A&A 381, 1080–1089 (2002)
DOI: 10.1051/0004-6361:20011532

Astronomy & Astrophysics

Bulk viscosity in superfluid neutron star cores

III. Effects of Σ^- hyperons

P. Haensel[1], K. P. Levenfish[2], and D. G. Yakovlev[2]

[1] N. Copernicus Astronomical Center, Bartycka 18, 00-716 Warszawa, Poland
[2] Ioffe Physical Technical Institute, Politekhnicheskaya 26, 194021 St.-Petersburg, Russia
e-mail: ksen,yak@astro.ioffe.rssi.ru

Received 16 October 2001 / Accepted 29 October 2001

Abstract. The bulk viscosity of neutron star cores containing hyperons is studied taking into account non-equilibrium weak process $\mathrm{n}+\mathrm{n} \rightleftharpoons \mathrm{p}+\Sigma^-$. The rapid growth of the bulk viscosity within the neutron star core associated with switching on new reactions (modified Urca process, direct Urca process, hyperon reactions) is analyzed. The suppression of the bulk viscosity by superfluidity of baryons is considered and found out to be very important.

Key words. stars: neutron – dense matter

1. Introduction

The bulk viscosity of matter in the cores of neutron stars has recently attracted great attention in connection with damping of neutron star pulsations and gravitational radiation driven instabilities, particularly – in damping of r-modes (e.g., Andersson & Kokkotas 2001). It is well known that the bulk viscosity is caused by energy dissipation associated with weak-interaction non-equilibrium reactions in a pulsating dense matter. The reactions and the bulk viscosity itself depend sensitively on the composition of matter.

In the outermost part of the outer neutron star core composed mainly of neutrons n with admixture of protons p, electrons e, and possibly muons μ bulk viscosity is mainly determined by the reactions of non-equilibrium modified Urca process,

$$\mathrm{n}+\mathrm{N} \rightarrow \mathrm{N}+\mathrm{p}+\mathrm{l}+\bar{\nu}_\mathrm{l}, \quad \mathrm{p}+\mathrm{N}+\mathrm{l} \rightarrow \mathrm{n}+\mathrm{N}+\nu_\mathrm{l}, \qquad (1)$$

where N stands for a nucleon (n or p), l is an electron or a muon, and ν_l is an associated neutrino. The problem of damping neutron star pulsations via modified Urca process in npe matter was analyzed long ago by Finzi & Wolf (1968) (although the authors did not introduce the bulk viscosity explicitly). The bulk viscosity in npe matter was calculated by Sawyer (1989a) and in npeμ matter by Haensel et al. (2000, hereafter Paper II).

Deeper in the core, at densities ρ of a few ρ_0 ($\rho_0 = 2.8\times10^{14}$ g cm^{-3} is the saturated nuclear matter density), direct Urca process may be open (Lattimer et al. 1991)

$$\mathrm{n} \rightarrow \mathrm{p}+\mathrm{l}+\bar{\nu}_\mathrm{l}, \quad \mathrm{p}+\mathrm{l} \rightarrow \mathrm{n}+\nu_\mathrm{l}. \qquad (2)$$

It produces the bulk viscosity, which is typically 4–6 orders of magnitudes higher than that due to modified Urca process. This bulk viscosity was studied by Haensel & Schaeffer (1992) for npe matter and by Haensel et al. (2000, hereafter Paper I) for npeμ matter. Note that the idea of strong enhancement of vibrational dissipation in the neutron stars via a weak process similar to direct Urca (beta decay and capture by quasinucleons in npe matter containing pion condensate) was put forward by Wang & Lu (1984). All the studies cited above are focused on not too young neutron stars which are fully transparent to neutrinos. We will also restrict ourselves to this case.

At about the same densities, hyperons may appear in the neutron star cores (first of all, Σ^- and Λ hyperons, and then Ξ^0, Ξ^-, Σ^+). To be specific, we will mainly consider Σ^- and Λ hyperons. Once appeared, the hyperons may also initiate their own direct Urca processes (Prakash et al. 1992) giving additional contribution to the bulk viscosity, nearly as high as that due to nucleon direct Urca process (2). However, direct non-leptonic hyperon collisions which go via weak interaction (with strangeness non-conservation) such as

$$\mathrm{n}+\mathrm{n} \rightleftharpoons \mathrm{p}+\Sigma^-, \qquad (3)$$

$$\Lambda+\mathrm{N} \rightleftharpoons \mathrm{n}+\mathrm{N}, \qquad (4)$$

are much more efficient. They may increase the bulk viscosity by several orders of magnitude above the

Send offprint requests to: P. Haensel,
e-mail: haensel@camk.edu.pl

direct-Urca level. The effect was analyzed by Langer & Cameron (1969) and Jones (1971, 2001a, 2001b). Analogous effect in quark matter was studied by several authors. The enhancement of the vibrational dissipation via non-lepton strangeness-changing quark collisions was considered by Wang & Lu (1984). The appropriate bulk viscosity was calculated by Sawyer (1989b), Madsen (1992), Goyal et al. (1994), and Dai & Lu (1996).

Calculation of the bulk viscosity limited by non-leptonic processes in hyperon matter is a complicated problem. There are a number of processes of comparable efficiency. The matrix elements can easily be calculated in the approximation of bare particles and exact SU(3) symmetry, and appear to be nonzero for some processes, e.g., (3), but are zero for the others, e.g., (4). However, experimental data on the lifetime of Λ in massive hypernuclei indicate (e.g., Jones 2001b and references therein) that process (4) (with N = n) is nearly as efficient as "bare-particle" process (3). Calculation of the matrix elements for "dressed" particles is complicated and model dependent; additional complications arise – even in the in-vacuum case – due to the SU(3) symmetry breaking (Savage & Walden 1997).

Another complication is introduced by superfluidity of neutron-star matter. It is well known that neutrons, protons and other baryons may be superfluid due to attractive part of strong baryon-baryon interaction. Superfluidity of neutrons and protons has been studied in numerous papers (as reviewed, for instance, by Yakovlev et al. 1999; Lombardo & Schulze 2001). Hyperons can also be in superfluid state as discussed, e.g., by Balberg & Barnea (1998). Critical temperatures $T_{\rm c}$ of baryon superfluidities are very sensitive to the model of strong interaction and to many-body theory employed in microscopic calculations. Their typical values range from 10^8 to 10^{10} K. They are density dependent, and they mainly decrease with ρ at densities higher than several ρ_0.

The effects of superfluidity of nucleons on the bulk viscosity associated with direct and modified Urca processes in npeμ matter were considered in Papers I and II. It was shown that the superfluidity may drastically reduce the bulk viscosity and, hence, the damping of neutron star pulsations.

In this paper we propose a simple solvable model of the bulk viscosity in hyperonic matter (Sect. 2) due to process (3) and study (Sect. 3) the effects of possible superfluidity of n, p, and Σ^- on this bulk viscosity. In Sect. 4 we discuss density and temperature dependence of the bulk viscosity in non-superfluid and superfluid neutron star cores.

2. Bulk viscosity of non-superfluid matter

2.1. Model

Consider non-superfluid hyperonic stellar matter in the core of a neutron star pulsating with a typical frequency $\omega \sim 10^3 - 10^4$ s^{-1}. In the presence of hyperons the contribution of direct Urca and modified Urca processes, (2) and (1), into the bulk viscosity may be neglected. It is sufficient to include the non-leptonic weak-interaction processes (3) and (4). For the sake of simplicity, let us take into account process (3) alone although we assume that matter may contain not only Σ^- but other hyperons. The advantage of this model is that it can be solved analytically. We will compare it with other models in Sect. 2.4.

