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Are radio pulsars strange stars ?

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Abstract. A remarkably precise observational relation for pulse core component widths of radio pulsars is used to derive stringent limits on pulsar radii, strongly indicating that pulsars are strange stars rather than neutron stars. This is achieved by inclusion of general relativistic effects due to the pulsar mass on the size of the emission region needed to explain the observed pulse widths, which constrain the pulsar masses to be $\leq 2.5\ M_{\odot}$ and radii ≤ 10.5 km.

Key words. pulsars: general – dense matter – equation of state: stars : neutron

1. Introduction

Radio pulsars are believed to be the most common manifestations of neutron stars, but it has not been possible so far to relate the voluminous data on radio pulses and their varied structure to the properties of neutron stars except through the arrival times of pulses. Here we make such a connection between pulse core component widths derived from very good quality radio data and the mass-radius ($M{-}R$) relation of neutron stars. This becomes possible only due to the inclusion of general relativistic effects of the stellar mass on pulsar beam shapes, which makes the stellar mass and radius relevant parameters in determining the pulse widths. We show that core component widths provide tight constraints on equations of state (EOS) of neutron stars. We compare our results with other similar attempts based, e.g., on the X-ray data. From our constraints it emerges that no neutron star EOS seem to be adequate, leading to the conclusion that pulsars are strange stars, i.e., ones composed of quarks of flavors u, d, and s (Alcock et al. 1986); and we examine it in light of similar recent suggestions.

2. Core component widths

A classification of radio pulse components into "core" and "conal" emissions has emerged which is based on various characteristics such as morphology, polarization, spectral index etc. of the pulses (Rankin 1983). Radio pulsars often show a three peaked pulse profile, the central component of which is identified as the core emission, as opposed to the outrider conal pair (Rankin 1990). By analysing the core components of many pulsars, especially the "interpulsars" which emit two pulses half a period apart in one pulse period, Rankin (1990) found a remarkable relation between the pulse width W and the pulsar period P (in seconds) for pulsars whose magnetic dipole and rotation axes are orthogonal, viz.

$$W = \frac{2^{\circ}\!.45}{\sqrt{P}} \quad \text{for} \quad \alpha = \pi/2. \tag{1}$$

Here α is the angle between magnetic and rotation axes. This relation (henceforth the Rankin relation) provides a fit to data within $\simeq$0.2% and the observations themselves have errors on the average of $\simeq$4%. Thus Eq. (1) is a rare example of an extraordinarily good fit. In addition, the Rankin relation has also been used (Rankin 1990) to predict α values for some other pulsars which are not interpulsars. These predicted values agree very well with determinations of α based on data about other components in the same pulsars (Rankin 1993). Thus its remarkable fit to the core component data is supported in addition by data on other pulsars. The Rankin relation in our view is one of the most reliable observational relations derived from the radio pulsar data.

The import of the currently accepted "polar cap model" of pulsar radio emission is that the radiation originates from the magnetic polar regions. The polar cap is defined on the stellar surface by the feet of the dipolar magnetic field lines which penetrate the "light cylinder", i.e. a cylinder of radius $cP/2\pi$ with rotation axis as its

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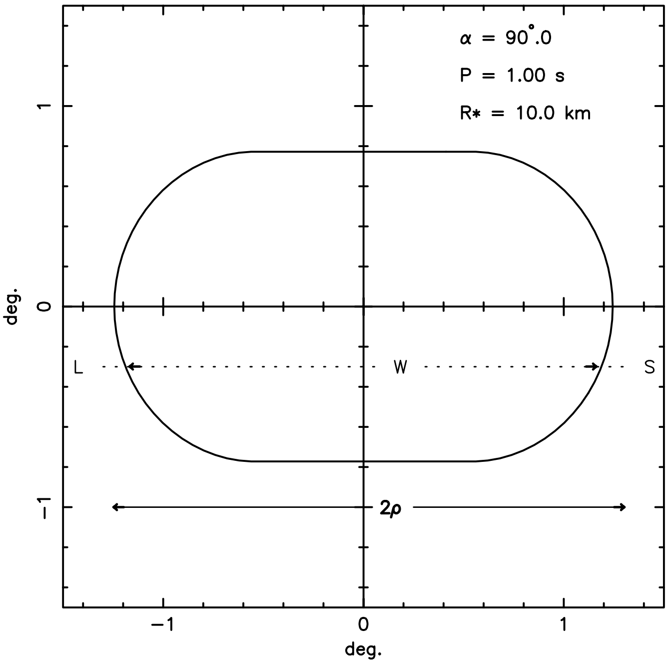

