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Stability of pulsation of the double-mode high-amplitude δ Scuti star AE Ursae Majoris

M. D. Pócs and B. Szeidl

Konkoly Observatory of the Hungarian Academy of Sciences, 1525 Budapest, Hungary
e-mail: `pocs@konkoly.hu; szeidl@konkoly.hu`

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Abstract. Stability of both the fundamental and first overtone oscillations of AE UMa was investigated by different methods which led to concordant results. The fundamental period of AE UMa has been essentially constant in the past 60 years consistent with the theoretical expectation (Breger & Pamyatnykh 1998). The reported fast period decrease (Hintz et al. 1997) is shown to be incorrect. The constancy of the fundamental period suggests that the star is in the post-main sequence evolutionary state as suggested by the evolutionary theories. The first overtone period is decreasing with a rate of $(1/P_1)(\mathrm{d}P_1/\mathrm{d}t) = -7.3\ 10^{-8}\ \mathrm{y}^{-1}$. The fact that the rate of period change for two modes is quite different indicates that non-evolutionary effects may also generate period changes. The changes in amplitude of the fundamental and first overtone were examined by comparing the least-squares amplitude solution for different segments of observations. Small long-term variations in the amplitudes have occurred in the past 25 years.

Key words. stars: oscillation – stars: variables: δ Sct – stars: individual: AE UMa

1. Introduction

In a recent paper, Breger & Pamyatnykh (1998) investigated the evolutionary period changes of δ Scuti stars. They carried out evolutionary model calculations both with and without convective core overshooting and compared the computed values of period changes with those observed. Although their results suggest that the observed changes in the periods cannot be fully explained by evolutionary effects, the very fast period change, measured for AE UMa ($-5\ 10^{-7}$ year^{-1}, Hintz et al. 1997) proved challenging to explain. The rate of its period decrease fitted Breger & Pamyatnykh's (1998) pre-MS models; however, they rejected this possibility for lack of evidence without precluding it.

Since accurate photoelectric and CCD maxima of the light variation of AE UMa were few and large gaps were present between the groups of published maxima, we suspected that the epoch numbers might be miscounted in calculating the rate of its period change.Therefore, we decided to reanalyze all the available observations.

The investigation of period changes of AE UMa may result in an interesting inference about the evolutionary effects. The star is a well-known double-mode pulsator; it oscillates radialy both in the fundamental and the first overtone modes. If the changes in the frequencies are constant or parallel and slowly decreasing, we have good reason to assume that they are essentially of evolutionary origin. On the other hand, if the changes in the periods of the two radial modes have different signs, then non-evolutionary physical effects likely play an important part in the frequency variations. Related to this, we will, in addition, investigate the stability of other observables.

The variability of AE UMa (BV 92, HIP047181, $\alpha_{2000} = 09^{\mathrm{h}}36^{\mathrm{m}}53^{\mathrm{s}}, \delta_{2000} = +44°04'.0$) was discovered by Geyer et al. (1955) on Bamberg plates. Tsesevich (1956) and Filatov (1960) observed the star, but could not determine the type of variability. (Tsesevich suggested an RR Lyrae-type while Filatov proposed a Cepheid-type variability.)

Tsesevich observed the star again visually and measured its brightness on old Moscow and Odessa archive photographic plates (Tsesevich 1973). He determined the period of the light variation and the type of variability as dwarf Cepheid. Tsesevich (1973) also showed that the star had significant light-curve variation. This fact aroused our interest as well as Broglia & Conconi (1975), and the star was then extensively observed at both the Merate and Konkoly observatories. The 1974 Konkoly observations have already made the determination of the beat period possible (Szeidl 1974).

Later, Rodríguez et al. (1992) observed the star in the Strömgren photometric system. AE UMa was also a program star of the Hipparcos project (ESA, 1997) and 112 photometric observations were published.

Send offprint requests to: B. Szeidl,
e-mail: `szeidl@konkoly.hu`

Recently, Hintz et al. (1997) investigated the star in more detail and published ten new accurate times of maximum light from CCD photometry. Several times of maximum light were reported by the BAV group (Braune et al. 1979, 1982; Huebscher et al. 1985, 1992; Agerer et al. 1999). The spectral type of AE UMa was classified by Götz & Wenzel (1961) as A9 in accordance with the type of variability. The star is listed in Garcia et al.'s (1995) catalogue as an SX Phoenicis star. Hintz et al. (1997), however, provided strong evidence (based on D. H. McNamara's data) against this classification, and have shown that AE UMa is a normal Population I, high-amplitude δ Scuti star.

In the following sections, we give an account of the photometry carried out at the Konkoly Observatory and present a detailed analysis of the available observations. Then, some conclusions are drawn about the stability of pulsation of the star and its evolutionary state.

2. The data

2.1. Photometric observations

Two excellent and extensive series of photoelectric observations of AE UMa have been published. Broglia & Conconi (1975) obtained 977 and 953 observations in the Johnson V and B bands, respectively during 7 nights in 1974. Rodríguez et al. (1992) carried out Strömgren photometry on 8 nights in 1987 and collected 229 observations in each colour band. These observations are well supplemented by the photometry of AE UMa carried out at the Konkoly Observatory.

