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Photon yields of energetic particles in the interstellar medium: An easy way to calculate gamma-ray line emission

E. Parizot[1] and R. Lehoucq[2]

[1] Institut de Physique Nucléaire d'Orsay, IN2P3-CNRS/Université Paris-Sud, 91406 Orsay Cedex, France
[2] DAPNIA/Service d'astrophysique, CEA-Saclay, 91191 Gif-sur-Yvette, France
e-mail: roller@discovery.saclay.cea.fr

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Abstract. A γ-ray line production calculation in astrophysics depends on i) the composition and energy source spectrum of the energetic particles, ii) the propagation model, and iii) the nuclear cross sections. The main difficulty for model predictions and data interpretation comes from the fact that the spectrum of the particles which actually interact in the ISM – the *propagated spectrum*, is not the same as the *source spectrum* coming out of the acceleration site, due to energy-dependent energy losses and nuclear destruction. We present a different approach to calculate γ-ray line emission, based on the computation of the total number of photons produced by individual energetic nuclei injected in the interstellar medium at a given energy. These photon yields take into account all the propagation effects once and for all, and allow one to calculate quickly the γ-ray line emission induced by energetic particles in any astrophysical situation by using directly their source spectrum. Indeed, the same photon yields can be used for any source spectrum and composition, as well as any target composition. In addition, these photon yields provide visual, intuitive tools for γ-ray line phenomenology.

Key words. cosmic rays – gamma rays: theory – nuclear reactions

1. Introduction

High energy astrophysics is experiencing a considerable development, notably through the operation of a new generation of gamma-ray observatories, both on the ground and onboard satellites. The analysis of gamma-ray emission from compact and diffuse sources is one of the most valuable ways to study high energy processes in the universe, and it is expected to put stronger and stronger constraints on the models in the near future. Among the processes of interest, the production of gamma-ray lines by energetic particles (EPs) interacting in the interstellar medium (ISM) has received increased attention in the last few years, in connection with the data of the Compton Gamma-Ray Observatory as well as the forthcoming satellite INTEGRAL. In addition to the study of the high energy sources, the study of EP interactions is important for the understanding of the EPs themselves, of which the Galactic cosmic rays (GCRs) are one of the main components.

Information about the acceleration sites and processes is also provided by the determination of the EP energy spectrum and chemical composition, which can be derived, in principle, from the measurement of gamma-ray line ratios and profiles. However, the information contained in the gamma-ray observational data relates to the spectrum and composition of the *propagated* particles (i.e. those who actually interact in the ISM), not the *source* particles. The difference arises from the fact that the EPs experience various types of interactions while they "propagate" from their source to the place where they produce gamma-ray lines. In particular, they lose energy through Coulombian interactions in a way which depends on both their energy and chemical nature, so that the propagated population of EPs is not identical to the source population, freshly coming out of the acceleration process.

Ideally, one would divide the whole process into three successive stages (e.g. Parizot & Lehoucq 1999): particle acceleration, propagation and interaction, with the gamma-ray production arising during the last stage only. In reality, of course, the reactions leading to gamma-ray production occur all the time, from the injection of the EPs into the ISM until they have slowed down to energies below the interaction thresholds. It is therefore necessary to sum the contributions of all the instants following acceleration (the gamma-ray emission occurring during acceleration itself can usually be neglected, except for the rarest, highest energy particles, which spent a long time in the accelerator). In steady state situations, this is equivalent to calculating the equilibrium distribution of EPs and integrating the relevant cross sections over this so-called propagated distribution.

Send offprint requests to: E. Parizot,
e-mail: parizot@ipno.in2p3.fr

From a technical point of view, the most difficult part consists in calculating the propagated spectrum, taking into account the energy losses and the energy-dependent escape time of the particles out of the confinement region, where the gamma-ray production is evaluated. This is what prevents a straightforward calculation of the expected gamma-ray line fluxes from the knowledge of the nuclear cross sections and the EP source distribution. Therefore, we propose here to work out this step once and for all, in the case of a steady state and a thick target, by calculating the integrated effect of energy losses on individual particles injected at any energy in the ISM. In Sect. 3, we shall justify the fact that the "propagation step" in a standard γ-ray line emission calculation can be "factorized out" and calculated separately, independently of the EP source spectrum and composition. This is true if the metallicity of the propagation medium does not exceed several tens of times the solar metallicity, as in most of the astrophysically sensible situations. As a result, we shall obtain the absolute γ-ray yields of energetic nuclei as a function of their initial energy, from which the γ-ray line emission induced by EPs of any spectrum and composition can be straightforwardly calculated. In addition to making γ-ray line calculations much easier, these absolute yields (or particle efficiencies for γ-ray line production) considerably help phenomenological interpretation of the observational data, as these yields only need to be convolved with the EP *source* spectra, rather than *propagated* ones.

2. Standard gamma-ray line calculations

In order to calculate the gamma-ray line emission in a given region of the ISM, one needs to integrate the nuclear excitation cross sections over the local flux of energetic particles, and sum all the contributions to each gamma-ray line. If i represents the projectile, j the target nucleus, and k the excited nucleus produced in the interaction between i and j, or equivalently the "photon species" emitted through nuclear de-excitation, the γ-ray line emission rate, in ph/cm^3/s, from all the $i + j \to k$ nuclear reactions, reads:

$$\begin{aligned}\frac{\mathrm{d}N_k}{\mathrm{d}t} &= \sum_{i,j} \int_0^{+\infty} N_i(E)\,[n_j \sigma_{i,j;k}(E) v(E)]\,\mathrm{d}E \\ &= \sum_{i,j} \int_0^{+\infty} \Phi_i(E) n_j \sigma_{i,j;k}(E) \mathrm{d}E, \qquad (1)\end{aligned}$$

where $N_i(E)$ is the spectral density of the projectiles i, in cm^{-3}(MeV/n)$^{-1}$, $\Phi_i(E) = N_i(E) \times v(E)$ is the corresponding flux, in cm^{-2} s^{-1}(MeV/n)$^{-1}$, $v(E)$ is the velocity of the projectiles (independent of the nuclear species, i, if the energy E is expressed in MeV/n), n_j is the number density of the target nuclear species j, and $\sigma_{i,j;k}$ is the cross section for the reaction $i + j \to k$. For instance, k might represent photons from the ^{12}C de-excitation line at 4.44 MeV, produced by the reaction $p + {}^{12}\mathrm{C} \longrightarrow {}^{12}\mathrm{C}^*$ or $\alpha + {}^{16}\mathrm{O} \longrightarrow {}^{12}\mathrm{C}^*$.

Assuming that the nuclear excitation cross-sections are known, the main challenge is to estimate the EP fluxes, for each nuclear species, in the gamma-ray source. This depends on the acceleration process at the origin of the injection of EPs in the ISM, and on what happens to the particles once they leave the accelerator, which involves the energy losses, the rate of escape from the region considered, and the particle destruction in inelastic nuclear processes. As far as the acceleration is concerned, it can be characterized here by the so-called *injection function*, $Q_i(E)$, which gives the number of particles of species i injected at energy E in the ISM (i.e. leaving the acceleration process and not being further accelerated afterwards), in cm^{-3} s^{-1}(MeV/n)$^{-1}$. The function $Q_i(E)$ will either be taken as the outcome of some particular acceleration model (e.g. diffusive shock acceleration), or phenomenologically postulated so as to reproduce some particular observation (e.g. from INTEGRAL data).

Most studies so far have assumed that the shape of the injection spectrum, $Q_i(E)$, was independent of the nuclear species, and could be re-written as $\alpha_i \bar{Q}(E)$, where the isotopic abundances α_i characterize the chemical composition of the EPs *at the source*. However, this simplification is not required and one will allow here for a different spectrum for each nuclear species, which is equivalent to an energy dependent EP composition.

In order to use Eq. (1) to calculate the gamma-ray emission produced in the region under consideration, one needs to derive the EP fluxes, $\Phi_i(E)$ or $N_i(E)$, from the injection functions, $Q_i(E)$, supposed known. The standard way to do this has been described in Parizot & Lehoucq (1999, and references therein) for the general case where the injection function as well as the conditions of propagation are time-dependent. It consists in solving the so-called propagation equation, which takes the following form in the stationary case:

$$\frac{\partial}{\partial E}(\dot{E}_i(E) N_i(E)) = Q_i(E) - \frac{N_i(E)}{\tau_i^{\mathrm{tot}}(E)}\cdot \qquad (2)$$

Here, $\dot{E}_i(E)$ is the energy loss function, in (MeV/n) s^{-1}, giving the rate of energy loss for nuclei of species i in the considered propagation medium, and $\tau_i^{\mathrm{tot}}(E)$ is the total "loss time" taking into account nuclear destruction and particle escape out of the region considered. It can be expressed in terms of the total inelastic cross sections for nuclei of species i in a medium made of j nuclei alone, $\sigma_{i,j}^{\mathrm{dest}}(E)$, and the mean escape time, $\tau_i^{\mathrm{esc}}(E)$, as:

$$\frac{1}{\tau_i^{\mathrm{tot}}(E)} = \frac{1}{\tau_i^{\mathrm{esc}}(E)} + \sum_j n_j \sigma_i^{\mathrm{dest}}(E) v(E), \qquad (3)$$

where n_j is as above the number density of nuclei of species j in the propagation medium.

