite positions. This is a difficult problem to overcome. In addition, it is also necessary to point out that such a reduction for calibration makes the determination of the mass of Uranus impossible because of the correlation between the scale factor and the mass. However, we can exploit the method while noting its weaknesses.

Recently Vienne et al. (2001a, 2001b) and Peng et al. (2002) discussed astrometric reduction of CCD observations of planetary satellites, when no astrometric stars are present in the frame. They think that the values of the calibration parameters given by a reduction using the positions of the satellites are difficult to interpret. Their further study shows that these values are affected by some errors, which are often neglected.

In our reduction the effects of stellar aberration and topocentric parallax have been incorporated into the positions derived from the orbital models, as we did in Paper I. From the differential corrections listed in Table 2 of Vienne et al. (2001a), we can see that the effects of light-travel time between satellites (maximum = 0′.001 for Uranus), relativistic deflexion from the Sun and the planet are small enough to be negligible. We have also noted that computation made by Vienne et al. in an extreme case (a standard CCD frame 400″ large) for inter-satellites measurements shows that the refraction effect can reach up to 1″ for zenith distance less than 70°. Therefore this correction has to be considered in the present work, unless

the zenith distance is less than 30°. As a result, we have used rigorous formulae given by Woolard & Clemence (1984) for the correction of separation and position angle. In practice, we have found that maximum correction for the refraction effect does not exceed 0′.03. Although this is a smaller quality, still astrometric consideration for it is obviously necessary.

The non-linearity of the projection of the celestial sphere on the tangential plane of the focal point requires a correction which can be as large as 0′.03 for fields of 240″ × 240″. Vienne et al. (2001b) have shown that this effect depends also strongly upon the declination of the observed bodies and can be approximated by $s^2 \tan \delta_c$. Our observations were made at a declination of -20° and the maximum separation between the satellites is less than 55″, so the corresponding correction never exceeds 0′.005. As a result, we may neglect this correction. However, we have published our observations as raw pixel coordinates so that the reader may perform a rigorous reduction if he wishes.

4. Analytical theory

The GUST86 analytical theory is firstly used in determining the scale and orientation of the CCD and secondly in the fitting to observations. In Paper I, we described several calibration methods suggested by previous authors. These methods include the “Double star method” used by Jones et al. (1989) and the “Globular cluster method” proposed by Colas & Arlot (1991). However, in Paper II we presented a more detailed analysis and discussion of the method called “Brighter moon calibration”. The use of the satellites themselves for the determination of calibration greatly simplifies the reduction of the observations (Harper et al. 1997), so we continue to prefer this technique in the present work.

An accurate ephemeris is most important for the “Brighter moon calibration” method. In Paper II, we suggested that in order to reduce the effect of systematic errors in a single analytical theory, the use of the multiple-theories method may be feasible. However, for the Uranian satellites, analysis shows that GUST86 remains the analytical theory with the best dynamical and internal consistency, and thus the multiple-theories method can not be used.

Table 4 gives observed-minus-computed residuals of inter-satellite positions, in separation (ΔS) and position angle ($S \Delta P$), for each satellite based on GUST86. Oberon is excepted, because it was used as the reference satellite.

5. Numerical integration

5.1. Integration method

Numerical integration has been used very effectively in generating ephemerides of natural satellites. A numerical integration can provide a powerful way for checking the validity of the analytical theory before making any fit to real observations (see, for example, Sinclair & Taylor 1985; Harper et al. 1989). In this paper, we explore the use of numerical integration in producing an ephemeris of the five major Uranian satellites, which

Table 2. Calibration parameters for each night of observations. Num gives the respective numbers of calibration satellites. Oberon was the reference satellite.

