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Statistical evaluation of the observational information on Ω_m and Ω_Λ

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Abstract. We undertake a critical evaluation of recent observational information on Ω_m and Ω_Λ in order to identify possible sources of systematic errors and effects of simplified statistical analyses. We combine observations for which the results have been published in the form of likelihood contours in the Ω_m, Ω_Λ plane. We approximate the contours by fifth order polynomials, and we then use the maximum likelihood method to obtain joint likelihood contours for the combined data. In the choice of statistical merits we aim at minimum loss of information rather than at minimum variance. We find that $\Omega_0 = \Omega_m + \Omega_\Lambda = 0.99 \pm 0.04 \pm 0.03$, where the first error is mainly statistical and the second error is systematical. In a flat Universe we find $\Omega_m^{flat} = 0.31 \pm 0.04 \pm 0.04$.

Key words. cosmology: observations – methods: statistical

1. Introduction

Our knowledge of the dynamical parameters of the Universe describing the cosmic expansion has improved rapidly over the last few years, starting with the epochal discovery of the large scale anisotropies of the CMB by COBE-DMR (Smoot et al. 1992), followed by the dramatic supernova Ia observations by the High-z Supernova Search Team (Riess et al. 1998) and the Supernova Cosmology Project (Perlmutter et al. 1998, 1999), and most recently by the measurements of the first acoustic peak in the CMB power spectrum in the first results from the balloon flights BOOMERANG (de Bernardis et al. 2000) and MAXIMA (Balbi et al. 2000; Hanany et al. 2000).

The list of other recent observations is very long, even if one restricts oneself to those having information on both the mass density parameter Ω_m and the density parameter Ω_Λ of vacuum energy. Recall that Ω_Λ is related to the cosmological constant Λ by

$$\Omega_\Lambda = \Lambda/3H_0^2. \quad (1)$$

Lineweaver (1998) and Tegmark (1999) have summarized and analyzed some 20 more observations of the CMB anisotropies (cf. their reference lists). Determinations of Ω_m and Ω_Λ have been reported from observations on the gas fraction in X-ray clusters (Evrard 1997), on X-ray cluster evolution (Bahcall & Fan 1998; Eke et al. 1998), on the cluster mass function and the Lyα forest (Weinberg et al. 1998), on gravitational lensing (Chiba & Yoshi 1998; Helbig 2000; Im et al. 1997), on the Sunyaev-Zel'dovich effect (Birkinshaw 1999; Carlstrom et al. 1999), on classical double radio sources (Guerra et al. 2000), on galaxy peculiar velocities (Zehavi & Dekel 1999), on the evolution of galaxies and star creation versus the evolution of galaxy luminosity densities (Totani 1997). The large scale structure and its power spectrum has been studied in the SSRS2 and CfA2 galaxy surveys (da Costa et al. 1994), in the Las Campanas Redshift Survey (Schectman et al. 1996), in the Abell-ACO cluster survey (Retzlaff et al. 1998), and in the 2dF Galaxy Redshift Survey (Peacock et al. 2001).

Some of the above information has already been used to constrain Ω_m and Ω_Λ, some of it could in principle be used that way, but has not been presented in a form readily useful to an analyst outside the observer teams. It must also be said that much is statistically weak, the analyses being simplified and the discussions of possible systematic errors absent.

Nevertheless, the list of large combined data analyses since 1999 is already long. Lineweaver (1999) combined the SN Ia data with CMB data, X-ray cluster data, cluster evolution data and double radio sources. Le Dour et al. (2000) analyzed only CMB data, whereas Tegmark et al. (2001) combined CMB data with IRAS LSS data. Tegmark & Zaldarriaga (2000a, 2000b) and Hu et al. (2000) combined BOOMERANG and MAXIMA data, Melchiorri et al. (2000) combined BOOMERANG and COBE data. Bridle et al. (2000) combined the CMB data with galaxy peculiar velocities and with the SN Ia data. The BOOMERANG, MAXIMA and COBE data

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have been combined with LSS and SN Ia data by Jaffe et al. (2000) and Bond et al. (2000), and combined with a different set of LSS data by Novosyadlyj et al. (2000a) and Durrer & Novosyadlyj (2000). In a sequence of papers we (Roos & Harun-or-Rashid 1998, 1999, 2000) combined much of the data quoted above having published an error on Ω_m and Ω_Λ, using simple χ^2 analysis. This of course implied believing in the errors and treating them as Gaussian.

The conclusion of all these partially overlapping analyses is that the Universe is consistent with being flat, $\Omega_0 = \Omega_\mathrm{m} + \Omega_\Lambda$ is near unity, and $\Omega_\Lambda/\Omega_\mathrm{m}$ is near 2. But the analyses differ in methods, in the treatment of errors and in their attention to possible systematic errors, so the results differ in the precision of these conclusions.

