

# The structure of radiative shock waves

## III. The model grid for partially ionized hydrogen gas

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**Abstract.** The grid of the models of radiative shock waves propagating through partially ionized hydrogen gas with temperature  $3000\text{ K} \leq T_1 \leq 8000\text{ K}$  and density  $10^{-12}\text{ gm cm}^{-3} \leq \rho_1 \leq 10^{-9}\text{ gm cm}^{-3}$  is computed for shock velocities  $20\text{ km s}^{-1} \leq U_1 \leq 90\text{ km s}^{-1}$ . The fraction of the total energy of the shock wave irreversibly lost due to radiation flux ranges from 0.3 to 0.8 for  $20\text{ km s}^{-1} \leq U_1 \leq 70\text{ km s}^{-1}$ . The postshock gas is compressed mostly due to radiative cooling in the hydrogen recombination zone and final compression ratios are within  $1 < \rho_N/\rho_1 \lesssim 10^2$ , depending mostly on the shock velocity  $U_1$ . The preshock gas temperature affects the shock wave structure due to the equilibrium ionization of the unperturbed hydrogen gas, since the rates of postshock relaxation processes are very sensitive to the number density of hydrogen ions ahead the discontinuous jump. Both the increase of the preshock gas temperature and the decrease of the preshock gas density lead to lower postshock compression ratios. The width of the shock wave decreases with increasing upstream velocity while the postshock gas is still partially ionized and increases as soon as the hydrogen is fully ionized. All shock wave models exhibit stronger upstream radiation flux emerging from the preshock outer boundary in comparison with downstream radiation flux emerging in the opposite direction from the postshock outer boundary. The difference between these fluxes depends on the shock velocity and ranges from 1% to 16% for  $20\text{ km s}^{-1} \leq U_1 \leq 60\text{ km s}^{-1}$ . The monochromatic radiation flux transported in hydrogen lines significantly exceeds the flux of the background continuum and all shock wave models demonstrate the hydrogen lines in emission.

**Key words.** shock waves – hydrodynamics – radiative transfer – stellar atmospheres

### 1. Introduction

Main indicators of shock waves in atmospheres of radially pulsating stars are enhanced hydrogen emission, double profiles of absorption lines and discontinuities in radial velocity curves. Most prominently these features are observed in W Vir (Lèbre & Gillet 1992), RV Tau (Gillet et al. 1989) and Mira type (Alvarez et al. 2000) pulsating variables. The ratio of the atmosphere scale height to the stellar radius in these stars is  $H/R \gtrsim 10^{-2}$  and the linear theory of stellar pulsation admits the existence of progressive waves propagating through the stellar atmosphere (Unno 1965; Pijpers 1993). The running waves must inevitably transform into shock waves due to non-linear effects and this conclusion is corroborated by hydrodynamic calculations of radial pulsations of low-mass late-type giants (Wood 1974; Fadeyev & Tutukov 1981; Bowen 1988; Fadeyev & Muthsam 1990).

Though the role of shock waves as the principal mechanism of mass loss from late-type stars is still disputable (Hoefner & Dorfi 1997) it is nevertheless undoubted that shock-driven gas flows are responsible for formation of circumstellar shells. It must be noted that the postshock velocity near the photosphere should not necessarily be greater than the local escape velocity, because periodic passage of shock waves distends the stellar atmosphere and, under some conditions, may lead to the gradual approach of the gas flow to the escape velocity (Willson & Hill 1979). Furthermore, the shock amplitude tends to increase while the shock wave propagates in a medium with decaying gas density.

The most important problem concerning the shock-driven mass loss is the rate of the shock decay during its passage through the stellar atmosphere. Various simplifying assumptions lead to estimates of mass

loss rates that differ from one another by many orders of magnitude (Wood 1979). Therefore, an analysis must be based on the self-consistent solution of the radiation transfer, fluid dynamics and rate equations. Unfortunately, the description of the shock wave passage in terms of the Lagrangean approach cannot be applied, since it does not allow the postshock relaxation zones to be explicitly resolved with respect to the spatial coordinate (Klein et al. 1976, 1978). Another approach is based on the assumption of steady-state gas flow. Applicability of this assumption for shock waves in stellar atmospheres is justified by the small width (including both the radiative precursor and the postshock relaxation zone) of the shock wave in comparison with the atmosphere scale height.