2.2. Matrix element of $\mathrm{n} + \mathrm{n} \to \mathrm{p} + \Sigma^-$

Let us start with the matrix element M in the "bare-particle" approximation. The process is described by two diagrams with the states of two neutrons interchanged. Accordingly, $M = M^{(\rm I)} + M^{(\rm II)}$, and ($\hbar = c = k_{\rm B} = 1$)

$$M^{(\rm I)} = \mathcal{A}\left[\bar{u}_{\rm p}\gamma_\lambda(1 + C\gamma_5)u_{\rm n}\right]\left[\bar{u}_\Sigma\gamma^\lambda(1 + C'\gamma_5)u_{\rm n'}\right],$$
$$M^{(\rm II)} = -\mathcal{A}\left[\bar{u}_{\rm p}\gamma_\lambda(1 + C\gamma_5)u_{\rm n'}\right]\left[\bar{u}_\Sigma\gamma^\lambda(1 + C'\gamma_5)u_{\rm n}\right]. \quad (5)$$

In this case u_i is a standard bispinor, $\bar{u}_i$ is its Dirac conjugate (i = n, n$'$, p, Σ; $\bar{u}_i u_i = 2m_i$, where m_i is a bare-particle mass), γ^λ is a Dirac's gamma-matrix, and

$$\mathcal{A} = -\frac{G_{\rm F}}{\sqrt{2}}\sin\theta_{\rm C}\cos\theta_{\rm C}. \quad (6)$$

Furthermore, $G_{\rm F} = 1.436 \times 10^{-49}$ erg cm^3 is the Fermi weak coupling constant; $\theta_{\rm C}$ is the Cabibbo angle ($\sin\theta_{\rm C} = 0.231$); $C = F + D$, $C' = F - D$, where $D \approx 0.756$ and $F \approx 0.477$ are the reduced symmetric and antisymmetric coupling constants (e.g., Prakash et al. 1992).

Using the standard technique in the limit of non-relativistic baryons we sum $|M|^2$ over particle spin states and obtain

$$\sum_{\rm spins}|M|^2 = 64\,|\mathcal{A}|^2\,\chi\,m_{\rm n}^2\,m_{\rm p}\,m_\Sigma, \quad \chi = (1 + 3\,CC')^2. \quad (7)$$

This expression coincides with that which can be deduced from the recent results of Jones (2001b). In the previous papers Jones (1971, 2001a) reported analogous expression but with $\chi' = (1 - 3\,CC')^2$ instead of χ. Numerically, replacing minus with plus makes a great difference due to almost total compensation of the terms in χ: $\chi \approx 0.001$ and $\chi' \approx 4.13$. Because of the strong compensation we cannot rely on the bare-particle approximation. Let us assume that a more evolved calculation based on dressed-particle technique will lead to the same Eq. (7) but with the value of χ renormalized by medium effects. Accordingly we will treat χ as a free parameter and, to be specific, we will set $\chi = 0.1$.

2.3. Non-equilibrium rate

Due to very frequent interparticle collisions, dense stellar matter almost instantaneously (on microscopic time scales) achieves a quasiequilibrium state with certain temperature T and chemical potentials μ_i of various particle species i. Relaxation to the full thermodynamic ("chemical") equilibrium lasts much longer since it realizes through much slower weak interaction processes.

In the case of process (3) the chemical equilibrium implies $2\mu_{\rm n} = \mu_{\rm p} + \mu_\Sigma$. In the chemical equilibrium the rates $[\,{\rm cm}^{-3}\ {\rm s}^{-1}]$ of the direct and inverse reactions of the process are balanced, $\Gamma = \bar\Gamma$. In a pulsating star, the chemical equilibrium is violated ($\Gamma \neq \bar\Gamma$) which can be described by the lag of *instantaneous* chemical potentials,

$$\eta = 2\mu_{\rm n} - \mu_{\rm p} - \mu_\Sigma. \qquad (8)$$

We adopt the standard assumption (e.g., Sawyer 1989a) that deviations from the chemical equilibrium are small, $|\eta| \ll T$. If so we can use the *linear approximation*

$$\Delta\Gamma \equiv \Gamma - \bar\Gamma = -\lambda\eta, \qquad (9)$$

where λ determines the bulk viscosity (Sect. 2.4). Our definition of λ is the same as in Sawyer (1989a). Thus defined, λ is negative.

Let us calculate the rate Γ of the direct reaction, ${\rm nn} \to {\rm p}\Sigma^-$, of the process. In the non-relativistic approximation we have ($\hbar = c = k_{\rm B} = 1$):

$$\Gamma = \int \frac{{\rm d}\boldsymbol{p}_{\rm n}}{2m_{\rm n}(2\pi)^3}\,\frac{{\rm d}\boldsymbol{p}'_{\rm n}}{2m_{\rm n}(2\pi)^3}\,\frac{{\rm d}\boldsymbol{p}_{\rm p}}{2m_{\rm p}(2\pi)^3}\,\frac{{\rm d}\boldsymbol{p}_\Sigma}{2m_\Sigma(2\pi)^3} \times \frac{1}{2}\sum_{\rm spins} |M|^2 f_{\rm n}\, f'_{\rm n}\,(1-f_{\rm p})(1-f_\Sigma)\,(2\pi)^4 \times \delta(\varepsilon_{\rm n} + \varepsilon'_{\rm n} - \varepsilon_{\rm p} - \varepsilon_\Sigma)\,\delta(\boldsymbol{p}_{\rm n} + \boldsymbol{p}'_{\rm n} - \boldsymbol{p}_{\rm p} - \boldsymbol{p}_\Sigma), \qquad (10)$$

where $\boldsymbol{p}_i$ is the particle momentum and ε_i is its energy. The symmetry factor $\frac{1}{2}$ before summation sign excludes double counting of the same collisions of identical neutrons; $f_i = \{1 + \exp[(\varepsilon_i - \mu_i)/T]\}^{-1}$ is the Fermi-Dirac function.

Evaluation of Γ is standard (e.g. Shapiro & Teukolsky 1983) and takes advantage of strong degeneracy of reacting particles in neutron star matter. The multidimensional integral is decomposed into the energy and angular integrals. All momenta $\boldsymbol{p}_i$ are placed on the appropriate Fermi spheres wherever possible. Introducing the dimensionless quantities

$$x_i = \frac{\varepsilon_i - \mu_i}{T}, \quad \xi = \frac{\eta}{T}, \qquad (11)$$

we can rewrite the reaction rate as $\Gamma = \Gamma^{(0)} I$, with

$$I = \left[\prod_{i=1}^{4} \int_{-\infty}^{+\infty} {\rm d}x_i\; f(x_i)\right] \delta\left(\sum_{i=1}^{4} x_i + \xi\right), \qquad (12)$$

where the blocking factors $(1 - f(x))$ are transformed into the Fermi-Dirac functions $f(x)$ by replacing integration variables $x \to -x$, and the typical reaction rate $\Gamma^{(0)}$ is defined as (in ordinary physical units)

$$\Gamma^{(0)} = \frac{4}{(2\pi)^5\,\hbar^{10}}\, G_{\rm F}^2\, \sin^2\theta_{\rm C}\, \cos^2\theta_{\rm C}\, \chi \times m_{\rm n}^{*2}\, m_{\rm p}^*\, m_\Sigma^*\, p_{{\rm F}\Sigma}\, k_{\rm B}^3\, T^3 \approx 2.15 \times 10^{38} \left(\frac{m_{\rm n}^*}{m_{\rm n}}\right)^2 \times \frac{m_{\rm p}^*\, m_\Sigma^*}{m_{\rm p}\, m_\Sigma} \left(\frac{n_\Sigma}{1\,{\rm fm}^{-3}}\right)^{1/3} T_9^3\, \chi\ {\rm cm}^{-3}\ {\rm s}^{-1}. \qquad (13)$$

In this case m_i^* is an effective baryon mass in dense matter, $p_{{\rm F}\Sigma}$ is the Fermi momentum of Σ^- hyperons, n_Σ is their number density, $T_9 = T/(10^9\,{\rm K})$. Note that in Eq. (13) we have used the angular integral calculated under the assumptions $p_{{\rm F}\Sigma} + p_{\rm Fp} < 2p_{\rm Fn}$ and $p_{{\rm F}\Sigma} < p_{\rm Fp}$ which are usually fulfilled in hyperonic matter ($p_{{\rm F}i}$ being Fermi momentum of particle species i).