Dataset	Numerical integration		GUST86		Num		
	δP (degree)	ρ (arcsec/pixel)	δP (degree)	ρ (arcsec/pixel)	Ariel	Umbriel	Titania
95.08.14	2.2935 ± 0.0063	0.249815 ± 0.000012	2.2945 ± 0.0058	0.249749 ± 0.000013	16	16	16
95.08.15	2.2133 ± 0.0050	0.249802 ± 0.000009	2.2149 ± 0.0052	0.249771 ± 0.000009	19	19	19
95.08.16	2.2340 ± 0.0076	0.249861 ± 0.000009	2.2334 ± 0.0077	0.249824 ± 0.000009	11	11	11
96.08.07	-3.3164 ± 0.0142	0.251309 ± 0.000016	-3.3447 ± 0.0128	0.251188 ± 0.000016	5	5	5
96.08.08	-3.1789 ± 0.0081	0.251596 ± 0.000008	-3.1897 ± 0.0080	0.251301 ± 0.000008	9	0	9
96.08.09	-3.2566 ± 0.0075	0.251254 ± 0.000014	-3.2581 ± 0.0075	0.251161 ± 0.000014	9	9	9
96.08.10	-3.3845 ± 0.0062	0.251326 ± 0.000008	-3.3690 ± 0.0068	0.251231 ± 0.000008	11	11	11
96.08.11	-3.1729 ± 0.0048	0.251254 ± 0.000009	-3.1595 ± 0.0047	0.251173 ± 0.000009	17	17	17
96.08.12	-3.3269 ± 0.0108	0.251136 ± 0.000018	-3.3071 ± 0.0110	0.251070 ± 0.000018	6	6	6
97.09.01	14.7824 ± 0.0122	0.251343 ± 0.000016	14.7934 ± 0.0122	0.251181 ± 0.000016	0	8	8
97.09.05	14.8314 ± 0.0082	0.251103 ± 0.000012	14.8248 ± 0.0083	0.251026 ± 0.000012	11	11	11

Table 3. The initial coordinates and velocities of the satellites in units of AU and AU/day respectively. The epoch is JED 2449948.5. The reference frame is the equator and ascending node of the equator of Uranus w.r.t. Earth's B1950.0.

	Miranda	Ariel	Umbriel	Titania	Oberon
X_0	0.2361733945D-02	-0.8797002368D-03	0.2306591996D-02	0.1726616737D-02	0.6793133718D-03
Y_0	-0.1708746181D-02	-0.9228496443D-03	-0.2201205161D-02	-0.4517485737D-03	0.2601083170D-02
Z_0	-0.1259666501D-05	-0.1118937248D-05	0.4560861795D-06	-0.1410787960D-05	0.5653119845D-05
\dot{X}_0	0.1632335411D-02	0.1232334544D-02	0.3124808798D-02	0.1089842773D-02	0.7860181204D-03
\dot{Y}_0	0.3495616918D-02	0.1710611378D-02	-0.2338011911D-02	0.1458592286D-02	-0.3646703001D-03
\dot{Z}_0	0.2426606830D-03	-0.1317287544D-05	0.7809170238D-05	0.9112033377D-06	-0.3680678777D-04

can also be used in our calibration calculation as an alternative to an analytical theory.

As concerns numerical integration methods, Hadjifotinou & Harper (1995) and Hadjifotinou & Ichtiaroglou (1997) studied the behaviour of two numerical integration methods, the 10th-order Gauss-Jackson backward difference and the 12th-order Runge-Kutta-Nyström method (RKN12(10)17M formulae of Brankin et al. 1989). They concluded that when using the first method, an instability occurs in the integration of the equations for the partial derivatives when the step-size is larger than $1/76$ of the orbital period of the innermost satellite. In contrast, the RKN method is stable and can achieve very good accuracy with step-sizes much larger than the critical step-sizes of the Gauss-Jackson method. Therefore the RKN method is preferable than Gauss-Jackson.

5.2. Main parameters and reference system used

Taylor (1998) computed the ephemerides of the five major Uranian satellites by numerical integration using all available Earth-based observations. His excellent work provided the newest physical parameters for the integration of the Uranian satellite system, the most of which have been retained in our calculation. The main parameters that we used are given as follows:

Mass of Uranus = $4.36587 \times 10^{-5} M_{\odot}$

Mass of Miranda: 7.050×10^{-7}

Mass of Titania: 3.839×10^{-5}

Mass of Oberon: 3.230×10^{-5}

$J_2 = 0.003365$

Equatorial radius of Uranus = 26 200 km.

These parameters come from fitting to all of the data, including those from the spacecraft mission, and are taken from Taylor (1998).

Mass of Ariel: 1.558×10^{-5}

Mass of Umbriel: 1.3497×10^{-5} .

The masses of Ariel and Umbriel come from Jacobson et al. (1992). The masses of the satellites given above are all expressed in units of the mass of Uranus.

Coordinates of the pole of Uranus:

$\alpha_p = 76^{\circ}59'69''$, $\delta_p = 15^{\circ}11'17''$.

The reference is the Earth's Mean Equator and Equinox of B1950.

$J_4 = -0.0000321$.

Both the pole of Uranus and J_4 were taken from French et al. (1988).

