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Period-doubling behavior in very slow classical cepheid models

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Abstract. In a search of nonlinear models for very long-period Cepheids, we found models with period-doubling behaviors. The feature is quite similar to that mentioned by Carson & Stothers (1984). Here, we report the main features of these models, and briefly discuss the origin of the period-doubling behavior.

Key words. stars: variables: Cepheids stars: variables: general-stars: oscillations

1. Introduction

Nonlinear pulsation models for long-period Cepheids are constructed to compare light curves with observations in the Galaxy and the Magellanic Clouds (Aikawa & Antonello 2000a, 2000b). Some of the models with very long periods show period-doubling behaviors. In the past, Carson & Stothers (1984) reported period-doubling-like behaviors in their very slow Cepheid models. They mentioned that the models showed some variabilities in light curves between cycles. Unfortunately, they did not report the details of these models, including linear non-adiabatic properties. We believe that the feature found in the present models is quite similar to that reported by Carson & Stothers (1984).

There is observational evidence of period-doubling behavior in very long-period Cepheids. Genderen (1983) reported that some very long-period Cepheids in the Magellanic Clouds showed large fluctuations in the light curve between cycles. A good example is HV 883 (period = 133 days). Figure 1 shows observed light curve of HV 883. The observational data are folded with a 266 day interval. We can see the alternation of slightly different light variation with a 133 day period, especially at light minima. Figure 2 shows the Lomb normalized periodogram (Lomb 1976; Press et al. 1992) of the star, obtained from the same data as Fig. 1. The main peak at 0.0075 corresponds to the main pulsation period of 133 days. We note that there are some significant sub-harmonic components, while the peak at the exact value of twice the main pulsation period is unclear. We suspect that the pulsation of HV 883 may be in a chaotic regime in the period doubling cascade (Buchler & Kovács 1987).

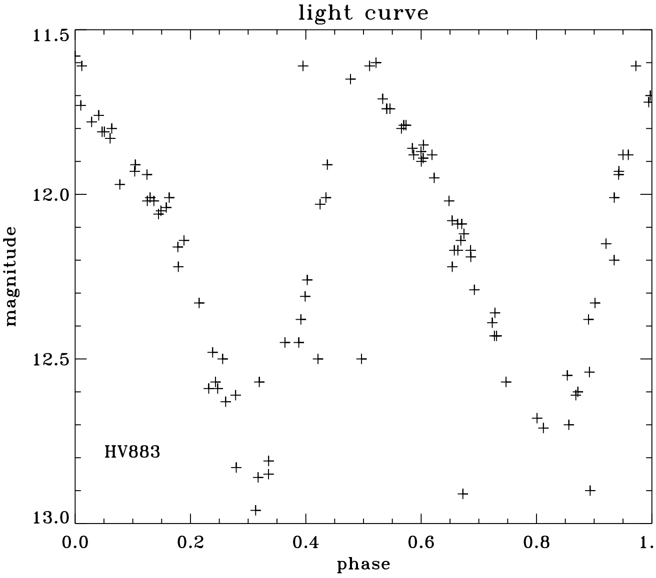

Fig. 1. Observed light curve of HV883. Ref.: Madore, B.F. 1975, ApJS, 29, 219; Eggen, O.J. 1977, ApJS, 34, 33; Martin, W.L. & Warren, P.R. 1979, SAAO Circulars, No. 4; Van Genderen, A. M. 1983, A&AS, 52, 423; Freeman, W.L., Grieve, G.R., & Madore, B.F. 1985, ApJS, 59, 311

Meanwhile Moskalik & Buchler (1991) found that some Cepheid models with moderately long periods (say, 25–40 days) showed period-doubling behaviors. Their detailed analysis shows that the sub-harmonic resonance between the fundamental and first overtone modes may cause the behavior.

