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Incompatibility of a comoving Lyα forest with supernova-Ia luminosity distances

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Abstract. Recently, Perlmutter et al. (1999) suggested a positive value of Einstein's cosmological constant Λ on the basis of luminosity distances from type-Ia supernovae (the "SN-method"). However, Λ world models had earlier been proposed by Hoell & Priester (1991) and Liebscher et al. (1992a,b) on the basis of quasar absorption-line data (the "Q-method"). Employing more general repulsive fluids ("dark energy") encompassing the Λ component, we quantitatively compare both approaches. Fitting the SN-data by a minimum-component model consisting of dark energy + dust (pressureless matter) yields a closed universe with a large amount of dust exceeding the baryonic content constrained by big-bang nucleosynthesis. The nature of the dark energy is hardly constrained. Only when enforcing a flat universe is there a clear tendency to a dark-energy Λ fluid and the "canonical" value $\Omega_{\rm M} \approx 0.3$ for dust. Conversely, a minimum-component Q-method fit yields a sharply defined, slightly closed model with a low dust density ruling out significant pressureless dark matter. The dark-energy component obtains an equation-of-state $\mathcal{P} = -0.96\epsilon$ close to that of a Λ-fluid ($\mathcal{P} = -\epsilon$). $\Omega_{\rm M} = 0.3$ or a precisely flat spatial geometry are inconsistent with minimum-component models. It is found that quasar and supernova data sets cannot be reconciled with each other via (repulsive ideal fluid+incoherent matter+radiation)-world models. Compatibility could be reached by drastic expansion of the parameter space with at least two exotic fluids added to dust and radiation as world constituents. If considering such solutions as far-fetched, one has to conclude that the Q-method and the SN-Ia constraints are incompatible.

Key words. cosmology: miscellaneous – cosmology: theory

1. Introduction

In Newtonian gravitational theory, confined to positive masses as active and passive sources, gravitation is an exclusively attractive type of interaction. In general relativity Einstein (1917) showed by introduction of the cosmological constant Λ that gravitation may be repulsive as well (if $\Lambda > 0$). Homogeneous and isotropic general-relativistic world models *including* the Λ-term have been reconsidered in recent years for a variety of reasons.

In the 1990's a tightly constrained world model including a Λ term (the so-called Bonn-Potsdam model; henceforth BP-model) was derived by fitting quasar absorption lines (Hoell & Priester 1991a,b; Hoell et al. 1994; Liebscher 1994; Liebscher et al. 1992a,b). The BP-model was based on the assumption that Lyα-forest quasar absorption lines situated between the Lyα and Lyβ emission lines reproduce the redshifts of comoving galaxy-populated shells (called "bubble walls" by the authors) in the universe. The authors considered as virtues that their model does not require dark matter (except the Λ-fluid) and has a long age ($3\,10^{10}$ years) providing ample time for galaxy formation. If fixing the matter content by the baryons predicted through big-bang nucleosynthesis, it leads to a large Hubble constant H_0. Large values of H_0 were claimed in the early days of the HST key project on the distance scale.

During the last few years also other groups jumped on the "Λ band wagon". One reason is the apparent need for a nearly flat ($\Omega \approx 1$) universe inferred from cosmic microwave background observations (Lineweaver 1998; de Bernardis et al. 2000). Since the Λ term in Einstein's field equation is equivalent to the presence of a fluid with an energy density constant in space and time it can be used to fill the universe with "missing energy". Nucleosynthesis constraints limit the amount of baryonic matter far below one in Ω units and peculiar-motion observations suggest that also the attractive dark matter falls short of making the universe flat (Zehavi & Dekel 1999).

Recently, the repulsive effect of $\Lambda > 0$ may have been detected more directly. Observations of supernovae of type Ia, taken as cosmic "standard candles", appear to require the presence of an accelerating component in the expanding universe (Riess et al. 1998; Perlmutter et al. 1999).

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In view of this added evidence for Λ, one may ask whether the seemingly pioneering Q-approach can be reconciled with the new data. However, since the original BP-model significantly differs from published fits of Λ models to the SN data (Perlmutter et al. 1999; from now on we use the term "SN models" for all models that fit the SN data) we employ a more general approach, which for our fits is nevertheless based on the same data sets as published BP and SN models. Rather than only taking Λ as representing a repulsive component, we allow for fluids which have in common that the equation of state of each fluid yields proportionality between energy density ϵ and pressure $\mathcal{P}$. This form of the equation of state includes the classical cosmological matter components dust, radiation and the Λ fluid, but also encompasses more general repulsive and attractive components.

2. Theoretical background

2.1. World models

We here briefly describe the framework of the multifluid world models, which will be fit to the SN and quasar data. Further details, in particular on their classification, are given in Thomas & Schulz (2001).

We assume the Cosmological Principle, i.e. homogeneity and isotropy of the universe, leading to the Robertson-Walker (RW) line element. Inserting this into Einstein's field equation leads to the Friedmann equations

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\epsilon + 3\mathcal{P}) \tag{1}$$

and

$$H^2(t) \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\epsilon - \frac{k}{a^2} \tag{2}$$

$H(t)$ is the Hubble parameter at time t, the vacuum velocity of light is set $c = 1$. $\epsilon = \sum \epsilon_i$ denotes the total energy density of the universe, which we consider as the added contributions of N ideal fluid components, each with an energy density ϵ_i. The energy density of each fluid is correlated to its pressure $\mathcal{P}$ via an equation of state $\mathcal{P} = \mathcal{P}(\epsilon)$, for which we assume

$$\mathcal{P} = (\gamma - 1)\epsilon. \tag{3}$$

The constants $\{\gamma_i\}$ specify different fluid components. This simple form encompasses the well known cosmic fluids like (e.g. Kolb & Turner 1990)

- dust ($\mathcal{P} = 0; \gamma = 1$);
- radiation and any other extremely relativistic matter ($\mathcal{P} = \epsilon/3; \gamma = 4/3$);
- the constant-density Λ-fluid ($\mathcal{P} = -\epsilon; \gamma = 0$);
- a network of slow cosmic strings ($\mathcal{P} = -\epsilon/3; \gamma = 2/3$) or
- domain walls ($\mathcal{P} = -(2/3)\epsilon; \gamma = 1/3$).