In Eq. (2), the injection function $Q_i(E)$ acts as a source term, while the equilibrium EP distribution $N_i(E)$ is what

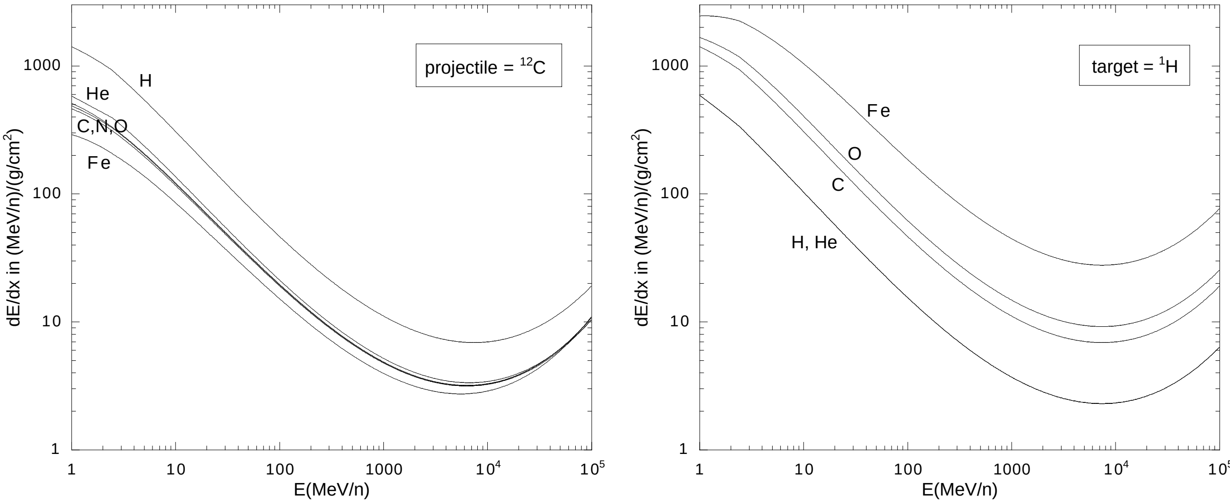

Fig. 1. Energy losses per unit grammage of matter passed through, in (MeV/n)/(g cm^{-2}): on the left, for a ^{12}C nucleus in various chemically pure propagation media indicated by the labels; on the right, for various nuclei indicated by the labels in a propagation medium made of pure Hydrogen.

we want to calculate. The formal solution of the stationary propagation equation reads:

$$N_i(E) = \frac{1}{|\dot{E}_i(E)|} \int_E^{+\infty} Q_i(E_{\rm in}) \mathcal{P}_i(E_{\rm in}, E) {\rm d}E_{\rm in}, \qquad (4)$$

where $\mathcal{P}_i(E_{\rm in}, E)$ can be interpreted as the survival probability, in the propagation medium considered, of a particle injected at energy $E_{\rm in}$ and losing energy down to energy E. It obviously depends on the total loss time at each energy between $E_{\rm in}$ and E and the energy loss function, and can be expressed as follows (see Parizot & Lehoucq 1999):

$$\mathcal{P}_i(E_{\rm in}, E) = \exp\left(- \int_{E_{\rm in}}^{E} \frac{{\rm d}E'}{\dot{E}_i(E') \tau_{{\rm tot},i}(E')} \right). \qquad (5)$$

3. Approached EP propagation universality

Equation (1) allows one to calculate the photon emission rate for any gamma-ray de-excitation line for which the production cross sections are known. It is based on the computation of the EP distribution function given by Eq. (4), which in turn requires the knowledge of the energy losses and the escape and destruction times.

3.1. Energy loss rates

Let us first consider the energy loss rate, $\dot{E}_i(E)$, for particles of species i. It can be expressed as a sum over the nuclear species present in the propagation medium:

$$\dot{E}_i(E) = \sum_j n_j \times \left.\frac{{\rm d}E}{{\rm d}x}\right|_j \times v(E), \qquad (6)$$

where $({\rm d}E/{\rm d}x)_j$ is the energy loss per unit grammage in a medium of pure j nuclei, in (MeV/n)/(g cm^{-2}). In this expression, $({\rm d}E/{\rm d}x)_j$ is a "universal function" of energy which can ideally be calculated from first physical principles or, failing that, extrapolated from laboratory measurements, while the astrophysics comes in the chemical composition, the degree of ionization and the density of the propagation medium, n_j. Some examples of energy loss functions used in this paper are shown in Fig. 1. They have been calculated using the program of J. Kiener (1994), based on the modified Bethe formula taking into account the effective charge of the projectile. This program implements the Ziegler tables for the various stopping powers, as corrected by Hubert et al. (1989) according to a semi-empirical procedure, where a new parameterization for the effective charge is deduced from a very large set of experimental stopping power values in the range 3–80 MeV/n. The typical error in the energy loss function is between 2% and 10%, i.e. less than the uncertainty about the nuclear excitation cross-sections.

The energy range of interest for the calculation of gamma-ray line emission is between a few MeV/n (corresponding to the nuclear excitation thresholds) and a few hundreds of MeV/n. At higher energy, the contribution of the EPs to the gamma-ray line emission is small, because of their reduced number (decreasing power-law source spectrum) as well as because they are destroyed by nuclear reactions before they reach the peaks of the nuclear excitation cross sections, as further discussed below.

In all the calculations presented here, the ambient medium is assumed neutral, which may not be appropriate for a number of astrophysical situations. However, one can estimate that the effect of an ambient ionized medium is small, except for the lowest energies. The reason why the energy losses depend on the ionization state of the propagation medium is that it is more difficult for an energetic ion to capture a free electron than to capture an electron from an atom at rest. Indeed, in the latter case, the orbital motion of the electron reduces the velocity difference between the energetic ion and the electron, and thereby

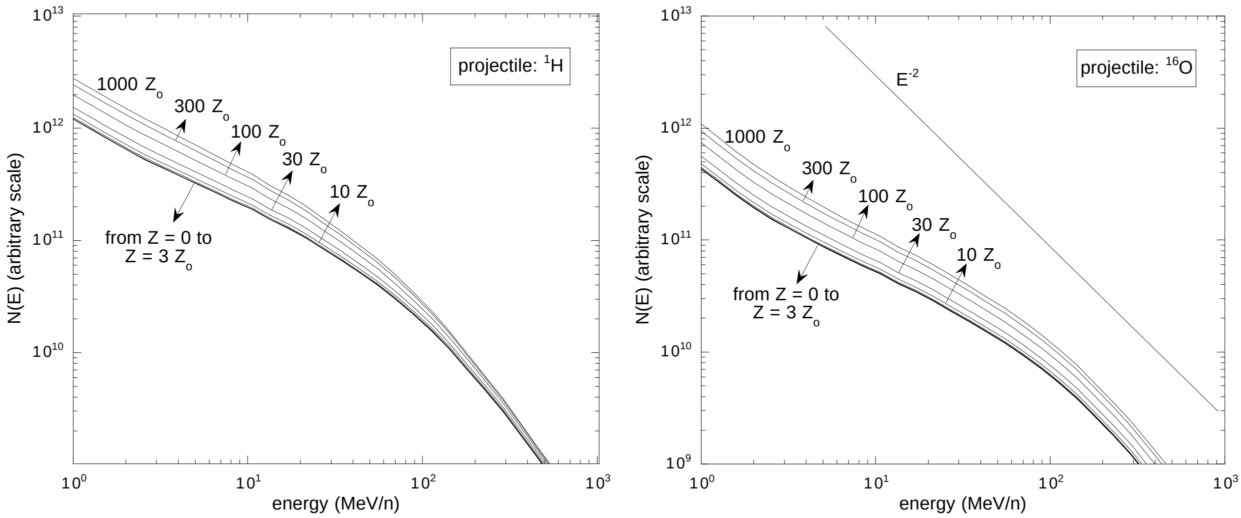

Fig. 2. Stationary energy spectra of ^{1}H (left) and ^{16}O (right) nuclei injected with a simple power-law spectrum in energy of index 2, after propagation in different media. The spectra corresponding to propagation media with a metallicity up to 10 Z_0 are virtually indistinguishable from the propagated spectrum in a metal-free gas.

facilitates capture. As a consequence, the equilibrium between electron stripping and electron capture depends on the ambient medium, and an energetic atom is on average more ionized when it travels through a plasma than through a neutral medium (Chabot et al. 1995a). This results in a higher effective charge, and thus a higher stopping power (or larger energy losses). However, when the projectile is too energetic, its relative velocity with even orbital electrons is too high for charge exchange to be efficient anyway. Therefore, the difference between an ionized and a neutral ambient medium becomes negligible, and the energy losses are almost identical. From the quantitative point of view, the stopping power of a plasma is higher than that of a neutral medium by a factor of about 40 below 100 keV/n, but only 2 or 3 at a few MeV/n, and their difference is negligible above 100 MeV/n (Hoffman et al. 1994; Chabot et al. 1995b). Therefore, we will assume throughout that the propagation medium is neutral.

