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Oscillations on the Sun in regions with a vertical magnetic field

I. Sunspot umbral oscillations

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Abstract. We have calculated the spectrum of eigenmodes of umbral oscillations. The eigenmodes of oscillations with periods from several tens of minutes (g-modes) to several tens of seconds (p-modes) have been found. It is shown that the 3-min umbral oscillations are p-modes modified by the magnetic field.

Key words. Sun: sunspots – Sun: oscillations – magnetohydrodynamics (MHD)

1. Introduction

In the beginning of the 70s the discovery of sunspot umbral oscillations (Bhatnagar et al. 1972; Giovanelli 1972; Beckers & Schultz 1972; Bhatnagar & Tanaka 1972) attracted attention of many theorists, as it seemed that the umbral oscillations could provide unique information about the sub-photospheric structure of sunspots.

For years, two basic approaches have been used to study oscillations in sunspots (see the reviews by Staude 1999 and Bogdan 2000). Starting from the study of Uchida & Sakurai (1975), the spectrum of eigenmodes of umbral oscillations has been calculated (Scheuer & Thomas 1981; Thomas & Scheuer 1982; Zhukov et al. 1987; Wood 1990, 1997; Hasan 1991; Cally & Bogdan 1993; Cally et al. 1994; Bogdan & Cally 1997; Gore 1997, 1998; Lites et al. 1998). In the second approach, Žugžda et al. (1983, 1984, 1987); Settele et al. (1999) (see also Gurman & Leibacher 1984) calculated the coefficient of the transmission for a longitudinal wave mode excited by broad-band noise from the deep layers of the convective zone. However, it follows from the paper of Zhukov & Efremov (1988) that these two approaches are essentially equivalent, provided the sunspot umbra contains a resonator from which wave energy can leak. It should be noted, however, that two factors affect the calculated transmission coefficient of the longitudinal wave mode: the partial reflection of the wave and the partial transformation of the longitudinal wave into transverse wave mode. These factors may result in the situation where the transmission coefficient of the longitudinal wave is a monotonous function of frequency (see Fig. 1 in the study of Zhukov & Efremov 1988, and also the study of Settele et al. 1999). In this case, the presence of a resonator for the transverse wave mode in the sunspot umbra can be indicated only by the frequency dependence of the coefficient of transformation of the longitudinal wave mode into the transverse mode (Fig. 4 in the study of Zhukov & Efremov 1988). Thereby, the resonance frequencies cannot always be determined only by calculation of the transmission coefficient of the longitudinal wave mode. That is why in the present study we use the first approach, which while being substantially simpler, unfortunately, does not provide detailed information about linear transformation of waves in a sunspot umbra.

The first and the most consistent attempt to give physical interpretation of the observed oscillations was made by Scheuer & Thomas (1981). They tried to calculate the spectrum of umbral eigenoscillations in sunspots by solving the complete linearized set of equations of magnetohydrodynamics. However, as the authors themselves noted, they were not able to find asymptotic solutions for this set of equations for $z \to -\infty$. In their calculations they used some conditions for sufficiently deep layers of the umbra which in fact did not take into consideration the presence of the convective zone. As a result, Scheuer & Thomas (1981) did not succeed in finding any eigenmodes trapped in the convective zone.

Here we assume that in sufficiently deep layers of umbra, the Alfvén speed is considerably less than the sound speed. In the weak field approximation ($v_{\mathrm{A}}/c \ll 1$), the full set of equations of magnetohydrodynamics proved to be divided into two independent subsystems whose asymptotes can be easily obtained (Sect. 3, see also Hasan & Christensen-Dalsgaard 1992 (for the isothermal

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atmosphere) and Cally & Bogdan 1993; Cally et al. 1994; Bogdan & Cally 1997 (for the polytropic atmosphere)) and, hence, the problem of calculation of the eigenmodes of the umbral oscillations can be completely defined.

The calculations made under the conditions on the infinity (for $z \to -\infty$) obtained in Sect. 3 have shown that the eigenmodes of the umbral oscillations display periods from several tens of minutes (g-modes) to several tens of seconds (p-modes), and that both the 5-min and 3-min oscillations are eigenmodes of the umbral oscillations.

