A&A 372, 583–587 (2001)
DOI: 10.1051/0004-6361:20010520


Astronomy & Astrophysics

# On the nature of the residual magnetic fields in millisecond pulsars

D. Konenkov[1,2] and U. Geppert[2]

[1] A.F. Ioffe Institute of Physics and Technology, Politechnicheskaya 26, 194021 St. Petersburg, Russia
[2] Astrophysikalisches Institut Potsdam, An der Sternwarte 16, 14482 Potsdam, Germany
e-mail: urme@aip.de

Received 5 March 2001 / Accepted 2 April 2001

**Abstract.** We consider the expulsion of proton fluxoids along neutron vortices from the superfluid/superconductive core of neutron star with weak ($B < 10^{10}$ G) magnetic field. The velocity of fluxoids is calculated from the balance of buoyancy, drag and crustal forces. We show, that the proton fluxoids can leave the superfluid core sliding *along* the neutron vortices on a timescale of about $10^7$ years. An alternative possibility is that fluxoids are aligned with the vortices on the same timescale. As the result, non–aligned surface magnetic fields of millisecond pulsars can be sustained for $\gtrsim 10^9$ years only in case of a comparable dissipation timescale of the currents in the neutron star crust. This defines upper limits of the impurity concentration in the neutron star crust: $Q \lesssim 0.1$ if a stiff equation of state determines the density profile.

**Key words.** stars: neutron – stars: magnetic fields

## 1. Introduction

There are observational evidences that the magnetic fields (MFs) of millisecond pulsars (MSPs) are stationary over the age of $\gtrsim 10^9$ years. The spin-down ages of many MSPs and the cooling ages of their white dwarf companions exceed $10^9$ years (Kulkarni 1986; Danziger et al. 1993; Kulkarni et al. 1991; Bell et al. 1995). MSP statistics indicates also, that the ages of MSPs must be greater than $10^9$ years (Bhattacharya & Srinivasan 1991 and references therein).

We assume that both protons and neutrons are in a superfluid state in the whole core (Baym et al. 1969). The softer the EOS and the more massive the NS, the larger is the probability that protons and neutrons in the central region of the core are in a normal state (Page 1998a), which implies a completely other core field evolution (see e.g. Goldreich & Reisenegger 1992). On the other hand, the millisecond pulsars are very old NSs. Their internal temperature is expected to be much less than any non–zero superfluid transition temperature (Page 2001, private communication), favouring the superfluid state in the entire core.

The superfluid phase transition takes place early in NS's life (Page 1998b) and the MF penetrates the core as an array of proton flux tubes (fluxoids), each of them carrying a quantum of the magnetic flux $\Phi_0 = hc/2e \approx 2 \times 10^{-7}$ G cm$^2$. Inside the fluxoids the MF is as high as $B_{\rm p} \sim 10^{15}$ G, and it decays outwardly exponentially with the London penetration depth $\lambda \sim 10^{-11}$ cm. The core superfluid rotates forming a discrete array of neutron vortices. The kernels of vortices and fluxoids consist of normal matter, their radii correspond to the coherence lengths $\xi_{\rm n}$ and $\xi_{\rm p}$, respectively.

Because of a superfluid drag effect the neutron vortices are also magnetized (Alpar et al. 1984). The MF inside each vortex is $B_{\rm n} \sim 10^{15}$ G. When a vortex crosses a fluxoid, the interaction of their MF results in a potential barrier, $E_{\rm m} \sim 10$ MeV per intersection (Ding et al. 1993, DCC hereafter), corresponding to a pinning force $F_{\rm m} \sim E_{\rm m}/\lambda \sim 10^6$ dyn per intersection.

The interaction energy due to density perturbations in the center of a fluxoid is $E_{\rm p} \sim 0.1$–1 MeV per intersection and causes a pinning force $F_{\rm p} \sim E_{\rm p}/\xi_{\rm n}$, which is in the same order of magnitude as $F_{\rm m}$ (Sauls 1989).