The term "dust" should not be confused with the solid-particle component of the interstellar medium, as is common in astronomy. We here follow the practice of relativists to designate non-relativistic ($\mathcal{P} = 0$) incoherent matter as dust. In particular, dust includes all baryonic matter and cold dark matter in its simplest form.

Apart from dust and radiation, the physical background of cosmic fluids with equation of state (3) is poorly understood. Therefore, in case of $\gamma \neq 1$ or $4/3$, we use the designation *exotic fluid*.

We confine the parameter of the equation of state to the range

$$0 \leq \gamma \leq 2 \;\text{ implying }\; c_{\mathrm{s}} \leq c \tag{4}$$

where the speed of sound c_{s} is restricted not to surpass the velocity of light c.

In standard world models, positive as well as negative values of the cosmological constant Λ have been considered. Since in the fluid description Λ corresponds to a constant density, it is suggestive to allow for positive *and* negative energy densities of other γ fluids in our phenomenological approach as well.

After introducing the normalized density Ω_i, the normalized scale factor $x(t)$ and the parameter of state α_i by the equations[1]

$$\Omega_i \equiv \frac{8\pi G}{3H_0^2}\epsilon_{0i}, \quad x(t) \equiv \frac{a(t)}{a_0} \quad \text{and} \quad \alpha_i \equiv 2 - 3\gamma_i.$$

Equations (1), (2) obtain the more suitable form

$$\ddot{x} = \frac{H_0^2}{2x}\sum_i \alpha_i \Omega_i x^{\alpha_i} \tag{5}$$

$$\dot{x}^2 = H_0^2\left(\sum_i \Omega_i x^{\alpha_i} + 1 - \sum_i \Omega_i\right). \tag{6}$$

To arrive at Eq. (6) one has to use the present-epoch ($t = t_0$) version of Eq. (2) to eliminate the curvature index k

$$k = a_0^2 H_0^2\left(\sum_i \Omega_i - 1\right). \tag{7}$$

Henceforth we call components with $\alpha > 0$ Λ-*like*, those with $\alpha < 0$ *dustlike* because of their dynamical *similarity* to the Λ-fluid ($\alpha = 2$) and cosmic dust ($\alpha = -1$), respectively.

According to Eq. (5), fluids obeying $\Omega\alpha > 0$ accelerate the cosmic expansion, we call them *repulsive*. Analogously, *attractive* fluids are characterized by $\Omega\alpha < 0$.

2.2. Method to fit the SN-Ia luminosity distances

We here assume that the corrections for extinction by intervening material, waveband width and shift (K correction) as well as peak magnitude via the shape of light-curve have been appropriately applied so that the SNe-Ia

[1] By H_0, a_0, and ϵ_{i0} we denote the values of $H(t)$, $a(t)$ and $\epsilon_i = \epsilon_i(t)$ at the present epoch $t = t_0$.

can be utilized as *relative* standard candles as discussed in Perlmutter et al. (1997, 1999). In a geometrical cosmological model, the result of light propagation between an observer at $z = 0$ receiving light from an emitter at redshift z is subsumed under the term "luminosity distance", which can be used like an Euclidean distance that leads to inverse-square light dilution. For a given model, parameterized by a set of fluid parameters $\{(\Omega_i, \alpha_i)\}$ luminosity distance or the corresponding distance module are functions of z. Since the absolute peak magnitude of the SNe is assumed to be known to a constant, one can compare the observed variation of apparent peak magnitude with z with the shape of theoretical distance-module curves given by

$$m \;=\; C + 5\,\log\left(\frac{1+z}{\sqrt{|\sum_i \Omega_i - 1|}}\left\{\begin{array}{ll}\sin(f) & \ldots\ k=1\\ f & \ldots\ k=0\\ \sinh(f) & \ldots\ k=-1\end{array}\right\}\right)$$

$$f \;=\; f(z) = \int\limits_0^z \frac{(1+z')^{-1}\sqrt{|\sum_i \Omega_i - 1|}\,\mathrm{d}z'}{\sqrt{\sum_i \Omega_i\left((1+z')^{-\alpha_i} - 1\right) + 1}}. \qquad (8)$$

Here m is the corrected apparent magnitude and C is a constant.

The supernova data set is given in Perlmutter et al. (1999). It contains redshift and magnitude data of 42 high redshift SN ($0.172 < z < 0.830$) and additionally of 18 low redshift SN ($0.014 < z < 0.101$) which are needed to calibrate the method (cf. Perlmutter et al. 1997).

We adopted this relatively homogeneous set as representative for a demonstration of the SN-method. Graphs and results of the measurements by Riess et al. (1998, 1999) show these to be pleasingly similar. Further SN luminosity distances are rapidly accumulating and will provide stronger constraints in the future. However, it is not our goal to look for the "ultimate" fitting model. Rather, we want to obtain a typical range of fitting models that include our more general fluids.

In order to obtain best-fit values for the fluid parameters we minimized the function

$$\chi^2 = \sum_{j=1}^{60} \frac{\left(m(z_j, \Omega_1, \ldots, \Omega_N, \alpha_1, \ldots, \alpha_N) - m_j\right)^2}{\sigma_j^2}. \qquad (9)$$

In contrast to the Q-method described below, we note that the apparent magnitude of the SNe only contains information about the *integrated* effect of cosmic evolution since the SN explosion occurred. Due to this smearing-out effect, the evaluation of parameters is not tightly constrained.