Another necessary condition for applicability of the steady-state assumption implies that the characteristic time scale of the shock wave decay is  $t_{\text{dec}} \ll d/U$ , where  $d$  and  $U$  are the width and velocity of the shock wave. In atmospheres of radially pulsating stars this condition is obviously fulfilled because emission lines indicating the shock wave are observed during the major part of the pulsation period (Abt 1954; Willson 1976) and in some stars this interval is as long as a half (Fox et al. 1984) or even 0.8 (Gillet et al. 1985) of the pulsation period.

Even in the framework of the steady-state assumption the problem of the shock wave structure in the partially ionized gas remains tremendously difficult, due to the strong coupling between the radiative precursor and the shock wave. Indeed, the occupation number densities of bound and free levels in both atoms and molecules strongly depend on the radiation field produced in the shock wave and at the same time must be determined as a solution of rate equations that are stiff on the relevant time scales. Fortunately, in stars with effective temperatures of  $T_{\text{eff}} \gtrsim 3000$  K, the gas is mostly in the atomic state and the principal dissipative processes in the postshock region are excitation and ionization of atoms.

In Paper I (Fadeyev & Gillet 1998) we proposed the method of global iterations that allows us to obtain the stable self-consistent solution of the equations of fluid dynamics, radiation transfer and rate equations for the steady-state plane-parallel shock wave propagating through the atomic hydrogen gas. In Paper II (Fadeyev & Gillet 2000) this method was employed for computations of the structure of shocks with upstream velocities of  $15 \text{ km s}^{-1} \leq U_1 \leq 70 \text{ km s}^{-1}$  and for the temperature and the density of the unperturbed hydrogen gas of  $T_1 = 6000$  K and  $\rho_1 = 10^{-10} \text{ gm cm}^{-3}$ , respectively. Results of these calculations are obviously insufficient for astrophysical applications since the temperature and the density in atmospheres of pulsating stars vary in wide ranges.

Below we discuss the structure of shock waves propagating through partially ionized hydrogen gas with temperature and density ranges within  $3000 \text{ K} \leq T_1 \leq 8000 \text{ K}$  and  $10^{-12} \text{ gm cm}^{-3} \leq \rho_1 \leq 10^{-9} \text{ gm cm}^{-3}$ , respectively. Basic equations and the method of their solution are described in Paper II. In comparison with our previ-

ous work, we improved the treatment of radiation transfer and extended the total spectral range to  $13 \leq \log \nu \leq 16$ . Furthermore, we employed the much more correct treatment of the spectral line radiation transfer which is of great importance for atomic level populations obtained as solution of rate equations. All calculations were done for the hydrogen atom with  $L = 4$  bound levels and a continuum.

## 2. General description of the shock wave structure

In the comoving fluid frame the radiation-modified Rankine-Hugoniot relations are written (Marshak 1958; Mihalas & Weibel Mihalas 1984) as