The strong interaction between neutron vortices and proton fluxoids led many authors to develop models which consider the MF evolution together with the NS spin–down. Srinivasan et al. (1990) and Jahan–Miri & Bhattacharya (1994) calculated the expulsion of the magnetic flux from the core assuming that fluxoids and vortices always move with the same velocity. DCC and Jahan–Miri (2000) calculated the magnetic evolution in more realistic models, in which other forces, acting on the

*Send offprint requests to*: D. Konenkov,
e-mail: dyk@astro.ioffe.rssi.ru

fluxoids (buoyancy, drag, tension) were taken into consideration. Konenkov & Geppert (2000, 2001) (KG00, KG01 hereafter) extended these models by taking into account the back–reaction of the crust ("crustal" forces) onto the rate of flux expulsion. All these authors reported the occurrence of a long–living ($\sim 10^{10}$ years) low MF component, which could be responsible for a non–decaying "residual" MF of MSPs.

In the present letter we calculate the flux expulsion timescale in the core of MSPs when the fluxoids slide *along* (parallel) to the neutron vortices, i.e., when the interpinning between vortices and fluxoids can be neglected. This possibility was first mentioned by Muslimov & Tsygan (1985). We show, that this timescale is about $10^7$ years.

## 2. Description of the model

Besides the force exerted by the neutron vortices, there exist other forces which act onto the fluxoids in the NS core. The buoyancy force, acting per unit length of the fluxoid, is given by (Muslimov & Tsygan 1985):

$$\boldsymbol{f}_{\mathbf{b}} = \left(\frac{\Phi_0}{4\pi\lambda}\right)^2 \frac{1}{R_{\mathrm{c}}} \ln\left(\frac{\lambda}{\xi_{\mathrm{p}}}\right) \boldsymbol{e}_{\mathrm{r}}, \quad (1)$$

where $R_{\mathrm{c}}$ is the radius of the NS core and $\boldsymbol{e}_{\mathrm{r}}$ is the unit vector in radial direction when spherical coordinates are used. The buoyancy force acts always radially outward.

The drag force, which arises due to the scattering of the relativistic electrons on the MF of the fluxoid, is given by (Harvey et al. 1986; Jones 1987):

$$\boldsymbol{f}_{\mathbf{v}} = -\frac{3\pi}{64}\frac{n_{\mathrm{e}} e^2 \Phi_0^2}{E_{\mathrm{F}}\lambda}\frac{\boldsymbol{v}_{\mathrm{p}}}{c}, \quad (2)$$

where $n_{\mathrm{e}}$ and $E_{\mathrm{F}}$ are the number density and the Fermi energy of electrons, $c$ is the speed of light, and $\boldsymbol{v}_{\mathrm{p}}$ the velocity of the fluxoid's motion through the core. The drag force acts opposite to the velocity of the fluxoid. This equation is valid only if collective effects are ignored (see DCC).

In MSPs the proton fluxoids are indeed expected to be unable to cut through the neutron vortices, and the motion of arbitrarily oriented fluxoids in the direction perpendicular to the rotational axis occurs only on the spin–down timescale of MSPs. However, the displacement of the fluxoid parallel to the neutron vortex does not change the pinning energy, thus the movement of the fluxoids parallel to the vortices is not restricted by the pinning force. There is a component of the buoyancy force, $\boldsymbol{f}_{\mathrm{b}||}$, which is not compensated locally by the pinning force (Fig. 1). Under the action of this non–compensated component, proton fluxoids can be either expelled from the core (line 1 in Fig. 1), or aligned with the neutron vortices (line 2 and 2′ in Fig. 1).