2.3. Q-method to fit the Lyα-forest

The basic idea underlying the Q-method is that the Ly-α-forest is caused by intersections of the quasar light with the boundaries of an inhomogeneous bubble structure of the (baryonic) matter (Hoell & Priester 1991a,b; Hoell et al. 1994; Liebscher 1994; Liebscher et al. 1992a,b). The differences Δz between the redshifts z of two absorption-lines are observed to be small, so that they are taken as infinitesimal. Then Δz can be directly converted into the model-dependent cosmic expansion rate $H(z)$ at z, the epoch of the twin absorption, if the distance between the absorbing bubble walls (the void diameter) is known. The BP group *assumed* a *constant* void diameter R_B in the *comoving* frame. With these presumptions (and using the abbreviation $y \equiv 1+z$), one finds, in close analogy to the Q-procedure (as described, e.g., in Liebscher et al. 1992a),

$$\Delta z^2 = R_\mathrm{B}^2 a_0^2 H_0^2\left(\sum_i \Omega_i y^{(2-\alpha_i)} + \left(1 - \sum_i \Omega_i\right) y^2\right). \qquad (10)$$

In the present case we minimize the function

$$\chi^2 = \sum_{j=1}^{14} \frac{\left(\Delta z^2(z_j, \Omega_1, \ldots, \Omega_N, \alpha_1, \ldots, \alpha_N) - {\Delta z_j}^2\right)^2}{\sigma_j^2}. \qquad (11)$$

The data set we used is given in Liebscher et al. (1992a). There are 12 absorption-line differences derived from the spectra of four quasars with $2.25 < z < 4.35$ together with the data of two nearby voids at $z = 0.08$ and $z = 0.03$, respectively.

In another work (Liebscher et al. 1992b) the BP-group published further absorption line data, however the cosmic parameters obtained from the enlarged data sample remain nearly the same.

The advent of space-based and ground-based high-resolution spectrographs led to Lyman forest data at lower z than available for the BP-group (Weymann et al. 1998; Christiani et al. 2000; Kim et al. 2001). However, we have refrained from extracting mean Δz values from these measurements, leaving this tedious task to defenders of the Q-method. It is shown below that the data set employed here leads to by far sufficient constraints for the Q-method to unveil strong differences with the SN-method.

3. Results of the fits

3.1. SN-Ia data

For the derivation of cosmic parameters, we considered various cosmic fluids given by (Ω, α) or $\{\Omega_i, \alpha_i\}$ in addition to incoherent dust with density Ω_M (the background radiation component usually could be neglected because of the low redshifts). The χ^2 values are not normalized and only meaningful within the same group of fits.

Case 1: Dust + exotic component. With only *one* arbitrary exotic fluid the best fit yields (ν denotes the degrees of freedom of the fit)

$$\begin{aligned}\Omega_\mathrm{M} &= 0.86 \pm 0.58\\ \Omega &= 1.38 \pm 0.73\\ \alpha &= 2 \pm 1.9\\ \chi^2 &= 102\ ,\ \nu = 58.\end{aligned}$$

Table 1. Results of the supernova-fits. $\chi^2_{\rm red} \equiv \chi^2/\nu$ denotes the reduced χ^2 with ν degrees of freedom. For the models below $\nu = 58$

Model	Parameter	$\chi^2_{\rm red}$	in Fig. 1
flat	$\Omega_{\rm M} = 0.30$	1.79	a
	$\Omega = 0.70$, $\alpha = 2$		
	$\sum \Omega_i = 1$ fixed		
Λ-fluid	$\Omega_{\rm M} = 0.86$	1.76	b
	$\Omega = 1.38$, $\alpha = 2$		
	$\alpha = 2$ fixed		
domain walls	$\Omega_{\rm M} = 0.65$	1.76	c
	$\Omega = 2.41$, $\alpha = 1$		
	$\alpha = 1$ fixed		
matter + CDM	$\Omega_{\rm M} = 0.3$	1.76	d
	$\Omega = 3.2$, $\alpha = 0.6$		
	$\Omega_{\rm M} = 0.3$ fixed		
matter	$\Omega_{\rm M} = 0.03$	1.76	e
	$\Omega = 3.3$, $\alpha = 0.5$		
	$\Omega_{\rm M} = 0.03$ fixed		

Interpreting the resulting $\alpha \approx 2$ as pointing towards Λ as dark energy component is premature. The amount of uncertainty in α suggests that nearly any Λ-like fluid can be fit to the data. For instance, when fixing $\alpha = 1$ we found no difference in χ^2:

$$\Omega_{\rm M} = 0.65 \pm 0.41$$
$$\Omega = 2.41 \pm 1.11$$
$$\chi^2 = 102 , \ \nu = 58.$$

With increasing α, the age of the universe in the best-fit models decreases ($\alpha = 1 \rightarrow < 20\,{\rm Gyr}, \alpha = 2 \rightarrow < 15\,{\rm Gyr}$) and concomittingly $\Omega_{\rm M}$ decreases.

Case 2: Dust fixed+free exotic component. The dust densities derived from the above fits are high compared to values usually derived with other methods (e.g. Zehavi & Dekel 1999). If one assumes lower dust densities, α decreases (albeit remaining positive) thereby weakening the tendency to Λ as repulsive fluid. For $\Omega_{\rm M} = 0.3$ (a typical result from observations of peculiar velocities) we found $\alpha = 0.6$, for $\Omega_{\rm M} = 0.03$ (closer to nucleosynthesis constraints for baryons) the result was $\alpha = 0.5$ (cf. Table 1).