$$\rho U = \mathcal{C}_0 \equiv \dot{m}, \quad (1)$$

$$\dot{m}U + P_g + P_R = \mathcal{C}_1, \quad (2)$$

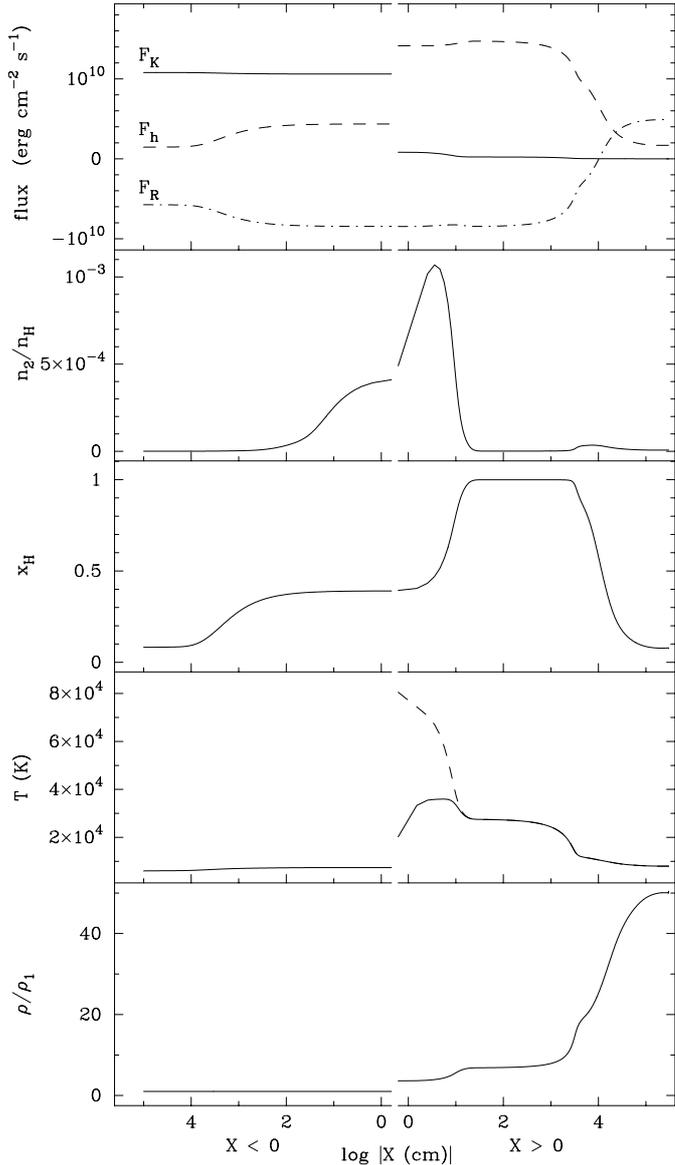
$$\frac{1}{2}\dot{m}U^2 + \dot{m}h + F_R + U(E_R + P_R) = \mathcal{C}_2, \quad (3)$$

where  $\rho$  is the gas density,  $U$  is the gas flow velocity,  $h$  is the specific enthalpy,  $P_g$  is the gas pressure,  $E_R$ ,  $F_R$  and  $P_R$  are the radiation energy density, radiation flux and radiation pressure, respectively.

In planar geometry the total energy flux  $\mathcal{C}_2$  given by relation (3) is constant along the spatial coordinate  $X$ , so that while the parcel of gas passes the shock wave the decrease of the flux of kinetic energy  $F_K = \frac{1}{2}\dot{m}U^2$  is balanced by changes in the enthalpy flux  $F_h = \dot{m}h$  and the radiation flux  $F_R$ . Throughout the shock wave the flux  $U(E_R + P_R)$  is significantly smaller than other terms of the left-hand side of relation (3) and can be ignored.

As in Paper II the shock wave model is represented by the comoving plane-parallel finite slab. The space coordinate  $X = 0$  is set at the viscous adiabatic jump which is treated as an infinitesimally thin discontinuous jump where hydrodynamic variables undergo an abrupt change. It is assumed that the space coordinate  $X$  of the gas element increases while the gas flows through the shock wave, so that in the preshock region  $X < 0$  and in the postshock region  $X > 0$ . The coordinates of the preshock and postshock outer boundaries are denoted as  $X_1$  and  $X_N$ , respectively. The upstream radiation flux emerging from the preshock outer boundary is negative, whereas the downstream radiation flux emerging from the postshock outer boundary is positive, that is,  $F_{R1} < 0$  and  $F_{RN} > 0$ .