The total flux of the Poynting vector through the crust–core interface is equal to the power of forces, acting upon the fluxoids in the core (for details see KG00):

$$\sum_{\text{fluxoids}} \int (\boldsymbol{f}_{\mathrm{b}||} + \boldsymbol{f}_{\mathrm{v}}) \cdot \boldsymbol{v}_{\mathrm{p}} \mathrm{d}l = -\frac{c}{4\pi} \int_{S_{\mathrm{core}}} [\boldsymbol{E} \times \boldsymbol{B}] \cdot \mathrm{d}\boldsymbol{S}_{\mathrm{core}}, \quad (3)$$

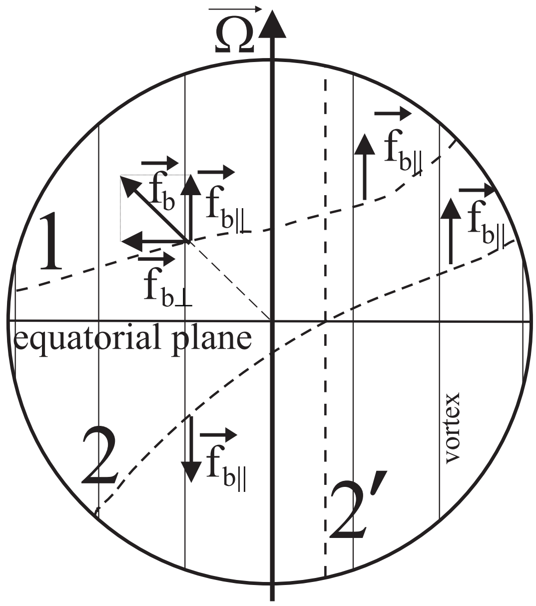


**Fig. 1.** The superfluid core of a NS rotates with the angular velocity Ω by forming array of neutron vortices (shown in the figure by thin vertical lines). Fluxoids are shown by dashed lines. The fluxoid 1, which does not intersect the equatorial plane of the NS, will be expelled from the core along the vortices under the action of noncompensated component of the buoyancy force $\boldsymbol{f}_{\mathrm{b}||}$, while its component $\boldsymbol{f}_{\mathrm{b}\perp}$ will be compensated by the vortex acting force. The fluxoid 2, which intersects the equatorial plane, will become aligned with the vortices into position 2′. The timescales of both processes are the same.

where the integral on the l.h.s. is taken over the length of a fluxoid, the summation runs over all fluxoids, and the integration on the r.h.s. is performed over the crust–core interface. For simplicity we assume that $f_{\mathrm{b}||} = f_{\mathrm{b}}$ (this simplification leads to an underestimation of the time of expulsion in the order of unity), that the number density of fluxoids is uniform, and calculate all quantities at the density of the crust–core interface. Thus we estimated the integral on the l.h.s. of Eq. (3) as $4/3 \cdot (f_{\mathrm{b}} + f_{\mathrm{v}}) v_{\mathrm{p}} N_{\mathrm{p}} R_{\mathrm{c}}$ (see KG00), where $N_{\mathrm{p}} = \pi B_{\mathrm{c}} R_{\mathrm{c}}^2/\Phi_0$ is the total number of fluxoids and $B_{\mathrm{c}}$ the strength of the mean core MF. Note, that Eq. (3) contains no force exerted by neutron vortices, and describes the motion of fluxoids along vortices.

We assume the MF to be axisymmetric, poloidal, and dipolar outside the NS. Thus, one can introduce the vector–potential $A = (0, 0, A_\phi)$, where $A_\phi = S(r,t)\sin(\theta)/r = B_0 R^2 s(r,t)\sin(\theta)/r$, $s$ is Stokes' stream function, normalized to $B_0 R^2$, $B_0$ and $R$ are the initial surface MF strength and radius of the NS, respectively. The integral on the r.h.s. of Eq. (3) is given by

$$\frac{c}{4\pi} \int_{S_{\mathrm{core}}} [\boldsymbol{E} \times \boldsymbol{B}] \cdot \mathrm{d}\boldsymbol{S}_{\mathrm{core}} = \frac{B_0^2 R^4}{6} \frac{\partial s(R_{\mathrm{c}},t)}{\partial t} \frac{\partial s(R_{\mathrm{c}},t)}{\partial r} \cdot \quad (4)$$