2. The basic equations and the model of umbra

Let us assume that the magnetic field is homogeneous and vertical ($\boldsymbol{H_0} = (0, 0, H_0)$). Then for an ideal gas, assuming that all small perturbations have the form $\sim \exp(k_x x + k_y y + \omega t)$, after linearization the basic set of equations of magnetohydrodynamics is reduced to two independent subsystems, one of which describes the propagation of Alfvén waves (here it will not be considered), and the other the propagation of magneto-atmospheric waves. It has the following form in a cartesian system of coordinates with the Z-axis directed upwards (Ferraro & Plumpton 1958)

$$v_{\mathrm{A}}^{2} \frac{\mathrm{d}^2 v_x}{\mathrm{d}z^2} + \left(\omega^2 - c^2 k_x^2 - v_{\mathrm{A}}^2 k_x^2\right) v_x + i k_x \left(c^2 \frac{\mathrm{d}v_z}{\mathrm{d}z} - g v_z \right) = 0, \quad (1)$$

$$c^{2} \frac{\mathrm{d}^2 v_z}{\mathrm{d}z^2} - g\gamma \frac{\mathrm{d}v_z}{\mathrm{d}z} + \omega^2 v_z + i k_x \left(c^2 \frac{\mathrm{d}v_x}{\mathrm{d}z} - g(\gamma - 1) v_x \right) = 0, \quad (2)$$

where $c = (\gamma R T(z))^{1/2}$ is the sound speed, γ the ratio of specific heats, g the gravity acceleration, which is assumed to be constant ($=0.274\,\mathrm{km\,s^{-2}}$) and $v_{\mathrm{A}} = (H_0^2/4\pi\rho)^{1/2}$ the Alfvén speed.

The system (1)–(2) is a rather complicated set of equations; its solutions have been studied in detail only for an isothermal atmosphere (Ferraro & Plumpton 1958; Lerroy & Schwartz 1982; Zhugzhda & Dzhalilov 1982).

In our opinion, at the present stage, until the origin of the umbral oscillations has been ultimately determined, the problem should not be complicated with attempts to describe the structure of the atmosphere of a sunspot umbra with all possible accuracy (for example, with the use of semi-empirical models). Therefore, in the present study we solve the set of Eqs. (1)–(2) numerically for the model of umbra similar to that used by Scheuer & Thomas (1981). We assume that our umbra model is a two-layer atmosphere. The sound speed and density in each layer are given as follows:

Layer 1 (Chromosphere – corona, $z > 0$)

$$c_1^2(z) = c_{01}^2 \left(1 + \delta \left[\tanh\left(\alpha \left(\frac{z}{z_1} - \beta \right) \right) - \tanh\left(-\alpha\beta \right) \right] \right),$$

$$\rho_1(z) = \rho_1(0) \frac{c_{01}^2}{c_1^2(z)} \left(\left(\frac{\xi}{\xi_0} \right)^{-\frac{1}{a^2}} \left(\frac{\xi^2 - a^2}{\xi_0^2 - a^2} \right)^{\frac{a^2+1}{2a^2}} \right)^{-\frac{g\gamma z_1}{\alpha c_\infty^2}},$$

where

$$\xi = \exp\left(\alpha \left(\frac{z}{z_1} - \beta \right) \right), \quad \xi_0 = \exp\left(-\alpha\beta \right),$$

$$a^2 = \frac{\delta - 1 + \delta \tanh(-\alpha\beta)}{\delta + 1 - \delta \tanh(-\alpha\beta)}$$

and

$$c_\infty^2 = c_{01}^2 \left[1 + \delta \left(1 - \tanh\left(-\alpha\beta \right) \right) \right].$$

Layer 2 (Convective zone, $z < 0$)

$$c_2^2(z) = c_{02}^2 \left(1 - \frac{z}{z_2} \right),$$

$$\rho_2(z) = \rho_2(0) \left(1 - \frac{z}{z_2} \right)^{\mu}, \quad \mu = \frac{g\gamma z_2}{c_{02}^2} - 1.$$

3. Asymptotic solutions of the basic equations

In most studies, zero boundary conditions were used in the calculation of the spectrum of eigenmodes of umbral oscillations. In fact, however, (see Zhukov & Efremov 1988; Cally & Bogdan 1993; Cally et al. 1994; Bogdan & Cally 1997; Lites et al. 1998; Settele at al. 1999), in order to correctly carry out the numerical integration of the set of Eqs. (1)–(2), it is necessary to know the asymptote of its solutions both in the corona (at $z \to +\infty$) and convective zone (at $z \to -\infty$).