On the upper panel of Fig. 1, we show the fluxes  $F_K$ ,  $F_h$  and  $F_R$  as a function of space coordinate  $X$  for the shock wave model with  $\rho_1 = 10^{-10} \text{ gm cm}^{-3}$ ,  $T_1 = 6000$  K and  $U_1 = 60 \text{ km s}^{-1}$ . For the sake of convenience we use the logarithmic scale along the spatial coordinate  $X$ , the preshock and postshock regions being represented by the left plot and the right plot, respectively. Below the upper panel of Fig. 1, we show as a function of  $X$  the fractional number density of hydrogen atoms in 2-nd state  $n_2/n_H$ , the hydrogen ionization degree  $x_H = n_e/n_H$ , the electron temperature  $T_e$  and the temperature of heavy particles (i.e. of neutral hydrogen atoms and hydrogen ions)  $T_a$ , and



**Fig. 1.** The structure of the shock wave with  $\rho_1 = 10^{-10} \text{ gm cm}^{-3}$ ,  $T_1 = 6000 \text{ K}$  and  $U_1 = 60 \text{ km s}^{-1}$ . On the upper panel the solid lines, dashed lines and dot-dashed lines represent the flux of the kinetic energy  $F_K$ , the flux of the enthalpy  $F_h$  and the radiation flux  $F_R$ . On lower panels are given the fractional number density of hydrogen atoms in the 2-nd state  $n_2/n_H$ , the hydrogen ionization degree  $x_H$ , the electron temperature  $T_e$  (solid lines) and the temperature of heavy particles  $T_a$  (dashed line), and the compression ratio  $\rho/\rho_1$

on the lowest panel we give the plot of the compression ratio  $\rho/\rho_1$ .

We assume that at the preshock outer boundary  $X_1$ , the gas temperature and the gas density are the same as those of the unperturbed medium that are denoted as  $T_1$  and  $\rho_1$ , respectively. At the same time, the upstream radiation flux emerging from the shock wake affects both the bound-level number densities and the ionization degree of hydrogen atoms. Therefore, the number densities of hydrogen atoms in  $i$ th state  $n_i$  and the number den-

sity of free electrons  $n_e$  at the preshock outer boundary are determined from the solution of the equations of statistical equilibrium. In our calculations we tried to set the preshock outer boundary  $X_1$  as far as possible from the discontinuous jump in order to diminish the perturbing influence of the shock wave radiation field on the preshock gas.

The most prominent changes that the gas flow undergoes in the preshock region are due to the absorption of the Lyman continuum radiation. In Fig. 1 the radiative precursor is revealed by the growth of the enthalpy flux  $F_h$  and due to the corresponding changes of the radiation flux  $F_R$ . The length of the radiative precursor  $\delta X_p$  is proportional to the mean free path of Lyman continuum photons and increases with decreasing density from  $\delta X_p \sim 10^3 \text{ cm}$  at  $\rho_1 = 10^{-9} \text{ gm cm}^{-3}$  to  $\delta X_p \sim 10^7 \text{ cm}$  at  $\rho_1 = 10^{-12} \text{ gm cm}^{-3}$ .

Within the radiative precursor the divergence of the Lyman continuum flux is negative, that is,

$$\nabla \cdot \mathbf{F}_{\text{Lyc}} = 4\pi \int_{\nu_1}^{\infty} (\eta_\nu - \kappa_\nu J_\nu) d\nu < 0, \quad (4)$$

where  $\eta_\nu$  and  $\kappa_\nu$  are the monochromatic emission and absorption coefficients,  $J_\nu$  is the mean intensity and  $\nu_1$  is the threshold frequency for ionization from the ground state of the hydrogen atom. Thus, the gas temperature increases due to absorption of the Lyman continuum radiation while the gas parcel approaches the discontinuous jump. As a result, the gas density  $\rho$  just ahead of the discontinuity exceeds its unperturbed value  $\rho_1$ , whereas the gas flow velocity  $U$  becomes smaller than  $U_1$ . The change of the gas density  $\rho$  and the gas flow velocity  $U$  in the radiative precursor depends not only on the upstream velocity  $U_1$  but also on the temperature and density of the unperturbed preshock gas. At  $\rho_1 = 10^{-10} \text{ gm cm}^{-3}$  and  $U_1 = 75 \text{ km s}^{-1}$ , the relative growth of the density within the radiative precursor ranges from  $\Delta\rho/\rho_1 \approx 0.03$  at  $T_1 = 3000 \text{ K}$  to  $\Delta\rho/\rho_1 \approx 0.07$  at  $T_1 = 7000 \text{ K}$ .