As far as the chemical composition is concerned, in most astrophysically relevant cases the propagation medium will be but the ISM, whose composition is relatively well known (Anders & Grevesse 1989). Now, although the heavy elements are of course crucial to the calculation of gamma-ray de-excitation line emission, the ISM is so much dominated by H and He nuclei that one can neglect all other elements in Eq. (6), and calculate the energy loss rate as if the ISM were made simply of 91% of H and 9% of He (by number). To demonstrate this, we have calculated the propagated (equilibrium) spectrum of the different energetic nuclei subject to energy losses in media of various metallicity. Results are shown in Fig. 2 for metallicities ranging from 0 (H and He only) to 1000 times solar. A significant change in the particle propagated spectrum can only be noticed for ambient metallicities larger than ten times the solar metallicity, Z_0. Since most of the astrophysically relevant media are not that rich in metals, we will assume that the propagation of the EPs is independent of metallicity. Note that even pure SN ejecta have a metallicity less than 30 times Z_0, so that even in such a metal-rich medium, neglecting the interaction of the EPs with the metals as they propagate through the ambient medium will lead to an error smaller than 20% on the propagated particle distribution, $N_i(E)$, and thus also on the gamma-ray line production rates.

3.2. Total inelastic cross sections and survival probabilities

The second crucial physical ingredient necessary to calculate the EP propagated spectra is the total "catastrophic loss time", $\tau_i^{\rm tot}(E)$, including both nuclear destruction and escape (see Eq. (3)). The latter will be neglected here, which amounts to say that we consider a distribution of EPs interacting with a thick target. This is well justified for the relatively low energy particles which are responsible for most of the gamma-ray line emission, as their range is of the order of $1\ {\rm g/cm}^2$, i.e. significantly less than the typical escape grammage of cosmic rays. One can thus replace the total catastrophic loss timescale, $\tau_i^{\rm tot}(E)$, by the destruction timescale, $\tau_i^{\rm D}(E)$.

Concerning the EP destruction through nuclear reactions, we use semi-empirical total inelastic cross sections from Silberberg & Tsao (1990). As for the energy loss function, we have just seen that the EP destruction time can be calculated as if the propagation medium were made of pure H and He, to an excellent approximation (even for relatively high ambient metallicities). In all of our calculations, the main uncertainty comes from the relatively poor knowledge of the nuclear cross sections, rather than from the use of a "universal" (metal-free) propagation medium.

The joint knowledge of $\dot{E}_i(E)$ and $\tau_i^{\rm tot}(E)$ allows one to calculate the survival probability of an EP injected at any energy $E_{\rm in}$ while it loses energy down to the energy E at which it interacts with the ISM to produce a

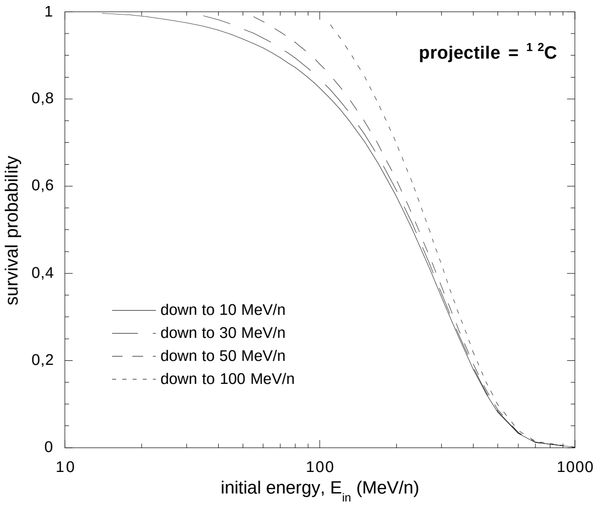

Fig. 3. EP survival probability down to 10, 30, 50 and 100 MeV/n, as a function of the injection energy, $E_{\rm in}$.

gamma-ray. It can be useful to define the energy loss timescale, $\tau_i^{\rm loss}(E)$ for an EP of type i as:

$$\tau_i^{\rm loss}(E) = E/|\dot{E}_i(E)|. \quad (7)$$

The survival probability given by Eq. (5) can then be rewritten as:

$$\mathcal{P}_i(E_{\rm in}, E) = \exp\left(- \int_E^{E_{\rm in}} \frac{\tau_i^{\rm loss}(E')}{\tau_i^{\rm D}(E')} \frac{{\rm d}E'}{E'} \right). \quad (8)$$

Figure 3 shows the results obtained in the case of a ^{12}C nucleus. As can be seen, particles injected at energies greater than 1 GeV/n have very little chance to survive long enough to reach energies of a few tens of MeV/n, where the nuclear excitation cross sections are maximum. As a consequence, their contribution to the total gamma-ray emission rate will be negligible, not mentioning the fact that higher energy particles are usually less numerous than lower energy ones, for the most common, power-law source spectra. This effect reminds us that nuclear excitation is only one of a number of competing nuclear processes affecting an energetic particle, so that the efficiency of gamma-ray line production depends on the balance between several cross sections.

4. Gamma-ray yields of individual EPs

We now make use of the results of the previous section to derive an alternate way to calculate gamma-ray line production from EPs. This idea is the following: since particle propagation is "universal" (i.e. independent of the physical conditions of the propagation medium, as long as metals can be neglected), there should be a way to work it out once and for all. This way is the following: instead of calculating the equilibrium EP distribution function (after propagation), and then integrate the nuclear cross section over this distribution, one can calculate the total photon yield, from injection to rest, of one EP of a given

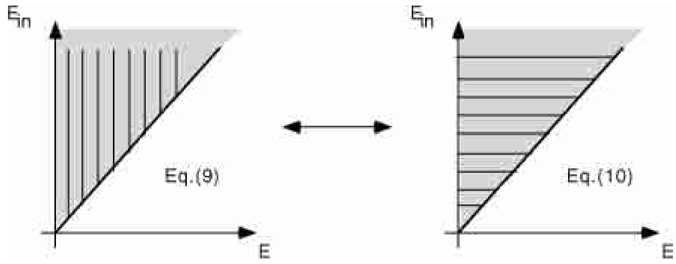

Fig. 4. Graphical demonstration of the equivalence between Eqs. (9) and (10): the shaded area is the integration domain, divided into vertical and horizontal slices, respectively.

kind thrown in the ISM at any given initial energy, independently of the other particles (i.e. independently of the global spectrum and composition of the EPs), and then integrate the individual photon yields over the source distribution function of each kind of EPs accelerated. The important point is that the integration is now over the *source* distribution function, which is known from acceleration models or postulated in a phenomenological study, rather than over the *propagated* distribution function, which no longer has to be calculated.

