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Letter to the Editor

Long-term collisional evolution of a dissipative particle disc perturbed by a giant-planet embryo

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Abstract. Recent works have shown that in a disc of planetesimals perturbed by a proto-Jupiter, the coupling between embryo's perturbations and inelastic collisions heats up the disc over several astronomical units in a few 10^5 years. Using a simulation of a disc made of hard-spheres suffering inelastic collisions, we performed long-term numerical integrations to determine if the energy dissipated in collisions may finally damp eccentricities and inclinations induced by the proto-Jupiter. It is shown that the coupling between mean-motion resonances and collisions induces different damping regimes as a function of the protoplanet's mass. A 15 $M_\oplus$ proto-Jupiter is more efficient than a 300 $M_\oplus$ one to stir the disc over longer timescales, due to the non-emptying of isolated first order mean-motion resonances.

Key words. solar system formation – planetesimals – giant-planet formation – Asteroid Belt – collisions

1. Introduction

It seems that inelastic collisions played an important role during planetary formation (Marzari & Scholl 2000; Stern & Weismann 2001; Charnoz et al. 2001, hereafter C01), especially when some proto-planets appear embedded in the planetesimal disc. Proto-planet's gravitational perturbations increase random velocities of neighboring planetesimals up to the proto-planet's escape velocity, that is far larger than planetesimal's escape velocity (Ida & Makino 1993; Petit et al. 2000). In return, planetesimals may collide among themselves in high velocity impacts, in which a fraction of the impact kinetic energy is dissipated (inelastic collisions). The gravitational stirring induced by a proto-planet may be substantially damped by the inelasticity of collisions.

The present letter follows C01 in which the effect of a very massive proto-planet, a proto-Jupiter of 15 $M_\oplus$, on a disc of particules suffering dissipative collisions, was studied. The study of inelastic collisions among planetesimals in a disc perturbed by one masive proto-planet has been only little studied over the last years, mainly because of strong numerical difficulties: on the one hand, statistical simulations cannot handle the complex dynamics induced by proto-planet's gravitational perturbations, on the other hand, direct simulations cannot handle a realistic number of particles with the whole relevant physics (fragmentation, gas-drag, close and distant perturbations). Thus, in the actual state of computer's capabilities, only some "academic" but non trivial models can be used to try to understand the role of inelastic impacts in a perturbed planetesimal disc over 10^5 to 10^6 years of evolution, as in Marzari & Scholl (2000).

Using models of discs made of colliding hard spheres, Thébault & Brahic (1999), and later C01, have begun a step-by-step study. Despite of their simplicity, these models have revealed some unexpected mechanisms. The most gravitationally perturbed regions, i.e. the mean-motion resonances and the immediate embryo's vicinity, redistribute random kinetic energy throughout the disc via multiple violent collisions like a heat conduction. In consequence, the perturbation that was initially confined only in the dynamically perturbed regions propagates far away from the perturber over several astronomical units, where random velocities are increased up to a few 100 m s^{-1}, well beyond planetesimal escape velocity. This mechanism, called "collisional diffusion", has been studied only during the first 10^5 years of evolution. Due to the loss of energy in collisions, the disc may be expected to cool-down on a long-term evolution. This is the subject of this letter. The questions we want to address are:

1. What is the cooling timescale of collisional diffusion?
2. How does it depend on the perturber's mass?
3. What are the surface density and random velocities of the system on a long-term evolution?

The numerical model is described in the first section. The long-term evolution of collisional diffusion is presented

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different values of the perturber's mass in the second section. The results are discussed in the last section.

2. The model

Our model, described in C01, is a direct three-dimensional simulation of a disc composed of 4000 massless particles with finite radii. They move in the gravitational field of a central body and of a massive secondary body, called hereafter the "perturber", on a circular orbit at 5.2 a.u. The perturber's mass is a free parameter. Three representative simulations are reported here: (simulations S1 and S2 respectively) an initially cold particle disc perturbed by 15 and a 300 $M_\oplus$ perturber, (simulation S3) an initially cold particle disc perturbed by a 15 $M_\oplus$ perturber, which mass is set to 300 $M_\oplus$ after 1.3×10^6 years, accounting for the abrupt increase of Jupiter's mass (Pollack et al. 1996). Simulations S1 and S2 are mostly discussed in this letter, as they show the essence of physical mechanisms at work. The initially cold disc is made of particles distributed randomly between 1.5 and 5.5 a.u., with a surface density decreasing by the power -1 of the distance to the central body. Initial eccentricities and inclinations are in the range 10^{-4} to 10^{-3} so that initial random velocities are of the order of 10 m s^{-1}, comparable to the escape velocity of kilometer-sized planetesimals. Particles suffer inelastic rebounds among themselves with a radial restitution coefficient of 0.3. The particle's radius is set to 5×10^{-4} a.u. in order to reproduce a realistic collision rate of ~1 collision per particle every 10^4 years. Of course, these particles are much bigger and much less numerous than the kilometer-sized planetesimals that were initially present between 1.5 and 5 a.u. (Weidenschilling 1977). However, as shown by Trulsen (1971), Brahic (1976) and illustrated recently in C01, the collisinal evolution of a disc containing a large number of small particles suffering inelastic rebounds may be simulated with a small number of large particles, if the optical depth (i.e. the collision rate) is the same in both systems. Despite of the simplicity of our model, each simulation takes several weeks on modern computers.