Case 3: Flat models. With no restriction for $\Omega_{\rm M}$ but considering only flat universes, the Λ-fluid turns out to be the best-fit exotic fluid:

$$\Omega_{\rm M} = 0.30 \pm 0.04$$
$$\alpha = 2 \pm 0.04$$
$$\chi^2 = 104 , \ \nu = 58.$$

Now, in contrast to case 1 the minimum of χ^2 is rather well defined and, statistically, there is no likely alternative to a dark-energy component close to the Λ fluid. We nevertheless emphasize that – when relaxing the assumption of flatness – closed world models lead to slightly better (lower χ^2) fits (compare Fig. 1).

Among the fluid parameter sets leading to world models compatible with the observations, we found models without a big bang (however, all best fit models start with a big bang). If we want to transfer the cosmological standard model of the formation of light elements to non-big-bang universes, we have to require an epoch characterized by physical conditions similar to those in the nucleosynthesis epoch of a successful big-bang model. One of the necessary conditions would be a temperature exceeding 10^9 K, which forces the minimum of the scale factor to fall short of $x_{\rm min} < 10^{-9}$. This condition rules out successfully fitting non-big-bang models.

However, the situation changes when allowing for more than one exotic fluid. For instance, we found flat models with two exotic fluids (plus dust) that fit the data and reach a minimum $x_{\rm min} < 10^{-9}$. A particular example is given by the following parameters:

$$\Omega_{\rm M} = 0.3$$
$$\Omega_\gamma = 5.9\,10^{-5}$$
$$\Omega_1 = -5.90003\,10^{-14} , \ \alpha_1 = -3$$
$$\Omega_2 = 1 - \Omega_{\rm M} - \Omega_\gamma - \Omega_1 , \ \alpha_2 = 1.5$$
$$\chi^2 = 102 , \ \nu = 58.$$

In this model we have also included the component of cosmic background radiation Ω_γ because of the desired high temperatures and the correspondingly small x (high z) at which the dynamical effect of the radiation component (or any classical relativistic component) is strongly enhanced, although negligible today.

3.2. Quasar data

Fitting the quasar data with a model consisting of dust and only one arbitrary exotic fluid we obtain

$$\Omega_{\rm M} = 0.0162 \pm 0.0003$$
$$\Omega = 1.100 \pm 0.001$$
$$\alpha = 1.87 \pm 0.01$$
$$\chi^2 = 10 , \ \nu = 11.$$

This model favours a low dust content and a repulsive fluid with properties close to that of a Λ-fluid. When attributing the dust component exclusively to the baryon content predicted by big-bang nucleosynthesis, it requires a large H_0. In comparison to the SN fits it appears most striking that the minimum of χ^2 is sharp and other Λ-like fluids are practically ruled out.

We attempted to fit models including cold dark matter, e.g. by fixing $\Omega_{\rm M} = 0.3$, but found no match with the data ($\chi^2 > 100\,000$). This statement no longer holds if more than one exotic fluid is invoked. In this more general case we found models when fixing $\Omega_{\rm M} = 0.3$, for parameters

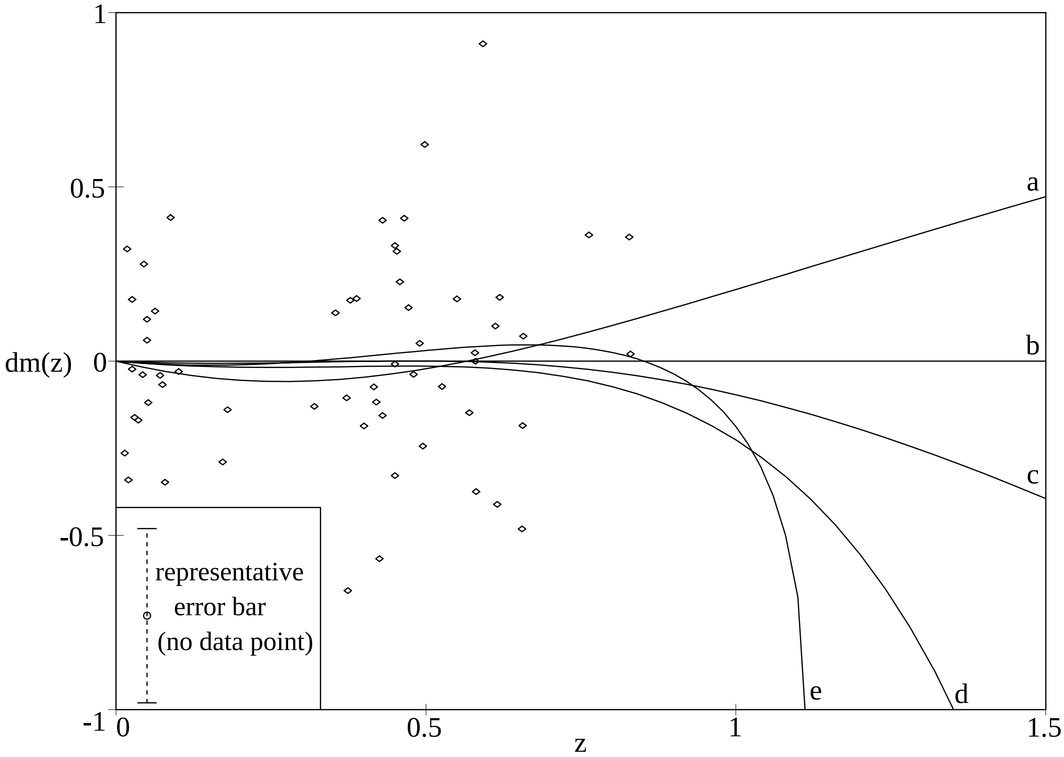

Fig. 1. SN world models of Table 1 relative to the best fit model (horizontal line). $dm(z) \equiv m_{\rm model}(z) - m_{\rm bestfit}(z)$. A typical error bar is shown in the insert (lower left). For individual errors of the data see Perlmutter et al. (1999)

of state of the exotic fluids in the range $\alpha_{1/2} < -1$ and $\alpha_{1/2} > 0$. One example is the model

$$\begin{aligned} \Omega_{\rm M} &= 0.3 \\ \Omega_1 &= -0.060 \ , \ \alpha_1 = -1.56 \\ \Omega_2 &= 1.879 \ , \ \alpha_2 = 1 \\ \chi^2 &= 11 \ , \ \nu = 11. \end{aligned}$$