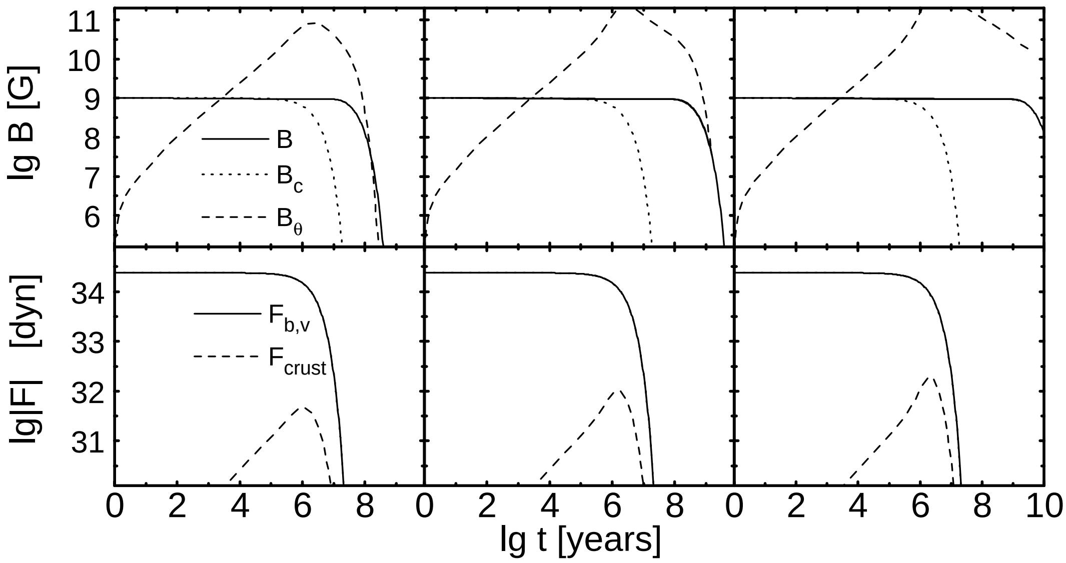


**Fig. 2.** The top panels show the evolution of MF strengths in the core ($B_\mathrm{c}$), at the surface ($B$), and at magnetic equator at the crust-core interface ($B_\theta$). The bottom panels show forces, acting on the fluxoids in the core of MSP. Left, middle and right columns correspond to $Q = 0.1, 0.01$ and $0.001$.

We introduce the total core forces $F_\mathrm{b,v} = f_\mathrm{b,v} \cdot 4R_\mathrm{c}/3 \cdot N_\mathrm{p}$, and the "crustal" force, which arises when the roots of the fluxoids are moved along the crust–core interface:

$$F_\mathrm{crust} = \frac{B_0^2 R^4}{6} \frac{1}{v_\mathrm{p}} \frac{\partial s(R_\mathrm{c}, t)}{\partial t} \frac{\partial s(R_\mathrm{c}, t)}{\partial r}. \quad (5)$$

Equation (3) can be rewritten as

$$F_\mathrm{b} + F_\mathrm{v}(v_\mathrm{p}) + F_\mathrm{crust}(v_\mathrm{p}) = 0. \quad (6)$$

The "crustal" force $F_\mathrm{crust}$ depends on $v_\mathrm{p}$ in a rather complicated way, since both $\partial s(R_\mathrm{c}, t)/\partial t$ and $\partial s(R_\mathrm{c}, t)/\partial r$ depend on $v_\mathrm{p}$. The evolution of the homogeneous MF within the core is governed by $v_\mathrm{p}$ too (for details see KG00). $F_\mathrm{crust}$ might be found by solving the induction equation in the NS crust ($R_\mathrm{c} < r < R$):

$$\frac{\partial s}{\partial t} = \frac{c^2}{4\pi\sigma} \left( \frac{\partial^2 s}{\partial r^2} - \frac{2s}{r^2} \right), \quad (7)$$

where $\sigma$ is the conductivity of NS crust. The crustal conductivity is determined by collisions of the electrons on impurities and phonons. For the phonon conductivity we use the numerical data given by Itoh et al. (1993), for the impurity conductivity we apply the analytical expression derived by Yakovlev & Urpin (1980). The magnitude of the impurity conductivity is determined by the impurity parameter $Q$, whose value is highly uncertain and has to be varied. The impurity conductivity dominates in the inner crust, where currents, generated by the flux expulsion, are expected to be localized.