3. Results

3.1. Two cooling regimes on a long-term evolution

Two cooling regimes, illustrated with bold lines in Fig. 1, have been identified for a low mass perturber and a high mass perturbers respectively, corresponding to simulations S1 and S2. Eccentricities and inclinations (e and i) of disc's particles are linked to random velocities ($V_{\rm r}$) via a simple relation: $V_{\rm r} \sim CV_{\rm k}\sqrt{e^2+i^2}$, where $V_{\rm k}$ is the local Keplerian velocity, and C a constant factor of the order of 1. For a 300 $M_\oplus$ perturber, there is a first rapid increase of eccentricities and inclinations within the first 10^4 years which corresponds to the propagation of collisional diffusion (described in C01). After the maximum is reached, the slope is inversed very rapidly, and the median value of $\sqrt{e^2+i^2}$ decreases exponentialy on a typical timescale of 4×10^5 years. After 10^6 years of evolution, eccentricities and inclinations decrease close to their initial value, about a few 10^{-3}. This decreasing phase is the cooling regime. For a 15 $M_\oplus$ perturber, the propagation of collisional diffusion lasts ~2×10^5 years. It is followed by a progressive cooling over a much longer time-scale, which is at least of the order of few millions years. Thus, a somewhat "paradoxical" behaviour is identified: *after* 5×10^5 *years of evolution, the disc stirred by the 15* $M_\oplus$ *body is more perturbed than the disc stirred by a 300* $M_\oplus$ *one.* These two significantly different behaviours are the consequence of the coupling between collisions and first order mean-motion resonances as explained below.

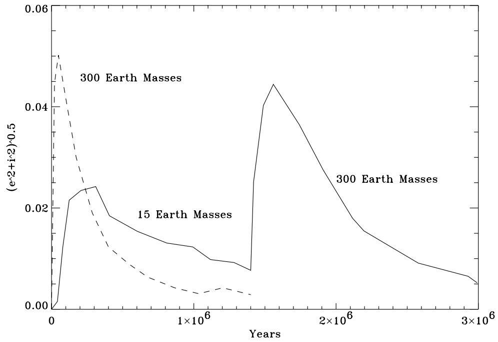

Fig. 1. Median value of particle's $\sqrt{e^2+i^2}$ between 2 and 4 a.u., as a function of time and for different values of the perturber's mass. Bold lines correspond to simulations S1 and S2. The dashed line corresponds to simulation S3.

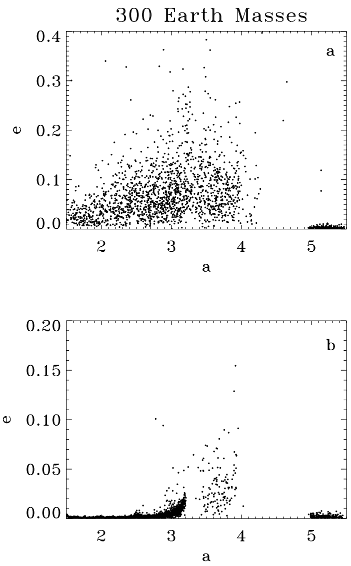

Fig. 2. Collisional evolution of the particle disc with a 300 $M_\oplus$ perturber at 5.2 a.u. at times: 7×10^4 years **a)** and 1.3×10^6 years **b)**, a and e stands for particle's semi-major axis and eccentricity.

