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Cosmology from cosmic microwave background and galaxy clusters

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Abstract. We present the results of analysis of constraints on cosmological parameters from cosmic microwave background (CMB) alone and in combination with the galaxy cluster baryon fraction assuming inflation–generated adiabatic scalar fluctuations. The CMB constraints are obtained using our likelihood approximation method (Bartlett et al. 2000; Douspis et al. 2001). In the present analysis we use the new data coming from MAXIMA and BOOMERanG balloon-borne experiments and the first results of the DASI interferometer together with the COBE/DMR data. The quality of these independent data sets implies that the C_ℓ are rather well known, and allow reliable constraints. We found that the constraints in the $\Omega - H_0$ plane are very tightened, favouring a flat Universe, that the index of the primordial fluctuations is very close to one, that the primordial baryon density is now in good agreement with primordial nucleosynthesis. Nevertheless degeneracies between several parameters still exist, and for instance the constraint on the cosmological constant or the Hubble constant are very weak, preferred values being low. A way to break these degeneracies is to "cross-constrain" the parameters by combining them with constraints from other independent data. We use the baryon fraction determination from X–ray clusters of galaxies as an additional constraint and show that the combined analysis leads to strong constraints on all cosmological parameters. Using a high baryon fraction (∼15% for $h = 0.5$) we found a rather low Hubble constant, values around 80 km s^{-1}/Mpc being ruled out. Using a recent and low baryon fraction estimation (∼10% for $h = 0.5$) we found a preferred model with a low Hubble constant and a high density content ($\Omega_{\rm m}$), an Einstein–de Sitter model being only weakly ruled out.

Key words. cosmic microwave background – galaxy clusters: general – cosmology: observations – cosmology: theory

1. Introduction

The determination of cosmological parameters is a central goal of modern cosmology. The measurements of the angular spectrum of the fluctuations in the CMB is one of the most promising techniques. Indeed the first measurements of fluctuations at the degree scales (Saskatoon Netterfield et al. 1997) has allowed us to put the first constraints on cosmological parameters (Lineweaver et al. 1997; Hancock et al. 1997). Probably the most spectacular result was that the open models could be reasonably excluded, while flat models were favoured (Lineweaver & Barbosa 1998b), essentially because of the location of the first peak. This result has been brilliantly confirmed by the first BOOMERanG and MAXIMA measurements (Hanany et al. 2000; de Bernardis et al. 2000). The data are now becoming of high quality and the detailed features expected in so-called inflationary models are now becoming apparent (the famous peaks) from independent measurements (BOOMERanG, MAXIMA, DASI). Alternative scenarios like topological defects models are almost entirely ruled out. The possibility of a partial contribution remains open (Bouchet et al. 2000) and will be difficult to entirely rule out, although inflationary models are clearly preferred. In this paper we analyse the constraints that can be set from the recent measurements obtained by BOOMERanG, MAXIMA, DASI combined with the COBE/DMR results.

However, it has been realised that a combination of different methods to constraint cosmological parameters was very welcome, not only because of the gain in precision, but more because most of cosmological tests present degeneracies and may constrain rather well only specific combinations of cosmological parameters but not each parameter individually. Combinations of constraints or measurements, often implemented as priors on some

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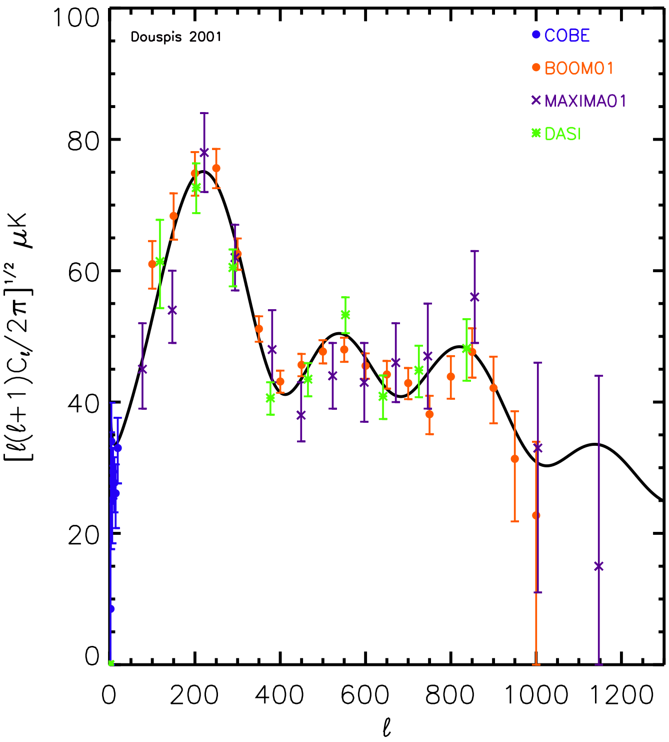