The equatorial plane of Uranus is chosen as the reference plane for the numerical integration. The integration equations were formulated in Cartesian coordinates with the x -axis directed at the ascending node of the equator of Uranus w.r.t. Earth's equator of B1950.0 (JED 2433282.423), and the origin at the center of mass of Uranus. For simplification of reduction, no planetary and solar perturbations were included in our numerical integration. From trial computation we found no significant loss of accuracy by omitting them. The position of Uranus itself was given from the ephemerides DE406.

5.3. Fitting of the numerical integration

A numerical integration was iteratively fitted to the GUST86 theory over a span of 128 days in order to obtain a first set of initial positions and velocities at the epoch JED 2449948.5,

Table 4. Statistics of O–C residuals, obtained from GUST86 and from numerical integration respectively, in arcseconds for the CCD astrometric observations for each of the Uranus satellites, Oberon excepted (used as the reference satellite). μ is the mean residual, σ the standard deviation about the mean. Nu is the number of observations used. The rejection level that we used was 0'.8.

Satellites	Nu	GUST86				Numerical integration			
		Position angle		Separation		Position angle		Separation	
		μ (")	σ (")	μ (")	σ (")	μ (")	σ (")	μ (")	σ (")
Ariel	114	-0.0006 ± 0.0036	0.0361	-0.0033 ± 0.0043	0.0455	-0.0011 ± 0.0034	0.0360	-0.0016 ± 0.0043	0.0447
Umbriel	113	0.0022 ± 0.0042	0.0429	-0.0047 ± 0.0044	0.0464	0.0048 ± 0.0041	0.0423	0.0054 ± 0.0043	0.0462
Titania	122	-0.0101 ± 0.0040	0.0424	-0.0050 ± 0.0039	0.0425	-0.0122 ± 0.0040	0.0427	0.0042 ± 0.0038	0.0411
Miranda	83	0.0183 ± 0.0091	0.0796	-0.0060 ± 0.0093	0.0842	0.0195 ± 0.0090	0.0798	-0.0039 ± 0.0092	0.0836

which is close to the middle of the time span covered by the observations.

Next, the numerical integration was iteratively fitted to the observations. The resulting initial positions and velocities are given in Table 3.

The RKN integration method calculates the step-size adaptively, but an initial estimate is required to start the process. A value of 0.0625 days was used. In the fitting of integration to the observations, good convergence was obtained after only three iterations. The observed-minus-computed residuals of inter-satellite positions from numerical integration, in separation (ΔS) and position angle ($S \Delta P$), are given also in Table 4. Oberon is excepted, as it was used as a reference. The rejection level that we used was 0'.8.

6. Analysis and discussion

The residuals shown in Table 4 include systematic orbital and observational errors (systematic and accidental). The systematic residuals probably arise from two causes. First, we used the two different ephemerides, which were produced by GUST86 and numerical integration respectively, for determining the calibration parameters. Second, the mean residuals mainly reflect the imperfection of the analytical theory. This implies that the use of the satellites themselves to calibrate the scale and orientation of the CCD frame, unavoidably introduces the systematic errors in the satellite positions. In order to illustrate this better, in Table 5 we deduced the mean residuals (μ ') and the standard deviations (σ ') of numerical integration – GUST86 about the mean, for Ariel, Umbriel, Titania and Miranda. The bias in fact demonstrates the imperfection of the analytical theory. However the small residuals about the mean (generally, less than 0'.01) in Table 5, show that GUST86 has still very good internal consistency.

In Table 4 the mean residuals and standard deviations of Miranda are about twice as large as those of other satellites (they reach the value of 0'.08). Miranda is a faint satellite close to a bright primary. Because of the proximity of Uranus and the unavoidable effect from halo light, the image of the faint satellite Miranda is generally difficult to measure. This effect has also been noted by Harper et al. (1997) for Mimas, the innermost of the major satellites of Saturn.

In addition, Table 5 shows that the poorest residuals are those of Titania, which really corresponds to systematic errors in the ephemeris. The theory of this satellite seems to need

Table 5. The mean residuals μ and standard deviations σ of the residuals (from numerical integration - GUST86) about the mean. Nu is the number of observations used.

Satellites	Nu	Position angle		Separation	
		μ (")	σ (")	μ (")	σ (")
Ariel	114	-0.0002 ± 0.0007	0.0075	-0.0089 ± 0.0010	0.0060
Umbriel	113	-0.0001 ± 0.0007	0.0073	0.0107 ± 0.0013	0.0080
Titania	122	-0.0043 ± 0.0010	0.0102	0.0147 ± 0.0019	0.0118
Miranda	83	-0.0003 ± 0.0009	0.0080	-0.0088 ± 0.0012	0.0072

Table 6. Statistics of O–C residuals from GUST86 in arcseconds calculated when using only Ariel, Umbriel and Oberon as calibration satellites.