We undertake here yet another combined study using the maximum likelihood method, and paying special attention to statistical arguments. In Sect. 2 we discuss statistical methods in general and present our method of analysis. In Sect. 3 we discuss the data chosen for our analysis, all of which have been published graphically as likelihood contours in the $\Omega_\mathrm{m}, \Omega_\Lambda$ plane. In Sect. 4 we give our estimates for the parameters, and in Sect. 5 we discuss some related analyses.

2. Statistical methods

The observational data generally contain information on a large set of parameters. The information about each single parameter is then obtained by fixing some parameters at known input values, and marginalizing over others. Note that it is quite misleading to report the values of each parameter in turn, always carrying out unconditional marginalizing over all the others, because then the same information has been reused many times. Already when marginalizing to obtain the value of the first parameter one has used all the information available.

There is only one remedy to this: if one is mainly interested in the values of a small subset of parameters, in our case two, one should describe their joint pdf by confidence contours (or surfaces or hypersurfaces) in the space of those parameters, marginalizing over less interesting ancillary parameters. The confidence range of the second parameter is then conditional on the range of the first parameter, and so on. In practice all the conditional confidence ranges are determined by the size of the orthogonal box circumscribed around the two- (or higher-) dimensional confidence contour.

In the present analysis we are only interested in the values of Ω_m and Ω_Λ, therefore we only use data for which the marginalization over ancillary parameters has already been carried out. Note that thereby we do use the full information of each observation.

In several observations it has been noted that some parameters are strongly correlated. If one marginalizes over one of a pair of correlated parameters, the likelihood function of the other one becomes quite broad. This is an effect we do not try to avoid, because it implies including one type of systematic error.

Let us make a few comments of statistical nature comparing the least squares or χ^2 method with the maximum likelihood method. The advantage of the least squares method is its simplicity, it is unbiased, and a goodness-of-fit value can be obtained by comparing the least squares sum with the number of degrees of freedom. The disadvantage is that it requires the pdf of the input data to be symmetric, preferably normal, but even then there is no guarantee that the final estimate will be normally distributed, except asymptotically. To form the least squares of very conflicting data is statistically meaningless.

In contrast, the log-likelihood functions of any data can be added, and if there are conflicts, due for instance to systematic errors, they will show up as several dips. No goodness-of-fit value can be obtained (thereby one does not risk statistically meaningless statements), but relative confidence levels can be defined. There are no restrictions to the symmetry or normality of the pdf of the input data. Asymptotically the maximum likelihood estimator attains normality faster than the least squares estimator.

When one compromises between different statistical merits, one may conclude that it is more desirable to achieve minimum loss of information than minimum variance. In the first case one wants to make sure that no other single number could contain more information about the parameter of interest than the estimate chosen. In the second case one feels that the smaller the variance, the more certain one is that the estimate is near the true value. If one opts for minimum loss of information, the maximum likelihood estimator is optimal in the asymptotic limit.

A problem that occurs in some of the data we use is that the pdf extends into an unphysical region, specifically the region $\Omega_\mathrm{m} < 0$. Even the region $\Omega_\Lambda < 0$ might be considered unphysical in this context. To simply ignore the unphysical region biases the pdf and produces a systematic error. The remedy to this is the method proposed by Feldman & Cousins (1998). We, however, cannot apply their method to the data we use, it has to be done at the time of original data analysis. We shall only give mention where it has been done, and where it should have been done.

Since none of the likelihood contours in the data we use look normal nor even symmetric, we clearly choose the maximum likelihood method. (The tool to use is actually not the likelihood function which is the product of individual pdf's, but the negative of the sum of their logarithms, or the log-likelihood function.) We approximate the contours by general fifth order polynomials of the form

$$P(\Omega_\mathrm{m}\Omega_\Lambda) = \Omega_\mathrm{m}^m \Omega_\Lambda^n, \quad m + n \leq 5. \tag{2}$$

There are then 20 terms in the polynomials, so we read off 20 points from the 1σ, 1.64σ, 2σ, 3σ contours and the best value, where available. Since we already know the approximate location of the globally favored region from all the previous studies, it is enough that we require our polynomial approximation to be good over that region. This fit region is defined by $0.15 \leq \Omega_{\rm m} \leq 0.50$ and $0.40 \leq \Omega_\Lambda \leq 0.88$, but the sample points are of course taken also from outside this region in order to obtain a well-behaved polynomial inside the region. Far away from it the polynomial approximation of course breaks down completely.