All best-fit models with one exotic fluid are nearly flat, whereas *exactly flat* (within the parameter errors) models were not found to fit to the data. Exactly flat models with *two exotic fluids* exist. Thereby, for each of the exotic fluids, the parameter of state must obey $\alpha_{1/2} \geq -0.58$. An example is given by the following parameters:

$$\begin{aligned} \Omega_{\rm M} &= 0.037 \\ \Omega_1 &= -0.082 \ , \ \alpha_1 = -0.58 \\ \Omega_2 &= 1 - \Omega_{\rm M} - \Omega_1 \ , \ \alpha_2 = 2 \\ \chi^2 &= 11 \ , \ \nu = 11. \end{aligned}$$

The ages of the universe in those fitting models that start with a big bang is high in comparison to those of SN-world models (∼35 Gyr for $H_0 = 65\,{\rm km\,s^{-1}/Mpc}$). For successful models with an exotic fluid and no big bang, the same holds as for the corresponding SN-models: they do not contain an epoch with $x_{\rm min} < 10^{-9}$, ruling out the possibility of light-element nucleosynthesis in the same way as in standard-big-bang universes.

Table 2. Results of the quasar-fits

Model	Parameter	$\chi^2_{\rm red}$	in Fig. 2
Λ-fluid	$\Omega_{\rm M} = 0.0122 \pm 0.0003$	0.92	D
	$\Omega = 1.073 \pm 0.001\ ,\ \alpha = 2$		
	$\alpha = 2$ fixed		
best fit	$\Omega_{\rm M} = 0.0162 \pm 0.0003$	0.91	B
	$\Omega = 1.100 \pm 0.001$		
	$\alpha = 1.87 \pm 0.01$		
matter	$\Omega_{\rm M} = 0.0323 \pm 0.0003$	1.17	C
	$\Omega = 1.220 \pm 0.001\ ,\ \alpha = 1.5$		
	$\alpha = 1.5$ fixed		
flat	$\Omega_{\rm M} = 0.037$	1.00	A
(example)	$\Omega_1 = -0.082\ ,\ \alpha_1 = -0.58$		
	$\Omega_2 = 1.045\ ,\ \alpha_2 = 2$		
	$\sum \Omega_i = 1$ and $\alpha_2 = 2$ fixed		
matter	$\Omega_{\rm M} = 0.3$	1.00	-
+ CDM	$\Omega_1 = -0.060\ ,\ \alpha_1 = -1.56$		
(example)	$\Omega_2 = 1.879\ ,\ \alpha_2 = 1$		
	$\Omega_{\rm M} = 0.3$ and $\alpha_2 = 1$ fixed		

3.3. Comparison of the results

For the case of world models consisting of dust and one exotic fluid, both methods agree in the requirement of a *repulsive* fluid leading to an *accelerated* universe even for low z. Within the framework of generalized ideal fluids

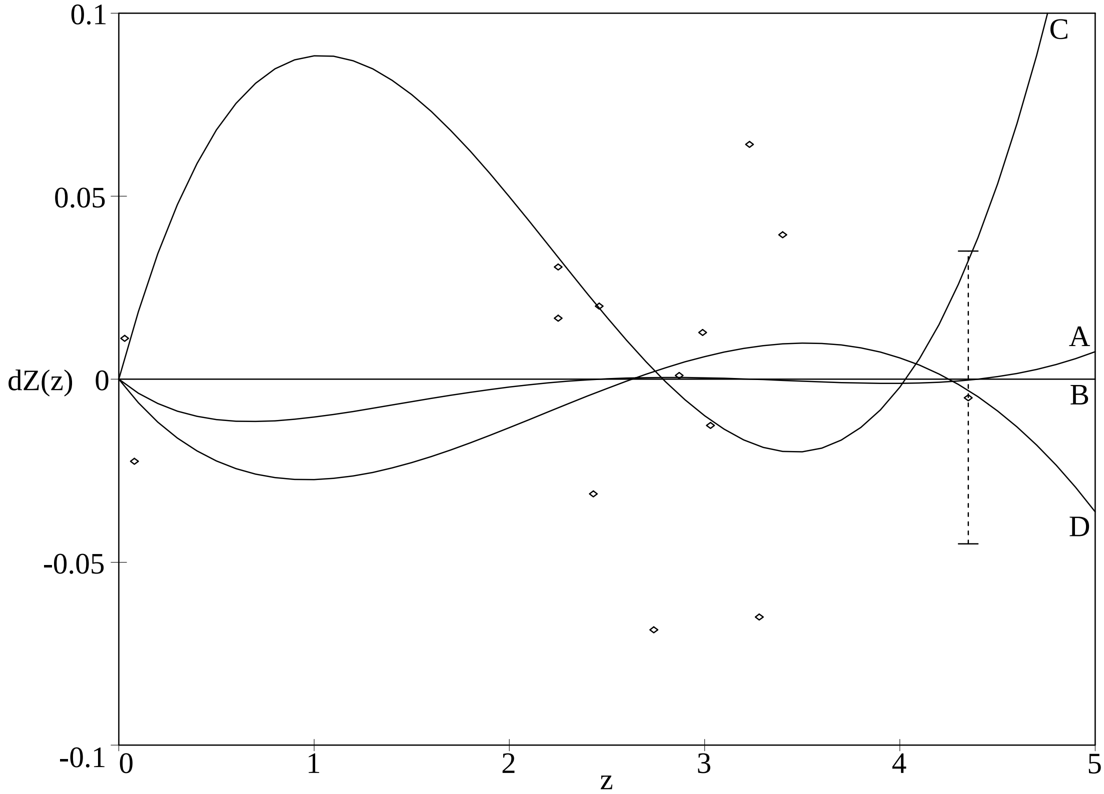

Fig. 2. Q-world models of Table 2 relative to the best fit model B ($\alpha = 1.87$). $dZ(z) \equiv \Delta z^2_{\mathrm{model}}(z) - \Delta z^2_{\mathrm{bestfit}}(z)$. Only one error bar, representative for all data points, is shown

presented in Thomas & Schulz (2001) such a repulsive fluid could be either Λ-like with positive energy density or dust-like with negative density (cf. Sect. 2.1). Both the SN luminosity distance and the quasar absorption-line method lead to a Λ-like exotic repulsive fluid. In both cases, the data themselves can be fit sufficiently well with models containing only one exotic component.