It should be noted that when the hydrogen gas is not completely ionized in the preshock region, the increase in the enthalpy flux is mostly due to ionization of hydrogen atoms. The rates of collisional transitions, both bound-bound and bound-free, are negligible in comparison with those of radiative transitions, so that the increase in  $F_h$  ahead of the discontinuous jump results mostly from the photoionization of hydrogen atoms.

Across the discontinuity, the flux of the kinetic energy  $F_K$  undergoes an abrupt decrease which is exactly balanced by the same abrupt increase of the enthalpy flux  $F_h$ . Because population number densities per unit mass  $n_i/\rho$  and  $n_e/\rho$  do not change across the discontinuity, the increase in the specific enthalpy is only due to the abrupt increase in the temperature of heavy particles  $T_a$  and the electron temperature  $T_e$ . Here we assume that the temperature of heavy particles just behind the discontinuous jump is given by the solution of the Rankine-Hugoniot

relations (1)–(3):

$$T_a^+ = T_a^- - x_H^- (T_e^+ - T_e^-) + \frac{1}{5} \frac{\dot{m} U^- \eta^2 - 1}{n_H^- k} - \frac{2}{5} \frac{F_R^+ - F_R^-}{n_H^- k U^-} - \frac{2}{5} \frac{(E_R^+ + P_R^+) \eta^{-1} - (E_R^- + P_R^-)}{n_H^- k}, \quad (5)$$

whereas the electron temperature increases due to adiabatic compression:

$$T_e^+ = \eta^{\gamma-1} T_e^-, \quad (6)$$

where  $\eta = \rho^+/\rho^-$  is the compression ratio across the discontinuous jump,  $\gamma = 5/3$  is the ratio of specific heats of the electron gas, and superscripts minus and plus refer to the variables defined ahead of and behind the discontinuous jump, respectively<sup>1</sup>.

The mean free path of photons in the vicinity of the discontinuous jump is several orders of magnitude larger than that of hydrogen atoms and the radiation flux will remain constant to a high order of accuracy, even if we take into account the finite thickness of the viscous adiabatic jump. In our calculations, the space interval between two adjacent cell centers located ahead of and behind the discontinuity was set equal to 1 cm for all models. Thus, the last two terms on the r.h.s. of (5) can be omitted without loss of accuracy because they refer to the difference between the upstream and downstream values of the radiation energy density, radiation flux and radiation pressure.

Excitation and ionization of hydrogen atoms behind the discontinuity are mostly due to radiative transitions. The timescale of photoexcitation of the lower bound levels is comparable with that of the electron temperature growth, whereas the photoionization of hydrogen atoms is much slower and begins with a substantial delay. The increase in the hydrogen ionization leads to a small increase in the enthalpy flux and, correspondingly, to the small decrease in the flux of kinetic energy because the radiation flux remains almost constant. However, the most prominent process accompanying hydrogen ionization is the increase in the compression ratio  $\rho/\rho_1$ . For the model presented in Fig. 1, the compression ratio increases from  $\rho/\rho_1 = 3.6$  just behind the discontinuity, to  $\rho/\rho_1 = 6.8$  in the layers where the hydrogen ionization reaches its maximum.

At larger distances from the discontinuity the kinetic energy of the gas flow initially stored as the ionization energy of the gas is converted into radiation. In Fig. 1 these layers are revealed as those of remarkable changes in  $F_h$  and  $F_R$ , accompanied by the growth of the compression ratio  $\rho/\rho_1$ . For models considered in the present study, the compression ratio reached at the postshock outer boundary ranges from  $1 < \rho_N/\rho_1 \lesssim 10^2$  depending mostly on

the upstream velocity  $U_1$  and less on the preshock temperature  $T_1$  and preshock density  $\rho_1$ .

