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Apsidal motion and light-time effect in eclipsing binaries HS Herculis and U Ophiuchi⋆

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Abstract. Several new times of minima, based on photoelectric observations, have been secured for two bright and well-known eclipsing binaries with eccentric orbits and with very similar orbital periods: HS Her ($P = 1\overset{\mathrm{d}}{.}637, e = 0.02$) and U Oph ($P = 1\overset{\mathrm{d}}{.}677, e = 0.003$). For HS Her, an apsidal motion with a period of 78 years is confirmed and a third body in an eccentric orbit with a period of 85 years is found. An analysis of all available eclipse timings of U Oph has confirmed the presence of an apsidal motion with one of the shortest known periods (20.1 yr) and a light-time effect with the improved period of 37.6 yr. The corresponding internal structure constants log k_2 were also derived. It is estimated that the tertiaries of both systems are detectable via speckle interferometry.

Key words. stars: binaries: eclipsing – stars: individual: HS Her – stars: individual: U Oph – stars: fundamental parameters

1. Introduction

The photometric study of triple and multiple stellar systems with apsidal motion of the eclipsing pair is a poorly studied field of celestial mechanics. In this paper, we report new results for our observational project with the primary purpose to monitor eclipsing binaries with eccentric orbits (Wolf et al. 1999; Wolf 2000). Here we analyse the observational data and rates of apsidal motion for two eclipsing systems, HS Her and U Oph, which are both relatively bright northern-hemisphere objects with a substantial light-time effect contribution. Both objects are frequently observed early-type eclipsing binaries, whose orbits have been known to be eccentric and to exhibit apsidal motion. Both systems are also included in the latest catalog of apsidal motion in double stars of Petrova & Orlov (1999).

2. Observations of minimum light

In order to enlarge the number of times of minimum light, new observations for both systems were carried out. New photoelectric photometry was obtained at two observatories with the aim to secure several new, well-covered primary and secondary minima for each variable:

- During two weeks in June 2001, UBV observations of both binaries were secured at the San Pedro Mártir Observatory (hereafter SPM), Baja California, Mexico. The 0.84-m Cassegrain reflector equipped with the photon-counting photometer Cuentapulsos (utilizing a RCA 31034 photomultiplier) was used;
- Two minima of HS Her were observed in 2001 in the private observatory of KH at Lelekovice, Czech Republic. A 0.35-m Newtonian telescope, equipped with a CCD camera SBIG ST-6V and a standard Cousins R filter was used.

Photoelectric observations at SPM were carried out differentially, relative to the following comparison stars: HD 156208 for U Oph and HD 174369 for HS Her. These observations consisted from 10-second integrations in each filter. They were carefully reduced to the standard UBV system and corrected for differential extinction using the

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⋆ Partly based on photoelectric observations secured at the San Pedro Mártir Observatory, Baja California, Mexico.

Table 1. New times of minimum light.