One should recognize that the observational likelihood surfaces in the $\Omega_{\rm m}, \Omega_\Lambda$ plane are not known with a very good resolution. Thus one cannot set very high requirements on the polynomial representation inside the 1σ contours. We have been checking that the polynomial is non-negative in the fit region, and that its minimum (i.e. of the negative log-likelihood function) is indeed at the observational best value, where reported. We have also been checking the location of the 0.33σ contour of the polynomial approximation, in order to verify that it is reasonably centrally located with respect to the 1σ contour and to the best value, when reported.

3. Data

Altogether we use six independent data sets meeting our criteria, grouped into SN Ia data, CMB data, LSS data and other data. But as we shall see, some of these data sets actually comprise several other important observations.

3.1. Supernova Ia data

The SN Ia observations by the High-z Supernova Search Team (HSST) of Riess et al. (1998) and the Supernova Cosmology Project (SCP) of Perlmutter et al. (1998, 1999) are well enough known not to require a detailed presentation here. The importance of these observations lies in that they determine approximately the linear combination $\Omega_\Lambda - \Omega_{\rm m}$ which is orthogonal to $\Omega_0 = \Omega_{\rm m} + \Omega_\Lambda$.

HSST use two quite distinct methods of light-curve fitting to determine the distance moduli of their 16 SNe Ia under study. Their luminosity distances are used to place constraints on five cosmological parameters: $h, \Omega_{\rm m}, \Omega_\Lambda, q_0$, and the dynamical age of the Universe, t_0. The MLCS method involves statistical methods at a more refined level than the more empirical template model. The moduli are found from a χ^2 analysis using an empirical model containing four free parameters. The MLCS method and the template method give moduli which differ by about 1σ. Once the distance moduli are known, the parameters h, $\Omega_{\rm m}$, Ω_Λ are determined by a maximum likelihood fit, and finally the Hubble constant is integrated out. (The results are really independent of h.) One may perhaps be somewhat concerned about the assumption that each distance modulus is normally distributed. We have no reason to doubt that, but if the iterative χ^2 analysis has yielded systematically skewed pdf's, then the maximum likelihood fit will amplify the skewness.

The authors state that "the dominant source of statistical uncertainty is the extinction measurement". The main doubt raised about the SN Ia observations is the risk that (part of) the reddening of the SNe Ia could be caused by intervening dust rather than by the cosmological expansion. Among the possible systematic errors investigated is also that associated with extinction. No systematic error is found to be important here, but for such a small sample of SNe Ia one can expect that the selection bias might be the largest problem.

The authors do not express any view about which method should be considered more reliable, thus noting that "we must consider the difference between the cosmological constraints reached from the two fitting methods to be a systematic uncertainty". We shall come back to this question later. Here we would like to point out that if one corrects for the unphysical region $\Omega_{\rm m} < 0$ using the method of Feldman & Cousins (1998), the best value and the confidence contours will be shifted slightly towards higher values of $\Omega_{\rm m}$. This shift will be more important for the MLCS method than for the template method, because the former extends deeper into the unphysical region.

The likelihood contours in the $\Omega_{\rm m}, \Omega_\Lambda$ plane (Riess et al. 1998, Fig. 6) correspond to 68.3%, 95.4% and 99.7% confidence, respectively (1σ, 2σ, 3σ).

Let us now turn to SCP, which studied 42 SNe Ia. The MLCS method described above is basically repeated, but modified in many details for which we refer the reader to the source. The distance moduli are again found from a χ^2 analysis using an empirical model containing four free parameters, but this model is slightly different from the HSST treatment. The parameters $\Omega_{\rm m}$ and Ω_Λ are then determined by a maximum likelihood fit to four parameters, of which the parameters $\mathcal{M}_B$ (an absolute magnitude) and α (the slope of the width-luminosity relation) are just ancillary variables which are integrated out (h does not enter at all). The authors then correct the resulting likelihood contours for the unphysical region $\Omega_{\rm m} < 0$ using the method of Feldman & Cousins (1998). The likelihood contours in the $\Omega_{\rm m}, \Omega_\Lambda$ plane (Perlmutter et al. 1999, Fig. 7) correspond to 68% (1σ), 90% (1.64σ), 95% (1.96σ), and 99% (2.58σ) confidence, respectively.

Since the number of SNe Ia is here so much larger than in HSST, the effects of selection and of possible systematic errors can be investigated more thoroughly. SCP quotes a total possible systematic uncertainty to $\Omega_{\rm m}^{\rm flat}$ and $\Omega_\Lambda^{\rm flat}$ of 0.05.