Fig. 1. Polar plot of the polar cap of a typical pulsar. The x and y co-ordinates are the magnetic longitude and lattitude respectively. The centre of the figure represents the radial direction passing through the dipole magnetic axis. Dotted line LS shows the locus of the line of sight as the pulsar rotates. See text for further details.

axis. c is the speed of light. Pulsar emission occurs in this "open field line" flux tube at an altitude r measured radially from the center of the star. We refer to the surface of emission as the *emission cap* which coincides with the polar cap when $r = R_*$ the stellar radius.

As shown in Fig. 1 the line of sight cuts the polar cap along the line LS. This will lead to a pulse of width W. If LS passes through the centre, then $W = 2\rho$, the longitudinal diameter of the polar cap. For interpulsars LS passes very close to the centre and hence $W \simeq 2\rho$. For a value of $\alpha \neq 90°$, 2ρ cannot be recovered from W alone. One also needs to know the displacement of LS from the centre, usually called the impact angle β. If polarization data is available in addition to W, then both β and 2ρ can be retrieved from observations. The core component width data used by Rankin (1990) pertains only to interpulsars. Therefore, in essence the width W in Eq. (1) is the longitudinal diameter 2ρ of the emission cap and is thus independent of α (Kapoor & Shukre 1998, henceforth KS). From the dipole geometry (Goldreich & Julian 1969, henceforth GJ) one finds

$$2\rho = \frac{2^\circ\!.49}{\sqrt{P}}\sqrt{\frac{r}{10\,\mathrm{km}}}. \tag{2}$$

On the assumption that the full emission cap participates in the core emission, agreement between Eqs. (1) and (2) immediately allows the conclusion that $r = 10$ km. This remarkable agreement has provided compelling evidence favouring the origin of the core emission from the stellar surface as well as the dipolar configuration of the stellar magnetic field (Rankin 1993). Note that a value of the stellar radius R_* really has not entered the considerations so far. However, 10 km is considered to be the cannonical value of R_*, and it is on this basis that r is identified with R_*.

3. General relativistic widths and constraints on pulsar mass and radius

In the analysis of radio pulse structure, if the role of the radius R_* has been insignificant, then it is even more so for the stellar mass M. Inclusion of effects due to the space-time curvature caused by pulsar's mass changes this as follows. The stellar gravitational field affects the dipole field geometry and also causes bending of the rays of the emitted pulsar radiation. The former tries to shrink the emission cap while the latter has the opposite effect of widening it. A detailed study of these effects has been done and described in KS. In summary, we give below an analytic but approximate version of how Eq. (2) is modified, i.e.,

$$2\rho = \frac{2^\circ\!.49}{\sqrt{P}}\sqrt{\frac{r}{10\,\mathrm{km}}}\, f_{\mathrm{sqz}}\, f_{\mathrm{bnd}}, \tag{3}$$

where the factors f_{sqz} and f_{bnd} are respectively due to squeezing of the dipole magnetic field and bending of light by the stellar gravitation and are given by

$$f_{\mathrm{sqz}} = (1 + \frac{3m}{2r})^{-\frac{1}{2}}, \quad f_{\mathrm{bnd}} = \frac{1}{3}(2 + \frac{1}{\sqrt{1 - \frac{2m}{r}}}), \tag{4}$$

where $m = \frac{GM}{c^2}$, i.e., $2m$ is the Schwarzshild radius. Eq. (2) is recovered in the limit $m = 0$.

In Eq. (3) the effects due to special relativistic aberration are not included. Since stellar gravitational effects are significant for $r \le 20$ m (KS) we consider only such emission altitudes here. Even for the 1.5 ms pulsar PSR 1929+214, therefore, aberration does not play a role in considerations here. In what follows, however, the calculations include all the effects completely, as in KS. For $M = 1.4\,M_\odot$ and $R_* = 10$ km, the net effect on the emission cap on the surface is a shrinking by ∼4% compared to the value in Eq. (2). Although small, this difference allows us to relate M and R_*, and as we shall see provides tight constraints on the pulsar EOS.