As it was mentioned above, the solution of the system of Eqs. (1)–(2) is well known for the isothermal atmosphere; in particular, the asymptotic solutions of this system at $z \to +\infty$ which are necessary for our purposes have the form (Leroy & Schwartz 1982):

$$v_z = K \frac{\frac{\gamma - 1}{\gamma} + K}{K^2 + K + \Omega^2} \, \mathrm{e}^{-kz},$$

$$v = -\mathrm{e}^{-kz}$$

and

$$v_z = \exp\left(\frac{z}{2H_\infty} \right) \exp\left(-i \frac{z}{2H_\infty} \sqrt{4\Omega^2 - 1} \right),$$

$$v = 0.$$

The following notations have been used here:

$$k = k_x, \quad K = kH_\infty, \quad \Omega = \frac{\omega H_\infty}{c_\infty} \quad \text{and} \quad v = i v_x,$$

where $H_\infty (= c_\infty^2 / g\gamma)$ is the scale height at $z \to +\infty$.

Let us find asymptotic solutions of the system (1)–(2) in the deep layers of umbra ($z \to -\infty$). As the sound speed and density increase with depth, it is obvious that in sufficiently deep layers of the umbra, the Alfvén speed becomes far smaller than the sound speed. Therefore, let us introduce a small parameter $\epsilon = v_{\rm A}(z_{\rm a})/c(z_{\rm a})$, where $z_{\rm a}$ is the depth, at which $\epsilon \ll 1$. Then seeking the solutions of this system in the form of the series:

$$v_x = {\rm e}^{\frac{1}{\epsilon}\int^z \lambda(\zeta){\rm d}\zeta}\left(v_{x0} + \epsilon v_{x1} + \epsilon^2 v_{x2} + \ldots\right), \quad (3)$$

$$v_z = {\rm e}^{\frac{1}{\epsilon}\int^z \lambda(\zeta){\rm d}\zeta}\left(v_{z0} + \epsilon v_{z1} + \epsilon^2 v_{z2} + \ldots\right), \quad (4)$$

substituting series (3)–(4) into the system (1)–(2), and equating terms of the order of $1/\epsilon^2$, we obtain:

$$v_{z0} = 0, \quad \lambda^2 \neq 0,$$

$$v_{z0} \neq 0, \quad \lambda^2 = 0.$$

For the first case, it is easy to find:

$$v_x \sim \sqrt{v_{\rm A}}\,{\rm e}^{\pm i\int^z \frac{\omega}{v_{\rm A}}{\rm d}z}. \quad (5)$$

It is not difficult to see that these solutions represent slow magnetoacoustic waves.

In the second case, the following equation for $v_{\rm z0}$ is obtained:

$$c^2\left(\omega^2 - c^2k^2\right)\frac{{\rm d}^2 v_{z0}}{{\rm d}z^2} + \left[c^2k^2\frac{{\rm d}c^2}{{\rm d}z} - g\gamma \times\left(\omega^2 - c^2k^2\right)\right]\frac{{\rm d}v_{z0}}{{\rm d}z} + \left[\left(\omega^2 - c^2k^2\right)^2 + \frac{(\gamma-1)g^2k^2}{\omega^2} \times(\omega^2 - c^2k^2) - \frac{gk^4c^2}{\omega^2}\frac{{\rm d}c^2}{{\rm d}z}\right]v_{z0} = 0. \quad (6)$$

As the solutions of the Eq. (6) for the layer 2 are well known (Nye & Thomas 1976; Evans & Roberts 1990) they will not be discussed here.

4. Dispersion relation

Now, for the problem to be formulated completely it is necessary to find a dispersion relation which specifies the spectrum of umbra eigenoscillations. For this purpose, conditions on the boundary of discontinuity at $z = 0$ must be found. The conditions on the boundary of the discontinuity in a medium with a vertical magnetic field has been considered in a number of studies (see, for example, Leroy & Schwartz 1982); they have the form

$$[v_z] = 0, \quad [\boldsymbol{H}] = 0, \quad [p] = 0. \quad (7)$$

Thus on the boundary of the discontinuity, the vertical component of the velocity, magnetic field, and pressure should be continuous.