From the mathematical point of view, the above idea amounts to a simple change of the order of two integrations. Indeed, combining Eqs. (1) and (4), one can rewrite the γ-ray emission rate as follows (specializing to one nuclear reaction for illustration):

$$\frac{{\rm d}N_\gamma}{{\rm d}t} = \int_0^{+\infty} {\rm d}E \int_E^{+\infty} {\rm d}E_{\rm in} \frac{n_0 \sigma(E) v(E)}{|\dot{E}(E)|} Q(E_{\rm in}) \mathcal{P}(E_{\rm in}, E), \quad (9)$$

where n_0 is the density of the propagation medium and $Q(E)$ is the EP *source* spectrum. Now, as is made obvious by Fig. 4, this expression can be re-written as:

$$\frac{{\rm d}N_\gamma}{{\rm d}t} = \int_0^{+\infty} {\rm d}E_{\rm in} \int_0^{E_{\rm in}} {\rm d}E \frac{n_0 \sigma(E) v(E)}{|\dot{E}(E)|} Q(E_{\rm in}) \mathcal{P}(E_{\rm in}, E). \quad (10)$$

Getting the source function, $Q(E_{\rm in})$, out of the integral over E, one obtains the following expression for the γ-ray emission rate (adding the contribution of all the reactions involved):

$$\frac{{\rm d}N_\gamma}{{\rm d}t} = \sum_{i,j} \int_0^{+\infty} Q_i(E_{\rm in}) \alpha_j \mathcal{N}_{i,j;\gamma}(E_{\rm in}) {\rm d}E_{\rm in}, \quad (11)$$

where $\alpha_j = n_j/n_0$ is the relative abundance of nuclei of species j in the target, and

$$\mathcal{N}_{i,j;\gamma}(E_{\rm in}) = \int_0^{E_{\rm in}} \frac{n_0 \sigma_{i,j;\gamma}(E) v(E)}{|\dot{E}(E)|} \mathcal{P}_i(E_{\rm in}, E) {\rm d}E. \quad (12)$$

The physical interpretation of $\mathcal{N}_{i,j;\gamma}(E_{\rm in})$ is straightforward: it is the number of photons of species γ produced in a target made solely of nuclei of species j, by one projectile of species i injected in the ISM at the energy $E_{\rm in}$, integrated over its entire life (i.e. from its injection until it has lost so much energy that it is below the nuclear excitation threshold). Note that the lower bound of the integral can be replace by the energy threshold of the cross sections. Now the interesting point is that the absolute photon yields, $\mathcal{N}_{i,j;\gamma}(E_{\rm in})$, can be calculated from physical quantities alone and is independent of astrophysics: as can be seen from Eqs. (12) and (5), it only depends on the nuclear cross sections and energy loss rates. These can be calculated or measured once and for all, and so is it for $\mathcal{N}_{i,j;\gamma}(E_{\rm in})$.

As anticipated, the great advantage of this formulation is that once the quantities $\mathcal{N}_{i,j;\gamma}(E_{\rm in})$ have been calculated, the actual γ-ray emission rate in a given astrophysical situation can be derived from Eq. (11) which gathers all the astrophysical information (namely the EP spectrum and composition, and the target composition), but which is now expressed in terms of the *source spectrum*, rather than the *propagated* one. To better understand the signification of this transformation, it suffices to compare Eqs. (1) and (11). We have replaced the propagated spectral density of the EPs, $N_i(E)$, by their injection function, $Q_i(E)$, and the cross sections $\sigma_{i,j;\gamma}$ by our absolute photon yields, $\mathcal{N}_{i,j;\gamma}$, which play the role of "effective cross-sections" (although their physical dimension is different) taking into account the propagation of the EPs in the ambient medium. It should be stressed that the individual photon yields behave as universal physical quantities and can be used with any source spectrum, any EP composition *and* any target composition.

Two comments are in order here. First, the above expression giving the photon yields $\mathcal{N}_{i,j;\gamma}(E_{\rm in})$ may seem to depend on the density, n_0, of the propagation medium (e.g. the ISM). This is actually not the case, as the energy loss rate appearing in the denominator is also proportional to this density. The second comment concerns the universality of EP propagation, which is crucial in the approach developed here. Indeed, in principle the EP energy loss rates and survival probabilities depend on the propagation medium, so that a different photon yield should be calculated for each propagation medium. These specific photon yields could still be used with any source spectrum and composition, but not with any target composition, as the latter is usually the same as that of the propagation medium. However, as shown in Sect. 3, the dependence of the energy loss rates and survival probabilities with metallicity is negligible in most situations, so that the photon yields $\mathcal{N}_{i,j;\gamma}$ can indeed be considered universal.

5. Results and emission rates reconstruction

5.1. Nuclear excitation cross sections

Before showing the individual photon yields calculated for the main expected gamma-ray lines in the ISM, we should say a word about cross sections, as they provide the major uncertainty in our results and most of them are extrapolated from relatively scarce experimental data. We have used the data from Ramaty et al. (1979), updated with more recent experimental data whenever possible (Dyer et al. 1981, 1985; Lang et al. 1987; Lesko et al. 1988; Kiener et al. 2001). The paper by Kiener et al. (2001, and private communication) has been used to derive a general procedure to extrapolate the data at high energy.

Table 1. Data relative to the fit of the cross section as given by Eq. (13). The reaction is given in the first column, with the names of the target and projectile and the energy of the resulting gamma ray in MeV. The second and third columns give the energy of the cross section peak and the corresponding value of the cross section.

reaction	$E_{\rm peak}$	$\sigma_{\rm peak}$
^{1}H+^{56}Fe = 0.847	13.0	861
^{4}He+^{56}Fe = 0.847	13.0	1290
^{1}H+^{20}Ne = 1.634	11.0	330
^{4}He+^{20}Ne = 1.634	3.19	232
^{1}H+^{24}Mg = 1.634	23.0	180
^{1}H+^{28}Si = 1.634	21.7	157
^{1}H+^{14}N = 2.313	10.0	106
^{4}He+^{14}N = 2.313	3.25	119
^{1}H+^{12}C = 4.438	11.0	317
^{4}He+^{12}C = 4.438	3.00	393
^{1}H+^{14}N = 4.438	21.7	291
^{4}He+^{14}N = 4.438	9.64	333
^{1}H+^{16}O = 4.438	22.5	156
^{4}He+^{16}O = 4.438	25.0	223
^{1}H+^{16}O = 6.129	13.0	163
^{4}He+^{16}O = 6.129	3.50	269
^{1}H+^{20}Ne = 6.129	19.0	108

Above the resonance peak of the cross sections, whenever the data is missing we assume that the cross section obeys a simple law in $a \times E^{-x} + b$ (above 20–30 MeV/n, say). The best fit of the data for the ^{12}C(p,pγ) reaction gives $\sigma(E) = 24337 \times E^{-1.74} + 4$ mbarn, where E is in MeV. For the $^{12}{\rm C}(\alpha, \alpha\gamma)$ reaction, one finds $\sigma(E) = 31400 \times E^{-1.51} + 7$ mb, and for the excitative spallation reaction $^{16}{\rm O}({\rm p}, {\rm p}\alpha)^{12}{\rm C}^{*}_{4.44}$, $\sigma(E) = 75951 \times E^{-1.98} + 4.3$ mb (Kiener 2001; private communication).

As a first approximation, we assume that the excitation cross sections of other nuclei (namely, ^{14}N, ^{16}O, ^{20}Ne and ^{56}Fe) obey the similar laws with the same value of the exponent x, i.e. 1.74 and 1.51 respectively for proton-induced and alpha-particle-induced excitations. The constant value at high energy, B, is simply taken as the value of B for the ^{12}C excitation cross sections, but scaled to the measured value of the cross section at the resonance peak (i.e. proportionally to the peak value). Finally, the value of A is obtained by imposing the continuity of the cross sections above the peak. The same procedure is applied to excitative spallation reactions (i.e. an exponent of 1.98 and a high energy value scaled proportionally to the peak value), although such an extrapolation is more problematic in this case, as the ^{16}O nucleus has an atypical structure with a large component of four α particles.

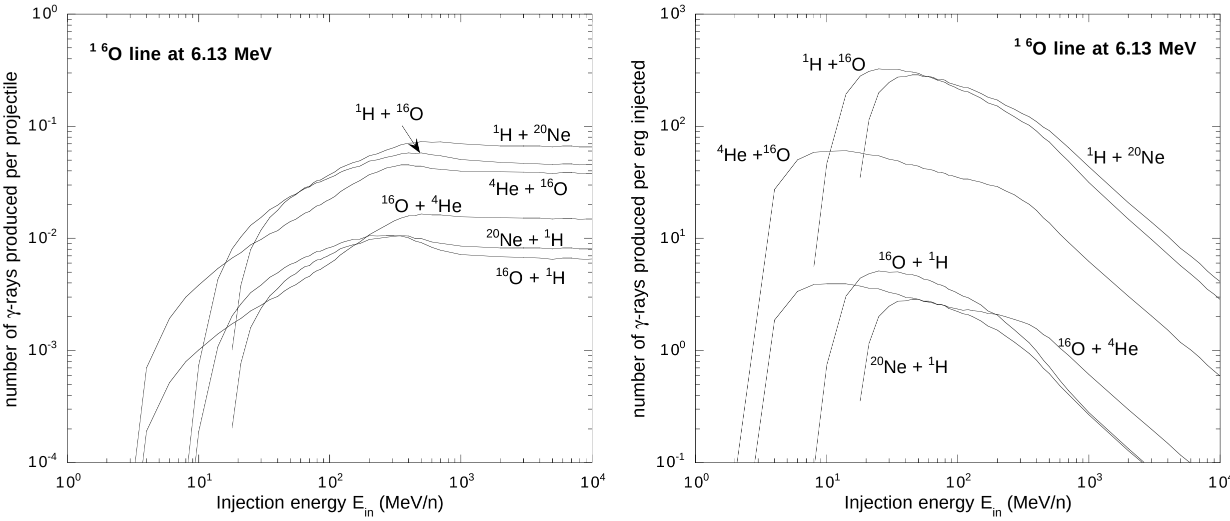

Fig. 5. On the left **a)**: absolute photon yields, $\mathcal{N}_{i,j;\gamma}$, in the 6.13 MeV line of ^{16}O through various channels, as a function of the injection energy of the projectile. The latter is the first nucleus appearing in the label, and the target is the second. On the right **b)**: gamma-ray production efficiency, in photon/erg, for the 6.13 MeV line of ^{16}O through various channels, as a function of the injection energy of the projectile.