System	JD Hel.- 2400000	Error [day]	Epoch	Method Filter	Reference Observatory
HS Her	49193.4252*	0.0010	5525.0	pe, *BV*	Agerer (1994)
	49202.4267*	0.0010	5530.5	pe, *BV*	Agerer (1994)
	49229.4499*	0.0003	5547.0	pe, *BV*	Agerer (1994)
	49509.4548	0.0018	5718.0	pe, *UBV*	Müysseroglu et al. (1996)
	49518.454	0.002	5723.5	pe, *B*	Diethelm (1994)
	49523.3757	0.0028	5726.5	pe, *UBV*	Müysseroglu et al. (1996)
	49545.4726	0.0010	5740.0	pe, *UBV*	Müysseroglu et al. (1996)
	49559.3891	0.0024	5748.5	pe, *UBV*	Müysseroglu et al. (1996)
	49811.5528	-	5902.5	pe, *V*	Hegedüs et al. (1996)
	49861.5010*	-	5933.0	pe, *BV*	Hegedüs et al. (1996)
	49929.4496*	0.0012	5974.5	pe, *BV*	Agerer & Hübscher (1996)
	49961.3763	0.0011	5994.0	pe, *BV*	Müysseroglu et al. (1996)
	49979.3943	0.0008	6005.0	pe, *V*	Agerer & Hübscher (1996)
	50313.4290	0.0003	6209.0	pe, *BV*	Agerer & Hübscher (1997)
	50688.4034	0.0005	6438.0	pe, *BV*	Agerer & Hübscher (1999)
	50945.4749	0.0002	6595.0	pe, *V*	Borkovits & Bíró (1998)
	50972.4946	0.0003	6611.5	pe, *V*	Borkovits & Bíró (1998)
	50981.5011	0.0003	6617.0	pe, *V*	Borkovits & Bíró (1998)
	51302.4332	0.0002	6813.0	CCD	Bíró & Borkovits (2000)
	51397.4074	0.0005	6871.0	pe, *BV*	Agerer et al. (2001)
	51681.509	0.001	7044.5	CCD	Bíró & Borkovits (2000)
	52063.8421	0.0001	7278.0	pe, *UBV*	this paper, SPM
	52068.7538	0.0001	7281.0	pe, *UBV*	this paper, SPM
	52097.4158	0.0003	7298.5	CCD, *R*	this paper, Lelekovice
	52151.4537	0.0003	7331.5	CCD, *R*	this paper, Lelekovice
U Oph	47670.4453**	0.000	1940.0	pe, *BV*	Mayer et al. (1991)
	47913.659	0.005	2085.0	pe, *H*	this paper, Hipparcos
	47914.501	0.005	2085.5	pe, *H*	this paper, Hipparcos
	48018.4958*	-	2147.5	pe, *BV*	Agerer (1990)
	48034.4285*	-	2157.0	pe, *BV*	Agerer (1991)
	48086.4277*	-	2188.0	pe, *BV*	Agerer (1991)
	48107.3977	-	2200.5	pe, *B*	Diethelm (1990)
	48476.4160	0.0005	2420.5	pe	Blättler (1991)
	48765.75180	0.00015	2593.0	pe, *V*	Caton & Burns (1993)
	49167.4789*	0.0003	2832.5	pe, *BV*	Agerer (1994)
	49895.4427	0.000	3266.5	pe	West (1996)
	49895.4431**	0.0004	3266.5	pe, *B*	Blättler (1995)
	49905.5077	0.0006	3272.5	pe, H_β	Jordi et al. (1996)
	52060.88441	0.00015	4557.5	pe, *UBV*	this paper, SPM
	52065.91583	0.00015	4560.5	pe, *UBV*	this paper, SPM
	52066.75762	0.00015	4561.0	pe, *UBV*	this paper, SPM
	52071.79043	0.00015	4564.0	pe, *UBV*	this paper, SPM

Notes: ** mean value of B and V measurements, ** corrected time of minimum.*

reduction program HEC 22 rel.13.2 (Harmanec & Horn 1998).

The CCD measurements in Lelekovice were flat-fielded via sky exposures taken at dusk or dawn. Several comparison stars were chosen on the same frame as the variables. No variations in the brightness of these stars exceeding the expected error of measurements (typically $\sigma \simeq 0.01$ mag) were detected. No correction for differential extinction was applied, due to the proximity of the comparison stars to the variable and the resulting negligible differences in air mass.

The new times of primary and secondary minima and their errors were determined using a least squares fit of the data and by the bisecting cord method. Only the bottom parts of the eclipses were used. The new times of minima are included in Table 1. In this table, the epoch is calculated from the light elements given in the text, the other columns being self-explanatory.

3. Apsidal motion and light-time effect analysis

The apsidal motion and the light-time effect in both systems were studied by means of an (O−C) diagram analysis. The deviation of the observed values $(O-C)_{obs}$ from the linear ephemeris is given by a superposition of the apsidal motion of the eccentric binary system and by the light-time effect caused by the presence of a third body:

$$(O-C)_{obs} = (O-C)_{aps} + (O-C)_{lte},$$

where $(O-C)_{aps}$ represents the influence of the apsidal motion and $(O-C)_{lte}$ is the contribution of the light-time effect. For a more accurate calculation of the apsidal motion rate, the method described by Giménez & García-Pelayo (1983), with equations revised by Giménez & Bastero (1995), was used. Our relation for the prediction of the times of minimum caused by the apsidal motion is given in Wolf & Šarounová (1995).