Fig. 1. The power plane: measured flat–band power estimates (COBE, BOOMERanG, MAXIMA and DASI) and our best fit model (by approximate maximum likelihood): $(H_0, \Omega_{\rm tot}, \lambda_0, \Omega_{\rm b}h^2, n, Q)$ = (30 km s^{-1}/Mpc, $1.3, 0.1, 0.019, 0.91, 23.0$ μK).

parameters in likelihood analysis, are currently used to obtain accurate values for the cosmological parameters. In this paper, we use as a first additional constraint the observed baryon fraction in clusters. This quantity has the advantage of not specifying one fundamental cosmological parameter in a direct way. Nevertheless, we obtained contours which are significantly tightened for all the parameters, breaking all the degeneracies. This allows us to investigate constraints on cosmological parameters without introducing any priors from measurements based on a standard candle hypothesis. In this paper we use $H_0 = 100\ h$ km s^{-1}/Mpc.

2. Constraints from CMB alone

2.1. CMB analysis

During the last decade (since the COBE/DMR results) different measurements of CMB anisotropies at different scales have been used to constrain the cosmology of our Universe (Hancock et al. 1997; Lineweaver et al. 1997; Dodelson & Knox 2000; Tegmark & Zaldarriaga 2000a,b; Ledour et al. 2000; Jaffe et al. 2001 and others). The last experiments (second generation) are sensitive to different scales. This gives homogeneous sets of data from a few degrees to a few arcmins, which cover two or three peaks in the power spectrum with good accuracy. We then expect the constraints to be stronger, especially if several experiments are consistent.

Table 1. Parameter space explored: $\Omega_{\rm tot} \equiv 1 - \Omega_\kappa$, where Ω_κ is the curvature parameter; $\lambda_0 \equiv \Lambda/3$; $\eta_{10} \equiv$ (baryon number density)/(photon number density) $\times 10^{10}$ (Note: $\Omega_{\rm b}h^2 = 0.00366\eta_{10}$); $n \equiv$ primeval spectral index; $Q \equiv \sqrt{(5/4\pi)C_2}$.

	H_0 km s^{-1}/Mpc	$\Omega_{\rm tot}$	λ_0	η_{10}	n	Q μK
Min.	20	0.7	0.0	2.78	0.70	10.0
Max.	100	2.0	1.0	11.94	1.30	30.0
step	10	0.1	0.1	0.83	0.03	1.0

Our method to derive parameters from CMB data has been presented in Bartlett et al. 2000 (BDBL), Douspis et al. 2001 (DBBL) and used in Le Dour et al. (2000) (LDBB)[1]. We use the maximisation technique rather than the marginalisation obtained by integrating over some parameters. Both techniques are equivalent when the probabilities are Gaussian and the model is linear in the parameters. This is far from being the case however when dealing with likelihood on the C_ℓ. The degeneracies imply that likelihood surfaces are sheets in a multidimensional space. In the presence of such a complex structure in the likelihood space, the marginalisation has the inconvenience that the model corresponding to the preferred parameters may actually lie outside of the actual allowed region! We present our results by means of contours in a two dimensional parameter space. This allows us to some extent to identify complex structure in the multidimensional parameter space, that might be hidden in the likelihood on one parameter. Goodness of fit has to be evaluated before addressing parameter estimation. The technique to derive the goodness of fit in our method is detailed in Douspis et al. (2001) (DBB). The basic approach is to use flat–band estimates as published (see Table 2 of LDBB) and apply our likelihood approximation, instead of a non-appropriate χ^2 minimisation. Our method remains as simple as a χ^2 fit but is less biased, as shown in DBBL. In this paper, we use the most recent results of MAXIMA (Lee et al. 2001), BOOMERanG (Netterfield et al. 2001) and DASI (Halverson et al. 2001). We also use the COBE/DMR data (Tegmark & Hamilton 1997) and refer to the data set as CBDM. We also upgraded our grid of models, allowing closed models and refining the step and range of the baryon content ($\Omega_{\rm b}h^2$), the total density ($\Omega_{\rm tot}$) and the spectral index (n). The parameter space explored is given in Table 1.