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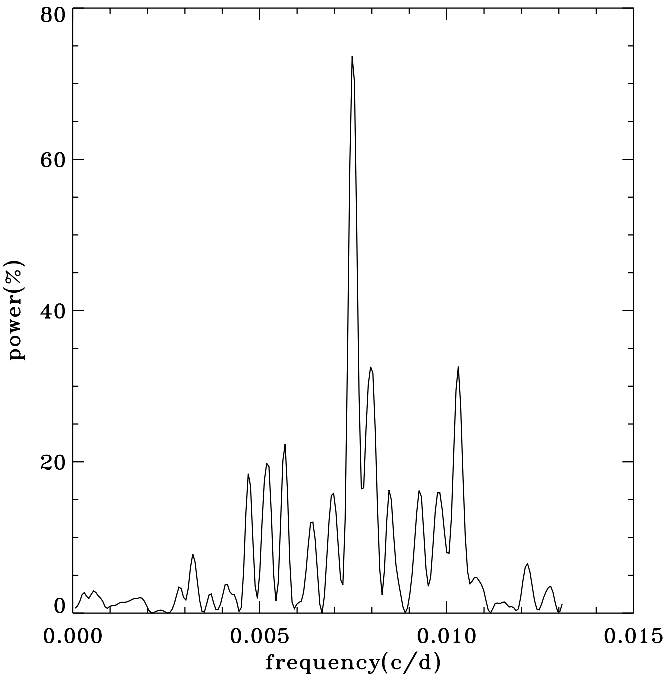

Fig. 2. The Lomb normalized periodogram of HV 883 obtained using the data quoted in Fig. 1. The abscissa is the frequency with a unit of cycles per days, and the ordinate is the spectral power normalized by the variance of the data. The unit of the spectral power is per cent

Here we summarize the characteristics of the variabilities in the present models, and briefly discuss the origin of these variablities.

2. Models

Nonlinear models were constructed with the code TGRID (Simon & Akiawa 1983). This code has a facility of rezoning adapted to temperature changes in the vicinity of the hydrogen ionization region, and gives smooth light curves. The adopted chemical composition was $X = 0.70$ and $Z = 0.02$, and the OP opacity (s92 380) supplied by the OP project (Seaton et al. 1994) and OPTFIT code (Seaton 1993) for fitting and smoothing of the opacity table are used. We entirely ignore the effects of convection in the envelope, and the diffusion approximation is used for radiative transfer. The Richtmyer-Morton formula of the artificial viscosity is applied with $C_q = 4$. Under the same assumptions, we construct static models and linear models as well as nonlinear simulations. We made numerous fine zoning models with about 250 zones.

The mass and luminosity are fixed, with $M = 12\ M_\odot$, and $\log L/L_\odot = 4.71$. We used the mass-luminosity relation with a convective core over-shooting type (Chiosi 1990). Then, a model sequence is constructed changing the values of the effective temperature. Table I shows the characteristics of the sequence along with the results of a linear nonadiabatic analysis of several radial modes. It is noted that only the fundamental mode is pulsationally unstable.

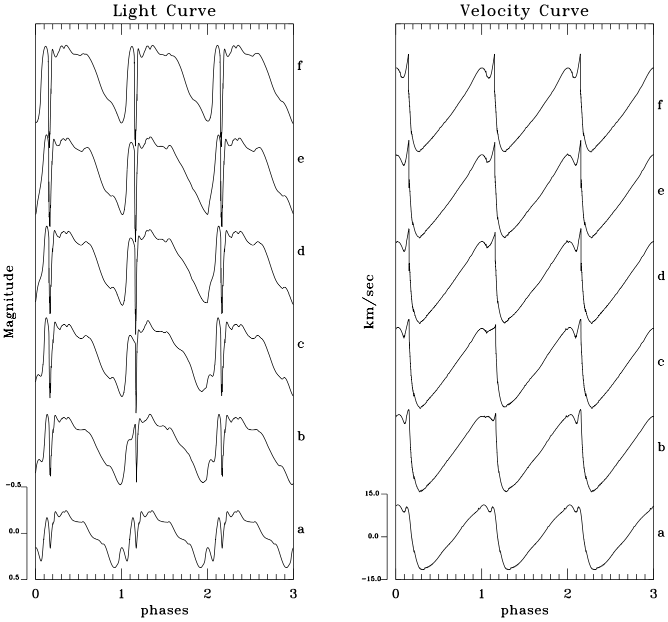

Fig. 3. Light (left) and velocity (right) curves at the photosphere of theoretical models. Each light curve is shifted vertically. Magnitude and velocity scales are indicated in the lower-left corner. The velocity is converted to the conventional scale. The curves are labeled by model names

3. Results and discussions

Nonlinear simulations are performed with an 8 10^5 integration step for each model. This covers a time interval of about 30 000 days (about 300 pulsation cycles), and oscillations are settled into steady states. Figure 2 shows light and velocity curves at the photosphere of the models. The phases are defined from the period of the limit cycle or a mean period for models with period-doubling behaviors.