If we compare the observations along the line defining a flat Universe, SCP finds $\Omega_\Lambda - \Omega_{\rm m} = 0.44 \pm 0.085 \pm 0.05$, whereas HSST finds $\Omega_\Lambda - \Omega_{\rm m} = 0.36 \pm 0.10$ for the MLCS method and $\Omega_\Lambda - \Omega_{\rm m} = 0.68 \pm 0.09$ for the template method. This comparison tells us that the template method is afflicted by systematic errors of its own. We choose the former since this method is basically the same as used by SCP. This is admittedly a selection bias of ours, but we shall account for it, at least partly, by applying the same systematic error of $\Delta\Omega_{\rm m}^{\rm flat} = \Delta\Omega_\Lambda^{\rm flat} = 0.05$ to HSST as to SCP.

SCP and HSST then agree within their statistical errors – how well they agree cannot be established since they are not completely independent. Part of the difference may be explained by selection bias in the smaller set of SNe Ia.

In Fig. 1 we show the confidence contours of the log-likelihood sum of the two SN Ia observations in our polynomial approximation, drawn only in the ranges of Ω_m and Ω_Λ that we sample. Along the flat line these observations determine $\Omega_\Lambda - \Omega_m = 0.45 \pm 0.13$. Note that our value is not obtained from the weighted mean of the SCP and HSST values, but from our log-likelihood sum.

There are many other types of observations which give complementary information in support of the SN Ia data. These observations have been summarized briefly by Perlmutter et al. (1998, 1999), and in more detail for instance by us (Roos & Harun-or-Rashid 1998, 1999, 2000). But there have also been gravitational lensing data in strong conflict with the SN Ia data. The best value in our Fig. 1 is excluded with 99.7% confidence by the joint optical (spiral galaxies) and radio data of Falco et al. (1998) (six gravitational lenses analyzed). However, the authors point out that these results depend on the choice of galaxy sub-type luminosity functions in the lens models. Subsequently Chiba & Yoshii (1999) have emphasized this point in an analysis with E/S0 luminosity functions that yielded a best fit mass density in a flat cosmology, finding $\Omega_\Lambda - \Omega_m = 0.4 + 0.2/-0.4$ in agreement with the SN Ia data.

More recently Helbig (2000) has shown preliminary results from the Cosmic Lens All-Sky Survey (CLASS) of radio lenses only. These results are still inaccurate, $-0.8 < \Omega_\Lambda - \Omega_m < 0.3$ at 95% confidence, but appear to be in strong conflict with the SN Ia data. It is still too early to say whether SNe Ia will have to come down to smaller values of $\Omega_\Lambda - \Omega_m$, or whether gravitational lensing will have to go up. In Sect. 5 we shall discuss how our combined fit changes when including the preliminary CLASS constraints.

3.2. CMB data

Before the advent of the balloon observations BOOMERANG (de Bernardis et al. 2000) and MAXIMA-1 (Balbi et al. 2000; Hanany et al. 2000), Lineweaver (1998) and Tegmark (1999) analyzed all the then existing CMB data in the form of multipole power spectra up to $\ell \simeq 800$. The parameter space is then very large, but some parameters really drop out and others can be handled in a simplified manner if their effect is significant only below or above $\ell \simeq 100$. In the analysis of Tegmark (1999), the following ten cosmological parameters are jointly constrained: Ω_k, Ω_Λ, the optical depth parameter τ, the amplitudes and slopes A_s, n_s, A_t, n_t of scalar and tensor fluctuations, and the physical matter densities $\omega_b, \omega_{cdm}, \omega_\nu$. Of these parameters only six are well constrained; the resulting 6-dimensional likelihood function is then integrated over remaining parameters, and it is stated to be highly non-Gaussian in some directions. We trust that the plotted marginalized 2-dimensional confidence limits in

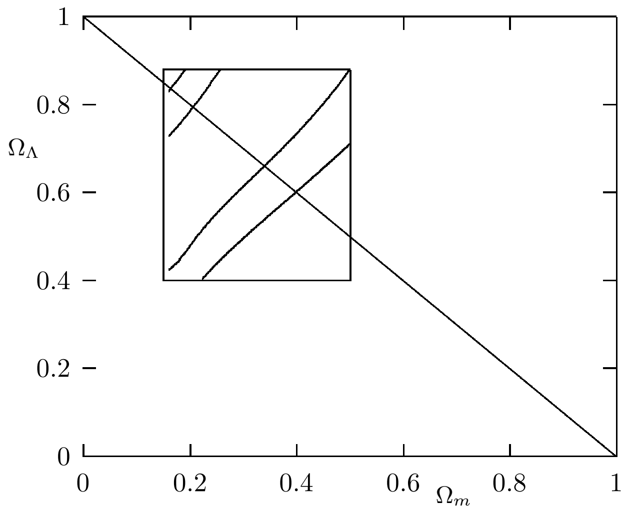

Fig. 1. The confidence contours of the log-likelihood sum of the two SN Ia observations (HSST and SCP). The curves correspond to 1σ and 2σ in the $(\Omega_m, \Omega_\Lambda)$-plane. The significance of the square is described in the text. The diagonal line corresponds to a flat cosmology.

the Ω_m, Ω_Λ plane are then realistic, and do not contain any imposed Gaussian form.