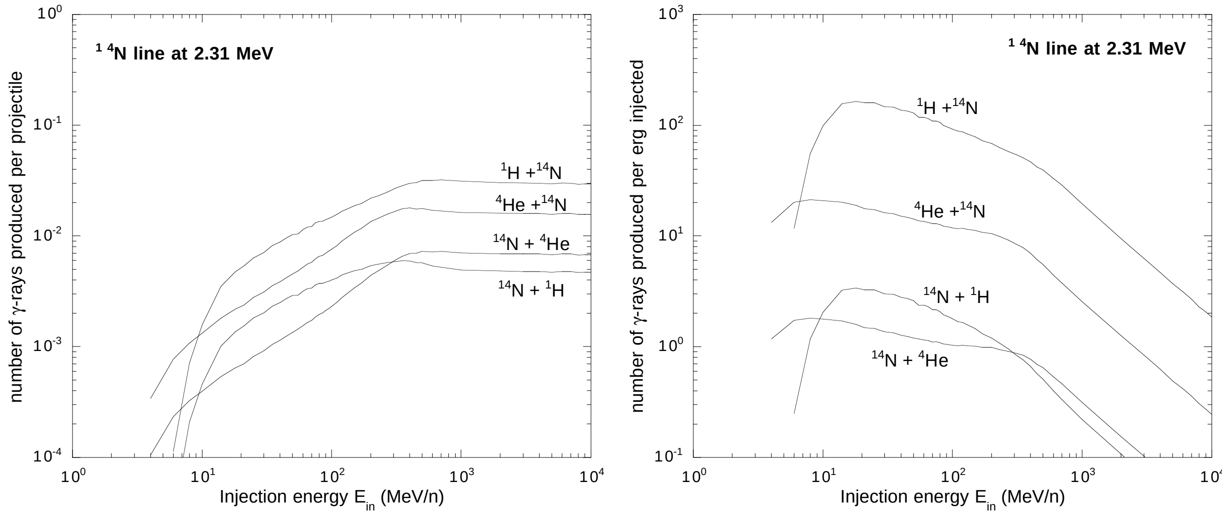

Fig. 6. Same as Fig. 5 for the ^{14}N line at 2.31 MeV.

The above procedure can be summarized by the following expression giving the cross section above the peak (at energy $E_{\rm peak}$ and with cross section $\sigma_{\rm peak}$), in terms of the values of the reference cross section at the peak, $\sigma^0_{\rm peak}$, and at high energy, σ^0_∞, given above:

$$\sigma(E) = \sigma_{\rm peak}\left[\left(1-\frac{\sigma^0_\infty}{\sigma^0_{\rm peak}}\right)\left(\frac{E}{E_{\rm peak}}\right)^{-x}+\frac{\sigma^0_\infty}{\sigma^0_{\rm peak}}\right]. \quad (13)$$

For completeness, we give the values of $E_{\rm peak}$ and $\sigma_{\rm peak}$ for the various cross sections considered in this paper in Table 1.

In general, the error on the excitation cross sections and thus on the gamma-ray yields is typically 10% whenever actual experimental data exist (this is the case for the two main lines of ^{12}C and ^{16}O), and of the order of 20% to 50% when the values are simply estimated or extrapolated. This is quite substantial, especially when our goal is to look at line ratios, in the hope to determine the composition of the EPs and/or the ambient medium from gamma-ray line measurements. These errors, unfortunately, cannot be lowered but by increasing the experimental effort at terrestrial accelerators. This is strongly recommended in order to make the most of the opening field of gamma-ray astronomy.

5.2. Photon yields for the ^{12}C, ^{14}N, ^{16}O, ^{20}Ne and ^{56}Fe γ-ray lines

In Fig. 5a, we show the absolute γ-ray yields, $\mathcal{N}_{i,j;\gamma}$, corresponding to the main ^{16}O line at 6.13 MeV, for various projectiles and targets.

The evolution of $\mathcal{N}_{i,j;\gamma}$ as the injection energy increases can be interpreted in the following way. Photon production begins when $E_{\rm in}$ becomes greater than the reaction

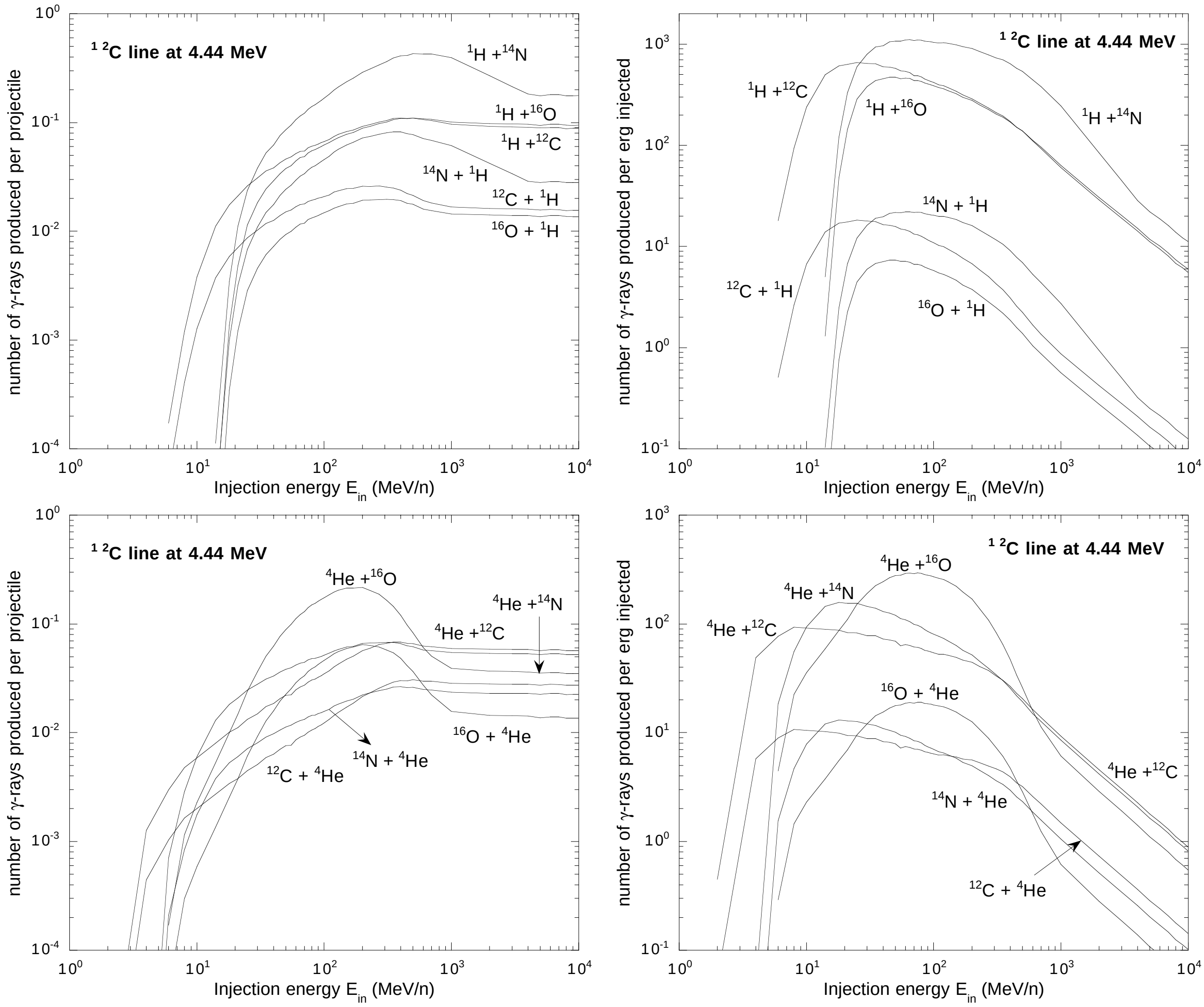

Fig. 7. Same as Fig. 5 for the ^{12}C line at 4.44 MeV, for reactions involving H nuclei (top) and He nuclei (bottom).

threshold. Then it increases sharply as $E_{\rm in}$ passes through the peak of the cross section, and increases more smoothly afterwards. As long as particle destruction or escape can be neglected, Eq. (12) makes it clear that the number of photons produced is an increasing function of $E_{\rm in}$, the upper bound of the integral. Physically, the particle produces γ-rays all the way as its energy goes down from $E_{\rm in}$ to below the reaction threshold. If it is injected at a higher energy, it will produce γ-rays for a longer time, integrating the cross section over a larger range of energy.

