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The centrifugal force reversal and X-ray bursts

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Abstract. Heyl (2000) made an interesting suggestion that the observed shifts in QPO frequency in type I X-ray bursts could be influenced by the same geometrical effect of strong gravity as the one that causes centrifugal force reversal discovered by Abramowicz & Lasota (1974). However, his main result contains a sign error. Here we derive the correct formula and conclude that constraints on the $M(R)$ relation for neutron stars deduced from the rotational-modulation model of QPO frequency shifts are of no practical interest because the correct formula implies a weak condition $R_* > 1.3R_\mathrm{S}$, where R_S is the Schwarzschild radius. We also argue against the relevance of the rotational-modulation model to the observed frequency modulations.

Key words. equation of state – relativity – stars: neutron – X-rays: bursts

1. Introduction

It is rather well established that type I X-ray bursts are caused by thermonuclear explosions in the material slowly accreted onto the surface of a neutron star. During the burst, X-ray emission is often quasi-periodic (e.g., Strohmayer et al. 1997a), possibly as a result of rotational modulation of the inhomogeneities in the burning matter. During the outburst the burning layer expands by a factor of about 10 (Joss 1978; Paczyński 1983) and according to Strohmayer et al. (1997b) the subsequent contraction accounts for the observed frequency shifts. Heyl (2000) noticed that in calculating these frequency shifts one should take into account various relativistic effects such as the Lense-Thirring frame-dragging and the Abramowicz-Lasota (Abramowicz & Lasota 1974, 1986; Barrabes et al. 1995) centrifugal-force reversal. Unfortunately Heyl used an unnecessarily complicated derivation of these effects and obtained an incorrect result. Below we obtain the correct formula and discuss under what assumptions it can be applied, if at all, to QPOs frequency shifts observed during type I X-ray outbursts.

2. Frequency shifts

Let us assume, as Heyl did, that as the burning region of the atmosphere (or a hot spot) expands and then contracts, the specific angular momentum of a fluid element that emits observed X-rays is conserved: $l \equiv -u_\varphi/u_t$, where u_φ, u_t are components of the four-velocity.

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In this case, one has a well defined problem to solve: to calculate the change in the angular velocity $\Delta\Omega$, knowing the change in radius Δr (that corresponds to the thickness of the atmosphere), and assuming that the specific angular momentum l is conserved:

$$\Delta\Omega = \left(\frac{\mathrm{d}\Omega}{\mathrm{d}r}\right)_{l=\mathrm{const.}} \Delta r. \tag{1}$$

Because the spin of a neutron star in the rotational-modulation model may be assumed to be small, all calculations can be made in a metric in which only first order rotational corrections to the Schwarzschild metric of a non-rotating, spherical star are considered. With this accuracy, the relevant metric components are (Hartle & Thorne 1968):

$$g_{tt} = -c^2\left(1 - \frac{2M}{r}\right), \tag{2}$$

$$g_{\varphi\varphi} = r^2 \sin^2\theta, \tag{3}$$

$$g_{t\varphi} = -c\frac{2J}{r}\sin^2\theta. \tag{4}$$

Here $M = GM_*/c^2$ is the mass of the star in geometrical units, and $J = GJ_*/c^3$ is its angular momentum, also expressed in geometrical units. From now on, all formulae in this paper are given with the same accuracy to linear terms in rotation. It will be convenient to define the radius of gyration $\tilde{r}$ and the angular velocity of frame dragging ω by

$$\tilde{r}^2 = -\frac{g_{\varphi\varphi}}{g_{tt}} = \frac{r^2\sin^2\theta}{1 - \frac{2M}{r}} \tag{5}$$

$$\omega = -\frac{g_{t\varphi}}{g_{\varphi\varphi}} = \frac{2J}{r^3}, \tag{6}$$

see e.g. Abramowicz et al. (1993) for covariant definition and discussion of the meaning of the radius of gyration.

Heyl (2000) attempted to solve the problem by using a method that was correct in principle, but by far unnecessary complicated: he transformed the metric (2)–(4) to a reference frame that corotates with the star, derived and integrated in this frame the geodesic equation, discussed its solution introducing the Coriolis force, and transformed the solution back to the non-rotating frame.

One may derive the desired result in only one line of calculations, realizing that since $\Omega = u^\varphi/u^t$, one gets from our definition of l and from Eq. (6):

$$\Omega = \frac{l}{\tilde{r}^2} + \omega, \tag{7}$$

to first order in rotational effects.

From this, just by direct differentiation (with l = const., θ = const.) one immediately arrives at,

$$\frac{\mathrm{d}\Omega}{\mathrm{d}r} = -2\frac{\Omega}{r}\left(1 - \frac{2M}{r}\right)^{-1}\left(1 - \frac{3M}{r} + \frac{J}{\Omega r^3}\right)\cdot \tag{8}$$

The sign of the (last) term proportional to J directly follows from the monotonic decrease of ω with radius.