The approximations made are claimed to reproduce the power spectrum to about 5% accuracy. Otherwise no systematic errors are discussed. But the input data show rather large scatter, so one might hope that by combining them, most of the systematic differences between them have been taken into account. One worry is that the best fitting models all fail to quite match the COBE DMR data (Tegmark 1999).

The likelihood contours in the Ω_m, Ω_Λ plane from this compilation (Tegmark 1999, Fig. 3) correspond to 68% (1σ) and 95% (1.96σ) confidence, respectively. No best value point is given.

The balloon observations BOOMERANG (de Bernardis et al. 2000) and MAXIMA-1 (Balbi et al. 2000; Hanany et al. 2000) have produced the first high-resolution, high signal-to-noise maps of the CMB from independent patches of the sky, and thereby derived the angular power spectrum with a better precision than was achievable in compilations of earlier observations. In both cases the so far published results represent a complete analysis of a limited portion of the data.

Given the multipole spectrum, Balbi et al. (2000) fit different 7-dimensional CDM models to some pixelization of the measured range of l, including the COBE DMR data. The jointly constrained cosmological parameters are $\tau, \Omega_b, \Omega_m, \Omega_\Lambda, n_s$ and C_{10}, the amplitude of fluctuations at multipole $\ell = 10$. For a seventh parameter they alternatively used h and $\Omega_b h^2$. Marginalizing over five parameters, the confidence range in the Ω_m, Ω_Λ plane were found. The first acoustic peak is the dominant feature in the power spectrum, the maximum occurring at $\ell = 197 \pm 6$ in BOOMERANG and at $\ell \simeq 220$ in MAXIMA-1. The position of the peak determines Ω_0 pretty independently of all other parameters; thus for a flat universe it determines Ω_m.

BOOMERANG (de Bernardis et al. 2000) jointly constrain a six-dimensional parameter space, comprising $h, \Omega_\mathrm{b}h^2, \Omega_\mathrm{m}, \Omega_\Lambda, n_\mathrm{s}$ and an overall normalization A. Subsequently they marginalize over four parameters to obtain the confidence range in the $\Omega_\mathrm{m}, \Omega_\Lambda$ plane plotted in their Fig. 3.

The difference in Ω_0 between the two BOOMERANG and MAXIMA-1 is rather large, since $\ell \simeq 200\Omega_0^{-1.58}$ for $\Omega_\mathrm{m} = 0.3$ and Ω_0 near 1 (Weinberg 2000). (In the literature one sometimes sees the relation $\ell \simeq 200\Omega_0^{-0.5}$ which is true only when $\Omega_\Lambda = 0$.) This difference clearly represents a systematic error which should be allowed to affect the total fit. All other systematic errors discussed have less influence on Ω_m and Ω_0. The likelihood contours in the $\Omega_\mathrm{m}, \Omega_\Lambda$ plane from MAXIMA-1 (Balbi et al. 2000) correspond to 68% (1σ), 95% (1.96σ), and 99% (2.58σ) confidence, respectively, which we can use. No best value point is given.

BOOMERANG has only published a coarsely pixelized likelihood surface of 95% (1.96σ) confidence (de Bernardis et al. 2000, Fig. 3), which would make our polynomial fit a poor approximation. Therefore we do not include BOOMERANG as an independent constraint. However, it is included together with the LSS constraint to be discussed in the next subsection, so the BOOMERANG information is not neglected.

A totally different approach is taken by Jaffe et al. (2000). The two data sets both have a calibration uncertainty, 20% for BOOMERANG and 8% for MAXIMA-1, which Jaffe et al. (2000) uses to adjust the power spectra so that the peaks are more similar in amplitude. The data can then be combined into multipole bands, and the goodness-of-fit improves considerably. However, since the relative importance of the two peaks is then altered, the combined data yields a significantly shifted Ω_m, and the originally visible systematic difference in Ω_m disappears. We fear that this leads to an underestimation of the Ω_m systematic error. As stressed before, we prefer to exhibit all systematic errors in order to achieve minimum loss of information.

3.3. LSS and other data

An important source of information on Ω_m and Ω_0 is the power spectrum of matter density fluctuations. Durrer & Novosyadlyj (1999) have chosen to study this for Abell-ACO clusters (Retzlaff et al. 1998), arguing that the power spectrum of clusters should better represent the observed Universe as a whole, rather than density fluctuations on the scale of galaxies. Making this choice may represent a bias implying ignoring some inherent systematic error, but this seems the only choice for us at the moment.