Models a and f have limit cycles. The other four models show period-doubling behaviors in both curves. These models settled into the period-doubling pulsation typically after about 50 pulsation cycles from the beginning of the simulations and continued the feature without noticeable changes. We thus believe that these are permanent features of the models. We notice that considerable variations are observed at light minimum and velocity maxima alternately within two cycles. It is noted that the observed fluctuations in the light curves of HV 883 also appear considerably at the phase of light minima, while the pulsation period is longer than those of the present models. Light curves in the present models have a sharp dip at light maxima. Probably this is due to pure radiative approximation on energy transfer in the envelope, and we expect that it will disappear in the models including convective energy transfer (Buchler 2000). Figure 3 shows the phase diagram for dynamical behaviors of the models. The period-doubling is noticeable at the velocity maxima of the contraction, and we note strictly periodic behavior even in the period-doubling models.

All the models in the sequence are recalculated with $C_q = 8$ to see the effects of the artificial viscosity on the

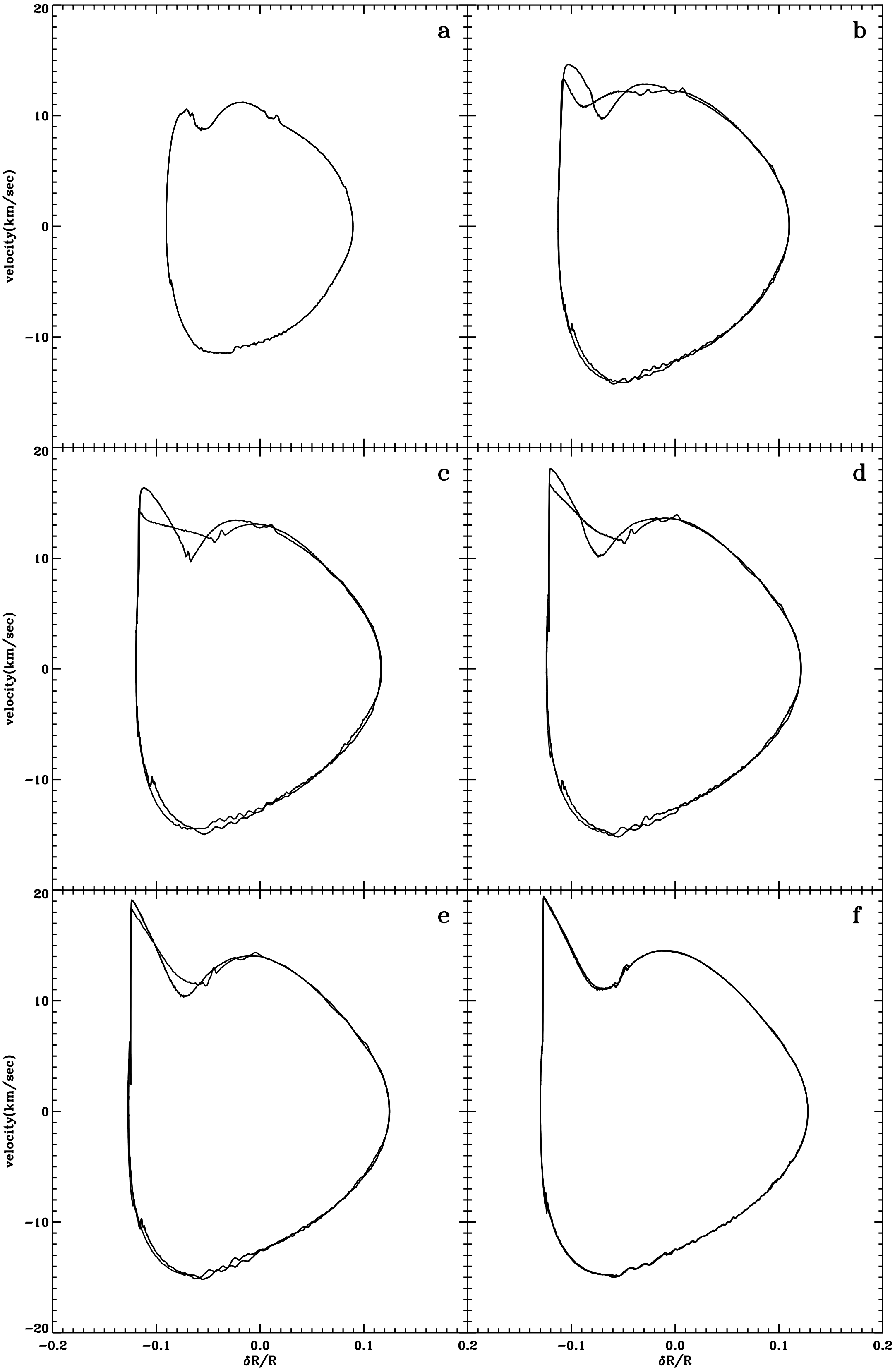

Fig. 4. Phase diagrams of theoretical models. For each model the x-axis is the relative displacement, and the y-axis is velocity at the photosphere. Data with the same time intervals in Fig. 2 are used for each model