The integral I, Eq. (12), is:

$$I = \frac{{\rm e}^\xi}{{\rm e}^\xi - 1}\,\frac{\xi}{6}\,\left(4\pi^2 + \xi^2\right). \qquad (14)$$

The rate $\bar\Gamma = \Gamma^{(0)}\bar I$ of the inverse reaction, $\Sigma^-{\rm p} \to {\rm nn}$, is obtained from Γ, Eqs. (10) and (12), by replacing $\xi \to -\xi$. Then for $|\xi| \ll 1$

$$\Delta\Gamma = \Gamma^{(0)}\,\Delta I, \quad \Delta I = \frac{2\pi^2}{3}\,\xi. \qquad (15)$$

Finally, from Eqs. (9) and (15) we obtain

$$|\lambda| = \frac{\Gamma^{(0)}}{k_{\rm B}T}\,\frac{\Delta I}{\xi}\cdot \qquad (16)$$

In non-superfluid matter $|\lambda_0| = 2\pi^2\Gamma^{(0)}/(3k_{\rm B}T)$.

2.4. Bulk viscosity

The bulk viscosity ζ_Σ due to the hyperon process (3) is calculated in analogy with that due to the modified or direct Urca process (Sawyer 1989a; Haensel & Schaeffer 1992). The result is

$$\zeta_\Sigma = \frac{C^2 n_{\rm b}^2}{|\lambda| B^2}\,\frac{1}{1+a^2}, \quad a \equiv \frac{\omega n_{\rm b}}{|\lambda| B}, \qquad (17)$$

where $n_{\rm b}$ is the number density of baryons, and

$$B = \frac{\partial\eta}{\partial X_\Sigma}, \quad C = n_{\rm b}\frac{\partial\eta}{\partial n_{\rm b}} = -\frac{1}{n_{\rm b}}\,\frac{\partial P}{\partial X_\Sigma}\cdot \qquad (18)$$

In this case P is the pressure and $X_\Sigma = n_\Sigma/n_{\rm b}$ is the fraction of Σ^- hyperons. The quantities B and C can be calculated numerically for a given equation of state.

The bulk viscosity depends on the frequency ω of neutron star pulsations. Using the results of Sect. 2.3 the dynamical parameter a can be written as

$$a \approx 6.09 \left(\frac{m_{\rm n}}{m_{\rm n}^*}\right)^2 \frac{m_{\rm p}}{m_{\rm p}^*}\,\frac{m_\Sigma}{m_\Sigma^*}\,\frac{\omega_4}{T_9^2\,\chi} \times \left(\frac{100\ {\rm MeV}}{B}\right)\left(\frac{n_{\rm b}}{1\,{\rm fm}^{-3}}\right)\left(\frac{1\,{\rm fm}^{-3}}{n_\Sigma}\right)^{1/3}, \qquad (19)$$

where $\omega_4 = \omega/(10^4\ {\rm s}^{-1})$. For typical values $T \sim 10^8$–10^9 K, $\omega_4 \sim 1$, $n_{\rm b} \sim 1\ {\rm fm}^{-3}$, $n_\Sigma \ll n_{\rm b}$, $m_i^* \sim 0.7\ m_i$, $B \sim 100$ MeV, $\chi \sim 0.1$ we have $a \gg 1$.Then we may use the *high-frequency limit* in which ζ_Σ is independent of B and inversely proportional to ω^2:

$$\zeta_\Sigma = \frac{2}{3\,(2\pi)^3}\,\frac{G_{\rm F}^2\, m_{\rm n}^{*2}\, m_{\rm p}^*\, m_\Sigma^*\, C^2 \chi\, p_{\rm F\Sigma}}{\hbar^{10}\omega^2} \times \sin^2\theta_{\rm C}\, \cos^2\theta_{\rm C}\, (k_{\rm B}T)^2$$
$$\approx 2.63\times 10^{30}\, T_9^2\, \omega_4^{-2}\, \chi \left(\frac{n_\Sigma}{1\,{\rm fm}^{-3}}\right)^{1/3} \times \left(\frac{m_{\rm n}^*}{m_{\rm n}}\right)^2 \frac{m_{\rm p}^*}{m_{\rm p}}\,\frac{m_\Sigma^*}{m_\Sigma}\left(\frac{C}{100\,{\rm MeV}}\right)^2 \ {\rm g\ cm^{-1}\ s^{-1}}. \quad (20)$$

If, due to interplay of parameters, $a \lesssim 1$ one can use more general Eq. (17). For instance, we would have $a \lesssim 1$ for the same parameters as above but at higher temperatures, $T \gtrsim 10^{10}$ K (Sect. 4). We could have $a \lesssim 1$ even below $\sim 10^{10}$ K if the phenomenological constant χ is higher than the adopted value $\chi = 0.1$.

In the absence of hyperons the bulk viscosity is determined by direct or modified Urca processes (Sect. 1). These processes are much slower than hyperonic ones. They can certainly be described in the high-frequency approximation in which partial bulk viscosities due to various processes are summed together into the total bulk viscosity (e.g., Papers I and II). Thus we will add contributions from direct and modified Urca processes whenever necessary in our numerical examples in Sect. 4.

Note that all the studies of bulk viscosity of hyperonic matter performed so far are approximate. The subject was introduced by Langer & Cameron (1969) who estimated dumping of neutron star vibrations but did not calculate the bulk viscosity itself. Jones (1971, 2001a) calculated effective Σ^- hyperon relaxation times and estimated the bulk viscosity but did not evaluate it exactly for any selected model of dense matter. Recently Jones (2001b) analyzed the bulk viscosity of hyperonic matter taking into account a number of hyperonic processes but also restricted himself to the order-of-magnitude estimates.

Our approach is also simplified since we take into account the only one hyperonic process (3) and neglect the others. Even in this case we are forced to introduce the phenomenological parameter χ (Sect. 2.2) to describe the reaction rate. The advantage of our model is that, once this parameter is specified, we can easily calculate the bulk viscosity (as illustrated in Sect. 4) and introduce the effects of superfluidity (Sects. 3 and 4). Technically, it would be easy to incorporate the contribution of process (4) as well as of other hyperonic processes (Sect. 1). However, for any new process we need its own phenomenological parameter (similar to χ) which is currently unknown. Generally, in the presence of several hyperonic processes, the bulk viscosity cannot be described by a simple analytical expression analogous to Eq. (17). Nevertheless, in the high-frequency limit the contributions from different processes are additive and it will be sufficient to add new contributions to that given by Eq. (20). Thus we prefer to use our simplified model rather than extend it introducing large uncertainties.