Figure 2 shows the variation of 2ρ with r for various values of M as labelled. The points where the Rankin line intersects the curve for a particular mass M gives for that M the altitude(s) where the core emission must originate.

Generally there are two intersection points, r_1 and r_2, such that $r_1 \le r_2$. In the limiting case $M = M_0$ the two points coalesce. For higher values of M there is no intersection. The mass M_0 is $2.48\,M_\odot$, which we take as $2.5\,M_\odot$. Thus we can conclude that core emission does not occur if $M > M_0$. Probably, this is an indication that *all* radio pulsars have masses $< M_0$ because the incidence

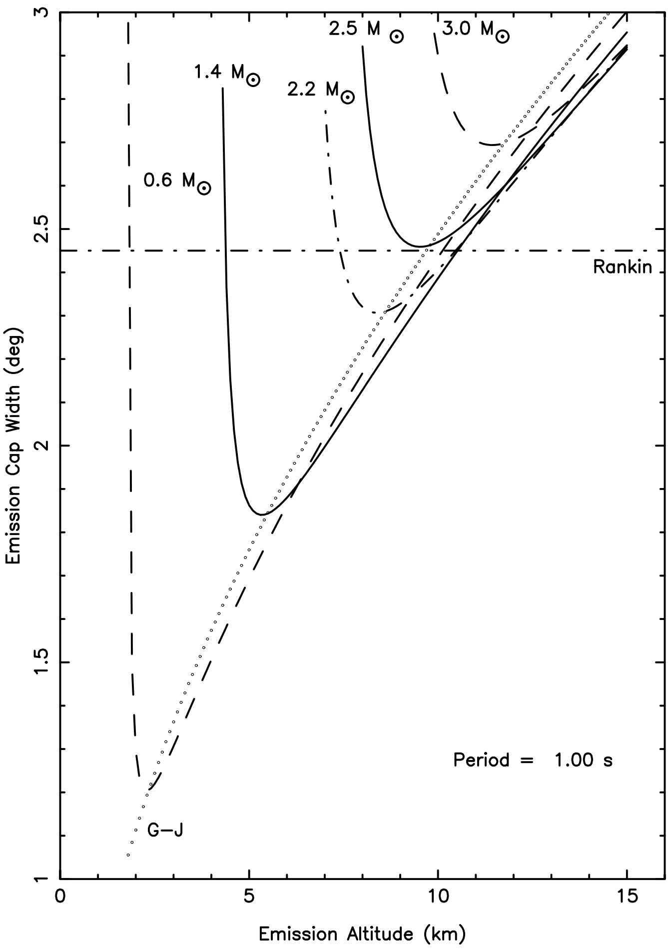

Fig. 2. Width 2ρ of the emission cap after inclusion of all special and general relativistic effects *vs.* the emission altitude r for various stellar masses as shown. Horizontal line is the Rankin relation of Eq. (1) and the dotted line labelled GJ is given by Eq. (2). For a different pulsar period, widths scale as $\sqrt{P}$ as in Eq. (3).