But when $E_{\rm in}$ increases further, there comes a time when the projectile has a large probability of being destroyed (through a nuclear reaction) or escaping from the region under study (in a thin target model), *before* its energy drops below the reaction threshold. In this case, the effective energy range over which the cross section is integrated is reduced from below, and the overall γ-ray yield starts to decrease. For large enough $E_{\rm in}$, the particle never reaches the most efficient energy range corresponding to the peak of the cross section. Since both the destruction and the excitation cross sections are roughly constant at high energy, the photon yield tends to an "asymptotic value" where increasing the injection energy only shifts upwards the energy range of activity of the EP but does not change the integrated photon yield. This asymptotic value merely depends on the ratio of the excitation and destruction cross sections.

Typically, for the main γ-ray lines to be expected in the ISM, one can see from Figs. 5 to 8 that several percent to up to 30% of the projectiles injected will produce a gamma-ray, with a peak of this number in the range $E_{\rm in} =$ 100–300 MeV/n.

While the decrease of $\mathcal{N}_{i,j;\gamma}$ at high energy is not very steep, it should be realized that the γ-ray production efficiency, defined as the number of photons produced per erg of projectiles injected, is falling down more quickly, as shown on the right sides of Figs. 5 to 8, for the same reactions as on the left sides. The corresponding curves give a visual representation of the most efficient energy range for an EP to produce a given γ-ray line. They can be thought of as simple phenomenological tools: a simple look at them gives an idea of the kind of source spectrum and composition required to reproduce any γ-ray line observational data. Note that contrary to what might have been naively

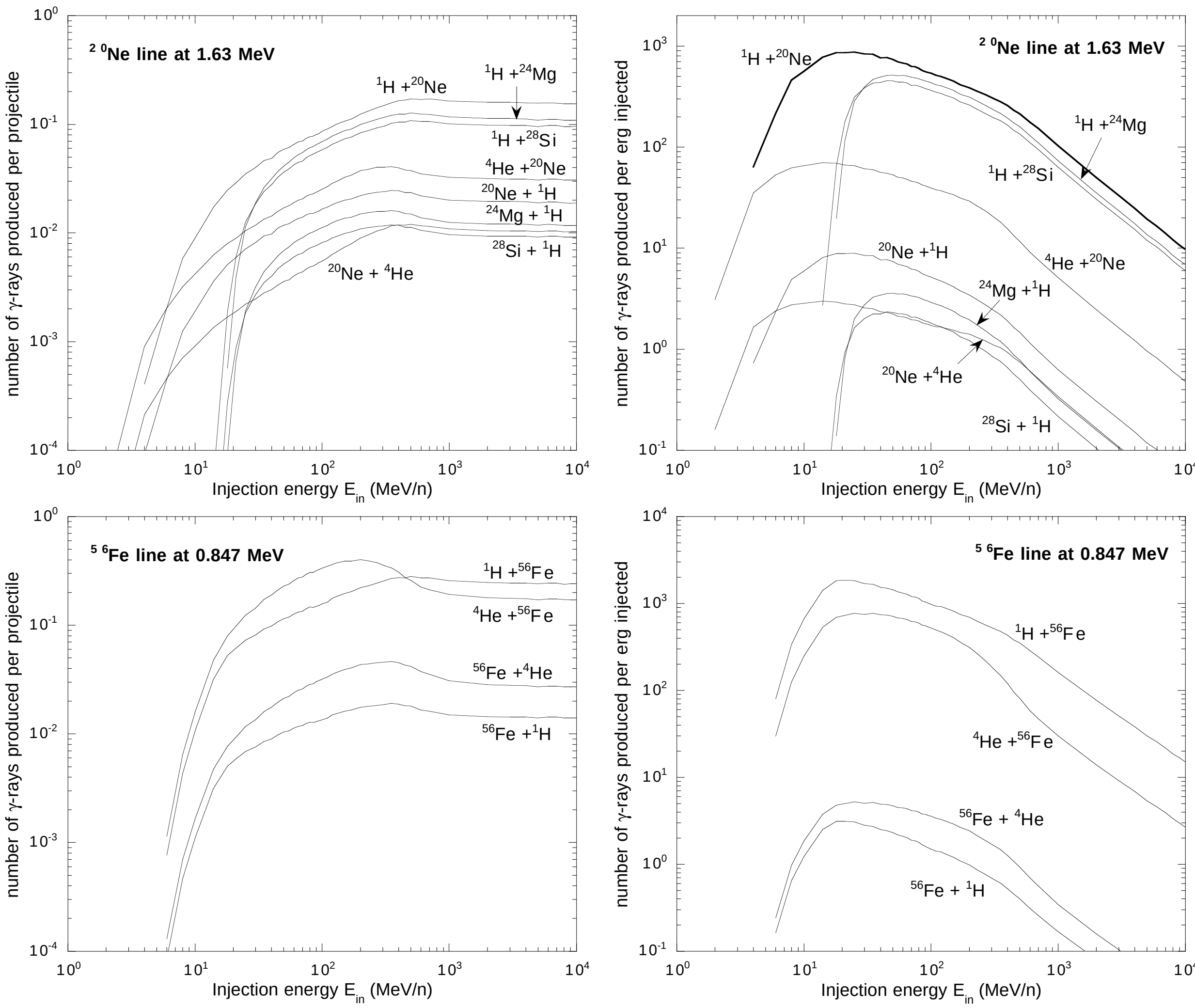

Fig. 8. Same as Figs. 5 for the ^{20}Ne line at 1.63 MeV and the ^{56}Fe line at 0.847 MeV.

expected, this range starts at an energy higher than the cross section peak, and extends to even higher energies. In other words, the most efficient way to produce gamma-rays in the ISM is to use EPs with energies between, say, 10 to 300 MeV/n.

5.3. Gamma-ray line emission synthesis

From a practical point of view, the quantities $\mathcal{N}_{i,j;\gamma}(E_{\rm in})$ also allow one to straightforwardly calculate the γ-ray line emission in a given astrophysical situation. It suffices to sum the contributions of each reaction involved, weighted according to the desired chemical abundances of both the source and the target. In other words, one can calculate the γ-ray line emission rate for *any* EP spectrum and composition in *any* medium (except maybe the most extremely metal-rich), without needing to worry about particle propagation and energy losses at all, as intended.

Such a weighting is illustrated in Figs. 9 and 10, where we show the γ-ray yields of H, He, C and O nuclei in a medium of solar metallicity. Note that although the γ-ray yields of C and O projectiles appear much higher than those of He (or H), they still have to be weighted by the relative abundances of the various projectiles among the EPs. The number of gamma-rays in the C and O lines produced by one EP injected in the ISM at energies above a few tens of MeV/n is typically between 10^{-5} and 10^{-4}.

An interesting result is the fact that, in addition to ^{12}C nuclei, ^{16}O nuclei are also rather efficient in producing the ^{12}C line at 4.44 MeV. In Fig. 11, we have also shown the ^{12}C*/^{16}O* emission line ratio for the three projectiles producing both of these lines, namely H, He and O. This can be used to estimate quickly the probable composition of EPs producing any observed ^{12}C*/^{16}O* line ratio.