We do not consider reionisation or gravitational waves nor neutrino contributions to the matter density $\Omega_{\rm m}$. Previous analyses showed that these parameters do not play an important role. The present analysis has been done with all the information available in the literature. Unfortunately, some information is still missing (like the window functions); some effects have thus not been taken into account but our previous investigation of these questions indicates that this lack of information is not critical.

[1] See `http://webast.ast.obs-mip.fr/cosmo/CMB` for complementary information.

The likelihood of each model is calculated as described in LDBB, and the best model is found by maximising the likelihood function over the explored space. The contours are defined in the full six–dimensional space with values of $\Delta \log(\mathcal{L}) = 1, 4, 9$ (dashed, in red), corresponding to the 1, 2 and 3 σ contours for a Gaussian distributions when projected onto one of the axes. Identically the 1, 2 and 3 σ contours in the two parameters space are defined by $\Delta \log(\mathcal{L}) = 2.3, 6.17, 11.8$ (filled, in blue), for Gaussian distribution. Since the likelihood is not actually Gaussian, the confidence percentages associated with our contours are actually not known; the technique is however standard practice.

We also would like to emphasise the effect of priors in the analysis. As it has been noticed in the recent literature (Jaffe et al. 2001; Balbi et al. 2000; Lange et al. 2000), the final results on the preferred parameters are very sensitive to priors. Most of the recent work set H_0 as a "ghost" parameter, using physical densities ($\Omega_\mathrm{b} h^2$, Ωh^2, $\Omega_\mathrm{dm} h^2, ...$) and deriving H_0 by:

$$h = \sqrt{\frac{\omega_\mathrm{dm} + \omega_\mathrm{b}}{1 - \Omega_k - \Omega_\Lambda}}.$$

Then the analysis proceeds by putting a prior on H_0 like $H_0 > 45$. The representation in one-dimensional plots does not allow us to know exactly what is the effect of such technique. We will show in Sect. 2.2 that such a prior puts more severe constraints on Ω_total, as is expected.

The study of the goodness of fit of "best" models is detailed in DBB. We apply the same technique on the new data and the best model plotted in Fig. 1. The goodness of fit expressed in terms of a generalised χ^2 is $\chi^2_\mathrm{gen} = 45.0/43$ where $43 = 49 - 6$ is the number of degrees of freedom corresponding to the number of experimental points minus the number of investigated parameters. This good value of the goodness of fit allows us to consider confidence intervals on cosmological parameters.

2.2. Constraining the cosmological parameters

The first Doppler peak is one of the main feature of the CMB angular power spectrum. It is clear now that different experiments indicate both a rise in the power at intermediate scale and a fall–off at smaller scales (higher ℓ). The position of the first acoustic peak is strongly related to the curvature of the Universe. A statistical analysis of the CMB data should then give some strong constraint on the total density (or curvature) of the Universe. The confidence intervals on the Ω_total parameter in different combinations ($(\Omega_\mathrm{total}, \Lambda), (\Omega_\mathrm{total}, H_0), (\Omega_\mathrm{total}, \eta_{10})$) are shown as 2-D contour plots in Fig. 2. The $\Omega_\mathrm{total} - H_0$ plane reveals the strongest constraint implied by CMB fluctuations the contours are almost reduced to a line in this plane. The main implication is the exclusion of almost all open cosmologies at a high confidence level. Preferred models are closed ($\Omega_\mathrm{total} \sim 1.3$) and relative maxima are present at lower values ($\Omega_\mathrm{total} \sim 1.05$). The models corresponding to the latter are in good agreement with a flat Universe and the inflationary scenario prediction. Within the 95% Gaussian confidence level (95% GCL hereafter), and allowing H_0 to be in the range $[20, 100]$ we find: $0.95 < \Omega_\mathrm{total} < 1.4$. Given our 2-D representation, it is easy to see that the closest models have the lowest value of H_0 (cf. Fig. 2). This is supplementary information that the 1-D reduction is unable to show.