The large increase in the postshock compression ratio  $\eta = \rho/\rho_1$  is due to the ionization of hydrogen gas and its subsequent radiative cooling. In order to roughly estimate the contribution of these effects we can write the energy conservation relation (3) with the omitted radiation energy density and radiation pressure terms as

$$\frac{U_1^2}{2} + \tilde{E}_{t1} + \tilde{E}_{in1} + \frac{P_{g1}}{\rho_1} + \frac{F_{R1}}{\dot{m}} = \frac{U^2}{2} + \tilde{E}_t + \tilde{E}_{in} + \frac{P_g}{\rho} + \frac{F_R}{\dot{m}}, \quad (7)$$

where  $\tilde{E}_t = \frac{3}{2}(n_H + n_e)kT/\rho$  is the specific energy in the translational degrees of freedom and  $\tilde{E}_{in}$  is the specific energy of excitation and ionization of hydrogen atoms. The left-hand side of (7) is written for the preshock outer boundary, whereas its right-hand side represents an arbitrary layer of the postshock region.

Relation (7) and the momentum conservation relation (2) can be rewritten as

$$\eta = 4 + 3 \frac{\tilde{E}_{in} - \tilde{E}_{in1}}{\tilde{E}_t} + 3 \frac{F_R - F_{R1}}{\dot{m} \tilde{E}_t}, \quad (8)$$

where we assumed that the specific energy in the translational degrees of freedom of the preshock gas is negligible in comparison with that of the postshock compressed gas, that is,  $\tilde{E}_{t1}/\eta \ll \tilde{E}_t$ .

The first term in the right-hand side of (8) is the limiting compression ratio of the atomic hydrogen gas at the discontinuous jump. The second term in the right-hand side of (8) describes the compression of the postshock gas due to excitation and ionization of hydrogen atoms. For the model shown in Fig. 1, the maximum hydrogen ionization degree is  $x_H = 0.99$  and the ratio of the specific energies is  $\tilde{E}_{in}/\tilde{E}_t \approx 2$ . Thus, the upper limit for the compression ratio beyond the layers of hydrogen ionization is  $\eta \approx 10$ .

However the most important contribution to the growth of the compression ratio comes from the third term in the right-hand side of (8). Indeed, for the model shown in Fig. 1 the ratio of the sum of radiative fluxes (note that  $F_{R1}$  is negative) to the flux of the translational energy of the gas at the postshock outer boundary is  $(F_{RN} - F_{R1})/(\dot{m} \tilde{E}_N) \approx 18$ . Therefore, the maximum compression ratio at the postshock outer boundary is  $\eta \approx 64$ .

### 3. Effects of the preshock temperature

Just behind the discontinuity, the temperature of electrons,  $T_e$ , is substantially lower than the temperature of heavy particles,  $T_a$ , and electrons acquire energy in elastic collisions with neutral hydrogen atoms and hydrogen ions. Because the elastic scattering crosssection of electrons and hydrogen ions is much larger than that of electrons and neutral hydrogen atoms, the efficiency of the energy exchange between heavy particles and free electrons is very

<sup>1</sup> In relation (5) we corrected the typeset errors which appeared in relation (15) of Paper II.