6. Analytical estimates

In an earlier work, Bykov & Bloemen (1994) calculated the average photon yield of a single energetic nucleus suffering from Coulombian energy losses (neglecting nuclear destruction). It is worth comparing our results with their analytical approximation, obtained with the Bethe-Bloch formula for energy losses, and extending their approached formula to situations where the excitation cross section

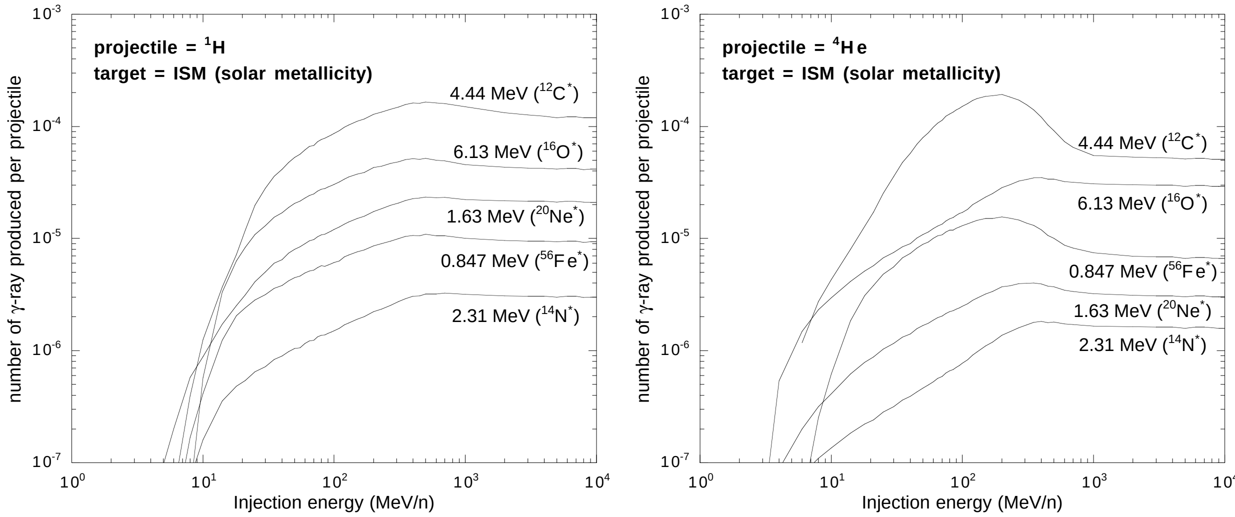

Fig. 9. Gamma-ray yields of a ^{1}H (left) and a ^{4}He (right) nucleus injected in a medium of solar metallicity, as a function of the injection energy. Contributions to various γ-ray lines are shown.

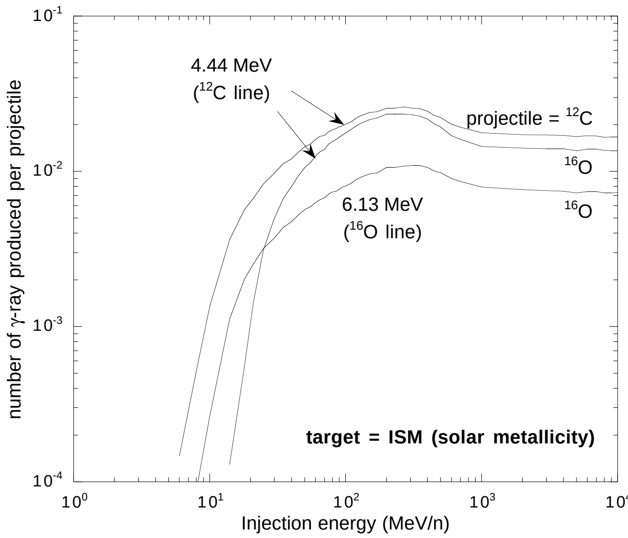

Fig. 10. Same as Fig. 9 for ^{12}C and ^{16}O projectiles.

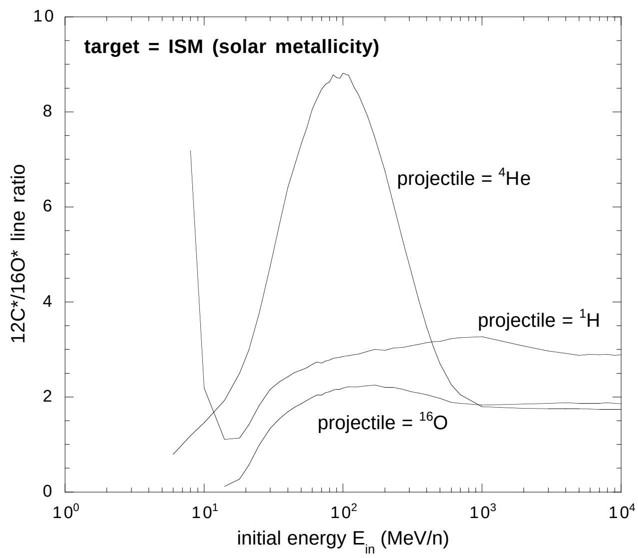

Fig. 11. ^{12}C*/^{16}O* emission line ratio produced by H, He and O nuclei injected into the ISM with solar metallicity, as a function of the injection energy.

follows a simple analytical law (such as that mentioned in Sect. 5.1), and where the nuclear destruction is important.

Depending on the EP injection energy, one can identify two opposite "regimes" where the estimation of the photon yields $\mathcal{N}_{i,j;\gamma}(E_{\rm in})$ can be simplified. From the physical point of view, this depends whether the particle energy losses can be neglected with respect to nuclear destruction, or vice versa. In Fig. 12, we have drawn the destruction cross section, $\sigma_{\rm D}(E)$, together with the energy loss cross section, $\sigma_{\rm loss}(E)$, defined by:

$$\sigma_{\rm loss}(E) = \frac{1}{n_0 v(E) \tau_{\rm loss}(E)} = \frac{|\dot{E}(E)|}{nvE}, \tag{14}$$

for energetic ^{1}H and ^{12}C nuclei. Note that with this definition, Eq. (12) can be rewritten in a simple way as:

$$\mathcal{N}_{\gamma}(E_{\rm in}) = \int_0^{E_{\rm in}} \frac{\sigma_{\rm prod}(E)}{\sigma_{\rm loss}(E)} \mathcal{P}_i(E_{\rm in}, E) \frac{{\rm d}E}{E}. \tag{15}$$

It can be seen in Fig. 12 that at low energy, destruction is negligible compared to energy losses, while the opposite is true at high energy. The exact transition energy depends on the nucleus considered, and ranges between 300 and 300 MeV/n. This is consistent with the results of Fig. 3, showing the transition of the survival probability from 1 to 0 around this energy.

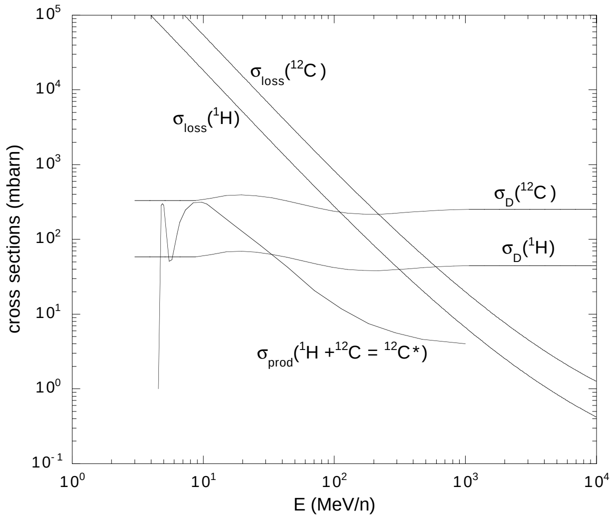

Fig. 12. Comparison of the total inelastic cross section (nuclear destruction) and the energy loss cross section for ^{12}C and ^{1}H. The ^{12}C excitation cross section is also shown for comparison.

6.1. Low energy limit

In the case when particle destruction can be neglected, one can set the survival probability, $\mathcal{P}_i(E_{\rm in}, E)$, equal to 1 in Eq. (15) (this is the case studied by Bykov & Bloemen 1994). The Bethe-Bloch formula for the energy losses of a nucleus $(Z,\ A)$ gives:

$$\sigma_{\rm loss}(E) = \frac{Z^2}{A} \times \frac{3}{2}\sigma_{\rm T}\ln(\Lambda) \times \frac{m_{\rm e}c^2}{E} \times \frac{c^2}{v^2}, \quad (16)$$

where $\sigma_{\rm T}$ is the Thomson cross section and $\ln\Lambda$ is the usual Coulomb logarithm (see e.g. Lang 1999). It is a slightly increasing function of energy ($\sim$7.3 at 10 MeV/n, and $\sim$12 at 1 GeV/n).