Novosyadlyj et al. (2000a) combine the power spectrum of Abell-ACO clusters with six independent constraints for the amplitude of the fluctuation power spectrum on different scales: from clusters at different redshifts, from quasar spectra, and from the bulk flow of galaxies in our vicinity (cf. their reference list). Very importantly, they also use the CMB value $\ell = 197 \pm 6$ for the multipole moment of the first acoustic peak from BOOMERANG which thus gets included into our data base. In addition they constrain the Hubble constant to be $h = 0.65 \pm 0.10$, a low but reasonable compromise value with a generous error. They constrain the baryon density to be $\Omega_\mathrm{b}h^2 = 0.020 \pm 0.002$ (95%CL) (Burles et al. 2001).

The MAXIMA-1 and BOOMERANG values for $\Omega_\mathrm{b}h^2$ are in stark contradiction to that value, but this conflict does not affect the value of Ω_0 noticeably. In fact, the outcome of these authors' multiparameter least squares fit to the cluster power spectrum yields a baryon density more similar to the MAXIMA-1 and BOOMERANG value, and yields a higher Hubble constant closer to present best determinations.

It is not clear whether any systematic errors are included by Novosyadlyj et al. (2000a). However, the fit errors are rather large anyway, e.g. $\Omega_\mathrm{m} = 0.37^{+0.25}_{-0.15}$ marginalized over all other parameters, so that the influence of systematic errors would be small. The information in Novosyadlyj et al. (2000b), Fig. 1a are the $1\sigma, 2\sigma$ and 3σ confidence contours in the $\Omega_\mathrm{m}, \Omega_\Lambda$ plane, obtained by marginalization over the other five parameters (and assuming that only one species of massive neutrinos contribute to the neutrino density parameter Ω_ν).

Our last constraint is based on the measurements of the coordinate distance to sources at redshifts between 1 and 2, which depends on the global values of the cosmological parameters, but which is independent of the power spectrum of density fluctuations, and of whether matter is biased relative to light. From a parent population of 70 powerful extended classical double radio galaxies Guerra et al. (2000) have studied a subset of 20, for which it was possible to estimate independently the mean and characteristic size, and thus the coordinate distance. There is one model parameter β in the theory in addition to Ω_m and Ω_Λ. The main systematic error is the model uncertainty in β, but it is shown to be unimportant compared to known statistical errors, and quite uncorrelated to Ω_m and Ω_Λ.

The information in Guerra et al. (2000), Fig. 11, are the 68% (1σ) and 90% (1.64σ) likelihood contours in the $\Omega_\mathrm{m}, \Omega_\Lambda$ plane, obtained by marginalizing over β. The favored region is quite large, so that this constraint is at present quite weak, adding only some preference for small Ω_m values. A best value is claimed at $\Omega_\mathrm{m} = 0$ and $\Omega_\Lambda = 0.45$. It is not clear to us how the unphysical region $\Omega_\mathrm{m} < 0$ has been treated, in any case not with the Feldman–Cousins (1998) procedure.

In Fig. 2 we show the confidence contours of the log-likelihood sum of the data sets discussed in this subsection and in the CMB subsection. We have plotted our polynomial approximation only in the ranges of Ω_m and Ω_Λ that we sample. As can be clearly seen, the likelihood function contains information mainly on Ω_0, but it also gives

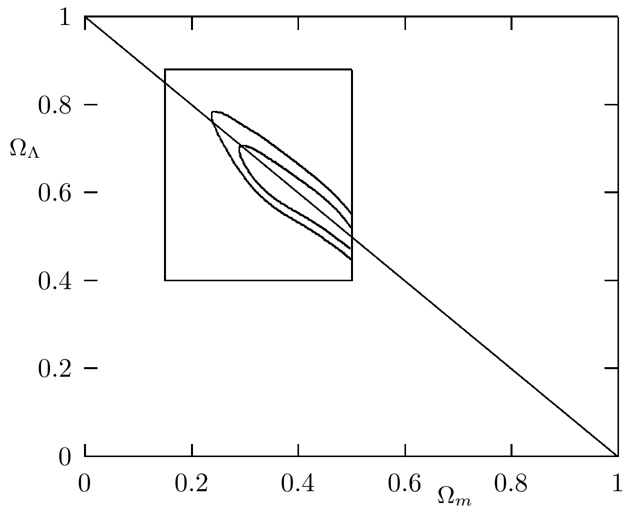

Fig. 2. The confidence contours of the log-likelihood sum of MAXIMA-1, early CMB, Double Radio Galaxies, and LSS (including BOOMERANG and various other independent constraints). The curves correspond to 1σ and 2σ in the $(\Omega_m, \Omega_\Lambda)$-plane. The significance of the square is described in the text. The diagonal line corresponds to a flat cosmology.

a rather conspicuous upper limit on the orthogonal combination $\Omega_\Lambda - \Omega_m$.