Taking into account the fact that in the corona (at $z \to +\infty$) the exponential decaying solution (v_{z1t}) and the solution corresponding to upwards propagated slow magnetoacoustic waves (v_{z2t}) should be taken, for layer 1 ($z > 0$) we have

$$v_z = T_1 v_{z1t}(z) + T_2 v_{z2t}(z)$$

(two linearly independent solutions of the set of Eqs. (1)–(2)).

In deep layers of the convective zone it is necessary to take the exponential decaying solution of the Eq. (6) (v_{z1b}) and the solution for downwards propagated slow magnetoacoustic waves (5) (v_{z2b}) and for the layer 2 ($z < 0$) we have

$$v_z(z) = B_1 v_{z1b}(z) + B_2 v_{z2b}(z)$$

(as well as the two solutions from four linearly independent solutions of the set of Eqs. (1)–(2)).

Thus, from the boundary conditions (7) the following set of equations for the definition of arbitrary constants T_1, T_2, B_1 and B_2 is obtained:

$$B_1 v_{z1b}(0) + B_2 v_{z2b}(0) = T_1 v_{z1t}(0) + T_2 v_{z2t}(0)$$

$$B_1 v_{1b}(0) + B_2 v_{2b}(0) = T_1 v_{1t}(0) + T_2 v_{2t}(0)$$

$$B_1 \phi_{1b}(0) + B_2 \phi_{2b}(0) = T_1 \phi_{1t}(0) + T_2 \phi_{2t}(0)$$

$$B_1 \psi_{1b}(0) + B_2 \psi_{2b}(0) = T_1 \psi_{1t}(0) + T_2 \psi_{2t}(0),$$

here

$$\phi = \frac{{\rm d}v_z}{{\rm d}z}, \quad \psi = \frac{{\rm d}v}{{\rm d}z}.$$

The determinant of the system is the dispersion relation which specifies the spectrum of the umbral oscillations. For a determination of eigenfrequencies, the set of Eqs. (1)–(2) was numerically integrated from $z = 35z_1$ up to $z = 0$ and from $z = -(40 \div 85)z_2$ up to $z = 0$.

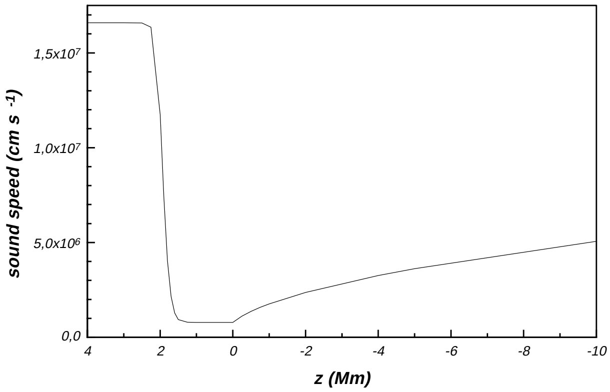

Fig. 1. Variation of the sound speed with height in the umbra.

5. The results of the calculations

To compare our results with observations, we adopt the following numerical values for the parameters in our umbral models: $H_0 = 1000\,G$, $\rho_0 = 5\times10^{-7}\,{\rm g\,cm^{-3}}$, $\gamma = 5/3$, $\alpha = 3.5$, $\beta = 4.$, $\delta = 220$, $c_{01} = 7.9\,{\rm km\,s^{-1}}$. The variation of the sound speed and density with height for the chosen

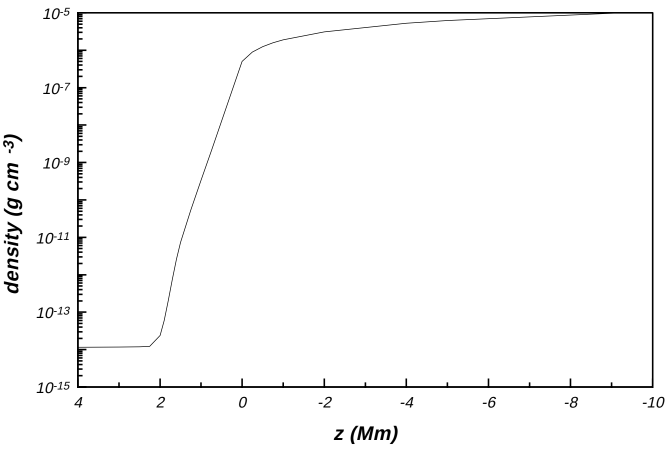

Fig. 2. Variation of the density with height in the umbra.