3. Bulk viscosity of superfluid matter

3.1. Baryon pairing in dense matter

Now consider the effects of baryon superfluidity on the bulk viscosity associated with process (3). According to microscopic theories (reviewed, e.g., by Yakovlev et al. 1999 and Lombardo & Schulze 2001) at supranuclear densities (at which hyperons appear in dense matter) neutrons may undergo triplet-state (^{3}P$_2$) Cooper pairing while protons may undergo singlet-state (^{1}S$_0$) pairing. As discussed in Sect. 1 microscopic calculations of the nucleon gaps (critical temperatures) are very model dependent. Current knowledge of hyperon interaction in dense matter is poor and therefore microscopic theory of hyperon pairing is even much more uncertain. Since the number density of hyperons is typically not too large it is possible to expect that such a pairing, if available, is produced by singlet-state hyperon interaction. Some authors (e.g., Balberg & Barnea 1998) calculated singlet-state gaps for Λ hyperons. We assume also singlet-state pairing of Σ^- hyperons and consider the bulk viscosity of matter in which n, p and Σ^- may form three superfluids. Since the critical temperatures $T_{\rm cn}$, $T_{\rm cp}$ and $T_{\rm c\Sigma}$ are uncertain we will treat these temperatures as arbitrary parameters.

Microscopically, superfluidity introduces a gap δ into momentum dependence of the baryon energy, $\varepsilon(\boldsymbol{p})$. Near the Fermi surface ($|p-p_{\rm F}| \ll p_{\rm F}$) we have

$$\varepsilon = \mu - \sqrt{\delta^2 + v_{\rm F}^2\,(p-p_{\rm F})^2} \quad \text{at } p < p_{\rm F},$$
$$\varepsilon = \mu + \sqrt{\delta^2 + v_{\rm F}^2 (p-p_{\rm F})^2} \quad \text{at } p \geq p_{\rm F}, \quad (21)$$

where $v_{\rm F}$ is the Fermi velocity. The gap δ is isotropic (independent of orientation of $\boldsymbol{p}$ with respect to the spin quantization axis) for singlet-state pairing but anisotropic for triplet-state pairing. Strict calculation of the bulk viscosity with anisotropic gap is complicated. We will adopt an approximate treatment of triplet-state pairing (with zero projection of total angular momentum of Cooper pairs onto the spin quantization axis) proposed by Baiko et al. (2001) for calculating diffusive thermal conductivity of neutrons. In this approximation the gap is artificially considered as isotropic in microscopic calculations but in the final expressions it is related to temperature in the same way as the minimum value of the anisotropic gap on the Fermi surface.

It is convenient to introduce the dimensionless quantities

$$\tau = \frac{T}{T_{\rm c}}, \quad y = \frac{\delta(T)}{T}, \quad z = {\rm sign}(x)\,\sqrt{x^2+y^2}. \quad (22)$$

For the singlet-state pairing (case A in notations of Yakovlev et al. 1999) the dependence of y on τ can be fitted as

$$y_{\rm A} = \sqrt{1-\tau}\left(1.456 - \frac{0.157}{\sqrt{\tau}} + \frac{1.764}{\tau}\right), \quad (23)$$

while for the triplet-state pairing (case B)

$$y_{\rm B} = \sqrt{1-\tau}\,\left(0.7893 + \frac{1.188}{\tau}\right)\cdot \qquad (24)$$

3.2. Superfluid reduction factors

We consider the effects of superfluidity on the bulk viscosity in the same manner as in Papers I and II and omit technical details described in these papers. Following Papers I and II we assume that all constituents of matter participate in stellar pulsations with the same macroscopic velocity (as in the first-sound waves). Then the damping of pulsations is described by one coefficient of bulk viscosity ζ. The effects of superfluidity are included by introducing superfluid gaps into the reaction rates, Γ and $\bar{\Gamma}$, Eq. (10), through the dispersion relations, Eq. (21). These effects influence mainly the only parameter λ in Eq. (17). Quite generally, we can write

$$\lambda = \lambda_0\, R, \qquad (25)$$

where λ_0 refers to non-superfluid matter, Eq. (16), and R is a factor which describes the superfluid effects. The latter factor depends on the three parameters, $R = R(y_{\rm n},\, y_{\rm p},\, y_\Sigma)$, which are the dimensionless gaps of neutrons, protons, and Σ^- hyperons. Obviously, $R=1$ if all these baryons are normal ($y_{\rm n}=y_{\rm p}=y_\Sigma=0$). Calculations show that one always has $R<1$ in the presence of at least one superfluidity.

Using Eqs. (17) and (25) we can write the hyperon bulk viscosity in superfluid matter in the form

$$\zeta_\Sigma = \frac{C^2 n_{\rm b}^2}{|\lambda_0| B^2 R}\,\frac{1}{1+a^2}, \quad a = \frac{\omega n_{\rm b}}{|\lambda| B} = \frac{a_0}{R}, \qquad (26)$$

where a_0 is the non-superfluid value of a given by Eq. (19).

In the high-frequency limit (Sect. 2.4), which is often realized in neutron star matter, we have $\zeta_\Sigma \propto \lambda$, i.e.,

$$\zeta_\Sigma = \zeta_0\, R, \qquad (27)$$

where ζ_0 is the bulk viscosity of non-superfluid matter, Eq. (20). Accordingly, superfluidity *suppresses* the high-frequency bulk viscosity. On the contrary, it *enhances* the static ($\omega=0$) bulk viscosity $\zeta_\Sigma \propto 1/\lambda \propto 1/R$. Moreover, superfluidity increases the dynamical factor a and widens thus the range of plasma parameters where the bulk viscosity operates in the high-frequency regime.

Under our assumptions superfluidity modifies only the integral ΔI in the factor λ given by Eq. (16). To generalize ΔI to the superfluid case it is sufficient to replace $x_i \to z_i$ in the all functions under the integral in Eq. (12). Then R can be written as

$$R = \frac{\Delta I}{\Delta I_0} = \frac{3}{\pi^2}\frac{\partial}{\partial \xi}\left[\prod_{i=1}^{4}\int {\rm d}x_i\, f(z_i)\right]\delta\left(\sum_{i=1}^{4} z_i + \xi\right) \qquad (28)$$

in the limit of $\xi \to 0$. Here ΔI_0 is the value of ΔI calculated for normal matter, Eq. (15).

We have composed a code which calculates R numerically in the presence of all three superfluids. The results will be presented in Sect. 4. Here we mention some limiting cases in which evaluation of R is simplified.

3.3. Superfluidity of protons or Σ^- hyperons

The cases in which either protons or Σ^- hyperons are superfluid are similar. Let, for example, neutrons and Σ^- be normal while protons undergo $^1{\rm S}_0$ Cooper pairing. Accordingly, $R=R_{\rm p}$ depends on the only parameter $y = y_{\rm Ap}$. For a strong superfluidity ($\tau = T/T_{\rm cp} \ll 1$, $y \gg 1$) the asymptote is

$$R_{\rm p} = \frac{3}{\pi^2}\sqrt{\frac{\pi y}{2}}\,\left(\frac{y^2}{2} + \frac{y}{2} + \frac{\pi^2}{6}\right)\,{\rm e}^{-y}. \qquad (29)$$

We have calculated $R_{\rm p}$ in a wide range of y and proposed the fit to the numerical data (with the maximum error $\lesssim$0.5%) which reproduces also the leading term of the asymptote, Eq. (29):

$$R_{\rm p} = \frac{a^{5/4} + b^{1/2}}{2}\,\exp\left(0.5068 - \sqrt{0.5068^2 + y^2}\right), \qquad (30)$$

where $a = 1 + 0.3118\, y^2$ and $b = 1 + 2.556\, y^2$. If Σ^- hyperons are superfluid instead of protons, the expressions for R are the same but $y = y_{\rm A\Sigma}$.