Table 1. Model characteristics

model	T_e	P_0	η_0	P_1	η_1	P_2	η_2	P_3	η_3
a	5227	81.4[a]	0.07[b]	48.4	−0.44	33.4	−1.13	24.5	−0.95
b	5127	87.4	0.13	51.2	−0.42	35.2	−1.11	25.8	−0.95
c	5077	90.5	0.17	52.6	−0.41	36.1	−1.10	26.5	−0.95
d	5027	93.9	0.21	54.1	−0.40	37.0	−1.09	27.2	−0.95
e	4977	97.4	0.24	55.7	−0.39	38.0	−1.07	28.0	−0.95
f	4927	101.0	0.28	57.3	−0.38	39.0	−1.06	28.7	−0.94

[a] The units are days.

[b] The growth rates are defined as $-4\pi\omega_i/\omega_r$.

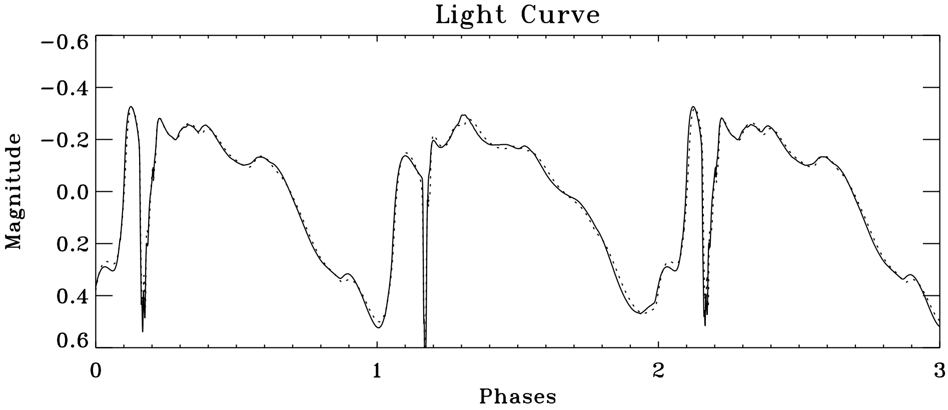

Fig. 5. Light curves of theoretical model c with different C_q values. Solid and dotted lines are for $C_q = 4$ and $C_q = 8$, respectively. Note that the period doubling behavior is not changed with such differences

period-doubling behaviors. We found that this change induced no serious effects on light and velocity curves, and also the transition from limit cycle to period-doubling occurred at the same value of the effective temperature. Figure 4 demonstrates this conclusion along with light curves of both calculations for model c.

We shall discuss the origin of the period-doubling bifurcation. Modal resonances may be excluded for two reasons: (1) no reasonable resonances are expected from period ratio, (2) only one pulsation mode is unstable. The period ratio of the fundamental and the first overtone modes is about 0.58 and is quit far from 2/3, which is one of the conditions found by Moskalik & Buchler (1991), and only the fundamental mode is pulsationally unstable in the linear regime.

The degree of nonadiabaticity is high in long-period Cepheids (Aikawa & Antonello 2000a,b). It is thus more likely that strong dissipations are responsible for the features. Takeuti (1988) suggested the importance of strong dissipation for the period-doubling bifurcation in stellar pulsation.

It is well known that the period-doubling bifurcation cascade is one of main routes to chaos. It is also known that in many systems the cascade is terminated on the way to chaos. We, thus, may have only strictly periodic oscillations with period-doubling behaviors, and this might be the case in the present sequence. Aikawa (1990) reported a similar termination of the cascade in stellar pulsation models.

Compared with the observed light curve (Fig. 1), the present models have shorter periods. Hydrodynamic models including effects of convection may have longer period models with period-bifurcation (Buchler 2000). It is also desirable to perform nonlinear simulations with different assumptions, and those calculations also help to clear artifacts of numerical schemes and machine dependencies on the present results.

The present models have metal-rich chemical compositions. Envelope models with metal-poor compositions will be necessary for Cepheids in Magellanic Clouds. A preliminary calculation with metal-poor compositions (Aikawa & Antonello 2000b) has not succeeded in finding models with period-doubling behavior.

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