The theory of the third body motion and the light-time effect analysis in eclipsing binaries was reviewed several times in the literature, see e.g. Irwin (1959), Mayer (1990), or Wolf et al. (1999). The observed semi-amplitude A of the light-time curve (in days) is

$$A = \frac{a_{12} \sin i_3}{173.15} \sqrt{1 - e_3^2 \cos^2 \omega_3}, \quad (1)$$

where a_{12} is semi-major axis of the relative orbit of the eclipsing pair around the common center of mass (in AU), i_3 is the inclination of the third-body orbit, e_3 is eccentricity and ω_3 the longitude of periastron of the third-body orbit. There are 10 independent variables to be determined in this procedure:

$(T_0, P_s, e, \dot{\omega}, \omega_0)$, for the apsidal motion and

$(A, T_3, P_3, e_3, \omega_3)$ for the light-time effect.

The stability of our reduction procedure was tested by solving for the light-time effect as well as for apsidal-motion parameters separately. The relation between the sidereal and the anomalistic period, P_s and P_a, is given by

$$P_s = P_a\,(1 - \dot{\omega}/360°)$$

and the period of apsidal motion by

$$U = 360° P_a/\dot{\omega}.$$

We have collected all reliable times of minimum light available in the literature as well as in the current databases of BAV and BBSAG observers or in the Besançon Double and Multiple Star Database. All photoelectric and CCD times of minima were used with a weight of 10 in our computation. Less accurate measurements were assigned weights of 5, 3 or 1. With the exception of the first photographic times of minima of HS Her, secured by Martynov (1940), no visual or photographic times of minima were used because of their large scatter. We tested the stability of the results with respect to our – somewhat arbitrarily chosen – weighting scheme. It turned out that the results for the well-covered phase curves of U Oph are insensitive to the weighting scheme used. For HS Her, the resulting parameters show some dependence on the weighting but this is mainly related to the less ideal distribution of available observations. For this reason, as well as for other reasons discussed below, the results for HS Her must be considered preliminary and less certain than those for U Oph.

4. HS Her

The detached eclipsing binary HS Her (HD 174714, BD+24°3552, SAO 86497, HIP 92478, PPM 107939, HV 10129, FL 2574; $\alpha_{2000} = 18^h50^m49.^s8$, $\delta_{2000} = +24° 43'11.''9$, $V_{max} = 8.^m5$; Sp. B5III+A4) is a rather frequently observed binary with an eccentric orbit ($e = 0.02$) and an orbital period of about 1.637 days. It was discovered to be a variable in 1934 by Martynov (1940) who derived the first light elements. The variability and binary nature of HS Her was discovered independently by Jacchia (1940) on Harvard plates. Later Martynov (1951) improved the light elements. The first spectroscopic study was published by Cesco & Sahade (1945). They obtained a single-lined radial velocity curve with the semiamplitude $K = 82.6 \pm 1.4$ km s^{-1} and the orbital eccentricity of $e = 0.05 \pm 0.02$. A photometric study of HS Her was presented by Hall & Hubbard (1971). They obtained photometric elements and absolute dimensions of the components and derived also the apsidal motion period of 15.5 years. Their photoelectric observations exhibit some asymmetries in the secondary eclipse. The resulting ephemeris for primary minimum is

$$\begin{aligned} \text{Pri.Min.} &= \text{HJD } 24\,37854.194 + 1.^d63743333 \times E \\ &\quad +0.^d017 \sin\left(\frac{360°\,E}{3450}\right). \end{aligned}$$