The boundary conditions are given by $s(R,t)/R = -\partial s(R,t)/\partial r$ at the surface which joins the MF inside the NS with the dipolar field in the vacuum outside, and by Eq. (6) at the crust–core interface. The initial condition is set by $B_0$ and the initial $s$–profile in the NS.

## 3. Results

In Fig. 2 we show the evolution of the MF strengths and of the forces acting on the fluxoids for $B_0 = B_{\mathrm{c}0} = 10^9$ G and an initial spin period $P_0 = 0.001$ s. The results in the left, middle and right column are obtained for an impurity parameter $Q = 0.1, 0.01$ and $0.001$, respectively. The computations were performed for a NS model, based on the Friedman-Pandharipande equation of state (EOS), with NS mass $M = 1.4\ M_\odot$, radius $R = 10.4$ km, and thickness of the crust $\Delta R = 934$ m (Van Riper 1991).

The velocity of the fluxoids remains almost constant ($\approx 7 \times 10^{-9}$ cm/s) during the MSP evolution for all values of $Q$ considered here. This is because in Eq. (6) $F_\mathrm{crust}$ can be neglected in comparison with $F_\mathrm{b}$ and $F_\mathrm{v}$ during the whole evolution. It is seen from Fig. 2, that $F_\mathrm{crust} \lesssim 0.01(F_\mathrm{b}, F_\mathrm{v})$ and the fluxoid velocity is determined only by the balance of buoyancy and drag forces. The expulsion timescale of the MF from the core, $R_\mathrm{c}/v_\mathrm{p}$, is $\lesssim 10^7$ years for all values of $Q$ in the crust, whereas the decay timescale of the surface MF is dependent on $Q$, and can be estimated by $10^7/Q$ years for the given EOS. Note, that the timescale of the alignment must be in the same order of magnitude as the expulsion timescale.

An interesting feature is the generation of a strong $\theta$–component of the MF at the crust–core boundary. Depending on $Q$ it may exceed the surface field strength by up to three orders of magnitude when the flux expulsion proceeds. This effect reflects the induction of currents close to the crust–core interface, which are the stronger the longer the characteristic decay time in the crust is.

We have performed similar computations for $B_0 = 10^8$ and $10^{10}$ G and values of the impurity parameter in the interval $10^{-4} < Q < 0.1$. We obtained the same results: the expulsion timescale of the flux from the core is $\lesssim 10^7$ years, the timescale of decay of the surface MF is $10^7/Q$ years.

That means that the expulsion timescale does not depend on the MF strength and $Q$ if $B_0 \leq 10^{10}$ G. This is because both $F_{\rm b}$ and $F_{\rm v}$ are proportional to the core MF strength, while $F_{\rm crust}$ is negligible. Our estimate for the flux expulsion timescale in case of MF strengths, characteristic for MSPs, is consistent with that of Jones (1987). Note, however, that for standard pulsars with $B \sim 10^{12}$–$10^{13}$ G, the expulsion timescale is dependent on the MF strength. This is because $F_{\rm crust}$, which is proportional to the $B_{\rm c}^2$, is then the dominating force, which hampers the expulsion of the flux from the core of canonical pulsars (KG00) and determines the expulsion timescale, balancing $F_{\rm b}$. Though our model simplifies the real magnetic configuration, we expect that it gives the correct estimate for the expulsion timescale *along* the rotational axis ($10^7$ years). In contradiction, the expulsion *across* that axis occurs on the spin–down timescale, which for MSPs exceeds $10^9$ years.