It is remarkable, that to first order in stellar rotation, an initially uniformly rotating shell continues to rotate uniformly as it expands or contracts. Our formula shows that near $r = 3M$ the shift in frequency $\Delta\Omega$ should be smaller than that predicted by Newtonian theory. This has correctly been noted by Heyl (2000), and it reflects the fact, known previously (Abramowicz & Prasanna 1990), that in the gravitational field of a *non-rotating body*, matter which moves on nearly circular orbits with constant angular momentum experiences no shear (in the sense that $\mathrm{d}\Omega/\mathrm{d}r = 0$) exactly at the location of the circular photon orbit ($r = 3M$), i.e., at the location where the centrifugal force reverses. Equation (8) shows that for matter moving on nearly circular orbits with constant angular momentum in the gravitational field of a (slowly) rotating body, zero shear occurs at some radius $r_0 < 3M$, contrary to Heyl's result, but in accordance, e.g., with the well known fact that the greater the angular momentum of the black hole, the smaller the radius of the (corotating) circular photon orbit $r_{\rm ph}$. (Note, however, that only for non-rotating bodies $r_0 = r_{\rm ph}$, in general this is not true, because for rotating bodies $r_{\rm ph}$ depends only on M and J, while r_0 depends, in addition, on Ω.)

In any case, the derivative of Ω in Eq. (8) is positive only for $r < 2.6M + [5I/(2R_*^2) - 1]$. For realistic neutron star models, the moment of inertia is lower than that of a uniform Newtonian sphere, $I < 2MR_*^2/5$, so the zero shear radius is less than the causality limit for neutron star radii, $R_* > 2.8M$ (Haensel et al. 1999).

3. Discussion

We may ask if the formula (8) is relevant to the problem of frequency shifts observed in QPOs during type I X-ray bursts and to the constraints on the mass-radius relation for neutron stars. For this, the Strohmayer et al. (1997b) model relating the QPO frequency to the neutron star's spin and the frequency shift to movements of the atmosphere must be correct, *and* during the expansion and contraction the specific angular momentum l of the QPO source should be conserved. We are skeptical on both counts.

Muno et al. (2001), find that the presence of a ~600 Hz quasiperiodic oscillation in the X-ray emission is correlated with radius expansion in X-ray bursts, but that the presence of a ~300 Hz QPO is not correlated. This in itself raises doubt as to the general validity of the rotational-modulation model of these QPOs. Further, we note that an independent confirmation of the rotational period of the neutron stars is needed before the frequency observed can be identified with the spin rate. Second, no long-lasting inhomogeneities have as yet been demonstrated in computations of the burst evolution. Third, even if the model is correct, there seem to be so many uncertainties involved in the motion of a hot spot in a stellar atmosphere, that we do not see how any frequency change can be reliably identified with the properties of the space-time metric. Suppose that no frequency change is observed. Would we claim that this is because the radius of the star, $r = R$ coincides with the zero-shear radius r_0, or should we rather suppose that the hot spot is stationary at the surface of the rotating star? Is the frequency decrease observed in 4U 1636-53 (Strohmayer 1999), due to $R < r_0$ or is some other effect involved (perhaps the hot spot is rising like a balloon)? Finally, we know of no atmospheric phenomenon on Earth which would resemble the proposed model–castellanus clouds do not show a systematic westerly bend with altitude, and large scale motions which can be ascribed to Coriolis forces, such as the jet stream or trade winds, are related to latitudinal motions of air masses, not vertical.

Is $l = -u_\phi/u_t$ conserved when the QPO is observed? In the case of axially symmetric, stationary motion of a perfect fluid, both $-u_t(P+\rho)/n$ and $u_\varphi(P+\rho)/n$, where P, ρ and n are respectively the pressure, energy and particle densities, are conserved (see, e.g., Bardeen 1973) and because l is simply their ratio, it is conserved as well. It is not clear what relevance this has to the frequency observed in X-ray bursts. After all, if the frequency is related to the stellar rotation it must be related to non-axisymmetry, breaking the basic assumption under which l is conserved for perfect fluids.

A naive reading of the suggestion that the X-ray burst frequency may be caused by a dark or bright fluid element conserving angular momentum as it changes altitude, suggests the motion of a hovercraft. This would be a particle free of azimuthal forces but supported vertically, and the relevant question would be the change of its azimuthal velocity or simply angular frequency, as seen by a distant observer, when the particle changes its radial position. The energy per unit rest mass $-u_t$ of the particle would not be conserved, but its angular momentum per unit rest mass $u_\varphi = l(1-2M/r)^{1/2}$ would, and hence, in this case,

$$\frac{\mathrm{d}\Omega}{\mathrm{d}r} = -2\frac{\Omega}{r}\left(1-\frac{2M}{r}\right)^{-1}\left[1-\frac{5M}{2r}+\frac{J(1-\frac{M}{r})}{\Omega r^3}\right], \qquad (9)$$

could be a better model for frequency shift than (8).

In conclusion, it seems unlikely that Heyl's idea can ever be used to constrain the mass-radius relationship of a neutron star. Unfortunately, it seems that the effect two of us discovered 27 years ago has yet to be observed.

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