4. Results

Adding up our polynomial approximations to the confidence contours of all the data discussed in the previous section, results in Fig. 3, where we show the location of the minimum, the 1σ and 2σ contours. From this figure one can read off the following results:

$$\Omega_m = 0.31^{+0.12}_{-0.09} \quad (3)$$
$$\Omega_\Lambda = 0.68 \pm 0.12, \quad (4)$$

or alternatively

$$\Omega_0 = 0.99 \pm 0.04 \quad (5)$$

$$\Omega_\Lambda - \Omega_m = 0.37^{+0.20}_{-0.23}. \quad (6)$$

Of these results, only the determination of Ω_0 is quite precise and worth detailed attention. We can conclude from it that a flat universe with $\Omega_0 = 1$ is very likely.

To the results in Fig. 3 we have to add some further quantifiable systematic errors which we evaluate as follows.

As mentioned earlier, Perlmutter et al. (1998, 1999) have quoted a total systematic error for $\Omega_m^{\rm flat}$ and $\Omega_\Lambda^{\rm flat}$ along the flat line of ± 0.05. We consider that the same error should be applied to the SN Ia data of HSST, where a similar evaluation did not give a significant result due to the limited sample of SNe Ia. As explained in Sect. 3.1, this is further motivated by the discord between the MLCS method and the template method. Displacing both the SN Ia contours by ± 0.05 along the flat line, we obtain a very small systematic error in the Ω_0 direction

$$\Delta_1(\Omega_0) = {}^{+0.012}_{-0.006}. \quad (7)$$

There are also two kinds of systematic errors inherent to our method of analysis. Firstly, we are reading off the coordinates of the confidence contours of the different observations with some finite precision. We estimate this precision to be

$$\Delta_2(\Omega_0) = 0.027. \quad (8)$$

Secondly, since we only use 20 points to fit the confidence contours of each observations, there is an arbitrariness in their choice; all we require is that the confidence contours should be well fitted by whichever polynomial. We have tested this polynomial arbitrariness and found that it results in the systematic error

$$\Delta_3(\Omega_0) = 0.01. \quad (9)$$

The quadratic sum of the errors in Eqs. (7), (8), (9) is then

$$\Delta_{\rm tot}(\Omega_0) = 0.03. \quad (10)$$

Thus our final result for Ω_0 is

$$\Omega_0 = 0.99 \pm 0.04 \pm 0.03, \quad (11)$$

where the first error is statistical and the second error systematical. Thus our total error is ± 0.05.

Let us now turn to the case of exact flatness, $\Omega_m = 1 - \Omega_\Lambda$. Along the flat line the SN Ia systematic error is

$$\Delta_1(\Omega_0)^{\rm flat} = \pm 0.025. \quad (12)$$

Our result is then

$$\Omega_m^{\rm flat} = 0.31 \pm 0.04 \pm 0.04, \quad (13)$$

where the first error is statistical and the second error systematical. Thus our total error here is ± 0.055. Note once again that this systematic error is not included in Fig. 3.

5. Discussion

The closest comparison we can make with other analyses of Ω_0 is that of Jaffe et al. (2000), who combine BOOMERANG, MAXIMA-1, COBE DMR, the SN Ia constraints of SCP (but not HSST), and another LSS input than we have used. Also, the earlier CMB data compiled by Lineweaver (1998) and Tegmark (1999), and the information from the double radio sources are not used. They find

$$\Omega_0 = 1.06 \pm 0.04, \quad (14)$$

to be compared with our result in Eq. (11). The difference in central value is easy to understand, and due to two causes. The main cause is the way BOOMERANG and MAXIMA-1 have been combined, as we explained already

Table 1. The observational data used in the fifth order polynomials are summarized.

Observation	*Reference*	*Source*
SN Ia: HSST	Riess et al. (1998)	Fig. 6
SN Ia: SCP	Perlmutter et al. (1999)	Fig. 7
CMB: MAXIMA-1	Balbi et al. (2000)	Fig. 3
CMB compilation	Tegmark (1999)	Fig. 3
Double Radio Gal.	Guerra et al. (2000)	Fig. 11
LSS	Novosyadlyj et al. (2000b)	Fig. 1a
LENSING	Helbig (2000)	Fig. 3

in the Data section. A small shift in the same direction is due to our inclusion of other input.The statistical errors are the same, but we have in addition a systematic error, part of which is not applicable to the Jaffe et al. (2000) analysis.