However, as Fig. 1 shows, the SN-method does not allow us to decide which equation of state describes the exotic fluid. A large part of the degeneracy in the exotic component is caused by the relatively low redshifts so far reached by SN observations. Therefore at first glance, the Q-method, reaching to comparatively high redshifts close to $z \approx 4.5$, appears to be superior to narrow down the exotic component. Caution has to be applied to this suggestive conclusion because the fits of both methods differ significantly.

Typically, the SN data lead to closed models with large dust densities Ω_{M} (allowing for cold dark matter). Despite the presence of a repulsive component, these models yield a relatively low age of the universe – roughly between 10 and 20 Gyr ($H_0 = 65\,\mathrm{km\,s^{-1}/Mpc}$). As pointed out above, the kind of accelerating component is hardly constrained: nearly any Λ-like component can be tuned to match the observations. Only when a flat geometry is required does this degeneracy break down and the SNe Ia point towards Λ as the dark-energy fluid.

On the other hand, the quasar models usually turn out to be nearly flat, require low dust densities, which exclude substantial amounts of cold dark matter. They lead to values of the age of the universe of at least 30 Gyr (H_0 as above). Furthermore, there is a strong tendency to Λ as repulsive component rather than something "more exotic".

Actually, the SN models are not compatible with the Q-method and vice versa. For illustration, the upper part of Fig. 3 shows the absorption-line redshift differences predicted by the SN models, the lower part shows the SN-luminosity residuals predicted by the Q-models. In order to fulfil the conditions set by both methods with one model, one is forced to consider at least two repulsive fluids. For example, the following model "SN+Q", which is also shown in Fig. 3, fits both data sets:

$$\begin{aligned} \Omega_{\mathrm{M}} &= 0.5 \\ \Omega_1 &= 2.44\ ,\ \alpha_1 = 0.8 \\ \Omega_2 &= -0.12\ ,\ \alpha_2 = -1.5 \\ \chi^2_{\mathrm{SN}} &= 102 \\ \chi^2_{\mathrm{Q}} &= 11. \end{aligned}$$

The kinematical differences of the SN-models and the Q-models are illustrated in Fig. 4, for which we have chosen the model "b" of Table 1 and "D" of Table 2 as representative examples for SN-models and Q-models, respectively. The figure shows $f(x) \equiv \dot{x}^2(x)$, e.g. the expansion rate of the models as a function of the scale factor x. In the range of the absorption line data ($0.2 < x < 0.35$) the Q-models expand slowly, as compared to the SN-models, leading to relatively small absorption line differences for given sizes of comoving bubbles. In most SN-models the same comoving bubbles would produce much larger differences of the lines (cf. Fig. 3, upper panel). SN models cannot fit a slow observed evolution of bubble-wall differences because the development of brightness of the supernovae for $0.5 < x < 1$ with redshift z forces a rapid expansion. In the Q-models the supernovae would simply be too dark.

The nearly static phase at $x \approx 0.2$ strongly contributes to the high ages of the Q-models.

3.4. Remark on void evolution

We attempted to reconcile the SN world models with a bubble-wall Lyα-forest via assuming various void evolution models. To this end we replaced the constant void-parameter $R_{\rm B}$ in the Q-method by an ansatz $R_{\rm B}(z) = a_1\, z^{b_1} + a_2\, z^{b_2}$. However, this approach yielded bad fits with at least ten times larger chisquares than the Q-fits.

This result does not appear surprising in view of the fact that a comoving Lyα-forest requires an extended nearly static epoch in the Q-fits (e.g. see Fig. 4). Forcing an agreement with the rapidly expanding SN models cannot even be achieved by our double-power-law ansatz.

4. Discussion

Unless we are invoking complicated multi-parameter models, the Q-models and the SN fits with world models composed out of $\mathcal{P} \propto \epsilon$ - fluids have been shown to be incompatible. One possibility would be that Robertson-Walker models themselves are inadequate approximations to an inhomogeneous universe. However, this unsolved problem reviewed by Krasinski (1997) is beyond the scope of the present work.

Taking for granted the standard view that RW models are fair approximations of the global behavior of the universe we confine the discussion to possible pitfalls inherent to the Q-approach or to the SN fitting procedure.

4.1. The Q-approach

The Q-approach rests on at least two assumptions: (i) voids and their "bubble walls" are stable structures, whose comoving average size is constant from $z = 0$ to at least $z = 4.4$; (ii) the locations of the bubble walls are uniquely determined by the quasar absorption lines selected by the Bonn-Potsdam group.

(i) During its early linear phase, the size of a small cosmic perturbation will be constant in a comoving frame (Padmanabhan 1993). However, the comoving size of the void will grow nonlinearly when its density contrast exceeds an appreciable fraction of unity (Friedmann & Piran 2000). Since current voids have a density contrast of about $\sim(-0.8)$, they have already reached the nonlinear stage (El-Ad & Piran 2000). Thus, it appears to be a critical point that the Q-method has not yet been applied *together* with a self-consistent model of void evolution.