The fourth plot in Fig. 2 presents the results in the $\eta_{10} - \Omega_\mathrm{total}$ plane. The constraints on η_{10} are becoming tight with the present-day data set compared to the analysis of first generation experiments (LDBB). They point toward a medium value of the baryon content which is in agreement with the BBN and light elements determination of η_{10}, unlike the analysis based on first BOOMERanG and MAXIMA results (Tegmark & Zaldarriaga 2000; Douspis 2000; Jaffe et al. 2001). The change in the preferred value of $\Omega_\mathrm{b} h^2$ is due to the improved measurements of the second and third peaks. In early 2000, BOOMERanG and MAXIMA experiments measured the power at scales corresponding to the second acoustic peak. The latter appears to be lower (relative to the first one) than expected in a model with a baryon content compatible with BBN predictions ($\Omega_\mathrm{b} h^2 \sim 0.02$, Burles et al. 2001). The relative heigh between the two peaks favours high values of η_{10} corresponding to $\Omega_\mathrm{b} h^2 \sim 0.03$. In spring 2001, BOOMERanG, DASI and MAXIMA teams published new analyse showing the possible existence of a third acoustic peak with the same height as the second. This disfavours high baryon models which predict a higher third peak (see Fig. 1 of LDBB). The actual value in accordance with our analysis is $4.0 < \eta_{10} < 7.0$ at 95% GCL and almosis independent of the value of H_0.

The spectral index of primordial fluctuations, n, is also a well-constrained parameter. We found $0.81 < n < 1.02$ at 95% GCL. This is in agreement with the inflationary predictions.

The remaining parameters are almost not constrained. In Fig. 2 we can see the different degeneracies between parameters. There is no constraint on H_0 from the CMB. We can also see that Ω_Λ can take almost any value: $0.0 < \Omega_\Lambda < 0.9$ at 95% GCL. Another contour plot in Fig. 2 shows a tight degeneracy between Ω_total and H_0. There again, it is obvious that the lower H_0, the higher Ω_total. Imposing $H_0 > 45$ km s^{-1}/Mpc results in a tight constraint on Ω_total: $0.9 < \Omega_\mathrm{total} < 1.2$ (99% GCL). Finally, the second plot in Fig. 2 points out the anticorrelation of the pair (n, η_{10}). Increasing n (lowering Q) will increase the height of the second peak relative to the first. On the other hand, increasing η_{10} will decrease the second peak.

It is clear that the CMB alone does not directly constrain very tightly Ω_total without further restriction (or prior) on H_0. Nevertheless it is important to keep in mind that the CMB imposes a very narrow region in the $\Omega_\mathrm{total} - H_0$ plane. Other parameters are not so well constrained. However, the baryon content is now becoming rather well constrained, in agreement with nucleosynthesis and light element abundances and the primordial index is