The photoelectric observations of AE UMa were carried out at the Konkoly Observatory from 1974 to 1998 and 6575 observations (1198 in U, 1947 in B, 1935 in V, 784 in R and 711 in I) were obtained over 41 nights. Between 1974 and 1986, the 60 cm Newtonian telescope near Budapest and the 50 cm Cassegrain telescope of the Piszkéstető mountain station were used. These telescopes were equipped with conventional unrefrigerated UBV photometers. In the years 1996–1998 the 1 m Ritchey-Chretien telescope of the mountain station was used with a photon-counting, electronically cooled photometer which operated close to the $UBV(RI)_{\mathrm{C}}$ photometric system. Throughout our photometry the star BD+44°1882 was used as the comparison and BD+44°1884 as a check star. Several differential observations were carried out between the comparison and check stars. These runs indicated that the errors of the single observations were less than 0.01 magnitudes in V, B and R and around 0.02 magnitudes in U and I on average nights. Since the night sky near Budapest was too bright, the observations with the 60 cm telescope were limited only to the B and V bands. A detailed description of the telescopes and the observations are published in a separate paper (Szeidl & Virághalmy 2000). The observations are also available in electronic form from the second author of this paper.

2.2. The times of maximum light

The earliest known epoch of maximum light of AE UMa was published by Tsesevich (1973) based on old archive plates of the Sternberg Institute, Moscow and Odessa Observatory. He also published four times of maximum, making use of his visual observations from the years 1956, 1963, 1971 and 1973.

Filatov (1960) estimated the star's brightness on the archive plates of the Tadjik Astronomical Observatory and announced five epochs of maximum. In fact, he misclassified the type of variability and deduced a wrong period; therefore the folded light curves provided erroneous epochs and they were irreconcilable with Tsesevich's data. In his discussion, Tsesevich (1973) already neglected Filatov's observations. We, too, regarded them as very uncertain and did not take them into account in the analysis.

Broglia & Conconi (1975) reported 19 epochs of brightness maximum in 1974. These are the mean values of the instants of B and V maxima, since they could not determine any systematic difference between them. Rodríguez et al.'s (1992) *uvby* photometry covered 13 maxima of light variation in 1987. Hintz et al. (1997) published ten times of maximum light obtained from CCD photometry in 1997.

The BAV group kept AE UMa on the list of program stars and has observed it visually between 1975 and 1992 and published seven epochs of maximum (Braune & Mundry 1982; Braune et al. 1979; Huebscher et al. 1985, 1992). Recently, the group observed the star photoelectrically and published one accurate time of maximum light (Agerer et al. 1999).

The published data are well-supplemented by the times of maximum light obtained from the Konkoly photometry.

The light curves of AE UMa have the typical asymmetrical shape of the large amplitude δ Scuti stars. The large variation in the amplitude and shape of its light curve makes the determination of the times of light maximum difficult. The observations near the top of the light curves were fitted with third order polynomials and a least-squares solution provided the times of maximum. The results, of course, depended on how many data points were involved in the calculation. Therefore, several trials were made at each observed maximum. The most reliable results could be achieved when only 5–6 data points were taken into account. This procedure also gave an estimate of the error of the determination of the instant of maximum light, which was around ±0.0004 day. The epochs of maximum light derived from the Konkoly observations are the mean values obtained from the B and V light curves. Although a phase lag between the times of maximum light in different colours may exist, it is, however, smaller than the error of the determination of an epoch. In this way 69 new times of maximum light could be derived. The new epochs, together with those found in the literature, are given in Table 1.

Table 1. Times of maximum of AE UMa ($C = 2442062.5824 + 0.08601707 \times E$)