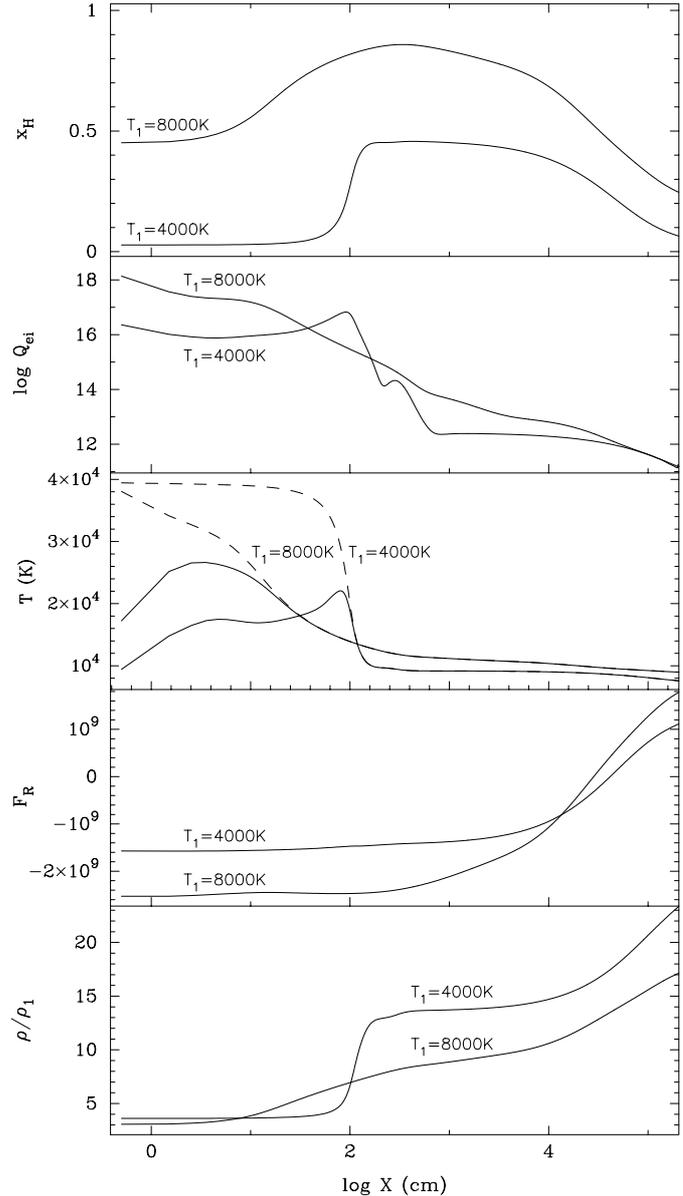
sensitive to the hydrogen ionization degree,  $x_{\text{H}}^-$ , ahead of the discontinuous jump. In particular, at  $x_{\text{H}}^- \gtrsim 10^{-2}$ , the postshock equilibration between the electron temperature  $T_{\text{e}}$  and the temperature of heavy particles  $T_{\text{a}}$  is due to elastic scattering of electrons by hydrogen ions and the equilibration rate very rapidly increases with increasing  $x_{\text{H}}^-$ . Also, in reverse, at  $x_{\text{H}} < 10^{-2}$ , free electrons gain most of the energy from elastic collisions with neutral hydrogen atoms and the rate of temperature equilibration is much slower. This process, however, is perceptible only in shock waves with weak ionization in the radiative precursor (that is, at upstream velocities  $U_1 < 30 \text{ km s}^{-1}$ ), the preshock gas temperature being  $T_1 < 4000 \text{ K}$ .

Thus, effects of the preshock gas temperature on the shock wave structure are mostly due to the preshock equilibrium hydrogen ionization. This is illustrated in Fig. 2 for shock wave models with  $T_1 = 4000 \text{ K}$  and  $T_1 = 8000 \text{ K}$ . In both cases, the preshock gas density and the upstream velocity are  $\rho_1 = 10^{-10} \text{ gm cm}^{-3}$  and  $U_1 = 40 \text{ km s}^{-1}$ , respectively, so that the postshock equilibration of  $T_{\text{e}}$  and  $T_{\text{a}}$  is due to elastic scattering of electrons by hydrogen ions. As is seen from the upper panel of Fig. 2, the hydrogen ionization degree just behind the discontinuous jump is  $x_{\text{H}} \approx 0.02$  at  $T_1 = 4000 \text{ K}$  and  $x_{\text{H}} \approx 0.46$  at  $T_1 = 8000 \text{ K}$ . As a result, at a preshock gas temperature  $T_1 = 8000 \text{ K}$ , the rate of the energy gain by electrons in elastic collisions with hydrogen ions is larger by a factor of 60 than that at  $T_1 = 4000 \text{ K}$ .

The increase in the preshock gas temperature  $T_1$  is accompanied by the stronger radiation flux emerging from the shock wave. This is, obviously, due to the higher hydrogen ionization degree in the postshock region. The more gradual changes in the postshock electron temperature and of the ionization degree in shocks with higher preshock temperature lead to smaller compression ratios  $\rho/\rho_1$  (see the lower panel of Fig. 2).