3.4. Superfluidity of protons and Σ^- hyperons

If neutrons are normal but protons and Σ^- hyperons are superfluid $R = R_{\rm p\Sigma}(y_{\rm p}, y_\Sigma)$ depends on $y_{\rm p} = y_{\rm Ap}$ and $y_\Sigma = y_{\rm A\Sigma}$. We have determined the asymptote of $R_{\rm p\Sigma}$ at large $y_{\rm p}$ and y_Σ. Let Y be the larger gap, $Y = \max\{y_\Sigma, y_{\rm p}\}$, and $y_0 = \min\{y_\Sigma, y_{\rm p}\}$. At $Y - y_0 \gg \sqrt{Y} \gg 1$ the asymptote reads

$$R_{\rm p\Sigma} = \frac{3}{\pi^2}\sqrt{\frac{\pi Y}{2}}\,{\rm e}^{-Y}\left[\frac{Y+1}{2}\sqrt{Y^2 - y_0^2} - \frac{y_0^2}{2}\ln\left(\frac{Y + \sqrt{Y^2 - y_0^2}}{y_0}\right)\right]. \qquad (31)$$

If $y_0 \to 0$ then Eq. (31) reproduces the leading term of the asymptote (29). To prove this one should consider Eq. (31) at $1 \ll \sqrt{Y} \ll (Y - y_0) \ll Y$ and expand the logarithm in Eq. (31) in powers of $\sqrt{Y^2 - y_0^2}/Y \ll 1$.

Equation (31) becomes invalid at $y_0 \to Y$. In this case $R_{\rm p\Sigma}(y_{\rm p}, y_\Sigma) \approx R_{\rm n}(Y)$, where $R_{\rm n}$ is described below.

3.5. Superfluidity of neutrons

Now let neutrons be superfluid while protons and Σ^- hyperons not. For a strong superfluidity ($\tau = T/T_{\rm cn} \ll 1$, $y = y_{\rm Bn} \gg 1$) we get

$$R_{\rm n} = \frac{6\,\gamma\, y}{\pi^2}\,{\rm e}^{-y}, \quad \gamma = \int_0^\infty\!\!\int_0^\infty \frac{{\rm d}q\,{\rm d}q'\,(q'^2 - q^2)}{{\rm e}^{\,q'^2} - {\rm e}^{\,q^2}} = 1.413. \qquad (32)$$

We have calculated $R_{\rm n}$ numerically in a wide range of y and proposed the fit (with the maximum error ∼0.2%):

$$R_{\rm n} = \left(0.6192 + \sqrt{0.3808^2 + 0.1561\, y^2}\right) \times \exp\left(0.7756 - \sqrt{0.7756^2 + y^2}\right) + 0.18766\, y^2 \times \exp\left(1.7755 - \sqrt{1.7755^2 + 4y^2}\right). \quad (33)$$

4. Results and discussion

4.1. Non-superfluid matter

For illustration, we use the equation of state of matter in the neutron star core, proposed by Glendenning (1985) in the frame of relativistic mean field theory. Specifically, we adopt case 3 considered by Glendenning in which the appearance of n, p, e, μ, Σ^-, and Λ is allowed. (Note a misprint: numerical values of the parameters b and c of the Glendenning (1985) model should be replaced as $b \to b/3$ and $c \to c/4$.) In this model, muons appear at the baryon number density $n_{\rm b} = 0.110$ fm^{-3} (at $\rho = 1.86 \times 10^{14}$ g cm^{-3}); Λ hyperons appear at $n_{\rm b} = 0.310$ fm^{-3} ($\rho = 5.51 \times 10^{14}$ g cm^{-3}); and Σ^- hyperons appear at $n_{\rm b} = 0.319$ fm^{-3} ($\rho = 5.69 \times 10^{14}$ g cm^{-3}). The density dependence of the fractions of various particles is shown in Fig. 9 of Glendenning (1985). Let us remind that saturation density of nuclear matter $\rho_0 \approx 2.8 \times 10^{14}$ g cm^{-3} corresponds to $n_{\rm b0} \approx 0.16$ fm^{-3}.

Figure 1 shows the partial bulk viscosities and the total bulk viscosity versus $n_{\rm b}$ at $T = 10^9$ K for stellar vibration frequency $\omega = 10^4$ s^{-1}. One can see three density intervals where the bulk viscosity is drastically different.

At *low densities*, $n_{\rm b} < 0.227$ fm^{-3}, the bulk viscosity is determined by modified Urca processes (Paper II). For $n_{\rm b} < 0.110$ fm^{-3} it is produced by neutron and proton branches of Urca process involving electrons (processes (1) with N = n or p and with l = e). At higher $n_{\rm b}$ muons are created and muonic modified Urca processes (1) (again with N = n or p but now with l = μ) introduce comparable contribution. Note that Eq. (29) of Paper II for the angular integral $A_{\rm pl0}$ of the proton branch of modified Urca process is actually valid at not too high densities, as long as $p_{\rm Fn} > 3p_{\rm Fp} - p_{\rm Fl}$. For higher densities, it is replaced with

$$A_{\rm pl0} = \frac{(4\pi)^5}{4p_{\rm Fp}^2} \left(\frac{3}{p_{\rm Fn}} - \frac{1}{p_{\rm Fp}}\right), \quad (34)$$

which was neglected in Paper II (this replacement has no noticeable effect on the values of bulk viscosity).

At *intermediate densities* (0.227 fm^{-3} $< n_{\rm b} <$ 0.319 fm^{-3}) the main contribution into the bulk viscosity comes from direct Urca processes (Paper I). As long as $n_{\rm b} < 0.293$ fm^{-3} the only one direct Urca process (2) operates with l = e while at higher $n_{\rm b}$ the other one with l = μ is switched on; it makes comparable contribution.

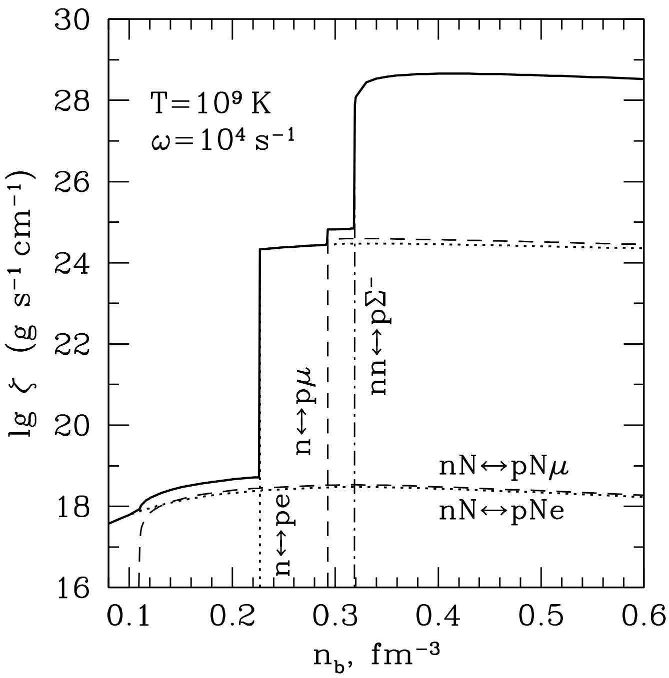

Fig. 1. Density dependence of partial bulk viscosities associated with various processes (indicated near the curves) at $T = 10^9$ K and $\omega = 10^4$ s^{-1} in non-superfluid matter. Dotted and dashed lines refer to Urca processes involving electrons and muons, respectively; dot-and-dashed line refers to hyperon process (3). Thick solid line is the total bulk viscosity.

We see that direct Urca processes at intermediate densities amplify the bulk viscosity by more than five orders of magnitude as compared to the low-density case.