(ii) In a comprehensive review, Rauch (1996) summarized the complicated history of analysis of the Lyα-forest. A general absence of "voids" in the "typical" forest (lines of column density below 10^{14} cm^{-2}) is claimed by statistical studies. Backed by numerical simulations of structure formation, the Ly-forest lines are considered to originate in an interconnected web of material sheets and filaments. These simulations utilize appreciable amounts of dark matter, which is at variance with the low-density baryonic Q-models. At low redshift, there appears to be a trend that the Lyα absorbers trace the large-scale distribution of galaxies (although absorption systems *within* voids were also found). However, the Q-approach, as proposed by the BP group, assumes that coincidence of galaxy sheets and absorber sheets has persisted since epochs at large redshifts. This contrasts with the current belief of strong evolution at relatively low redshift.

Summarizing, there is disagreement between the BP-group and other workers in this field about the origin of the Lyman-forest in a cosmological context.

4.2. SNe-Ia as standard candles

In the Perlmutter et al. (1999) sample, the evidence for an accelerating component could vanish if there were a systematic effect making SNe at high redshift dimmer by $\sim$0.25 mag relative to those at low z. Such a dimming might be caused by (i) material between the SNe Ia and the earth. Alternatively (or in addition) the possibility of (ii) intrinsic differences between high-z and low-z SNe has to be considered. (i) Since SNe with obvious reddening are not used for deriving luminosity distances Aguirre & Haiman (2000) discussed the possibility of "neutral" intergalactic solid particles making SNe appear fainter without reddening their spectrum. It turned out to be a *theoretical possibility*, which, if causing the total SNe dimming, should show up in future measurements of the cosmic far infrared background. (ii) Systematic differences in the spectra of low-z and high-z SNe have not been found. Comparison of light-curves even showed the widening with z ("time dilation") due to cosmological expansion (Leibundgut et al. 1996). There was concern about the significantly shorter rise time of high-z compared to low-z SNe (Riess et al. 1999), which was, however, narrowed down to $<2\sigma$ *agreement* by Aldering et al. (2000). Despite a controversy regarding the progenitors and early stages of their explosion history, the empirically observed homogeneity seems so far to be fairly well established. Some key tests are nevertheless still to be made, one of which is a significant extension of the sample to $z \sim 1.3$. A potential shape of the magnitude-z curve as for a transition from decelerated to accelerated expansion could hardly be mimicked by systematic or evolutionary effects (Livio 2000).

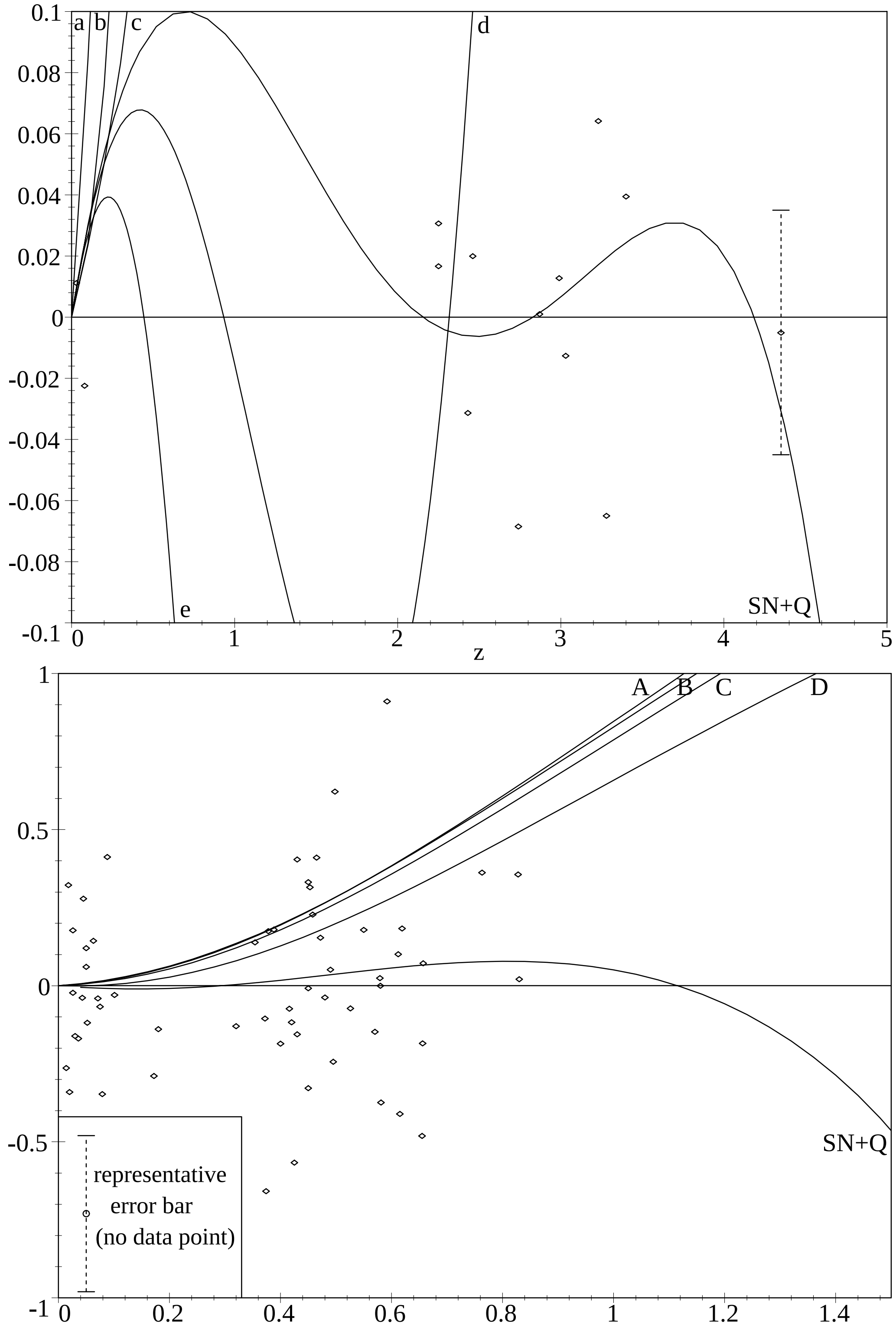

Fig. 3. The upper diagram shows the predicted absorption-line differences of the SN world models of Table 1 (cf. Fig. 2). In the lower diagram the differences of the supernova luminosities (relative to the best fit; cf. Fig. 1) predicted by the Q-models of Table 2 are drawn together with the observed data. The curves "SN+Q" represent the model of Sect. 3.3 that fits to both data sets