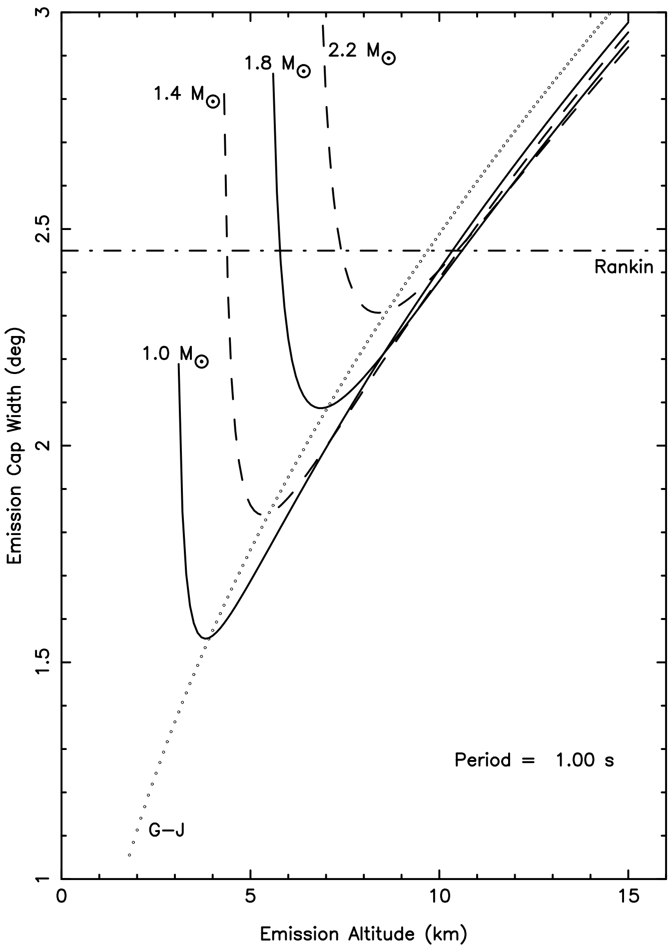

Fig. 3. Same as Fig. 2 but for $1.0\,M_\odot < M < 2.2\,M_\odot$.

of core emission among radio pulsars is $\sim$70% (Rankin 1990). Thus

$$M \leq M_0 \simeq 2.5\ M_\odot. \tag{5}$$

This constraint, though of interest, is not useful since observationally all masses seem to be well below it.

The second constraint involves R_*. The lowest altitude at which any emission can occur is R_*. Therefore for values of M below M_0,

$$R_* \leq r_1 \ \text{ and/or } \ r_2. \tag{6}$$

Since r_1 and r_2 depend on M we get a constraint on the pulsar mass-radius relation from the inequalities 6.

For all masses, values of r_1 are almost same as $2\,m$ and if taken seriously would imply that pulsars are black holes. We therefore consider only r_2. Values of r_2 range from 10.2 to 10.6 km for masses between 0.6 to 2.5 $M_\odot$. For masses between 1 $M_\odot$ and 2.2 $M_\odot$, values of r_2 remarkably enough are not very sensitive to M and are all close to 10.5 km as seen in Fig. 3. Again, pulsar masses are observationally seen to be well covered by the range 1.0–2.2 $M_\odot$ and so we can take 10.5 km as the upper limit for all M. Lower values of r_2 occur for lower values of M and their inclusion will only tighten the constraint further since for all EOS a decrease in mass implies an increase in radius. The Rankin relation thus leads us to the second constraint

$$R_* \ \leq \ 10.5\,\text{km}, \tag{7}$$

which is applicable to radio pulsars which show core emission, and, as remarked earlier, to most probably all pulsars.

4. Constraints and neutron star EOS

We have searched earlier works for neutron star $M-R$ relations. For about 40 EOS $M-R$ plots were available. Very conservatively dropping some among them which are now replaced by modern versions, we have selected the 22 listed in Table 1. For the additional six in Table 2 only the maximum masses ($M_{\max}$) allowed by the EOS and the associated radii are available (Salgado et al. 1994).

For all EOS in Table 2, radii are larger than 10.5 km for $M = M_{\max}$ and thus also for lower values of M. Therefore

Table 1. EOS for which $M-R$ plots are available. For meaning of $M_{\rm min}$ and $M_{\rm max}$ see text.

SN	EOS Name	$M_{\rm min}$ ($M_\odot$)	$M_{\rm max}$ ($M_\odot$)	Plot Ref.[1]
1	A	0.35	1.65	BBF,PC
2	B	0.35	1.40	BBF
3	M	- -	- -	PC
4	L	- -	- -	BBF,PC,BLC
5	WFFAU	0.45	2.15	BLC
6	WFFUU	2.10	2.20	PC,LRD
7	WFFUT	1.65	1.85	BBF
8	FPS	1.60	1.80	PC,BLC
9	HV	- -	- -	W
10	HFV	- -	- -	W
11	G^{π}_{300}	- -	- -	W
12	Hyp	- -	- -	LRD
13	BPAL12	1.35	1.45	LRD
14	BBB1	1.65	1.75	LRD
15	BBB2	1.70	1.90	LRD
16	EOS1	1.50	1.55	BLC
17	EOS2	1.70	1.75	BLC
18	RH	0.15	0.90	HWW
19	RHF	0.15	0.95	HWW
20	APR1	- -	- -	BBF
21	APR2	2.15	2.2	BBF
22	K^-	- -	- -	LRD