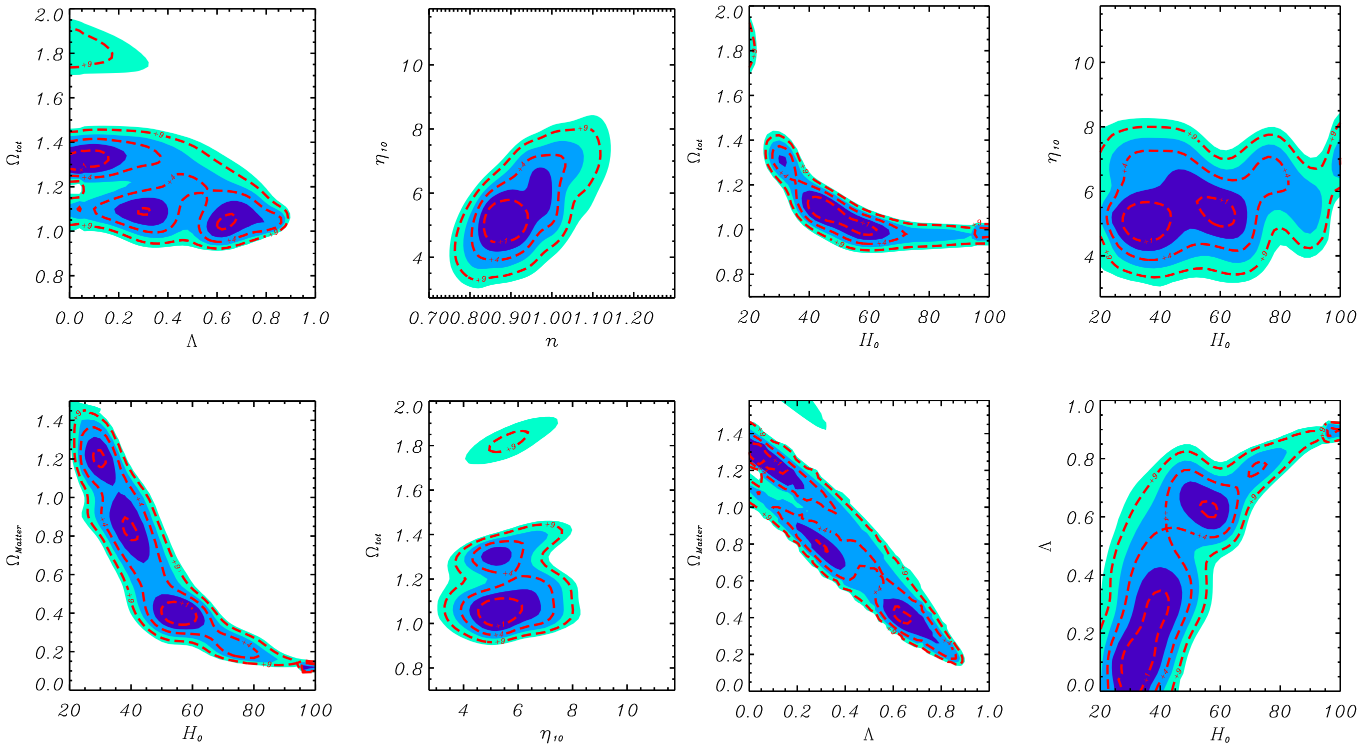

Fig. 2. Different contour plots from the analysis of set CBDM. The dashed red line defines the 68, 95 and 99% GCL when projected on the axis. The blue filled contours are the corresponding confidence intervals in 2 dimensions.

slightly below 1, close to the value predicted by inflation. It is therefore a fundamental success of Cosmology that the CMB alone does allow such tight constraints. However, it is not yet possible to infer robust constraints on other fundamental parameters like H_0, Ω_Λ or $\Omega_{\rm m}$. It is therefore vital to have other means to measure these fundamental parameters.

3. Combined analysis

Given the fact that CMB implies degeneracies between the fundamental cosmological parameters H_0, Ω_Λ and $\Omega_{\rm m}$, it is interesting to look for combinations with constraints that do not directly measure any of these parameters. Primordial nucleosynthesis is one such possibility; we have checked that this does not change much the constraints established in the above section. An other possibility that we investigate below is to use the baryon fraction inferred from cluster observations.

3.1. Baryon fraction in clusters

Clusters provide fundamental observations for cosmology. Historically, they provided the first evidence for the existence of dark matter. Furthermore they now provide us with the best evidence that this dark matter is non-baryonic. Indeed, clusters are the only structure for which the total mass, baryon mass and stellar mass can be evaluated simultaneously in a reliable way. Cosmological applications of clusters have been revived with new fundamental tests which provide a global constraint on the cosmological parameters: the baryon fraction in clusters reliably trace the ratio between baryons and the total mass in the Universe:

$$f_{\rm b} = \Gamma \frac{\Omega_{\rm b}}{\Omega_0}$$

where Γ has been estimated from numerical simulations and is close to 0.92 in the outer part of clusters (Frenk et al. 1999). Provided that $\Omega_{\rm b}$ can be estimated from primordial nucleosynthesis and the observed abundance of light elements, this argument can be used to estimate a robust upper limit on Ω_0 (White et al. 1993). In this paper we combine CMB and the sole baryon fraction constraint. This is interesting because the baryon fraction does not constrain by itself any of the cosmological parameters, so that in principle it could leave unchanged the conclusions obtained from the CMB alone.

3.2. Combining constraints

There are some dispersions among values published for the gas fraction in clusters, which mainly reflect the difference in the mass estimator used. Using mass estimators based on numerical simulations a median baryon fraction of $0.048h^{-3/2} + 0.014$ ($\pm 10\%$) was recently found by Roussel et al. (2000), consistently with previous investigations (Arnaud & Evrard 1999). More recently the standard value has been revisited by Sadat & Blanchard (2001), showing that the actual baryon fraction could actually be lower. They concluded that a baryon fraction as