3.2. Disc cooling with a 300 $M_\oplus$ perturber

After 1.3×10^6 years of evolution, eccentricities and inclinations are divided by 10 compared to their value at the end of the collisional diffusion at 7×10^4 years (Fig. 2), the

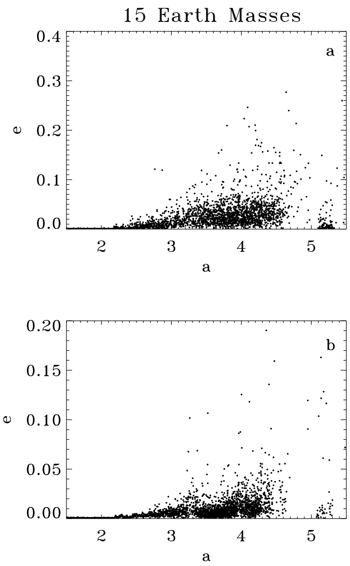

Fig. 3. Collisional evolution of the particle disc with a 15 $M_\oplus$ perturber at 5.2 a.u. at times: 2×10^5 years **a)** and 1.3×10^6 years **b)**, a and e stands for particle's semi-major axis and eccentricity

region extending beyond 4 a.u. is efficiently cleared. This region is the most unstable because of numerous overlapping mean motion resonances induced by the 300 $M_\oplus$ perturber at 5.2 a.u. This region is depopulated in only a few 10^5 years: particles deflected by the perturber suffer a decrease of their semi-major axes when their eccentricities increase due to the conservation of Jacobi's energy ($\sim 1/2a + \sqrt{a(1-e)^2}$, see Hayashi et al. 1977). These particles collide preferentially in the inner part of the disc and migrate coherently inwards. The most striking feature is the quasi-total clearing of the 2:1 mean motion resonance (at 3.27 a.u.) in about 10^6 years. The regions of overlapping and mean-motion resonances are the only sources of eccentric particles in the system. Thus, they are the "energy tank" of collisional stirring, as shown in C01 and in Thébault & Brahic (1999). Due to this clearing, the only source of random kinetic energy is lost. As a consequence, the disc may cool down to its initial state due to collision's inelasticity. At the end of the simulation, all particles have suffered about one hundred collisions. The mechanism by which mean-motion resonances are cleared via collisions has been analytically studied by Franklin et al. (1980) and numerically studied by Hanninen & Salo (1992) in a different context. On the one hand, the resonance region is fed with neighboring particles via viscous diffusion, on the other hand, resonant particles are removed from the resonance region as soon as they suffer one collision. As a consequence, a mean-motion resonance is depopulated (or "opened") if the typical collision time of a resonant particle is shorter than the time necessary, for neighboring particles, to random walk over the resonance's width via viscous diffusion. In the case of the 2:1 mean-motion resonance, this criterion is quantified in the following form: the resonance opens if particles eccentricities close the resonance's edges (e_0) are lower than a threshold value $e_\mathrm{t} = q^{5/9}$ where q is the perturber's mass divided by the central-body mass (Franklin et al. 1980). Thus, $e_\mathrm{t} = 0.02$ in our case. Note that this analytical estimate does not take into account the coupling between the forced eccentricity in resonance and particle's eccentricities close to the resonance's edges, which are considered as two independant parameters. In a first-order approximation, one may expect e_0 to be a fraction of the resonant forced eccentricity, i.e proportional to $q^{1/3}$ (Franklin et al. 1980). Thus, the ratio e_0/e_t may be proportional to $q^{-2/9}$, which decreases with q. This crude estimate shows qualitatively that more massive perturbers are more likely to open a gap at resonance than lighter perturbers. In our simulations, particle's eccentricities close to the 2:1 resonance inner edge (e_0) become comparable or lower than e_t after 5×10^5 years. At this time, the 2:1 mean-motion resonance begins to significantly empty. After 1.3×10^6 years of evolution, about 1% of the initial number of particles remain inside the resonance. Particles remaining outside this region suffer multiple dissipative collisions and have their eccentricities and inclinations progressively damped down to a floor value imposed by distant interactions with the perturber.

3.3. Disc cooling with a 15 $M_\oplus$ perturber

Particles eccentricities are presented in Fig. 3b after 1.3×10^6 years of evolution. The 2:1 and 3:2 mean-motion resonances are located at 3.2 and 3.9 a.u. Compared to the state of the disc at its maximum excitation, eccentricities and inclinations are only divided by a factor 2 to 3. The main differences with the 300 $M_\oplus$ case are the following: (i) resonances at 3.3 and 3.9 a.u. are not empty; (ii) the disc is more perturbed, by a factor 2 to 3, in eccentricities. In both cases (15 and 300 $M_\oplus$), the perturber's vicinity is empty. But, in the 15 $M_\oplus$ case, resonances are not cleared. Thus, the 2:1 and 3:2 resonances act as secondary sources of random kinetic energy and maintain the disc in a residual excited state. Indeed, the threshold eccentricity for depopulating the 2:1 resonance is $e_\mathrm{t} = 4 \times 10^{-3}$. Particle eccentricities oustside the resonance (e_0) are larger than e_t (Fig. 3b) thus the 2:1 resonance cannot be cleared. This is especially efficient beyond 3.3 a.u. because of collisional coupling of non-resonant regions with the nearby 2:1 and 3:2 mean-motion resonances. Until 1.3×10^6 years the number of particles inside the 2:1 mean-motion resonance is very stable and ranges from 60 to 80 in our simulation. There are about 30% less particles in the 3:2 resonance, which has a smaller width. These resonances are in a steady state in which the rate of incoming and outcoming particles balances. In conclusion, because of the low mass of the perturber, mean motion resonances are not powerful enough to be opened. The disc being endlessly fed with eccentric particles, it cannot cool down completely. In this case, on a long-term evolution, random velocities are controlled by the coupling between resonant interactions with collisions.