Reporting in Eq. (15) and assuming that the excitation cross section is constant, one obtains the result of Bykov & Bloemen (1994) in the non-relativistic limit ($E \sim m_{\rm p}v^2/2$):

$$\mathcal{N}_\gamma(E_{\rm in}) = \frac{A}{Z^2}\frac{m_{\rm p}}{6m_{\rm e}\ln\Lambda}\frac{\sigma_{\rm prod}}{\sigma_{\rm T}}\left(\frac{v}{c}\right)^4, \quad (17)$$

where the cross section can then be considered as a function of energy, although this amounts to moving $\sigma_{\rm prod}(E)$ outside the integral in Eq. (15), which is of course improper in principle.

The above formula can be improved by using an approached expression for the nuclear excitation cross-section, which allows an analytical integration of Eq. (15) (with $\mathcal{P}_i(E_{\rm in}, E) = 1$). By approximating the cross sections as $\sigma_{\rm prod}(E) = a \times E^{-x} + b$ above the peak (the values for a and b being deduced from Eq. (13) and Table 1), with a linear connection from the threshold energy, $E_{\rm th}$, to the peak energy, $E_{\rm peak}$, one obtains:
for $E_{\rm th} < E \le E_{\rm peak}$,

$$\mathcal{N}_\gamma(E_{\rm in}) = \frac{A}{Z^2}\frac{2\sigma_{\rm peak}}{9\sigma_{\rm T}\ln\Lambda}\frac{(E_{\rm in}-E_{\rm th})^2}{m_{\rm e}c^2m_{\rm p}c^2}\frac{2E_{\rm in}+E_{\rm th}}{E_{\rm peak}-E_{\rm th}}, \quad (18)$$

and for $E > E_{\rm peak}$,

$$\mathcal{N}_\gamma(E_{\rm in}) = \mathcal{N}_\gamma(E_{\rm peak}) + \frac{A/Z^2}{3\sigma_{\rm T}\ln\Lambda} \times \left[\frac{4E^2}{m_{\rm e}c^2m_{\rm p}c^2}\left(\frac{aE^{-x}}{2-x}+\frac{b}{2}\right)\right]_{E_{\rm peak}}^{E_{\rm in}}, \quad (19)$$

where we have used the non-relativistic relation between v and E, which is justified for the energy range under consideration.

6.2. High energy limit

In the other limit, at high energy, the destruction time is so small compared to the energy loss time that the particles will be destroyed before they have lost any significant amount of energy. We can thus assume that the gamma-ray emission occurs at a constant energy, namely the injection energy. The problem is solved straightforwardly in terms of the production and destruction timescales (or cross sections):

$$\mathcal{N}_\gamma(E_{\rm in}) = \frac{\tau_{\rm D}}{\tau_{\rm prod}} = \frac{\sigma_{\rm prod}}{\sigma_{\rm D}}. \quad (20)$$

Indeed, if one particle is injected at time $t = 0$, the number of remaining particles at time t is $N(t) = \exp(-t/\tau_{\rm D})$, and the gamma-ray production rate is ${\rm d}N_\gamma/{\rm d}t = N(t)/\tau_{\rm prod} = \exp(-t/\tau_{\rm D})/\tau_{\rm prod}$. Integrating over time, from $t = 0$ to ∞, gives the above result.

6.3. Accuracy of the analytical formulæ

The approached formulae (18) and (19) give an approximation of the photon yields at low energy, which may be noted $\mathcal{N}_{\gamma,{\rm LE}}(E)$, while Eq. (20) is accurate for the high energy limit, $\mathcal{N}_{\gamma,{\rm HE}}(E)$. One can then propose an approached formula valid in the whole energy range:

$$\mathcal{N}_\gamma(E) = [\mathcal{N}_{\gamma,{\rm LE}}(E)^{-1} + \mathcal{N}_{\gamma,{\rm HE}}(E)^{-1}]^{-1}. \quad (21)$$

In Fig. 13, the various above approached formulæ are compared with the results of the previous section for the reactions $^{12}{\rm C} + ^1{\rm H} \to ^{12}{\rm C}^*$ and $^{16}{\rm O} + ^4{\rm He} \to ^{16}{\rm O}^*$. As can be seen, the various approximations give reasonably good results in their respective energy range. However, it should be stressed that except for the high energy, the accuracy of our formulæ depends essentially on the accuracy of the cross section modeling. Since the real cross sections do *not* follow the simple analytical expression used in Sect. 6.1, we cannot expect the analytical photon yields calculated in this section to be accurate to more than a factor of two or so (as we could observe from the whole data set). This can be very problematic when considering gamma-line ratios. Therefore, we strongly recommend to use the results of Sect. 5 instead, which are also uncertain, but only insofar as the cross sections are not know. They thus provide the best estimates of individual gamma-ray yields given the present knowledge on the excitation cross sections.

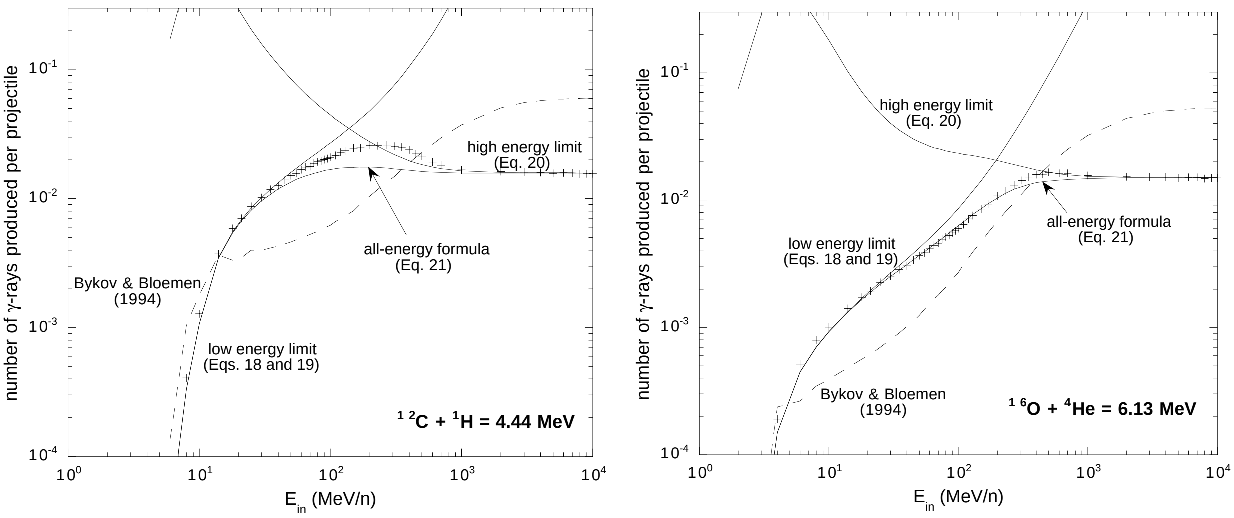

Fig. 13. Comparison of the analytical approximations for the individual gamma-ray yields with the results of Sect. 5 (plus signs), for two of the main reactions. The formula of Eq. (17) is also shown as a dashed line.

7. Summary

In this paper, we have presented an easier way to calculate gamma-ray line emission from energetic particle interactions in the ISM. It is based on a simple mathematical transformation whose physical interpretation has been given and which allows one to work with the source spectrum of the EPs rather than the propagated spectrum. Therefore, one does not need to worry about energy-dependent and nucleus-dependent energy losses of the particles, nor about their nuclear destruction in-flight, as they are taken into account once and for all through the calculation of absolute photon yields. The latter are the number of photons produced in each of the nuclear de-excitation lines by a given nucleus injected in the ISM at a given initial energy. These photon yields have been given here for various projectiles contributing to the main ^{12}C, ^{14}N, ^{16}O, ^{20}Ne and ^{56}Fe de-excitation lines. Numerical tables and electronic versions of the results are available from the authors upon request. These photon yields are to be used instead of the nuclear excitation cross-sections, and might be thought of as 'effective cross-sections' taking into account the specific effects of particle propagation in the ISM. They can be used to calculate the γ-ray line emission induced by EPs with any spectrum and any composition in any medium with a metallicity lower than a few tens of the solar metallicity.

In addition to simplifying the calculation of gamma-ray line emission, the individual EP gamma-ray yields also provide a direct, intuitive tool to analyze gamma-ray line data from a phenomenological point of view, and construct an EP source spectrum and composition which could reproduce the intensity of the gamma-ray emission and the various line ratios. The results presented here correspond to a thick target model, which is relevant to most astrophysical situations for EPs of energy lower than a few hundreds of MeV/n. However, the same formalism can be used to calculate the total EP photon yields in a target with any escape length, be it energy-dependent or not. Once these yields have been calculated once, they can be used in any situation with the same escape length, for any particle spectrum and any EP and target compositions.

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