Another photometric study was carried out by Martynov & Lavrov (1972), who derived absolute dimensions of the system and suggested the apsidal motion period of 110–130 years. The photometric elements were later revised by Giuricin & Mardirossian (1981), who estimated $M_1 = 6.5\ M_\odot$, $M_2 = 1.9\ M_\odot$, $R_1 = 3.0\ R_\odot$, $R_2 = 1.7\ R_\odot$. Next linear light elements were presented by Martynov (1985)

$$\begin{aligned} \text{Pri.Min.} &= \text{HJD } 24\,40146.6095 + 1.^d637435 \times E, \\ \text{Sec.Min.} &= \text{HJD } 24\,40147.4967 + 1.^d637434 \times E. \end{aligned}$$

Finally, Khaliullina & Khaliullin (1992) published new times of minima and estimated an apsidal motion period $U = 92 \pm 14$ years, using all published photoelectric data. They also presented the following light elements with two cosine terms:

$$\begin{aligned} \text{Pri.Min.} &= \text{HJD } 24\,47382.42318 + 1.^d6374344 \times E \\ &\quad +0.^d00988 \cos\omega + 0.^d00007 \cos 2\omega, \\ \text{Sec.Min.} &= \text{HJD } 24\,47383.24190 + 1.^d6374344 \times E \\ &\quad -0.^d00988 \cos\omega + 0.^d00007 \cos 2\omega, \end{aligned}$$

where $\omega = 234.^\circ91 + 0.^\circ01753 \times E$.

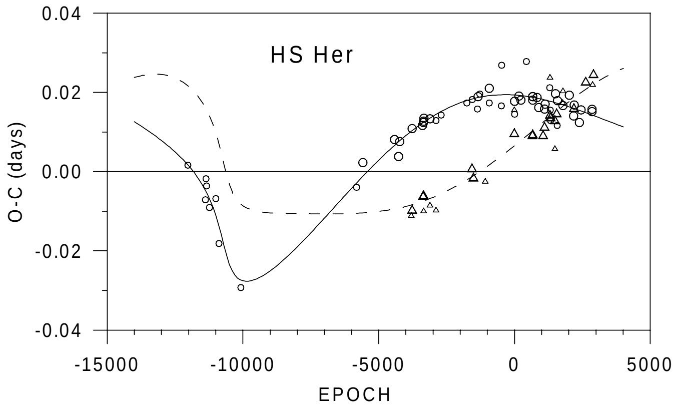

Fig. 1. The O–C diagram for the times of minima of HS Her. The full and dashed curves represent predictions for the primary and secondary eclipses, respectively. The individual primary and secondary minima are denoted by circles and triangles, respectively. Larger symbols correspond to the photoelectric measurements which were given higher weights in the calculations.

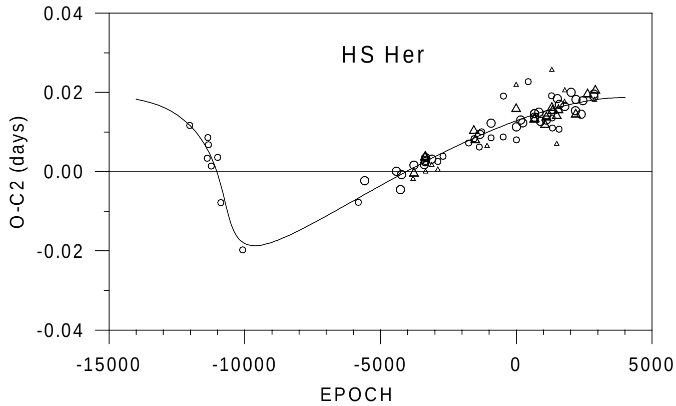

Fig. 2. The (O−C)$_2$ diagram for the times of minima of HS Her after substraction of the apsidal motion. The curve represents a light-time effect for the third body orbit with a period of 86 years and an amplitude of about $0\overset{\mathrm{d}}{.}02$. The individual primary and secondary minima are denoted by circles and triangles, respectively.