Year	Max.time	Det.	Cycle	O–C	Source
	2400000+				
1937	28632.3980	pg	−156134	+.0048	(1)
1946	31875.1220	pg	−118435	−.0287	(2)
1950	33379.2560	pg	−100949	+.0108	(2)
1956	35601.1880	pg	−75118	+.0359	(2)
	35604.3370	vis	−75081	+.0022	(1)
	35607.1730	pg	−75048	−.0003	(2)
1957	35981.2020	pg	−70700	+.0264	(2)
1963	38106.4020	vis	−45993	+.0027	(1)
1971	41059.3680	vis	−11663	+.0027	(1)
1973	41773.2230	vis	−3364	+.0020	(1)
1974	42062.5832	pe	0	+.0008	(3)
	42065.5959	pe	35	+.0029	(4)
	42065.6778	pe	36	−.0012	(4)
	42068.3432	pe	67	−.0023	(4)
	42068.4302	pe	68	−.0014	(4)
	42068.5203	pe	69	+.0027	(4)
	42068.6029	pe	70	−.0007	(4)
	42068.6871	pe	71	−.0025	(4)
	42069.3808	pe	79	+.0031	(4)
	42069.4651	pe	80	+.0013	(4)
	42069.5473	pe	81	−.0025	(4)
	42069.6363	pe	82	+.0005	(4)
	42086.4965	pe	278	+.0014	(4)
	42086.5787	pe	279	−.0025	(4)
	42087.4390	pe	289	−.0023	(4)
	42087.5263	pe	290	−.0011	(4)
	42087.6155	pe	291	+.0021	(4)
	42095.5298	pe	383	+.0029	(3)
	42095.6123	pe	384	−.0007	(3)
	42103.3513	pe	474	−.0032	(4)
	42106.4523	pe	510	+.0012	(3)
	42119.5252	pe	662	−.0005	(3)
	42121.5025	pe	685	−.0016	(3)
	42122.3628	pe	695	−.0015	(4)
	42122.4484	pe	696	−.0019	(4)
	42128.2968	pe	764	−.0026	(3)
	42128.3872	pe	765	+.0017	(3)
	42128.4727	pe	766	+.0012	(3)
	42128.5557	pe	767	−.0018	(3)
	42133.4627	pe	824	+.0022	(3)
	42133.5442	pe	825	−.0023	(3)
	42134.4055	pe	835	−.0012	(3)
	42147.3933	pe	986	−.0019	(3)
	42148.4295	pe	998	+.0021	(3)
	42148.5117	pe	999	−.0018	(3)
	42159.4365	pe	1126	−.0011	(3)
	42161.4145	pe	1149	−.0015	(3)
1975	42453.5306	pe	4545	+.0006	(3)
	42453.6137	pe	4546	−.0023	(3)
	42460.4989	pe	4626	+.0015	(3)
	42532.4070	vis	5462	−.0006	(5)
1976	42830.6280	pe	8929	−.0008	(3)
	42837.5120	pe	9009	+.0018	(3)
	42838.4591	pe	9020	+.0027	(3)
	42866.4960	vis	9346	−.0019	(5)
	42869.3377	pe	9379	+.0012	(3)
	42869.4205	pe	9380	−.0020	(3)
1977	43162.5708	pe	12788	+.0021	(3)
1981	44633.4626	pe	29888	+.0020	(3)
	44633.5440	pe	29889	−.0026	(3)
	44633.6309	pe	29890	−.0017	(3)
	44634.4046	pe	29899	−.0022	(3)
	44634.4902	pe	29900	−.0026	(3)
	44634.5810	pe	29901	+.0022	(3)
	44692.4709	pe	30574	+.0026	(3)
	44696.3430	vis	30619	+.0039	(6)
	44696.4260	vis	30620	+.0009	(6)
	44696.5200	vis	30621	+.0089	(6)
1983	45355.4902	pe	38282	+.0023	(3)
	45355.5727	pe	38283	−.0012	(3)
	45382.3228	pe	38594	−.0024	(3)
	45382.4104	pe	38595	−.0008	(3)
	45382.4997	pe	38596	+.0025	(3)
	45382.5807	pe	38597	−.0026	(3)
1985	46114.3320	vis	47104	+.0015	(7)
1986	46468.4601	pe	51221	−.0026	(3)
	46468.5468	pe	51222	−.0020	(3)
1987	46855.6279	pe	55722	+.0023	(8)
	46856.5729	pe	55733	+.0011	(8)
	46856.6561	pe	55734	−.0017	(8)
	46857.6017	pe	55745	−.0023	(8)
	46857.6925	pe	55746	+.0025	(8)
	46858.6382	pe	55757	+.0020	(8)
	46859.6666	pe	55769	−.0018	(8)
	46878.4181	pe	55987	−.0020	(8)
	46878.5064	pe	55988	+.0003	(8)
	46878.5946	pe	55989	+.0025	(8)
	46884.5262	pe	56058	−.0011	(8)
	46884.6117	pe	56059	−.0016	(8)
	46886.5907	pe	56082	−.0010	(8)
1992	48683.3170	vis	76970	+.0007	(9)
1996	50151.4564	pe	94038	+.0008	(3)
	50151.5384	pe	94039	−.0032	(3)
	50152.3170	pe	94048	+.0012	(3)
	50152.4862	pe	94050	−.0016	(3)
	50152.5756	pe	94051	+.0017	(3)
1997	50458.8815	CCD	97612	+.0009	(10)
	50458.9636	CCD	97613	−.0031	(10)
	50459.8240	CCD	97623	−.0028	(10)
	50459.9113	CCD	97624	−.0015	(10)
	50467.7388	CCD	97715	−.0016	(10)
	50467.8236	CCD	97716	−.0028	(10)
	50490.3607	pe	97978	−.0022	(3)
	50505.6697	CCD	98156	−.0042	(10)
	50505.7595	CCD	98157	−.0004	(10)
	50505.8461	CCD	98158	+.0001	(10)
	50516.7676	CCD	98285	−.0025	(10)
	50554.4432	pe	98723	−.0024	(3)
1998	50813.3550	pe	101733	−.0020	(3)
	50813.4408	pe	101734	−.0022	(3)
	50813.6151	pe	101736	+.0001	(3)
	50813.6985	pe	101737	−.0026	(3)
	50848.4540	pe	102141	+.0021	(3)
	50848.5391	pe	102142	+.0011	(3)