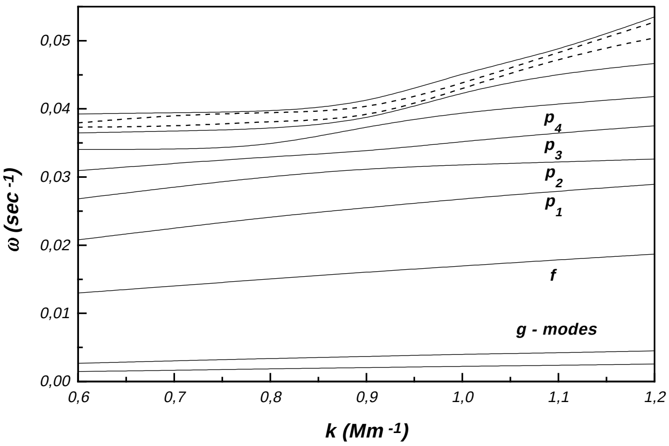

Fig. 3. Diagnostic diagram for the sunspot umbral oscillations.

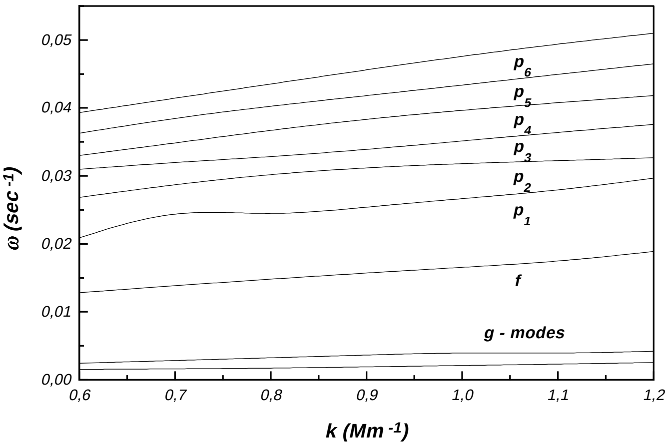

Fig. 4. Diagnostic diagram for the umbral oscillations without magnetic field.

values of the parameters and for $z_1 = 500\,\mathrm{km}$ is presented in Figs. 1 and 2.

The diagnostic diagram for the umbral oscillations presented in Fig. 3 was calculated for

$$c_{01} = c_{02}, \quad \rho_1(0) = \rho_2(0) = \rho_0 \quad \text{and} \quad z_2 = 250\,\mathrm{km}.$$

We can see from Fig. 3 that in the sunspot umbra, as well as on the Sun on the whole, there exist both g-modes

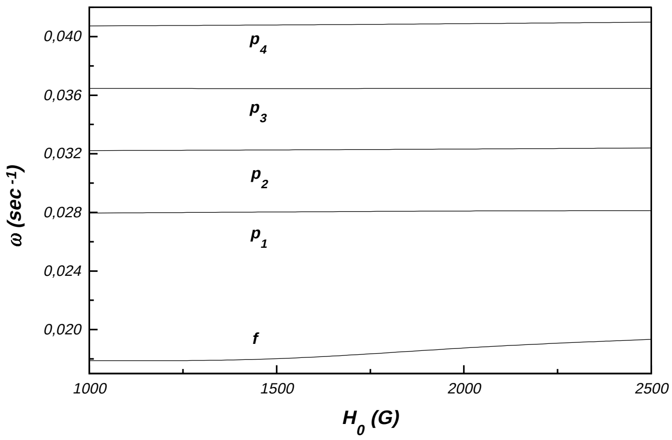

Fig. 5. Variations of frequencies of the f and p_n- modes ($n \leq 4$) with the magnetic field for $k = 1.1\,\mathrm{Mm}^{-1}$.