Possible apsidal motion or light time effect in the system HS Her was discussed again by Todoran (1992), Bastian (1993) and Todoran & Agerer (1994). All photoelectric times of minimum light published in Khaliullina & Khaliullin (1992), Todoran (1992, his Table 1), Caton & Burns (1993), Hegedüs et al. (1996), Agerer & Hübscher (1996, 1997, 1999), Borkovits & Bíró (1998), Bíró & Borkovits (2000), as well as Agerer et al. (2001) were incorporated in our analysis. A total of 85 times of minimum light were used, including 39 secondary eclipses.

Adopting the orbital inclination derived from the light curve solution of $i = 88\overset{\circ}{.}5$ (Martynov et al. 1988), the apsidal motion elements can be computed. The parameters found and their internal errors of the least squares fit (in brackets) are given in Table 2. In this table $P_{\rm s}$ denotes the sidereal period, $P_{\rm a}$ the anomalistic period, e represents the eccentricity and $\dot{\omega}$ is the rate of periastron advance (in degrees per cycle or in degrees per year). The zero epoch is given by T_0 and corresponding position of the periastron is represented by ω_0.

The (O−C) residuals for all times of minimum with respect to the linear part of the apsidal motion equation are shown in Fig. 1. The predictions, corresponding to the fitted parameters, are plotted as full and dashed curves for primary and secondary eclipses, respectively. After subtracting the contribution of the apsidal motion, the (O−C)$_2$ diagram of the light-time effect of Fig. 2 can be obtained. This (O−C)$_2$ curve is only partially covered by accurate times of minima so that the third-body period P_3 of 85.7 years remains only a preliminary one.

Table 2. Apsidal motion and third-body orbit parameters of HS Her and U Oph.

Element	HS Her	U Oph
T_0 [HJD]	24 47382.4104 (3)	24 44416.38565 (12)
$P_{\rm s}$ [days]	1.63743125 (7)	1.67734578 (4)
$P_{\rm a}$ [days]	1.6375254 (8)	1.6777293 (7)
e	0.020 (3)	0.0030 (2)
$\dot{\omega}$ [deg cycle^{-1}]	0.0207 (6)	0.0823 (7)
$\dot{\omega}$ [deg yr^{-1}]	4.62 (13)	17.92 (16)
ω_0 [deg]	232.0 (2.5)	172.3 (2.5)
U [years]	78.0 (0.3)	20.1 (0.2)
P_3 [days]	31 000 (800)	13 750 (70)
P_3 [years]	85.7 (2.2)	37.6 (0.2)
A [days]	0.0187 (8)	0.0101 (3)
e_3	0.80 (0.07)	0.185 (5)
ω_3 [deg]	199.3 (1.2)	152.3 (1.4)
T_3 [JD]	24 29900 (500)	24 50530 (50)

5. U Oph

The double-lined detached eclipsing binary U Oph (also HR 6414, HD 156 247, BD+1°3408, SAO 122 226, HIP 84 500, PPM 163 749, FL 2127, WDS 17165+0113, ADS 10 428 A; $\alpha_{2000} = 17^{\rm h}16^{\rm m}31\overset{\rm s}{.}7$, $\delta_{2000} = +1°12'38\overset{''}{.}0$, $V_{\max} = 5\overset{\rm m}{.}9$; Sp. B5V) is one of the brightest and best-known binary systems with a slightly eccentric orbit ($e = 0.003$). It was discovered to be variable by Gould in 1871. The early history is given in detail in the *Gesichte und Literatur der Veränderlichen Sterne (GuL)*. Parenago (1949) determined the first precise light elements

$$\text{Pri.Min.} = \text{HJD } 24\ 08279.643 + 1\overset{\rm d}{.}6773460 \times E.$$

Photometry and light-curve analysis have been reported by Huffer & Kopal (1951), Eaton & Ward (1973) and Koch & Koegler (1977). Spectroscopic orbits have been determined by Plaskett (1919), Abrami (1958) and Pearce (1960). Popper & Carlos (1970) obtained radial-velocity curves with semiamplitudes of $K_1 = 179.6 \pm 1.9$ km s^{-1} and $K_2 = 201.8 \pm 2.9$ km s^{-1}. The history of investigation of this binary can be found in Koch & Kogler (1977).