Table 1. continued

Year	Max.time	Det.	Cycle	O–C	Source
	50848.6212	pe	102143	−.0028	(3)
	50849.4815	pe	102153	−.0026	(3)
	50849.5688	pe	102154	−.0014	(3)
	50862.3840	pe	102303	−.0027	(11)
	50872.2809	pe	102418	+.0022	(3)
	50872.3634	pe	102419	−.0013	(3)
	50872.4481	pe	102420	−.0026	(3)
	50872.5394	pe	102421	+.0027	(3)
	50899.3729	pe	102733	−.0012	(3)
	50899.4570	pe	102734	−.0031	(3)
	50902.2976	pe	102767	−.0010	(3)
	50902.3819	pe	102768	−.0028	(3)
	50903.3321	pe	102779	+.0013	(3)
	50903.4192	pe	102780	+.0023	(3)
	50903.5009	pe	102781	−.0020	(3)

References: (1) Tsesevich (1973); (2) Filatov (1960); (3) Present paper; (4) Broglia et al. (1975); (5) Braune et al. (1979); (6) Braune et al. (1982); (7) Huebscher et al. (1985); (8) Rodríguez et al. (1992); (9) Huebscher et al. (1992); (10) Hintz et al. (1997); (11) Agerer et al. (1999).

3. The analysis

3.1. The Fourier analysis

The investigation of the stability of the pulsation of a star can be approached in different ways. One of the methods most commonly used is Fourier analysis of the observations obtained at different seasons. The study of the variations in the frequencies and Fourier parameters may provide useful information.

The distribution of the observations of AE UMa was uneven, therefore we divided the data into five separate sets. Each set contained observations of several years. The first set contained the Konkoly data of 1974–1977 and the observations of Broglia & Conconi (1975) in 1974. Since both photometric series were made in the same system and the same comparison star was used, the data were simply pooled without any correction. The second set was composed only of the Konkoly observations from 1981–1983.

Rodríguez et al.'s (1992) observations of 1987 were included in the third set. These data were supplemented with a few (67 in V and in B) observations obtained at Konkoly in 1986. Rodríguez et al. carried out their photometry in the Strömgren system and used another comparison star. Although there are refined methods for overcoming the zero point differences between different sets of observations (see e.g. Paparó et al. 1996), in our case, transformation problems may arise due to use of different photometric systems. From the Fourier decomposition of the by and BV data the zero point differences

$$\Delta V_{\text{Konkoly}} - \Delta y_{\text{Rodríguez et al.}} = -0.572$$

and

$$\Delta B_{\text{Konkoly}} - \Delta b_{\text{Rodríguez et al.}} = -0.456$$

could be derived, but because of the difficulties mentioned above, the 1986–1987 data set is not homogeneous and could not be used for accurate determination of the amplitudes. The frequency determination, however, is not sensitive to small systematic differences, and for that purpose, the combined data sets of Rodríguez et al. and the Konkoly photometry could be used. For this set, the Fourier amplitudes were derived solely from the observations of Rodríguez et al. (1992).

Table 2. Frequencies for different segments of observations

Years	f_0	f_1
1974–1977	11.62558	15.03120
(B, V)	2	2
1981–1983	11.62560	15.03123
(B, V)	2	2
1986–1987	11.62556	15.03121
$(B, V\&\ b, v)$	3	3
1989–1993	11.62560	—
(HIP)	2	
1996–1998	11.62560	15.03125
(B, V)	2	2

The fourth set contains the Hipparcos photometry of AE UMa carried out between December 1989 and March 1993. Unfortunately, this photometry was made in a special photometric system and cannot be easily combined either with UBV or $uvby$ observations. The Hipparcos data can only be used for frequency analysis. The fifth set of data comprises the 1996–1998 Konkoly observations.

Multifrequency analysis was performed with the *MUFRAN* (*MUltiFRequency ANalysis*) program package (Kolláth 1990). *MUFRAN* is a collection of methods for period determination, sine fitting for observational data and graphics routines for visualization of the results.

The first step in the analysis was to find the frequencies of the fundamental (f_0) and first overtone (f_1) pulsation for each data set. The results are presented in Table 2. The frequencies (except for the fourth set) are the mean values obtained from the yellow and blue observations. It is interesting to note that the Hipparcos data have not provided a definite value of the overtone frequency. The errors have been estimated using the criterion that one standard error in the frequency should give a difference of not more than 0.02 in phase for the extreme time intervals.