Finally, at *high densities* ($n_{\rm b} > 0.319$ fm^{-3}), according to the results of Sect. 2, the bulk viscosity increases further by about four orders of magnitude under the action of non-leptonic process (3) involving Σ^- hyperons. These values of the bulk viscosity are in qualitative agreement with those reported by Jones (2001b). If our model of bulk viscosity were more developed and incorporated the contributions of process (4) involving Λ hyperons then the high-density regime would start to operate at somewhat earlier density, at the Λ hyperon threshold, $n_{\rm b} = 0.310$ fm^{-3}. The associated bulk viscosity is expected to be of nearly the same order of magnitude as produced by Σ^- hyperons (Jones 2001b). Actually, in the presence of hyperons, some contribution into the bulk viscosity comes from modified and direct Urca processes involving hyperons (e.g., Prakash et al. 1992; Yakovlev et al. 2001). This contribution is not shown in Fig. 1. It is expected to be smaller than the contributions from nucleon modified and direct Urca processes (1) and (2) displayed in the figure.

Figure 1 refers to one value of temperature, $T = 10^9$ K, and one value of the vibration frequency, $\omega = 10^4$ s^{-1}. Nevertheless one can easily rescale ζ to other T and ω in non-superfluid matter in thehigh-frequency regime. For

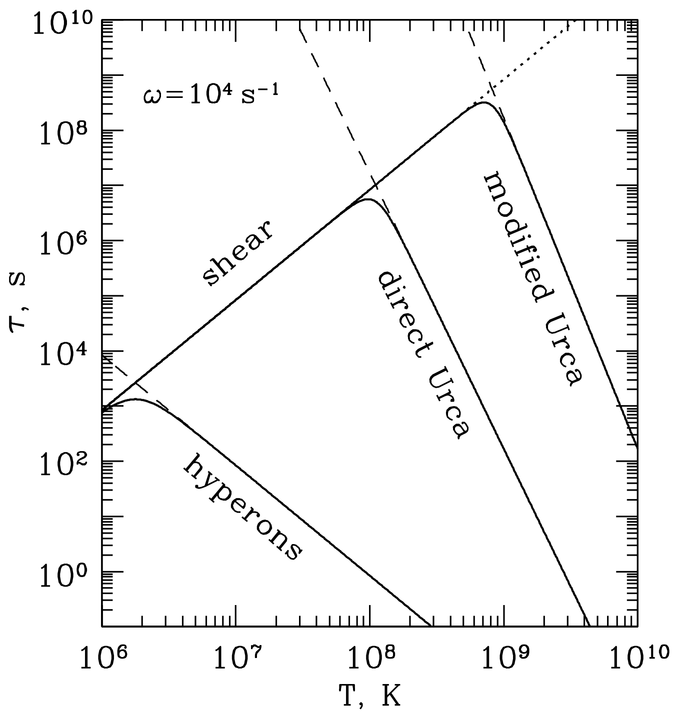

Fig. 2. Schematic representation of temperature dependence of viscous relaxation time scale τ in non-superfluid neutron star cores with different compositions of matter at stellar vibration frequency $\omega = 10^4$ s^{-1}. Three dashed lines show the relaxation due to high-frequency bulk viscosity associated either with modified Urca processes, or with direct Urca processes, or with hyperonic processes. The dotted line presents the relaxation due to shear viscosity. Solid lines refer to the total (bulk+shear) viscous relaxation for the three regimes.

the modified Urca processes (M), direct Urca processes (D), and hyperonic process (Σ) we obtain the estimates:

$$\zeta_{\rm M} \sim \frac{5\times 10^{18}\, T_9^6}{\omega_4^2}, \qquad \zeta_{\rm D} \sim \frac{5\times 10^{24}\, T_9^4}{\omega_4^2},$$
$$\zeta_\Sigma \sim \frac{10^{30}\, T_9^2\, \chi}{\omega_4^2} \;\; {\rm g\ cm^{-1}\ s^{-1}}. \qquad (35)$$

The difference in magnitudes and temperature dependence of ζ comes evidently from the difference of corresponding reaction rates. It can be explained by different momentum space restrictions (different numbers of particles, absence or presence of neutrinos) in these reactions (e.g., Yakovlev et al. 2001).

Now we can estimate viscous dissipation time scales τ of neutron star vibrations. A standard estimate based on hydrodynamic momentum-diffusion equation yields $\tau \sim \rho\mathcal{R}^2/\zeta$, where $\mathcal{R} \sim 10$ km is a radius of the neutron star core, and ρ is a typical density. For the leading processes of three types in the high-frequency regime we have

$$\tau_{\rm M} \sim \frac{10\,\omega_4^2}{T_9^6}\ {\rm yrs}, \quad \tau_{\rm D} \sim \frac{100\,\omega_4^2}{T_9^4}\ {\rm s}, \quad \tau_\Sigma \sim \frac{0.001\,\omega_4^2}{T_9^2\,\chi}\ {\rm s}. \qquad (36)$$

Schematic representation of the temperature dependence of these time scales is shown in Fig. 2 by dashed lines. Sharp difference of the dissipation time scales comes from different magnitudes of bulk viscosities in various processes. In particular, the presence of hyperons in the non-superfluid neutron star core results in a very rapid viscous dissipation of stellar pulsations (Langer & Cameron 1969; Jones 1971, 2001a, 2001b).

Great difference of possible bulk-viscosity scales is in striking contrast with the shear viscosity limited by interparticle collisions. The shear viscosity η should be rather insensitive to composition of matter being of the same order of magnitude as in *npe* matter (Flowers & Itoh 1979), i.e., $\eta \sim 10^{18} T_9^{-2}$ g cm^{-1} s^{-1}. It is independent of the pulsation frequency ω. The damping time of stellar pulsations via shear viscosity in a non-superfluid stellar core is $\tau_{\rm shear} \sim 10\, T_9^2$ yrs. It is shown in Fig. 2 by the dotted line. This damping dominates at low T while the damping by bulk viscosity dominates at higher T. The total viscous damping time τ ($\tau^{-1} \sim \tau_{\rm bulk}^{-1} + \tau_{\rm shear}^{-1}$) is displayed in Fig. 2 by the solid lines (for the three high-frequency bulk-viscosity damping regimes). One can easily show that damping by bulk viscosity associated with modified Urca processes dominates at $T \gtrsim 10^9\, \omega_4^{1/4}$ K. For direct Urca processes it dominates at $T \gtrsim 10^8\, \omega_4^{1/3}$ K, and for hyperonic processes at $T \gtrsim 3\times 10^6\, \omega_4^{1/2}$ K.

Finally, let us mention the validity of high-frequency bulk viscosity regime. As follows from Eq. (17) it is valid as long as $a \gtrsim 1$, i.e., $\omega \gtrsim \omega_{\rm c}$, where the threshold frequency $\omega_{\rm c} \sim |\lambda| B/n_{\rm b}$. From Eq. (19) for the hyperon bulk viscosity we have $\omega_{\rm c}^\Sigma \sim 500\,\chi\, T_9^2$ s^{-1}. Using the results of Papers I and II we obtain $\omega_{\rm c}^{\rm M} \sim 5\times 10^{-9}\, T_9^6$ s^{-1} and $\omega_{\rm c}^{\rm D} \sim 5\times 10^{-3}\, T_9^4$ s^{-1} for modified and direct Urca processes. Therefore, we always have the high-frequency regime for modified and direct Urca processes at typical temperatures $T \lesssim 10^{10}$ K and pulsation frequencies $\omega \sim 10^4$ s^{-1}. The same is true for hyperon bulk viscosity excluding possibly the case of very hot plasma, $T \sim 10^{10}$ K. Notice that in the low-frequency (static) limit $\zeta \propto 1/|\lambda|$ and the temperature dependence of the bulk viscosity is inverted with respect to the high-frequency case.