[1]References are same as in the reference section at the end: BBF-Benhar et al. (1999); PC-Psaltis & Chakrabarty (1999); BLC-Balberg et al. (1999); LRD-Li et al. (1999b); W-Weber (1999); HWW-Huber et al. (1998).

we consider now the 22 remaining EOS in Table 1. Since high precision is not called for, or, is available, we have read off from the published plots the mass range for which $R_* < 10.5$ km. These values are listed in Table 1 as $M_{\rm min}$ – the mass for which $R = 10.5$ km and $M_{\rm max}$ – the maximum mass allowed by the EOS. Where the EOS does not permit $R_* < 10.5$ km for any mass, only dashes appear for $M_{\rm min}$ and $M_{\rm max}$. There are 8 such EOSs and they are not favored by inequality (7).

Because of the accurately determined masses for the Hulse-Taylor binary system (i.e., 1.44 and 1.39 $M_\odot$) (Thorsett & Chakrabarty 1999), for the remaining EOS we impose an additional condition that their mass range allow the value 1.4 $M_\odot$. The inequality 7 selects out the softer EOS. By imposing this condition based on observations we are in effect demanding that the EOS should not be so soft as to have $M_{\rm max} < 1.4\,M_\odot$ or so stiff that $M_{\rm min} > 1.4\,M_\odot$. This further reduces the number of acceptable EOSs by 11. The remaining three are : A, WFFAU and BPAL12.

The core width constraints in conjunction with the observational information on pulsar masses have thus reduced the viable netron star EOS number from 28 to 3.

Table 2. EOS for which only $M_{\rm max}$ and its radius R are available.

SN	EOS[1] Name	$M^1_{\rm max}$ $M_\odot$	$R(M_{\rm max})^1$ km
1	HKP	2.83	13.68
2	Glend1	1.80	11.15
3	Glend2	1.78	11.29
4	Glend3	1.96	11.30
5	DiazII	1.93	10.93
6	WGW	1.97	10.97

[1]Names and values are from Salgado et al. (1994).

The EOS APR1 is an updated version of the EOS A. APR2 is APR1 with relativistic corrections included. Since both APR1 and APR2 do not survive the constraints we can drop also the EOS A from the short list. In addition, based on general restrictions following from the glitch data, Balberg et al. (1999) have disqualified the EOS A and WFFAU. We are thus left with the choice of BPAL12 or some variant of it as the only viable modern EOS.

We have considered only the non-rotating neutron star models because most pulsars are slow rotators. But inclusion of rotation (or magnetic field) will not change the situation because, in that case, for a given mass one expects larger radii on general physical grounds.

It should be noted that similar attempts using the pulsar timing data (glitches) and X-ray source data (quasi-periodic oscillations) do not provide such stringent constraints and are also not so selective of the EOS (Psaltis & Chakrabarty 1999; van Kerkwijk et al. 1995). Also, our constraints are not dependent on uncertainties in theoretical models, i.e., of accretion disks, and rely on very simple and fundamental assumptions.

Our constraints make crucial use of the Rankin relation and the assumption that the core emission emanates from the full polar cap. *It will be of great interest to re-evaluate both of these independently.* The database presently available is presumably more voluminous than in 1990 because the number of known pulsars has more than doubled since then and it can be used to further fortify the Rankin relation. On the other hand it would be worthwhile also to check the assumption of the participation of the full cap by some independent means. Non-dipolar magnetic field components have been invoked in the past in various contexts (Arons 2000; Gil & Mitra 2000). Our analysis crucially hinges on the Rankin relation, which in turn makes crucial use of the dipole nature of the field. The existence of non-dipolar components has been studied by Arons (1993) and he has concluded against their presence. We take the view that the remarkable agreement of the Rankin relation actually provides evidence for the dipolar nature of the field and strongly indicates the absence of non-dipolar components and also of propagation effects affecting the core emission.