The observations of the different sets were fitted with the formula

$$\Delta m = a_0 + \sum_{i,j} a_{i,j} \sin(2\pi f_{i,j} + \varphi_{i,j}) \tag{1}$$

by least-squares solution. Owing to the non-linear interaction between the two principal modes, frequencies which are linear combinations of $f_0 = 1/P_0$ and $f_1 = 1/P_1$ will be:

$$f_{i,j} = |if_0 \pm jf_1| \tag{2}$$

Table 3. Amplitudes for different segments of observations

Years	$a_{1,0}(V)$	$a_{0,1}(V)$	$a_{1,0}(B)$	$a_{0,1}(B)$
1974–1977	0.217	0.034	0.279	0.043
	3	3	3	3
1981–1983	0.217	0.040	0.283	0.049
	5	5	5	5
1986–1987	0.211	0.035	0.257[1]	0.043[1]
	2	2	1	1
1996–1998	0.210	0.040	0.271	0.048
	3	3	3	3

[1]Remark: From Strömgren b photometry.

where the integers i and j are subject to the constraints: $0 \leq i \leq 5$ and $0 \leq j \leq 1$. (Several test runs indicated that the residuals with $i > 5$ and $j > 1$ higher order fits did not decrease significantly.)

As the amplitudes $a_{1,0}$ and $a_{0,1}$ can give us a good measure of the stability of pulsation, they are given in Table 3 for the different sets of the blue and yellow observations. In the third set, Rodríguez et al.'s observations were taken into account only for the determination of amplitudes.

The results clearly show that both the amplitudes and the frequencies were subject only to small changes, during a time base of 25 years. A more rigorous treatment of the changes in the frequencies and in the phase modulation amplitude is given in the following subsections.

3.2. The O–C diagram

The Fourier analysis has the definite advantage that all the photometric observations are included into the frequency analysis. However, it often happens that the observations are not available, only the times of observed maxima are published. In this case, the construction of the O–C diagram may furnish the dominant frequency with better accuracy. The times of maximum light found in the literature and obtained from the Konkoly photometry are given in Table 1. The O–C values were calculated with the formula

$$C = \text{JD}2442062.5824 + 0\overset{\text{d}}{.}08601707 \times E$$

and are presented in Fig. 1. (Filatov's data were disregarded in the figure.)

The O–C values obtained from the photoelectric and CCD observation were fitted by a quadratic polynomial and the least squares solution resulted in the following formula ($n = 112$ maxima, time interval 24 years):

$$\text{O} - \text{C} = (-2.5 \pm 3.0)\ 10^{-4} + (0.62 \pm 1.90)\ 10^{-8} \times E$$
$$-(0.15 \pm 0.19)\ 10^{-12} \times E^2$$

which indicated that the period of the star was essentially constant in the last quarter of the century. The corrected period is

$$P_0 = 0\overset{\text{d}}{.}086017076 \pm (19\ 10^{-9})$$

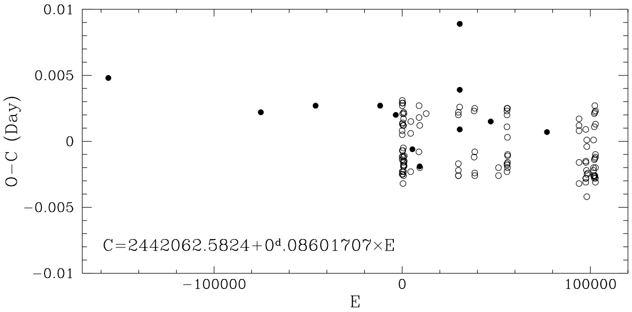

Fig. 1. O–C vs. epoch number diagram for the fundamental mode of AE UMa. Open circles denote photoelectric and CCD observations, dots indicate visual and photographic observations

or the frequency

$$f_0 = 11.6255986 \pm (26\ 10^{-7}),$$

in accord with the results of the Fourier analysis.

If we take into account all the O–C values with the exception of Filatov's data (124 maxima, time interval 61 years) we arrive at the quadratic polynomial fit:

$$\text{O} - \text{C} = (2.2 \pm 2.6)\ 10^{-4} - (1.71 \pm 0.53)\ 10^{-8} \times E$$
$$+(0.053 \pm 0.053)\ 10^{-12} \times E^2$$

which also favours a constant period over a time interval of more than half a century.

$$P_0 = 0\overset{\text{d}}{.}086017053 \pm (6\ 10^{-9}).$$

Owing to the double-mode nature of the star, the oscillation of the times of maximum light with the modulation period P_{m} manifests itself as a scatter on the O–C diagram. Moreover, if the distribution of the times of maxima in the modulation cycles is not uniform, it may falsify the value of both the linear and quadratic term (e.g. most of the maxima observed in 1997 and 1998 happened to be in the phase interval of the modulation cycle where the maxima came earlier than the mean epoch or the deviation of Tsesevich's early epoch from the mean value may have a strong influence over the parameters of the fit).

The O–C diagram can be constructed only for the dominant fundamental period (frequency). Table 2, however, suggests that the overtone frequency, at least in the last quarter of the century, has not changed significantly. The Fourier spectrum of the O–C values derived from the photoelectric and CCD maxima is presented in Fig. 2. The sharp line at $f_{\text{m}} = 3.40560$ indicates that the modulation frequency indeed has not varied significantly and the first overtone frequency is $f_1 = f_0 + f_{\text{m}} = 15.03120$ cycle/day.