Several studies of the period of U Oph have been made since its discovery as an eccentric eclipsing binary.

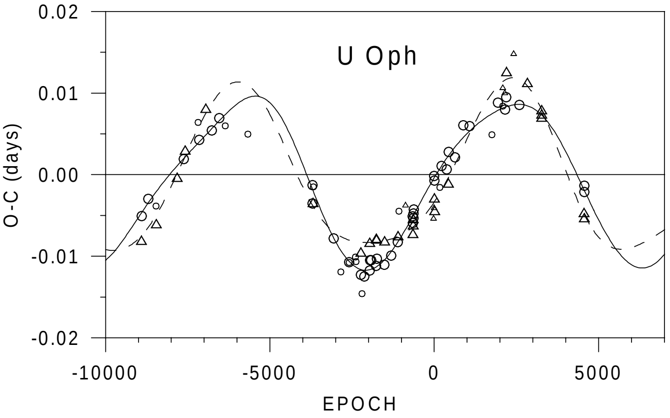

Fig. 3. The (O−C) diagram for U Oph. See legend for Fig. 1.

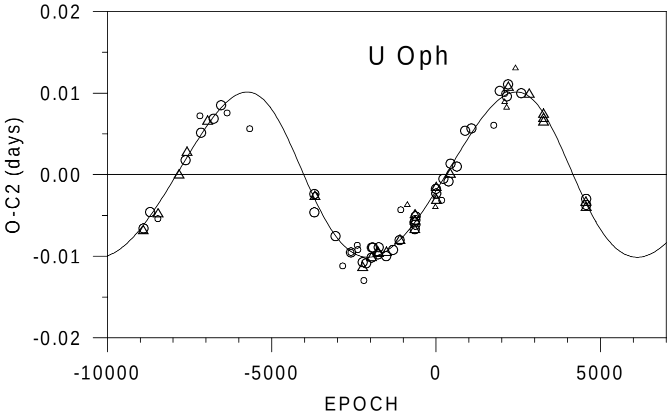

Fig. 4. The $(\mathrm{O-C})_2$ diagram for the times of minima of U Oph after substraction the terms of apsidal motion. The individual primary and secondary photoelectric minima are denoted by circles and triangles, respectively.

Frieboes-Conde & Herczeg (1973) derived a light-time effect with two possible periods of 49.3 or 55.25 years. Later, Panchatsaram (1981) derived a light-time effect with a substantially shorter period of 27.55 years. The most detailed and comprehensive period analysis is that by Kämper (1986). Using differential correction procedures, he obtained a light-time effect with a period of 38.7 ± 0.2 years and revealed a short-period apsidal motion with the period of $U = 20.7$ yr in a slightly eccentric orbit ($e = 0.0031 \pm 0.0003$). The internal structure constant $k_2 = 0.0059$ was also given.

Absolute dimensions of the components of U Oph were derived by Holmgren et al. (1991), who obtained $M_1 = (4.93 \pm 0.05)\ M_\odot$, $M_2 = (4.56 \pm 0.04)\ M_\odot$, $R_1 = (3.29 \pm 0.06)\ R_\odot$ and $R_2 = (3.01 \pm 0.05)\ R_\odot$. Moreover, this system is known to have a small relativistic contribution to the observed apsidal motion rate. For this reason, U Oph is a case of special interest from the point of view of the orbit rotation. The relativistic apsidal motion in this binary was discussed by Claret (1997), who found excellent agreement of observational data with the theoretical prediction based on stellar models. Recently, photometric and polarimetric observations of U Oph were obtained by Eritsian et al. (1998). Since the above-mentioned papers were published, several new times of minima have been obtained which allowed us to reduce the observational uncertainties.