A rather similar analysis is that of Durrer & Novosyadlyj (2000), who find $\Omega_{\rm m} + \Omega_\Lambda \approx 1.06$ and $\Omega_k \approx -0.06$, where our definition of Ω_0 corresponds to $\Omega_{\rm m} + \Omega_\Lambda + \Omega_k = 1$. On the flat line these authors find $\Omega_{\rm m} = 0.35 \pm 0.05$. We also note that these authors have taken into account all the observational constraints we used in our previous analyses (Roos & Harun-or-Rashid 1998, 1999, 2000), and which we therefore did not refer to explicitly here.

One further constraint which we did not make use of here is the position of the peak in the matter power spectrum of quasars at $z \approx 2$ as observed by Roukema & Mamon (2000). The reason for the omission is that their likelihood contours in the $\Omega_{\rm m}, \Omega_\Lambda$ space are so jagged that our polynomial approximations just cannot reproduce them. To quote one result, they find

$$\Omega_{\rm m}^{\rm flat} = 0.30 \pm 0.15 \qquad (15)$$

in good agreement with us. The error here is so large that the inclusion of this result would not have changed our conclusions.

As we mentioned in Sect. 3.1, Helbig (2000) (Fig. 3) has plotted preliminary lensing constraints from the Cosmic Lens All-Sky Survey (CLASS) of radio lenses, which appear to be in strong conflict with the SNe Ia data. A joint analysis is statistically quite meaningless, but excluding one or the other is a biased choice. Since we have so far included the SNe Ia data and excluded the lensing data, let us now include both. The result is shown in Fig. 4. One notes that the best value then moves to $\Omega_{\rm m} = 0.34^{+0.11}_{-0.03}$, $\Omega_\Lambda = 0.63^{+0.04}_{-0.10}$, or alternatively $\Omega_0 = 0.97^{+0.05}_{-0.04}$, $\Omega_\Lambda - \Omega_{\rm m} = 0.29^{+0.05}_{-0.18}$. We note that this best value is excluded by the SNe Ia data at 1σ CL, and it is excluded by the lensing data at 97% CL.

We conclude from Sect. 4 that Ω_0 is equal to unity to within ± 0.05 and that $\Omega_{\rm m}^{\rm flat} = 0.31$ to within ± 0.055.

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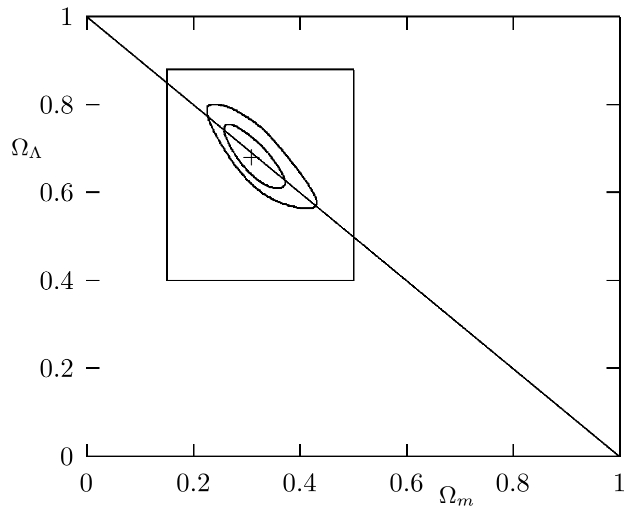

Fig. 3. Figures 1 and 2 combined. The "+" marks the best fit: $(\Omega_{\rm m}, \Omega_\Lambda) = (0.31, 0.68)$. The diagonal line corresponds to a flat cosmology.

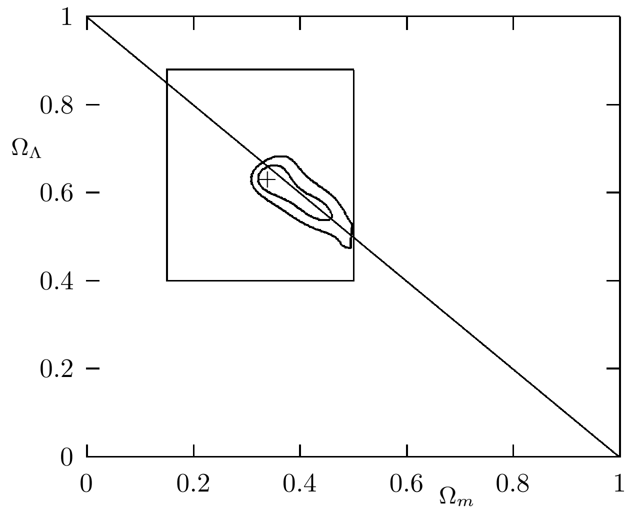

Fig. 4. LENSING (Helbig 2000) is combined with Fig. 3. The "+" marks the best fit: $(\Omega_{\rm m}, \Omega_\Lambda) = (0.34, 0.63)$. The diagonal line corresponds to a flat cosmology.

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