The investigations carried out so far have not given information about the rate of change in the overtone frequency. Therefore, we addressed the problem in another way. Assuming that the fundamental and the overtone period, as well as the modulation amplitude,

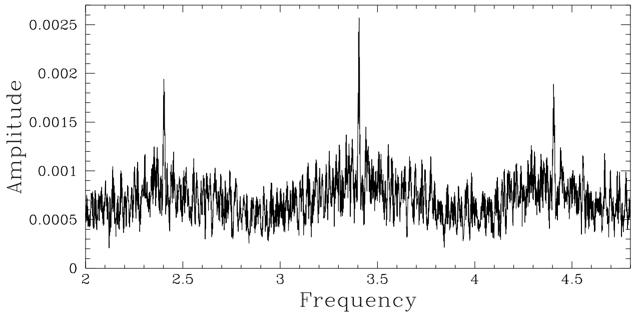

Fig. 2. Fourier spectrum of the photoelectric and CCD O–C values

have changed linearly over time, the O–C's of the photoelectrically observed and the CCD maxima were fitted by the following formula:

$$\mathrm{O-C} = a + b\times(t-T_0) + c\times(t-T_0)^2 + [d + e\times(t-T_0)] \times \sin[f + g\times(t-T_0) + h\times(t-T_0)^2] \quad (3)$$

where T_0 denotes the starting epoch (T_0 = JD2442062.5824) and $a = \Delta T_0$ is the correction to the starting epoch; $b = \Delta P_0/P_0$, where ΔP_0 is the correction to the fundamental period at $t = T_0$; $c = (1/2)\times(\mathrm{d}P_0/P_0\mathrm{d}t)$ is the rate of change in the fundamental period; d is the amplitude of the phase modulation cycle at $t = T_0$; e is the rate of change in the phase modulation amplitude; f is the phase of the modulation cycle at $t = T_0$; $g = 2\pi f_\mathrm{m}$, where $f_\mathrm{m} = f_1 - f_0$ is the phase modulation frequency; $h = (1/2)\times 2\pi\times(\mathrm{d}f_\mathrm{m}/\mathrm{d}t)$ gives information about the rate of change of the modulation frequency.

The least-squares solution of Eq. (3) results in the following:

$a = 0.00012 \pm 0.00015$ d

$b = (1.27 \pm 0.11)\ 10^{-7}$

$c = (0.47 \pm 1.23)\ 10^{-11}\ \mathrm{d}^{-1}$

$d = 0.00258 \pm 0.00023$ d

$e = (-0.36 \pm 0.36)\ 10^{-7}$

$f = 0.178 \pm 0.071$ rad

$g = 21.3973 \pm 0.0012\ \mathrm{rad}\cdot\mathrm{d}^{-1}$

$h = (0.17 \pm 0.18)\ 10^{-7}\ \mathrm{rad}\cdot\mathrm{d}^{-2}$.

The main conclusions are as follow: The fundamental period ($P_0 = 0.086017059 \pm 1\ 10^{-9}$) seems to be essentially constant,

$$\begin{aligned} P_0^{-1}\frac{\mathrm{d}P_0}{\mathrm{d}t} &= (0.95 \pm 2.46)\ 10^{-13}\,\mathrm{d}^{-1} \\ &= (0.35 \pm 0.90)\ 10^{-8}\ \mathrm{y}^{-1}. \end{aligned}$$

The total amplitude of time oscillation of maxima (phase modulation) is $2d = 0.0052$ day $= 7.4$ minute, which seems to increase with a rate of $-0.36\ 10^{-7} = -1.1\ \mathrm{s}\times\mathrm{y}^{-1}$. This value is, however, close to the error of its determination. The modulation frequency is equal to $f_\mathrm{m} = 3.40549 \pm 0.00019\ \mathrm{d}^{-1}$ or the modulation period $P_\mathrm{m} = 0.2936438 \pm 0.0000014$ days at $t = T_0$. The rate of change in the modulation frequency is $\mathrm{d}f_\mathrm{m}/\mathrm{d}t = (0.54 \pm 0.56)\ 10^{-8}\ \mathrm{d}^{-2}$ or, in the modulation period, $\mathrm{d}P_\mathrm{m}/\mathrm{d}t = (-0.47 \pm 0.49)\ 10^{-9}$.

We can obtain the first overtone frequency and its rate of change:

$$f_1 = f_0 + f_\mathrm{m} = 15.03109 \pm 0.00020\ \mathrm{d}^{-1}$$

and

$$\frac{\mathrm{d}f_1}{\mathrm{d}t} = \frac{\mathrm{d}f_0}{\mathrm{d}t} + \frac{\mathrm{d}f_\mathrm{m}}{\mathrm{d}t} = (0.54 \pm 0.56)\ 10^{-8}$$

or

$$\begin{aligned} P_1^{-1}\frac{\mathrm{d}P_1}{\mathrm{d}t} &= (-0.36 \pm 0.37)\ 10^{-9}\ \mathrm{d}^{-1} \\ &= (-0.13 \pm 0.14)\ 10^{-6}\ \mathrm{y}^{-1}. \end{aligned}$$

This result indicates that the first overtone period maybe slowly decreasing. The definite answer to this problem is given in the following subsection.