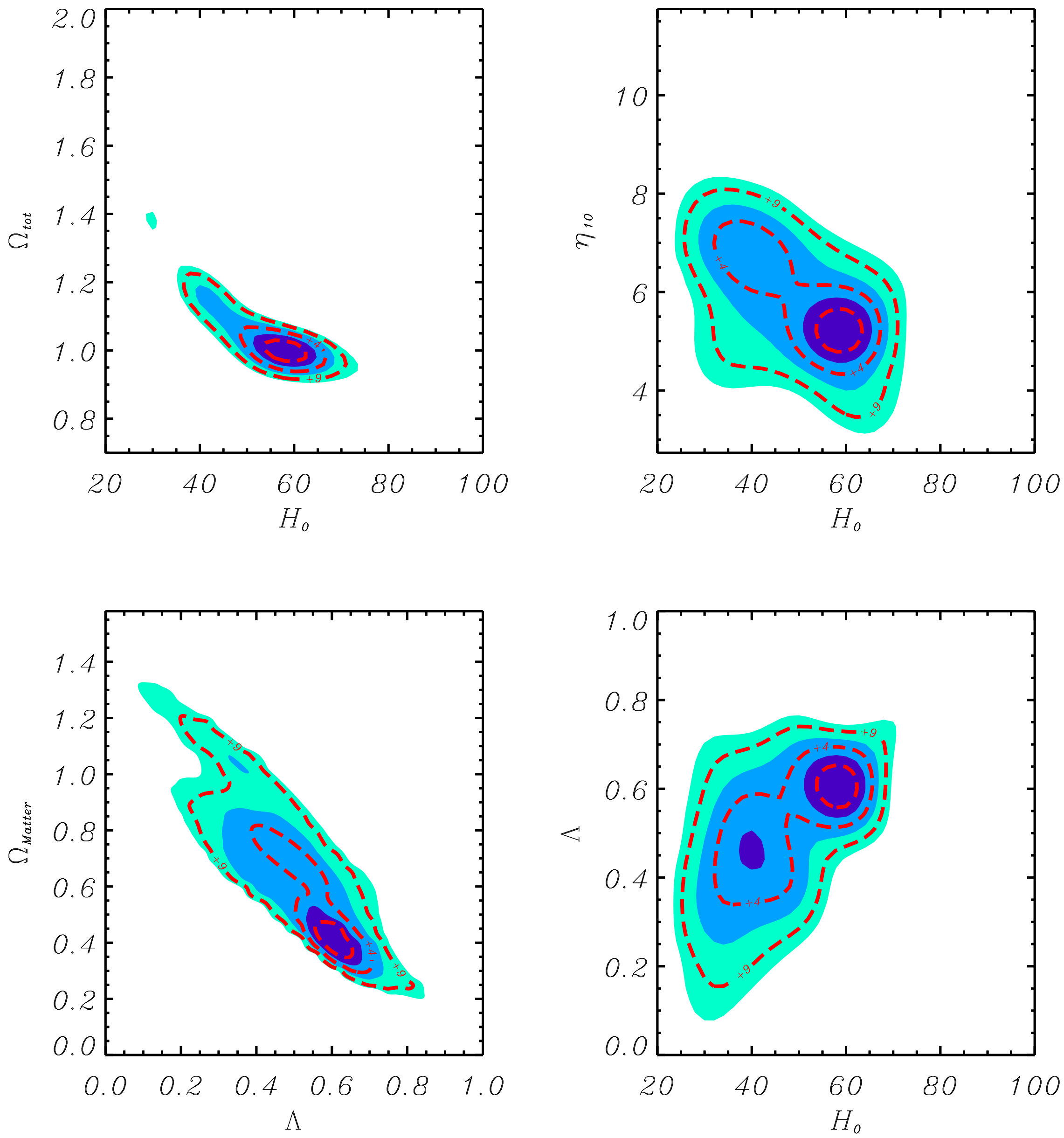

Fig. 3. Combined analysis of CMB and high baryon fraction. See Fig. 2 for the definition of contours.

low as $0.031h^{-3/2} + 0.012$ ($\pm 10\%$) is still consistent with the X–ray data on clusters.

3.2.1. The high baryon fraction case

As we have mentioned previously, we have chosen to investigate the combination of the baryon fraction f_b with the CMB data. The baryon fraction is an interesting case because its value can be measured in a rather direct way through observations of X-ray clusters provided the assumption that all the baryons are actually seen (assuming no large quantity of dark baryons). It is worth noticing that without an additional constraint on Ω_b, there is no trivial reason why the additional information on f_b should further constrain the various cosmological parameters. However, it happens that the additional constraint from the baryon fraction restricts significantly the various contours (see Fig. 3). This is very clear from the $H_0 - \eta_{10}$ plane: the area of the contours is much reduced, leading to a preferred model which is around $H_0 \sim 60$ km s^{-1}/Mpc and $\eta_{10} \sim 5$. The other two-dimensional plots also reveal that all the contours are significantly reduced: actually most of the one sigma contours are of the order of the grid size so that best model and one sigma ranges should be used with caution. It is highly interesting that the cosmological model is now very well constrained: $\Omega_\lambda \sim 0.6$, $\Omega_\mathrm{tot} \sim 1$ (implying $\Omega_\mathrm{m} \sim 0.4$, $H_0 \sim 60$ km s^{-1}/Mpc, $\eta_{10} \sim 5$, $n \sim 0.85$). As we mentioned, the allowed range has to be interpreted with caution, but we notice that the preferred value of the Hubble constant is rather low and that higher values like 80 km s^{-1}/Mpc lie uncomfortably outside of the preferred region.