## 4. Discussion

If in MSPs the dissipation timescale of crustal currents is less than $10^9$ years, their surface MF would vanish. Alternatively, the surface MF of MSPs can be aligned with their rotational axis on timescales of $<10^9$ years. Both effects would make MSPs unobservable as radiopulsars. The observed non–aligned long–living surface MFs of MSPs can be sustained only if the dissipation timescale of currents in the NS crust is $\gtrsim 10^9$ years. Because of large differences in the thickness of the crust, for a soft EOS the decay timescale of the crustal MF is roughly given by $10^6/Q$ years, while for stiff EOS that relation reads $10^8/Q$ years (Urpin & Konenkov 1997). Thus, for a soft EOS and a thin crust the impurity parameter must be as low as $Q \lesssim 10^{-3}$, while for a stiff EOS with a thick crust $Q$ can be much higher ($Q \lesssim 0.1$).

The models of flux expulsion, developed by DCC, KG00, KG01, and Jahan-Miri (2000) for standard pulsars, give an inadequate low velocity of fluxoids at the late evolutionary stages, resulting in the appearance of long–living residual MF components. The main reason is that the equations of balance of forces, which determine the velocity of fluxoids (see Eq. (31) in DCC, Eq. (11) in Jahan-Miri 2000, Eq. (6) in KG00), are written in scalar form, whereas the forces are vectors. These equations describe quantitatively only the evolution of the component of the fluxoid velocity directed perpendicular to the vortices. We now rediscuss the papers KG00, KG01 and show, that this is a good approximation for two evolutionary stages. I) In relatively young ($t < 10^4$ years) standard radiopulsars with $B \sim 10^{12}$–$10^{13}$ G this velocity component is determined by the balance of vortex acting force $F_{\rm n}$ and the drag force $F_{\rm v}$, and is about $10^{-8}$–$10^{-7}$ cm/s (KG00). The force exerted by fast outward moving neutron vortices exceeds the buoyancy force. Thus we expect that the component of $v_{\rm p}$ perpendicular to the rotational axis would also exceed the parallel component of $v_{\rm p}$. II) Later, when $F_{\rm n} \ll (F_{\rm b}, F_{\rm crust})$, $F_{\rm b}$ is balanced by $F_{\rm crust}$ thus determining the expulsion timescale. Then, the fluxoids are almost not affected by the vortices when they are moving towards the crust–core interface across them. At this stage the bulk of the flux is expelled from the core, and we conclude that the timescale of expulsion was estimated correctly.

When almost all magnetic flux is expelled from the core, the vortex acting force is balanced by the perpendicular component of buoyancy, thus hampering further flux expulsion *across* the vortices. However, expulsion may proceed *along* the vortices on a timescale of $\sim 10^7$ years. The appearance of the residual MF in the above cited papers seems to be a qualitatively wrong result. We do not expect any unaligned magnetic flux to remain in the cores of old ($10^9$–$10^{10}$ years) NSs with low ($B \leq 10^{10}$ G) surface MFs.

## 5. Conclusion

The basic idea of this paper is that the motion of proton fluxoids parallel to the neutron vortices is not restricted by the pinning force, since the displacement of (even pinned) fluxoid parallel to the vortex does not change the pinning energy. This approach is basically different from those which consider the occurrence of residual fields by the balance of buoyancy and vortex acting forces at the late stage of NS evolution.

The velocity of fluxoids in the superfluid core of the NS with low MF ($B \leq 10^{10}$ G) is calculated. It has been shown, that the fluxoids escape from the core or become aligned with the neutron vortices on a timescale of $10^7$ years, determined by the balance of the buoyancy and drag forces. This timescale is independent of the MF strength and of the crustal conductivity. The surface MF of the MSPs can be maintained on the timescale of $\gtrsim 10^9$ years only if the dissipation timescale of the crustal currents is also $\gtrsim 10^9$ years. Therefore, an estimate of the impurity parameter $Q$ is possible. Once the NS core matter behaves according a soft EOS this gives a limit of $Q < 0.001$, for a stiff EOS $Q < 0.1$, counteracting the slower field decay in a thicker NS crust. Because of the relatively weak MF in MSPs neither the spin–down, nor the "crustal" force play an important role for the flux expulsion.