3.4. Growing proto-Jupiter versus initial Jupiter

To account for the rapid growing of Jupiter when its gaseous enveloppe collapses (Pollack et al. 1996) the mass of the perturber was changed from 15 to 300 $M_\oplus$ in simulation S3 after 1.3×10^6 years of evolution. Results are presented in dashed line in Fig. 1. Right after the mass increase, a new collisional diffusion takes place, but its intensity is lower and the cooling timescale is longer by a factor 2 to 3 than in simulation S1. This is the consequence of a different distribution of particles in simulations S1 and S3 when Jupiter appears. In simulation S3, particles that where initially between 4 and 5 a.u. have been pushed below 4 a.u. due to the influence of the 15 $M_\oplus$ perturber. Thus when Jupiter appears, the region between 3 and 4 a.u. is densely populated (by a factor $\sim$2 compared to the region around 2.5 a.u.), whereas the disk is strongly depleted beyond 4 a.u. Consequently there is much less material available in the chaotic region to be scattered away, which results in a weaker heating of the disc. Because of high particle surface density between 3 and 4 a.u., the timescale to empty the 2:1 and 3:2 resonances are longer in simulation S3 than in simulation S1. In addition, particles between 3 and 4 a.u. on highly eccentric orbits, beeing very numerous, their statistical weight is large when computing the median eccentricity between 2 and 4 a.u. Those effects results in a longer cooling timescale, but the final state of simulation S3 is very close to simulation S1, appart from slightly larger eccentricities, but that decrease constantly and still more rapidly than in a disk perturbed by a 15 $M_\oplus$ body.

4. Discussion and conclusion

Possible implications of this work to the actual early solar system are difficult to evaluate because of the necessary simplicity of the model. During one million years of evolution, all particles have suffered few tens of violent collisions with impact velocities greater than 50 m s^{-1} (C01). Studies of solid material disruption (Fujiwara et al. 1977; Benz & Asphaug 1999) agree that kilometer-sized planetesimals may be catastrophically disrupted into smaller fragments only after one of these collisions. The typical size of bodies in the asteroid-belt region when the proto-Jupiter appeared is not known, and it may be possible that they would have grown large enough to survive violents impacts of few 100 m s^{-1}. Thus, one can wonder if the mechanisms we have identified may have played a role in the early solar system. This should be investigated in the future with more refined models including fragmentation. However, the present model allowed us to identify some new basic mechanisms occuring in a perturbed particle disc:

1. The system can completely cool only if first-order mean-motion resonances open, which is possible if the perturber's mass is comparable to the actual mass of Jupiter. According to actual models of Jupiter's formation (Pollack et al. 1996), a massive solid core of 10 $M_\oplus$ to 30 $M_\oplus$ appears first via runaway growth. In this case, the present study suggests that mean motion resonances might heat-up the Asteroid Belt over millions of years, which is unfavorable for planetary accretion.
2. As a consequence, a few 10 $M_\oplus$ perturber might be able to stir the particle disc on longer timescales than a few 100 $M_\oplus$ embryo due to the coupling between distant encounters with inelastic collisions.
3. The region stirred by mean-motion resonances may be large compared to the resonance's dynamical width, because of multiple violent collisions of resonant eccentric particles. Since stirring is unfavorable for planetary accretion (Wetherill & Stewart 1989), it should be taken into consideration as soon as a 10 $M_\oplus$ giant-planet embryo appears for planetary accretion.
4. In the case of a 300 $M_\oplus$ perturber, collisions are efficient to deplete the region extending beyond 3.2 a.u. (Fig. 2). This might probably solve a problem raised by Lecar et al. (1992) who showed that gravitational perturbations from Jupiter and Saturn cannot eject bodies located between 3.2 and 3.6 a.u. from the Asteroid Belt. At the end of the simulation, there are about 10 times fewer bodies between 3.2 and 3.6 a.u. than between 2.8 and 3.2 a.u. About a same order ratio is observed in the actual Asteroid Belt.

In the future, a simple fragmentation model, gas-drag and size distribution will be included in our model.

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