All photoelectric times of minimum light given in Plavec et al. (1960, their Table 10), Popovici (1970, 1971), Pohl & Kizilirmak (1970, 1972, 1975), Kizilirmak & Pohl (1971, 1974), Scarfe et al. (1973), Batten & Scarfe (1977), Scarfe & Barlow (1978), Aslan et al. (1981), Panchatsaram (1981, his Table 1), Kämper (1986, his Table I), Agerer (1990, 1991, 1994), Blättler (1991, 1995) and Jordi et al. (1996) were incorporated in our calculation. The epochs in Table 1 were computed according to the ephemeris of Aslan et al. (1981)

$$\text{Pri.Min.} = \text{HJD } 24\ 44416\overset{\text{d}}{.}3856 + 1\overset{\text{d}}{.}677346 \times E.$$

Using the Hipparcos photometry (Perryman et al. 1997), we were able to derive two additional times of minimum light. A total of 85 photoelectric times of minimum light were used in our analysis, with 32 secondary eclipses among them. The orbital inclination was adopted to be $i = 88\overset{\circ}{.}3$ based on the photometric analysis (Holmgren et al. 1991). The computed apsidal motion elements and derived light-time effect parameters and their internal errors of the least squares fit are given in Table 2. The O−C diagrams are shown in Figs. 3 and 4.

6. Internal structure constant

Observations of eccentric binary systems allow us to determine the internal structure constant k_2, which is related to the variation of the density within the star and is an important parameter of stellar evolution models. It is best studied in binary systems with eccentric orbits that show apsidal motion. The period of rotation of the periastron in eccentric eclipsing binaries does not allow us to derive the individual internal stellar constant of the component stars.

The observational average value of $k_{2,\mathrm{obs}}$ is given by the relation

$$k_{2,\mathrm{obs}} = \frac{1}{c_{21} + c_{22}} \frac{P_\mathrm{a}}{U} = \frac{1}{c_{21} + c_{22}} \frac{\dot{\omega}}{360}, \tag{2}$$

where c_{21} and c_{22} are functions of the orbital eccentricity, fractional radii, the masses of the components and the ratio between rotational velocity of the stars and Keplerian velocity (Kopal 1978). Taking into account the value of the eccentricity and the masses of the components, one has to subtract from $\dot{\omega}$ a relativistic correction $\dot{\omega}_\mathrm{rel}$ (Levi-Civita 1937; Giménez 1985)

$$\dot{\omega}_\mathrm{rel} = 5.45 \times 10^{-4} \frac{1}{1 - e^2} \left(\frac{M_1 + M_2}{P} \right)^{2/3}, \tag{3}$$

where M_i denotes the individual masses of the components in solar units. The values of $\dot{\omega}_\mathrm{rel}$ and the resulting mean internal structure constants $k_{2,\mathrm{obs}}$ are given in Table 3. Theoretical values $k_{2,\mathrm{theo}}$ according to available theoretical models for the internal stellar structure along the main

Table 3. Adopted basic physical properties of the binary components and the apsidal-motion constants and characteristics of the tertiary components derived by us.

Parameter	Unit	HS Her	U Oph
M_1	$M_\odot$	4.7 (5)	4.93 (5)
M_2	$M_\odot$	1.6 (5)	4.56 (4)
r_1		0.259 (3)	0.262 (4)
r_2		0.152 (5)	0.240 (4)
Source		Hall & Hubbard (1971)	Holmgren et al. (1991)
$\dot{\omega}_{\rm rel}$	deg cycle^{-1}	0.00134	0.00173
$\dot{\omega}_{\rm rel}/\dot{\omega}$	%	2.0	8.2
log $k_{2,\rm obs}$		−2.289	−2.183
log $k_{2,\rm theo}$		−2.31	−2.188
$f(M)$	$M_\odot$	0.0230	0.0040
$M_{3,\rm min}$	$M_\odot$	1.08	0.747
K	km s^{-1}	3.3	2.7
$P_{\rm s}/P_3$		0.000 05	0.000 12

sequence computed by Claret & Giménez (1992) for a variety of masses and chemical compositions are also given.