Many uncertainties affect the estimation of $Q$, both for the crust of isolated and accreting neutron stars. However, it is very unlikely that the impurity parameter is as small as $10^{-3}$. De Blasio (1998, 2000) argues that $Q$ is relatively large in that layers of the crust of isolated NSs, where the chemical composition changes; in average the impurity parameter should be larger than $10^{-3}$. The calculations of the chemical composition and impurity content of the crusts of accreting NSs show that $Q$ can be even as high as $\sim 100$ (Schatz et al. 1999). Since it is very likely that MSPs went through a phase of intense accretion, this indicates – under reserve that the suppositions of our model are fulfilled – that at least not too soft EOS should describe the state of NS core matter. Such a conclusion has been drawn also from the observation of kilohertz quasi-periodic oscillations in several low mass X-ray binaries (Kluźniak 1998) as well as from the consideration of the magneto–rotational and thermal evolution of isolated NSs with crustal MFs (Urpin & Konenkov 1997; Page et al. 2000).

Clearly, the scenario will be qualitatively changed when a portion of the core's volume remain in the normal state or the forces acting on the fluxoid (Eq. (2)) have to be modificated due to collective effects.

*Acknowledgements.* The authors are grateful to an anonymous referee for useful comments. A discussion with Andreas Reisenegger is gratefully acknowledged. The work of D. K. was supported by a scholarship of the Alexander von Humbold–Stiftung, and by RFBR grant 00-02-04011.

## References

Alpar, M., Langer, S., & Sauls, J. 1984, ApJ, 282, 533
Baym, G., Pethick, C., & Pines, D. 1969, Nature, 224, 673
Bell, J., Kulkarni, S., Bailes, M., Leitch, E., & Lyne, A. 1995, ApJ, 452L, 121
Bhattacharya, D., & Srinivasan, G. 1991 in Neutron Stars: Theory and Observation, ed. J. Ventura, & D. Pines (Kluwer Academic Publishers)
Danziger, I., Baade, D., & della Valle, M. 1993, A&A, 276, 382
De Blasio, F. V. 1998, MNRAS, 299, 118
De Blasio, F. V. 2000, A&A, 353, 1129
Ding, K., Cheng K., & Chau, H. 1993, ApJ, 408, 167
Goldreich, P., & Reisenegger, A. 1992, ApJ, 395, 250
Harvey, J., Ruderman, M., & Shaham, J. 1986, Phys. Rev. D., 33, 2084
Itoh, N., Hayashi, H., & Koyama, Y. 1993, ApJ, 418, 405
Jahan-Miri, M. 2000, ApJ, 532, 514
Jahan-Miri, M., & Bhattacharya, D. 1994, MNRAS, 269, 455
Jones, P. 1987, MNRAS, 228, 513
Kluźniak, W. 1998, ApJ, 509, L37
Konenkov, D., & Geppert, U. 2000, MNRAS, 313, 66
Konenkov, D., & Geppert, U. 2001, MNRAS, accepted [astro-ph/0103060]
Kulkarni, S., Djorgovski., S., & Klemola, A. 1991, ApJ, 367, 221
Kulkarni, S. 1986, ApJ, 306L, 85
Muslimov, A., & Tsygan, A. 1985, SvAL, 11, 80
Page, D. 1998, in Neutron Stars and Pulsars, ed. N. Shibazaki, N. Kawai, S. Shibata, & T. Kifune (Universal Academy Press: Tokyo), 183.
Page, D. 1998, in The Many Faces of Neutron Stars, ed. A. Alpar, R. Buccheri, & J. van Paradijs (Kluwer Academic Publishers), 539
Page, D., Geppert, U., & Zannias, T. 2000, A&A, 360, 1052
Sauls, J. 1989, in Timing Neutron Stars, ed. H. Ogelman, & E. P. J. van den Heuvel (Dordrecht: Kluwer), 48
Schatz, H., Bildstein, L., & Cumming, A. 1999, ApJ, 524, 1014
Srinivasan, G., Bhattacharya, D., Muslimov, A., & Tsygan, A. 1990, Curr. Sci., 59, 31
Urpin, V., & Konenkov, D. 1997, MNRAS, 292, 167
Van Riper, K. 1991, ApJS, 75, 449
Yakovlev, D., & Urpin, V. 1980, Sov. Astron., 24, 303