and p-modes of the eigenoscillations. We have computed two first g-modes, f and several first p-modes with periods from several tens of minutes to several tens of seconds. For comparison, Fig. 4 presents the diagnostic diagram calculated for the same values of parameters only without the magnetic field ($H_0 = 0$). The comparison of these diagnostic diagrams indicates that the presence of a sufficiently strong magnetic field weakly affects the f, p_1, p_2 and p_3-modes, however, it results in new modes emerging in the region of high frequencies. In Fig. 3, new modes caused by the presence of a strong magnetic field are marked by dotted lines. Apparently the presence of the strong magnetic field results in splitting of the p-modes with large n ($n > 4$) into two modes, one of which represents a slow MHD wave, and the other a fast MHD wave. The problem of high-frequencies p-mode splitting in a strong magnetic field of the sunspot umbra undoubtedly requires special investigation. As for the impact of the magnetic field on the f and several first p-modes, the strength of the magnetic field does not affect them appreciably, as we can see from Fig. 5.

The calculated eigenperiods of the umbral oscillations (see Fig. 3) for two values of the wave number (i.e., in fact, for two sunspots of different size) and the observed periods of the oscillations taken from the study of Thomas et al. 1984) are presented in Table 1. We can see from Table 1 that despite the rather arbitrary choice of some parameters for the sunspot umbra in our calculations (e.g., the parameter α, which essentially specifies the height of the transition region, or the magnitude z_2 which, as our calculations have shown, strongly influences the spectrum of oscillations), the calculated values of the periods are generally close to those observed. It thereby follows from the obtained results that both the 5-min and 3-min oscillations in the sunspot umbra are the eigenmodes of umbral oscillations. It is necessary to note, however, that the 3-min umbral oscillations are high-frequencies p-modes (with $n > 4$) appreciably modified by the magnetic field.

Table 1. Calculated and observed periods of the umbral oscillations.

mode	$k = 0.9\,\mathrm{Mm}^{-1}$ (s)	$k = 1.1\,\mathrm{Mm}^{-1}$ (s)	Obs. (s)
g_2	51.5 (min)	44.0 (min)	-
g_1	28.4 (min)	24.9 (min)	-
f	392	352	366
p_1	246	225	301
p_2	201	195	270
p_3	186	172	197
p_4	168	154	171
p_5	164	139	155

6. Conclusion

It follows from the asymptotic solutions of the system (1)–(2) (Sect. 3) that energy can leak from the sunspot umbra to the corona and deep layers of a sunspot in the form of slow MHD waves. The radiation of the waves to the corona and deep layers of a sunspot implies that the eigenfrequencies of the umbral oscillations must be complex ($\omega = \omega_{\mathrm{r}} + i\omega_{\mathrm{i}}$) (see Zhukov & Efremov 1988; Cally & Bogdan 1993; Bogdan & Cally 1997), and therefore the results obtained in the present paper are reliable provided that $\omega_{\mathrm{i}}/\omega_{\mathrm{r}} \ll 1$, i.e. the leakage of the wave energy from the umbral resonator is sufficiently small. In this case, as shown for example by Zhukov (2001), the leakage of the wave energy from the umbral resonator ($\omega_{\mathrm{i}} \neq 0$) should result in minor change of ω_{r}, which as a first approximation can be neglected. Moreover, the initial assumption $\omega_{\mathrm{i}} = 0$ accepted while calculating eigenfrequencies for g, f and p-modes can be shown to be actually valid, taking into account conditions at infinity ($z \to \pm\infty$) derived in Sect. 3. However, it follows from the study of Zhukov & Efremov (1988), in which the spectrum of eigenoscillations of the sunspot umbra was calculated (in the approximation of an incompressible medium, which, generally speaking, is applicable only to the f-mode), that the radiation of the wave energy from sunspots can be substantial (see also Cally & Bogdan 1993; Bogdan & Cally 1997) and therefore it is very important to take into account the leakage of wave energy from the sunspot umbra in calculations. In addition to that, in order to achieve the best consistency between calculations and observations, it is necessary to calculate the spectrum of the umbral oscillations for the various z_2 which dramatically affect both f and several first p-modes.

We note in conclusion, that since open boundary conditions are acquired for the umbra, the umbral oscillations (at least within the 3-min band) may be excited by the mechanism suggested by Žugžda et al. (1983). We consider, however, that generation of oscillations in sunspots constitutes a rather complicated problem, which will require substantial effort in order to be resolved.

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