As one can see, the agreement between the theoretical and observed value of the constant of interval structure is perfect for U Oph and satisfactory for HS Her. We warn in this connection that the masses of HS Her are still a subject of larger uncertainties since they were estimated on the basis of the primary spectrum and the mass function only.

7. Mass of the third body

The parameters of the orbits of third bodies which we derived allow us to calculate the mass function $f(M)$ of the triple systems

$$f(M) = \frac{M_3^3 \sin^3 i_3}{(M_1 + M_2 + M_3)^2} = \frac{a_3^3 \sin^3 i_3}{P_3^2} = \frac{1}{P_3^2}\left[\frac{173.15\,A}{\sqrt{1 - e_3^2 \cos^2 \omega_3}}\right]^3,$$

where P_3 is the period of the third-body (in years) and M_i are the masses of components. The systemic radial velocity of the eclipsing pair has an amplitude of

$$K = \frac{A}{P_3} \frac{5156}{\sqrt{(1 - e^2)(1 - e^2 \cos^2 \omega_3)}}.$$

Assuming a coplanar orbit ($i_3 = 90°$) we can obtain a lower limit for the mass of the third component $M_{3,\rm min}$. This value, as well as the mass function $f(M)$, $a_3 \sin i_3$ and the amplitude of the systemic radial velocity K are also given in Table 3 for both systems studied.

The acceleration of the rate of apsidal motion caused by the presence of the third body is negligible in both systems due to the relatively long period P_3 of the third body orbit.

8. Conclusions

We derive updated apsidal motion elements for two eccentric eclipsing binaries by means of an (O−C) diagram analysis. In the case of HS Her, we have confirmed the apsidal motion at a rate $\dot{\omega}_{\rm obs} = 4.62 \pm 0.13$ deg year^{-1}. A third body in an eccentric orbit with a period of 85 years and a minimum mass of 1.1 $M_\odot$ is found.

For U Oph, our resulting orbital eccentricity and the period of apsidal motion is in good agreement with the elements previously obtained by Kämper (1986). U Oph is the system with one of the shortest known period of apsidal motion of about 20 years. Nevertheless, the scatter of the (O−C) values seems to be relatively large for a detached eclipsing system. Subtracting the influence of apsidal motion, no other phenomenon (e.g. presence of a fourth body) can be simply derived. The differences are still larger than the published standard errors of many observed times of minimum.

A small group of triple eccentric eclipsing binaries with detectable apsidal motion and light-time effect was listed in Wolf et al. (2001, their Table 4). These excellent laboratories of celestial mechanics deserve a continuous monitoring. More high-accuracy timings of these systems are necessary in the future in order to enlarge the time span for a better analysis of the apsidal motion, especially in HS Her, where the light-time effect is only poorly determined. For HS Her, it is also highly desirable to obtain new, high-dispersion and high-S/N spectroscopic observations and apply modern disentangling methods to them to obtain the radial-velocity curves of both binary components and, therefore, derive accurate masses for this important system.

In passing, we note that in both systems, the distant third components should be detectable via speckle interferometry. Using the minimum masses, derived periods and Keplers third law, one can estimate the semi-major axis of the wide orbit to be larger than 8084 $R_\odot$ for HS Her, and larger than 5243 $R_\odot$ for U Oph. For the Hipparcos parallaxes of both systems (Perryman et al. 1997) this translates to angular separations of 0.″076 and 0.″13 for HS Her and U Oph, respectively. To the best of our knowledge, HS Her is not known to be a speckle-interferometric binary. On the contrary, U Oph is the brighter component of the visual binary ADS 10428. Its companion is a G0 star at a separation of 20.″7. The attempts to resolve the closer component (discussed here) via speckle-interferometry resulted in negative results, giving only the following upper limits: separation less than 0.″038 in 1985.5 and less than 0.″06 in 1991.6 (see Hartkopf et al. 2001). Note that according to our results, the system was close to periastron during these observations. Systematic speckle-inerferometric observations of both systems are therefore very desirable and could lead to the determination of all basic physical properties of these astrophysically interesting systems.

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