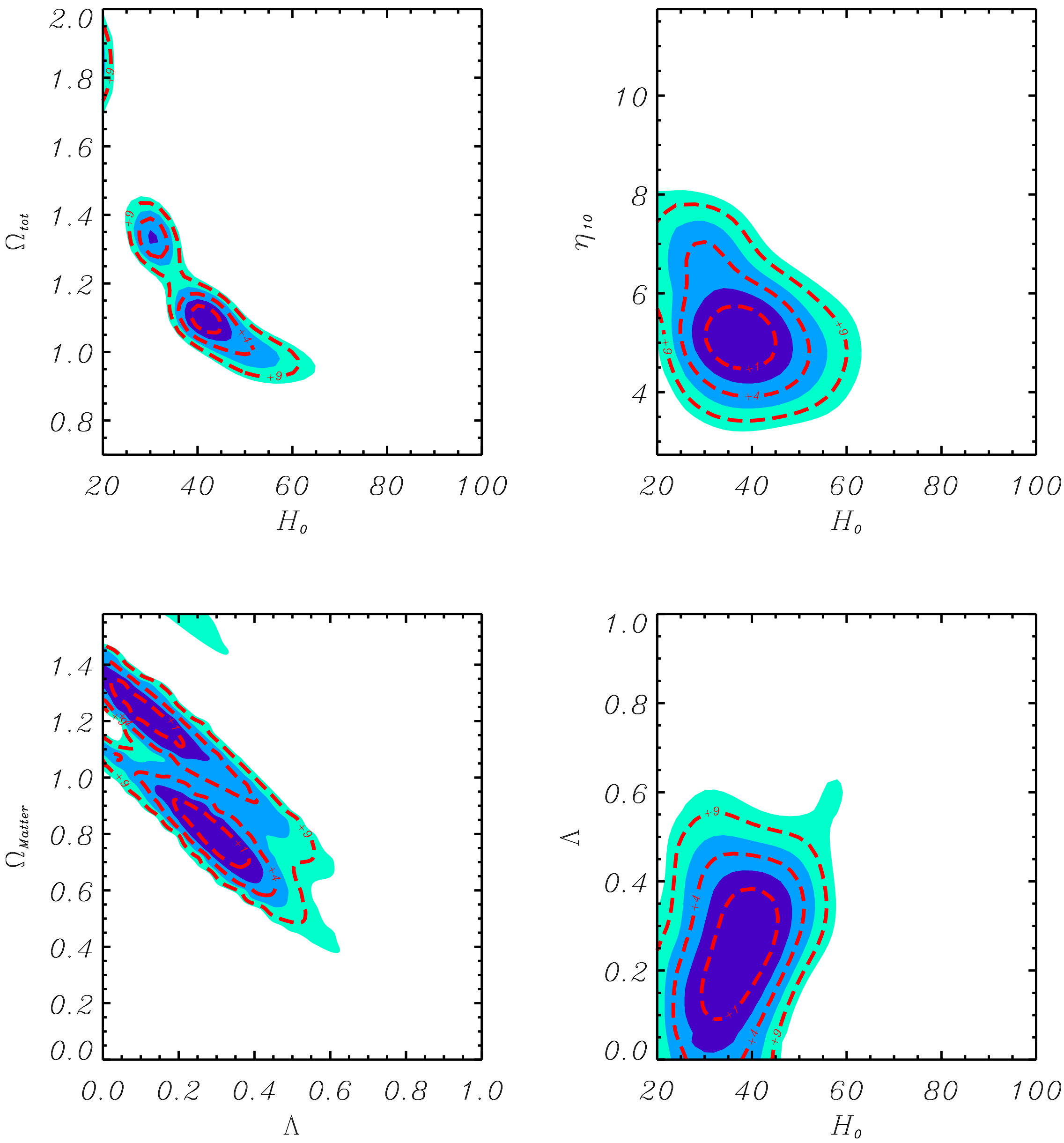

Fig. 4. Combined analysis of CMB and low baryon fraction. See Fig. 2 for the definition of contours.