5. Are radio pulsars strange?

In so far as our constraints hold, can we then conclude that BPAL12 is *the* neutron star EOS? Actually BPAL12 is used as an extreme case for illustrative purpose and can hardly be called a realistic netron star EOS (Bombaci 2000). In fact our present knowledge of the neutron star EOS is very far from final. Present theoretical uncertainties in these EOS relate to the very high density regime ($\rho \sim 10^{15}\,\mathrm{gm\,cm^{-3}}$) and are small in terms of pressure. For our constraint, however, these small changes in pressure are significant and can lead to very different radii R_* (See Figs. 2 and 3 in Benhar et al. 1999). The best we can do is to glean from the trend which is visible in the EOS that include the microphysics in the best possible way, i.e., those based on relativistic quantum field theory (Salgado et al. 1994; Prakash et al. 1997), rather than those in which nucleon interactions are described using potentials (as in the BPAL series). These are the EOS in Table 2 and none among these theoretically most advanced EOS are favored by our constraints. (This is also true of similar EOS described in Prakash et al. 1997.) Extrapolating on this trend it would seem that no neutron star EOS can satisfy the inequality (7). This in turn implies that pulsars are not neutron stars[1] and leaves us with the only alternative conceivable at present, that pulsars are strange quark stars. We discuss this next.

Some stars considered so far to be neutron stars have been proposed to be actually strange stars on two counts. The proposals for Her X-1 (Dey et al. 1998), 4U 1820-30 (Bombaci 1997), SAX J1808.4-3658 (Li et al. 1999a), 4U 1728-34 (Li et al. 1999b) are based on the compactness of stars being more than a neutron star can accomodate. From an entirely different viewpoint PSR 0943+10 has been proposed to be a bare strange star (Xu et al. 1999). This last proposal implies that all pulsars showing the phenomenon of drifting subpulses may be bare strange stars.

Pulsars being strange stars fits well with our constraints. Whether pulsars are bare strange stars, strange stars with normal crusts or the newly proposed third family of ultra-compact stars (Glendenning & Kettner 2000) is difficult to decide at present. For the relatively better-studied strange stars, the new EOS for strange stars give radii $\simeq$7 km as opposed to $\simeq$8 km given by earlier EOS based on the MIT bag model (Dey et al. 1999). Xu et al. (1999) propose that pulsars showing the phenomenon of drifting sub-pulses are bare strange stars. Our constraints apply to pulsars showing core emission. However, the core emission and drifting of subpulses which is a property of the conal emission (Rankin 1993; see also Xu et al. 1999) are not mutually exclusive. Therefore the proposal that pulsars are bare strange stars can be extended to all pulsars. Many issues, such as differences between bare strange stars and those with normal crusts etc. remain to be answered, although some answers have been proposed. We do not repeat here this discussion (Xu et al. 1999; Madsen 1999) except to state that our core width constraints are one more independent indication that pulsars are strange stars.

The source SAX J1808.4-3658 has been proposed to be a strange star on the basis of its compactness. In the analysis of Psaltis & Chakrabarty (1999) it was demonstrated that the presence of multipole components relaxes the amount of compactness required, such that the star could be a neutron star. In our analysis also, existence of multipoles (however ad hoc) would dilute our conclusion of pulsars being strange stars. It thus seems that existence of multipoles or the strange star nature of hitherto considered neutron stars are two mutually exclusive choices. At present it is very difficult to choose between them. More work on strange stars may in future elucidate this, but introduction of multipoles brings in so many parameters that how their existence could be proved from observations is unclear. The multipoles would also rob the Rankin relation of its beauty and turn its remarkable observational agreement into a mystery.

[1] Recently, based on general and well-accepted principles it has been shown (Glendenning 2000) that it is possible to have small radii for neutron stars, but none of the known EOS show this. Interestingly, for a radius <10.5 km the maximum mass turns out to be $2.5\,M_\odot$, in close agreement with the inequality (5).

6. Summary

In summary, the empirical formula of Rankin (1990) describing the opening angle of the pulsar beam emitting the core emission when compared to theoretically calculated value leads to a constraint that pulsar masses should be $\leq 2.5\ M_\odot$ and radii $\leq$10.5 km. This comes about due to the inclusion of general relativistic effects of the mass of the star on the pulsar beam size. For observationally reasonable pulsar masses a comparison with mass-radius relations of neutron star EOS shows that most of the EOS are ruled out, implying that pulsars are strange stars and not neutron stars, unless our understanding of the neutron star EOS is revised.

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