4.2. Superfluid reduction

As discussed in detail in Papers I and II superfluidity of nucleons can strongly suppress the bulk viscosity produced by direct and modified Urca processes. Now let us use the results of Sect. 3 and illustrate superfluid suppression of hyperon bulk viscosity.

Figure 3 shows this suppression at $n_{\rm b} = 0.48$ fm^{-3} and $\omega = 10^4$ s^{-1}. We present partial bulk viscosities produced by hyperonic processes, as well as by direct and modified Urca processes. The straight dot-and-dashed lines are the partial bulk viscosities in non-superfluid matter. The striking difference of these bulk viscosities is discussed in Sect. 4.1. Solid and dashed lines show partial bulk viscosities in matter with superfluid protons ($\lg T_{\rm cp}[{\rm K}] = 9.3$) and neutrons ($\lg T_{\rm cn} = 8.5$). At $T \gtrsim 10^{10}$ K the high-frequency approximation for the hyperon bulk viscosity is violated. One can see the tendency of inversion of the

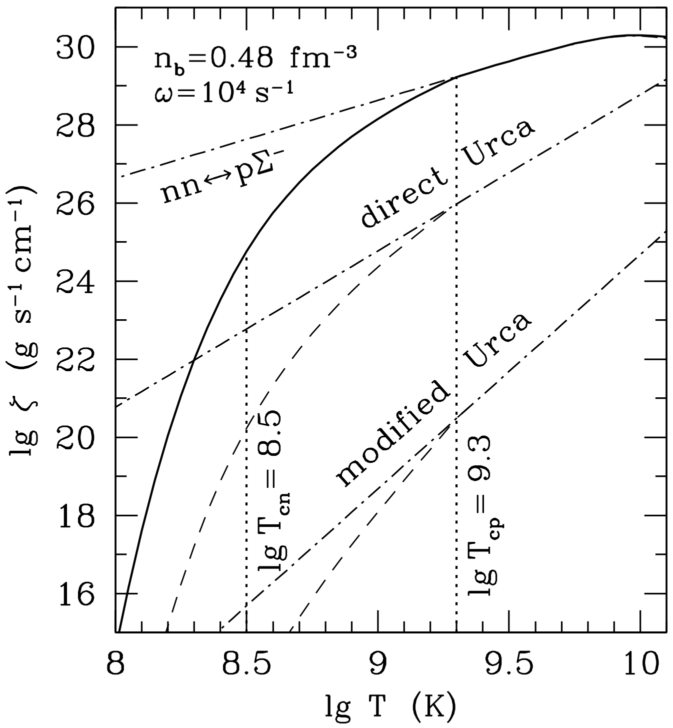

Fig. 3. Temperature dependence of bulk viscosity at n_b = 0.48 fm^{-3} and $\omega = 10^4$ s^{-1} in the presence of proton superfluidity with $\lg T_{cp} = 9.3$ and neutron superfluidity with $\lg T_{cn} = 8.5$. Dot-and-dashed lines (from up to down): partial bulk viscosities due to hyperonic, direct Urca and modified Urca processes, respectively, in non-superfluid matter. Associated solid and dashed lines: the same bulk viscosities in superfluid matter. Vertical dotted lines show $\lg T_{cn}$ and $\lg T_{cp}$.

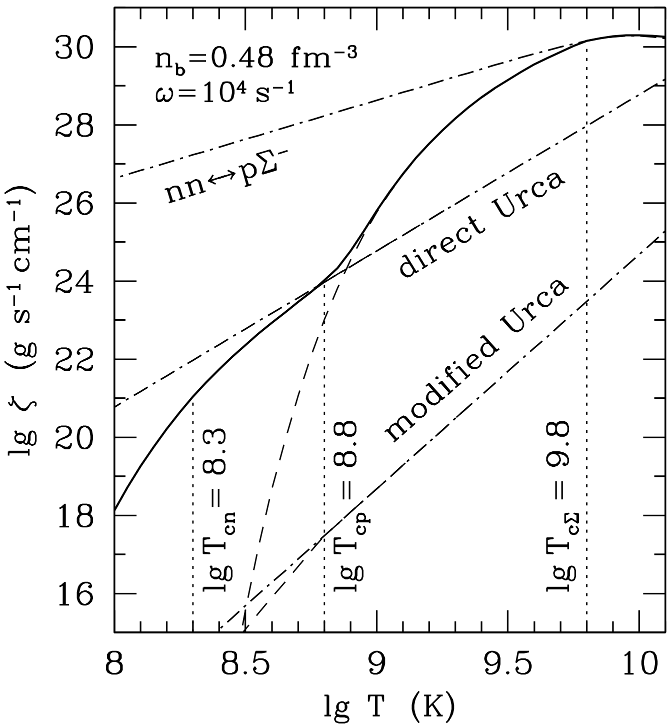

Fig. 4. Same as in Fig. 2 but in the presence of proton, neutron and Σ^- superfluids with $\lg T_{cp} = 8.8$, $\lg T_{cn} = 8.3$ and $\lg T_{c\Sigma} = 9.8$.

temperature dependence of ζ at $T \sim 10^{10}$ K associated with the transition to the low-frequency regime (Sect. 4.1). At $T < T_{cp}$ superfluidity reduces all partial bulk viscosities. In the temperature range $T_{cn} < T < T_{cp}$, where protons are superfluid alone, all the three partial bulk viscosities are suppressed in about the same manner. This is natural (e.g., Yakovlev et al. 1999) since the reactions responsible for the partial bulk viscosities contain the same number of superfluid particles (one proton). Indeed, there is one proton in hyperonic reaction (3) and direct Urca reaction (2), as well as in the neutron branch N = n of modified Urca reactions (1). At lower temperatures, $T < T_{cn}$, where neutrons become superfluid in addition to protons, the suppression is naturally stronger and becomes qualitatively different for different partial bulk viscosities since the leading reactions involve different numbers of neutrons. Evidently, the suppression is stronger for larger number of superfluid particles (as well as for higher critical temperatures T_c).

Figure 4 exhibits the same temperature dependence of the partial bulk viscosities, as Fig. 3, but in the presence of superfluidity of n, p, and Σ^- ($\lg T_{cp} = 8.8$, $\lg T_{c\Sigma} = 9.8$, $\lg T_{cn} = 8.3$). One can see that superfluidity of Σ^- hyperons strongly reduces the partial bulk viscosity associated with hyperonic process. As a result, at $T \lesssim 10^9$ K the total bulk viscosity is determined by direct Urca processes. In this regime one should generally take into account the contribution from direct Urca processes with hyperons (Sects. 1, 4.1). However, under the conditions displayed in Fig. 4 this contribution can be neglected.

Therefore, sufficiently strong superfluidity of baryons may reduce the *high-frequency* bulk viscosity by many orders of magnitude. This reduction will suppress very efficient viscous damping of neutron star pulsations in the presence of hyperons (Sect. 4.1). Accordingly, tuning critical temperatures T_c of different baryon species one can obtain drastically different viscous relaxation times.

Note that relaxation in superfluid neutron star cores may also be produced by a specific mechanism of *mutual friction* (e.g., Alpar et al. 1984; Lindblom & Mendell 2000, and references therein). If the neutron star core is composed of n, p, e (and possibly μ), this mechanism requires superfluidity of neutrons and protons, as well as rapid stellar rotation. The fact that the (conserved) particle currents are, in the case of a mixture of superfluids, not simply proportional to the superfluid velocities, implies non-dissipative *drag* (called also *entrainment*) of protons by neutrons. Dissipation (mutual friction) is caused by the scattering of electrons (and muons) off the magnetic field induced by proton drag within the neutron vortices. The relaxation (damping) time associated with mutual friction, τ_{mf}, depends on the type of stellar pulsations and the physical conditions within the superfluid neutron star core, in particular – on the poorly known superfluid drag coefficient. Its typical value τ varies from $\sim$1 s to $\sim$10^4 s.