3.3. The Fourier phase diagram

The Fourier phase method is possibly the most reliable determination of the frequencies of a double-mode pulsating star. If we fix the amplitudes and frequencies in the Fourier decomposition (in Eqs. (1) and (2)), then the variations in the , $\varphi_{1,0}$ and $\varphi_{0,1}$ phases reflect the changes in the fundamental and first overtone frequencies. The advantage of the method is that the reduced number of free parameters makes the use of short segments of observations possible. Test runs showed that small changes in the amplitudes did not significantly affect the determination of the phases and the zero point differences could easily be taken into account by leaving a_0 as free parameter.

The Fourier phase diagrams of AE UMa for both frequencies are shown in Fig. 3. The quadratic fits result in the following frequencies:

$$\begin{aligned} f_0 &= (11.625601 \pm 0.000002) \\ &\quad -(4 \pm 4)\ 10^{-10}(t - T_0) \end{aligned}$$

and

$$\begin{aligned} f_1 &= (11.031184 \pm 0.000008) \\ &\quad +(3 \pm 2)\ 10^{-9}(t - T_0) \end{aligned}$$

or the periods:

$$\begin{aligned} P_0 &= (0.086017058 \pm 15\ 10^{-9}) \\ &\quad +(3 \pm 3)\ 10^{-12}(t - T_0) \end{aligned}$$

and

$$\begin{aligned} P_1 &= (0.066528358 \pm 35\ 10^{-9}) \\ &\quad -(1.3 \pm 0.9)\ 10^{-11}(t - T_0). \end{aligned}$$

Hence, the fundamental mode frequency is constant and the first overtone frequency increases.

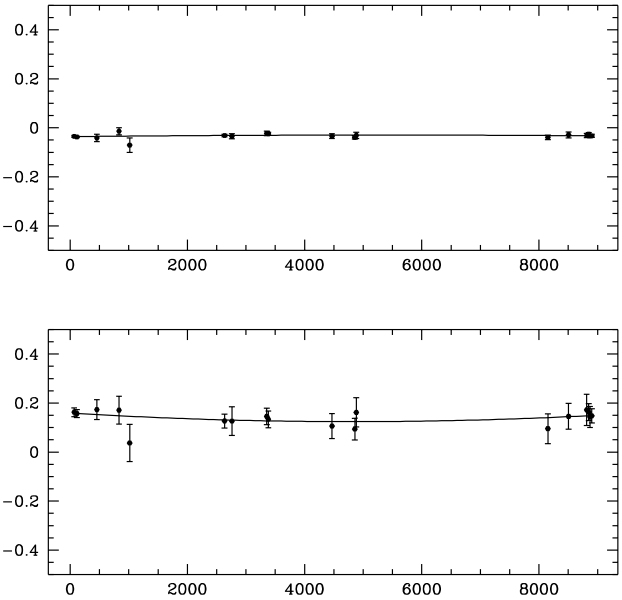

Fig. 3. Fourier phase diagrams for the fundamental mode frequency (upper panel) and for the overtone frequency (lower panel). The ordinate scales are the phases $\varphi_{1,0}$ and $\varphi_{0,1}$ respectively; the abscissa is time in day $JD - 2442000$

4. Discussion

4.1. Amplitudes

Amplitude and frequency variations have been discovered in various classes of pulsating variable stars. Significant variation in the amplitudes and frequencies of low-amplitude δ Scuti stars seems to be quite common (see e.g. Breger 1990a; Handler et al. 1998; Rodríguez et al. 1998). This phenomenon can be well explained by modulation of very closely spaced frequencies or by interaction of resonant modes. In the case of low amplitude δ Scuti stars, however, a number of frequencies are excited with amplitudes of similar order. The question arises: how stable are the strongly excited fundamental and first overtone radial oscillations of δ Scuti stars? Recent investigations have revealed that the high-amplitude δ Scuti stars (HADS) are also characterized by small but significant amplitude variations. Walraven et al. (1992) found that the amplitudes of the fundamental and first overtone pulsation of AI Vel were significantly larger in the past and they have begun to increase once more. Rodríguez et al. (1997) confirmed that the amplitude of the main frequency of AN Lyn varies on a long time scale, and changes in amplitude are also present in the secondary frequencies. Recently, Arentoft & Sterken (2000) investigated the behaviour of the high-amplitude δ Scuti star V1162 Ori in detail and showed that the light curve of the star undergoes amplitude changes on a short time scale.

The photoelectric observations of AE UMa cover 25 years and are suitable for an investigation of long-term variability in the amplitudes. The changes in the amplitude of the fundamental and first overtone were examined by comparing the least-squares amplitude solution for each segment of observations in both B and V. The amplitudes $a_{1,0}$ and $a_{0,1}$ can be determined with adequate accuracy and are presented in Table 3. Our results show that the amplitude of the fundamental and first overtone oscillations of AE UMa have undergone very minor changes and the star's pulsation exhibits in general high stability.