Satellites	Nu	Position angle		Separation	
		μ (")	σ (")	μ (")	σ (")
Ariel	114	-0.0019	0.0349	0.0042	0.0444
Umbriel	113	-0.0005	0.0414	0.0038	0.0451
Titania	122	-0.0052	0.0522	0.0029	0.0492
Miranda	83	-0.0034	0.0802	0.0028	0.0831

further improvement. In order to find out whether Titania as a calibration satellite introduced a systematic error, we rerun the calibration without Titania. Table 6 gives the O–C residuals after the re-reduction only for GUST86. The results seem to indicate that the use of Titania for calibration might introduce a systematic error of about 0'.01, which is consistent with the results given in Table 5. In connection with that we had to run a separate calibration determination for each night, as pointed out in Sect. 3. However, after this investigation, we still prefer to include Titania in our calibration. Otherwise, for some observing nights there would be very few calibration satellites (as shown in Table 2), and this might result in unreliability in determining the calibration parameters.

7. Conclusions

During the apparitions of 1995–1997, we successfully used the 1.56 m astrometric reflector at the Sheshan station of Shanghai observatory to acquire a total of 864 observations of the major Uranian satellites. As far as we aware, apart from our observations, only Veiga & Vieira Martins (1999) have made observations during this period. These observations can be

useful to researchers wishing to make further investigation of the analytical theories, especially GUST86.

Our calculations have shown that the observations of Uranian satellites using CCD are much more accurate than using traditional photographic astrometry. This was also proved in the case of Saturnian satellites, by the comparisons of Harper et al. (1997, 1999). A fit of the analytical theory GUST86 to the real observations has proven that GUST86 is a good analytical theory with good dynamical consistency. Our analysis has shown that GUST86 can provide a high-accuracy ephemeris that is the most appropriate choice for analyzing observations of the Uranian satellites.

As pointed out previously, the disadvantage of using the satellites themselves for determining calibration parameters comes mainly from the fact that systematic errors of satellite ephemerides affect their derived positions. In order to give an estimation of these errors, an ephemeris generated by numerical integration was also used for calibration and fitting. This has shown that the systematic errors in GUST86 are generally less than $0''.01$.

The program of CCD astrometry of Saturnian and Uranian satellites using the 1.56 m astrometric reflector at the Sheshan station was inaugurated in 1994. Since then, a significant number of high-quality CCD observations has been obtained. We intend to continue this program in spite of our funding difficulties. Next year, a back-illuminated chip with the large size of 2048×2048 pixels will be installed on the telescope at Sheshan, giving a field of view of approximately $500'' \times 500''$. This will be large enough to include a sufficient number of reference stars as calibration. The use of the new CCD device would provide the possibility of developing new investigations. Significant improvement in the accuracy of satellite astrometry can be expected in the near future.

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Appendix A: The catalogue of raw observations

The catalogue of the raw data is available in electronic form at the CDS via anonymous ftp to [cdsarc.u-strasbg.fr](ftp://cdsarc.u-strasbg.fr) (130.79.128.5) or via <http://cdsweb.u-strasbg.fr/cgi-bin/qcat?J/A+A/391/775>.

Table A.1. An extract of the raw measured pixel coordinates from the dataset made in 1995, the raw coordinates (X_c , Y_c) in pixel of the focal point (C) for all the frame equal to (512, 512).

UTC of mid-exposure						Measured		Residuals		
Yr.	M	D	h	m	s	Sat.	X	Y	X	Y
1995	8	14	12	31	3	1	400.69	581.39	0.24	0.27
1995	8	14	12	31	3	2	384.68	599.57	0.31	0.44
1995	8	14	12	31	3	3	563.45	518.28	0.19	0.20
1995	8	14	12	31	3	4	431.98	444.83	0.20	0.22
1995	8	14	12	31	3	5	424.57	555.50	0.17	0.14
1995	8	14	12	37	3	1	408.45	587.85	0.17	0.20
1995	8	14	12	37	3	2	392.43	606.18	0.22	0.33
1995	8	14	12	37	3	3	571.36	524.53	0.18	0.18
1995	8	14	12	37	3	4	439.77	451.33	0.22	0.19
1995	8	14	12	37	3	5	433.01	562.25	0.07	0.15

In Table A.1 we give an extract of the raw data in the form of pixel coordinates from the catalogue. These are apparent topocentric data; no correction has been made for the effects of refraction, stellar aberration or parallax. The date and time of the mid-exposure are given in Universal Time.

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