One can expect that similar mechanisms may operate in the superfluid hyperon core of a rapidly rotating neutron star. If so these mechanisms will produce efficient damping of stellar pulsations. Note, however, that theoretical description of mutual friction is complicated and contains many uncertainties.

5. Conclusions

We have proposed a simple solvable model (Sect. 2) of the bulk viscosity of hyperonic matter in the neutron star cores as produced by process (3) involving Σ^- hyperons. We have analyzed (Sect. 3) the hyperonic bulk viscosity in the presence of superfluids of neutrons, protons, and Σ^- hyperons. We have presented illustrative examples (Sect. 4) of the bulk viscosity in non-superfluid and superfluid neutron star cores using the equation of state of matter proposed by Glendenning (1985). In particular, we emphasized the existence of three distinct layers of the core (outer, intermediate and inner ones), where the bulk viscosity in non-superfluid matter is very different (in agreement with the earlier results of Jones 1971, 2001a, 2001b). This leads to very different viscous damping times of neutron star vibrations for different neutron star models (the presence or absence of hyperons; the presence or absence of direct Urca process). If we used another equation of state of hyperonic matter the threshold densities n_b of the appearance of muons and hyperons, and the fractions of various particles would be different but the principal conclusions would remain the same. As seen from the results of this paper and Papers I and II, the high-frequency bulk viscosities in all three layers may be strongly reduced by superfluidity of baryons. A strong superfluidity may smear out large difference of the bulk viscosities in different layers. In addition, it relaxes the conditions of the high-frequency regime.

Our consideration of the bulk viscosity in hyperonic matter is approximate since we include only one hyperonic process (3) (Sects. 1, 2.1, 2.4) characterized by one phenomenological constant χ. It would be interesting to undertake microscopic calculations of χ. Analogous problem of quenching the axial-vector constant of weak interaction in dense matter has been considered recently by Carter & Prakash (2001). It would also be important to determine analogous constants for other hyperonic reactions (e.g., for (4)) in the dressed-particle approximation. This would allow one to perform accurate microscopic calculations of the bulk viscosity of hyperonic matter.

In Sect. 4.1 we have presented simple estimates of typical bulk viscosities and associated damping time scales of neutron star vibrations in different non-superfluid neutron star models. Let us stress that the actual decrements or increments of neutron star pulsations have to be determined numerically by solving an appropriate eigenvalue problem taking into account various dissipation and amplification mechanisms (e.g., bulk and shear viscosities; mutual friction; gravitational radiation) in all neutron star layers, proper boundary conditions, etc. (e.g., Andersson & Kokkotas 2001). In principle, the vibrational motion of various superfluids may be partially decoupled. If so our analysis of superfluid suppression of the bulk viscosity must be modified (Sect. 3.2). Nevertheless, the presented estimates and the theory of superfluid suppression show that one can reach drastically different conclusions on the dynamical evolution of neutron star vibrations by adopting different equations of state in the neutron star cores (with hyperons or without), different superfluid models and neutron stars models (different central densities, allowing or forbidding the appearance of hyperons and/or operation of direct Urca processes). We expect that the results of this paper combined with the results of Papers I and II will be useful one to analyze this wealth of theoretical scenarios.

Note added at the final submission. After submitting this paper to publication we became aware of the paper by Lindblom & Owen (2001) devoted to the effects of hyperon bulk viscosity on neutron-star r-modes. The authors analyzed the bulk viscosity taking into account two hyperonic reactions, Eqs. (3) and (4), and superfluidity of Σ^- and Λ hyperons (but considering non-superfluid nucleons). Their treatment of the bulk viscosity in non-superfluid matter is more general than in the present paper since they include the reaction (4). They treat the superfluid effects using simplified reduction factors which is less accurate (a comparison of analogous exact and simplified reduction factors is discussed, for instance, by Yakovlev et al. 1999). The principal conclusions on the main properties of the bulk viscosity in hyperonic non-superfluid and superfluid matter are the same.

Acknowledgements. Two of the authors (KPL and DGY) acknowledge hospitality of N. Copernicus Astronomical Center in Warsaw. The authors are grateful to M. Gusakov who noticed a new form of the angular integral, Eq. (34). This work was supported in part by the RBRF (grant No. 99-02-18099), and the KBN (grant No. 5 P03D 020 20).

References

Alpar, M. A., Langer, S. A., & Sauls, J. A. 1984, ApJ, 282, 533
Andersson, N., & Kokkotas, K. D. 2001, Int. J. Mod. Phys., D10, 381
Baiko, D. A., Haensel, P., & Yakovlev, D. G. 2001, A&A, 374, 151
Balberg, S., & Barnea, N. 1998, Phys. Rev., C 57, 409
Carter, G. W., & Prakash, M. 2001, Phys. Lett. B, submitted [nucl-th/0106029]
Dai, Z., & Lu, T. 1996, Z. Phys., A335, 415
Finzi, A., & Wolf, R. A. 1968, ApJ, 153, 835
Flowers, E., & Itoh, N. 1979, ApJ, 230, 847
Glendenning, N. K. 1985, ApJ, 293, 470
Goyal, A., Gupta, V. K., Pragya, & Anand, J. D. 1994, Z. Phys., A349, 93
Haensel, P., & Schaeffer, R. 1992, Phys. Rev. D, 45, 4708
Haensel, P., Levenfish, K. P., & Yakovlev, D. G. 2000, A&A, 357, 1157 (Paper I)
Haensel, P., Levenfish, K. P., & Yakovlev, D. G. 2001, A&A, 372, 130 (Paper II)

Jones, P. B. 1971, Proc. Roy. Soc. Lond., 323, 111
Jones, P. B. 2001a, Phys. Rev. Lett., 86, 1384
Jones, P. B. 2001b, Phys. Rev. D, 64, 084003
Langer, W. D., & Cameron, A. G. W. 1969, Astrophys. Space Sci., 5, 213
Lattimer, J. M., Pethick, C. J., Prakash, M., & Haensel, P. 1991, Phys. Rev. Lett., 66, 2701
Lindblom, L., & Mendell, G. 2000, Phys. Rev. D, 61, 104003
Linblom, L., & Owen, B. J. 2001, Phys. Rev. D, submitted, [astro-ph/0110558]
Lombardo, U., & Schulze, H.-J. 2001, in Physics of Neutron Star Interiors, ed. D. Blaschke, N. Glendenning, & A. Sedrakian (Springer, Berlin), 31
Madsen, J. 1992, Phys. Rev. D, 46, 3290
Prakash, M., Prakash, M., Lattimer, J. M., & Pethick C. J. 1992, ApJ, 390, L77
Sawyer, R. F. 1989a, Phys. Rev. D, 39, 3804
Sawyer, R. F. 1989b, Phys. Lett. B, 233, 412
Savage, M. J., & Walden, J., 1997, Phys. Rev. D, 55, 5376
Shapiro, S. L., & Teukolsky, S. A. 1983, Black Holes, White Dwarfs and Neutron Stars (Wiley-Interscience, New-York)
Wang, Q. D., & Lu, T. 1984, Phys. Lett. B, 148, 211
Yakovlev, D. G., Levenfish, K. P., & Shibanov Yu. A. 1999, Physics-Uspekhi, 42, 737 [astro-ph/9906456]
Yakovlev, D. G., Kaminker, A. D., Gnedin, O. Y., & Haensel, P. 2001, Phys. Rep., 354, 1