4.2. Frequencies

The stability of the fundamental and first overtone frequencies have been investigated by different methods in the previous section, which led us to the conclusion that the fundamental oscillation is very stable and the rate of its change is less than the error of its determination. However, the different methods produce result with both signs for period changes more often for increasing period. We feel confident stating, however, that the rate of change of the fundamental period is smaller than 10^{-12} d/d. Thus the fast period decrease previously reported (Hintz et al. 1997) is shown to be incorrect and the observed period change is now reconciled with the theoretical expectation (Breger & Pamyatnyk 1998). The essentially constant fundamental period suggests that AE UMa is in the post-main sequence evolutionary state.

On the other hand, the investigations show that the first overtone period is definitely decreasing with a rate of $(1/P_1)(\mathrm{d}P_1/\mathrm{d}t) = -2.0\ 10^{-10}\ \mathrm{d}^{-1} = -7.3\ 10^{-8}\ \mathrm{y}^{-1}$. Although the constant fundamental period is consistent with evolutionary theories (Breger & Pamyatnyk 1998), the changes in the first overtone period cannot similarly be explained by evolution.

The situation becomes more interesting if we compare the period changes of AE UMa and AI Vel. Walraven et al. (1992) showed that AI Vel had a constant fundamental mode period while the radial overtone period increased at a rate of $(1/P_1)(\mathrm{d}P_1/\mathrm{d}t) = 4.1\ 10^{-10}\ \mathrm{d}^{-1} = 14.9\ 10^{-8}\ \mathrm{y}^{-1}$, the opposite of the AE UMa case.

The example of both AI Vel and AE UMa clearly shows that stellar evolution is not the only mechanism which can produce changes in the periods of radial pulsation. Breger (1990b, 1993) discussed the problem of changes in the period of δ Scuti stars in general and emphasized that "for an individual star the conversion of observed period changes into stellar evolution rates (e.g. radius changes) has to be applied with caution". The problem was more rigorously treated by Rodríguez et al. (1995), Breger & Pamyatnykh (1998) and Breger (1999). They compared the observed period changes of HADS with those expected from stellar evolutionary model calculations in the lower part of the Instability Strip and came to the conclusion that the observed period changes could not be caused by stellar evolution alone, and non-evolutionary effects also play an important part in the period changes of δ Scuti stars. The non-evolutionary effects, and especially the effect of period changes due to non-linear mode interactions, were discussed by Breger & Pamyatnykh (1998) in detail. Cox (1998) investigated the non-parallel period changes of the RR Lyrae star V53 in the globular cluster M15 (Paparó et al. 1998) and noted that δ Scuti stars are also good candidates for this kind of anomalous period changes because they "have ages old enough to affect their surface helium composition structure". This remark may be especially relevant to AI Vel and AE UMa.

One of the most interesting problems concerning high-amplitude δ Scuti stars is whether, apart from the radially excited fundamental and first overtone pulsations, higher order radial or non-radial oscillations are also excited. The importance of detecting the other pulsation modes in this type of star is that the larger the number of identified modes, the greater are the number of constraints imposed on pulsation and evolution theory. Walraven et al. (1992) found, indeed, two previously undiscovered periodicities which were tentatively identified as the third and fifth overtones and possibly a non-radial overtone as well. Garrido & Rodríguez (1996) have pointed out that other high-amplitude δ Scuti stars (SX Phe and DY Peg) are also pulsating with previously unknown, additional frequencies and this behaviour might be a common phenomenon among the HADS. Musazzi et al. (1998) also proved the existence of additional small-amplitude oscillation in some stars. At the same time, both Musazzi et al. (1998) and Garrido & Rodríguez (1996) emphasize that the detection of additional frequencies is a delicate problem and needs high quality data and prudent discussion. Notwithstanding, we hope that a careful analysis of other high-amplitude δ Scuti stars might reveal hidden periodicities. The observations of AE UMa in 1996–1997–1998 provide an accurate data set long enough for the search of possible further frequencies. After pre-whitening with the principal frequencies and their linear combinations, no trace of any frequency above the significance level could be detected in the 0–50 cycle/day range. The amplitude of the peaks was below 0.007 mag. The slightly higher signal at low frequencies is the consequence of the small night-to-night zero-point drifts. A least-squares solution ($0 \leq i \leq 5; 0 \leq j \leq 1$) produces residual rms deviations of 0.022 for the B and 0.020 for the V light curves that are significantly larger (at least a factor of two) than the observational errors. This fact may hint at the existence of other frequencies. This question deserves further investigation and CCD observations of higher accuracy are needed.

RV Ari and BP Peg are also double-mode HADS and resemble AE UMa and AI Vel almost in every respect. A similar study for those stars could also provide information on the nature of double-mode high-amplitude δ Scuti stars and some clues to the problems discussed herein.

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