5. Summary and conclusions

We have compared the methods to derive world models from the kind of luminosity distance-redshift relation given by SNe Ia observations (the SN approach) and, on the other hand, from Lyα-forest lines, when one assumes that these arise in galaxy-populated walls and filaments around cosmic voids (the Q-approach). While the original authors essentially focused on world models with Λ plus

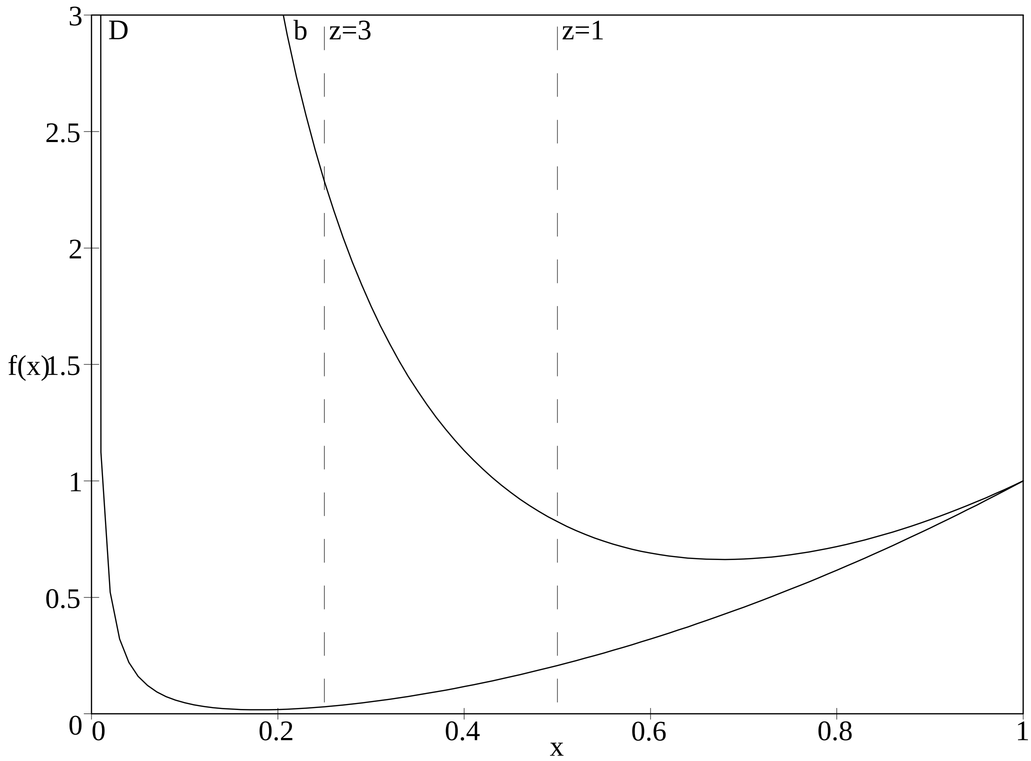

Fig. 4. $f(x) \equiv \dot{x}^2(x)$ for the Q-model "D" (cf. Table 2) and the SN-model "b" (cf. Table 1). Supernova data points are mainly located within the range $0.5 < x < 1$, the employed quasar Ly-forest lines arise between $x = 0.2$ and $x \approx 0.35$

normal incoherent matter (and radiation, which, however, only plays a role at dense stages of the universe, e.g. near a big bang) we allowed for a more general class of cosmic fluids only restricted by an equation of state $\mathcal{P} \propto \epsilon$.

However, even within this more general approach, world models fit to SNe Ia cannot be reconciled with those fit to the quasar Lyα redshift distribution[2]. A minimum-component Q-model tends to a baryonic universe with correspondingly low $\Omega_{\rm M}$ and to Λ as a repulsive component. It is therefore incompatible with a number of independent observations that suggest a universe with dark-matter content $\Omega_{\rm CDM} \sim 0.3$. Structure formation is an open issue in the Q-scenario, although the large ages of the Q-models combined with a long period of low expansion may ease galaxy formation.

Simple dust+dark energy models fit to the SN-data tend to be closed and require non-baryonic dark matter. There is no strong case for Λ as the dark-energy component unless the models are forced to be flat. Generally, there is a wide range of world models fitting the SN-Ia data, which can be reconciled with a dark-matter dominated, attractive $\Omega_{\rm M}$, a nearly flat universe, structure formation and current age constraints.

Despite apparent independent support for the SN-method, it is not our intention to weigh the pros and cons of either method. The main conclusion is that there is a deep incompatibility between the Q- and the SN method, even if allowing for rather general world models.

[2] Only less reasonable multi-parameter models can be adjusted to fit both approaches.

This suggests that at least one of the two methods is doubtful or based on systematic errors requiring critical scrutiny of the underlying assumptions (see Sects. 2.2 and 2.3). For the Q-method, it is mainly the bold identification of the low-column-density Lyman forest to represent a comoving "cell structure" of the galaxy distribution (Sect. 4.1). For the SN-method, although more strongly based on *empirical* calibration, there are still suspicions regarding some remaining systematic effect (Sect. 4.2).

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