3.2.2. The low baryon fraction case

As mentioned above, a baryon fraction significantly lower than current estimates is still consistent with, and actually preferred by, existing data on X-ray clusters. Using this low value for the baryon fraction leads similarly to much restricted contours. However the latter differ from the previous ones (see Fig. 4): the preferred value for η_{10} is again close to 5; the preferred value of the Hubble constant is now very low ($H_0 \sim 40$ km s^{-1}/Mpc). Although such low values have been argued for from time to time (Bartlett et al. 1995), it is worth noticing that values such as $H_0 \sim 60$ km s^{-1}/Mpc are still at the edge of the 3 sigma contour and for this reason cannot be entirely ruled out. The preferred model is $\Omega_\lambda \sim 0.3$, $\Omega_{\rm tot} \sim 1.1$ (implying and $\Omega_{\rm m} \sim 0.8$, $H_0 \sim 40$ km s^{-1}/Mpc, $\eta_{10} \sim 5.$, $n \sim 0.85$).

Finally, we also investigated the additional constraint that can be inferred from the evolution of the abundance of X-ray clusters: the evolution of the abundance of X-ray clusters is known to be a powerful constraint on the matter density of the universe (Oukbir & Blanchard 1992). The measurement of the temperature of a flux-limited sample of X-ray clusters has allowed the determination of the abundance of clusters at $z \sim 0.3$–0.4 (Henry 1997; Henry 2000). Using the updated estimates of local abundance of X–ray clusters a significant level of evolution is found, indicating a high-density Universe (Blanchard et al. 2000).

The constraint provided by the addition of the evolution of cluster abundance is essentially a constraint on $\Omega_{\rm m}$. Therefore it is rather natural that this additional constraint by itself will not change the previous contours much as the value found by Blanchard et al. (2000),

$\Omega_{\rm m} \sim 0.8$, is almost identical to that provided by the previous analysis. We can anticipate that using analysis leading to lower $\Omega_{\rm m}$ with the higher baryon fraction will not change much the contours presented in the previous section.

4. Conclusions

The new CMB data provide impressive evidence in favour of the standard inflationary picture for structure formation with non-baryonic dark matter. They also brilliantly confirm previous evidence for a nearly flat Universe. In this paper, we have actually shown that the data provide a very narrow allowed region in the $H_0 - \Omega_{\rm tot}$ plane. However, it is somewhat frustrating that despite the high quality of the data, degeneracies among the fundamental cosmological parameters ($\Omega_{\rm tot}, \Omega_\lambda$ and H_0) cannot be broken and that constraints on other parameters are relatively weak. It should also be realised that given the dependency on the power spectrum shape and the possible contribution at some level from topological defects, it will be extremely difficult to obtain entirely reliable constraints on these parameters from CMB alone. Actually, even with the precision anticipated for the Planck mission, the degeneracy problem may not be solved. It is therefore vital to develop new tests that provide additional constraints. The supernova Hubble diagram provides such a test, which has be widely advertised. However, the previous use of the Hubble diagram has always been polluted by systematic effects, and therefore we believe that a coherent picture will be firmly established only from further evidence. Clusters offer such an additional possibility. We have therefore investigated the complementary information coming from clusters. The most interesting constraint has been found, rather surprisingly, to be the baryon fraction. Indeed with this single additional constraint we found that nearly all the cosmological parameters are well specified. Using a classical value for the baryon fraction of ∼15% for $h = 0.5$ we obtain as the best model: $\Omega_\lambda \sim 0.6$, $\Omega_{\rm tot} \sim 1$, $H_0 \sim 60$ km s^{-1}/Mpc, $\eta_{10} \sim 5$, $n \sim 0.85$. Using a more recent result from Sadat & Blanchard (2001), who infer a lower baryon fraction when several systematics are corrected for, we found that the best model is $\Omega_\lambda \sim 0.3$, $\Omega_{\rm tot} \sim 1.1$, $H_0 \sim 40$ km s^{-1}/Mpc, $\eta_{10} \sim 5.$, $n \sim 0.85$ implying a high density parameter $\Omega_{\rm m} \sim 0.8$ consistent with the determination from cluster abundance evolution by Blanchard et al. (2000). Although the Einstein–de Sitter model is at the edge of the 3 sigma contour, it is probably premature to rule it out on this basis. It is remarkable that in both analyses we found that the preferred primordial baryon content is narrowly constrained and consistent with primordial nucleosynthesis. Identically the primordial index is found to be of the order of 0.85 in both cases.

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