In Fig. 3 we give the plots of the final compression ratio  $\rho_N/\rho_1$  which is reached at the postshock outer boundary  $X_N$  in shock wave models with  $\rho_1 = 10^{-10} \text{ gm cm}^{-3}$ . It should be noted that the final postshock gas density asymptotically tends to its limiting value  $\rho_\infty$  with  $X \rightarrow \infty$  and because in our study the shock wave model is represented by the finite slab, the compression ratio  $\rho_N/\rho_1$  gives only the lower estimate for the limiting value  $\rho_\infty/\rho_1$ . In our calculations we tried to set the postshock outer boundary as far as possible from the discontinuous jump. Unfortunately, the convergence of global iterations is very sensitive to the spatial coordinate of the postshock outer boundary and for too large values of  $X_N$ , the iterations diverge. Thus, the coordinate  $X_N$  was determined for each model from trial calculations as a compromise between the requirement of the postshock region and demands of convergence and accuracy. For this reason, the filled circles in Fig. 3, representing the models with the same value of  $T_1$ , do not lie on smooth curves.

At a fixed upstream velocity,  $U_1$ , the Mach number  $M_1 = U_1/a_1$  decreases with increasing  $T_1$  due to the temperature dependence of the adiabatic sound speed  $a_1$  at

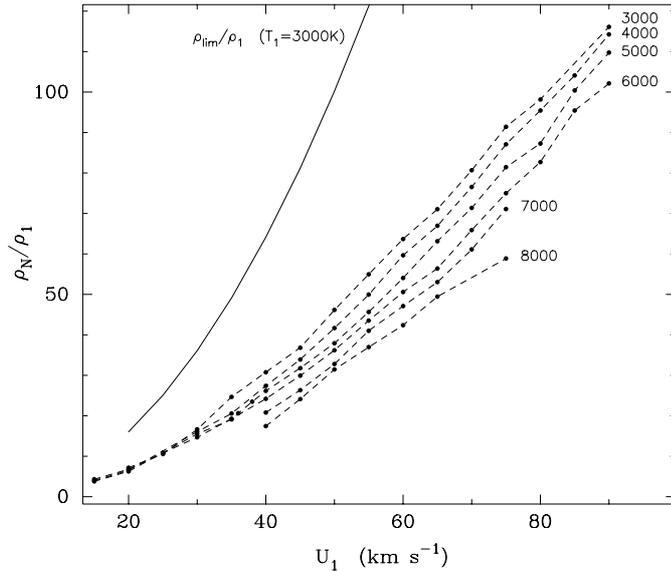


**Fig. 2.** The postshock hydrogen ionization degree  $x_{\text{H}}$ , the rate of energy gain by electrons in elastic collisions with hydrogen ions per unit mass  $Q_{\text{ei}}$ , the electron temperature  $T_{\text{e}}$  (solid lines) and the temperature of heavy particles  $T_{\text{a}}$  (dashed lines); the radiation flux  $F_{\text{R}}$  and the compression ratio  $\rho/\rho_1$  as a function of distance from the discontinuous jump in shock wave models with  $\rho_1 = 10^{-10} \text{ gm cm}^{-3}$ ,  $U_1 = 40 \text{ km s}^{-1}$ ,  $T_1 = 4000 \text{ K}$  and  $T_1 = 8000 \text{ K}$

the preshock outer boundary. In a hydrogen gas with negligible ionization ( $\rho_1 = 10^{-10} \text{ gm cm}^{-3}$ ,  $T_1 \lesssim 7000 \text{ K}$ ) the adiabatic sound speed is  $a_1 \propto \sqrt{T_1}$  and the compression ratio

$$\rho_{\text{lim}}/\rho_1 = \gamma M_1^2 \quad (9)$$

corresponding to the isothermal shock wave is inversely proportional to  $T_1$ . On the other hand, as is seen from Fig. 3, the final compression ratio  $\rho_N/\rho_1$  at the postshock outer boundary also decreases with increasing preshock



**Fig. 3.** The compression ratio  $\rho_N/\rho_1$  at the postshock outer boundary as a function of upstream velocity  $U_1$  at  $\rho_1 = 10^{-10}$  gm cm $^{-3}$  and  $3000$  K  $\leq T_1 \leq 7000$  K. Each sequence of models with the same preshock temperature  $T_1$  is represented by filled circles connected by dashed lines. Each curve is labeled with  $T_1$ . The solid line shows the compression ratio of the isothermal shock wave propagating through the gas with  $T_1 = 3000$  K

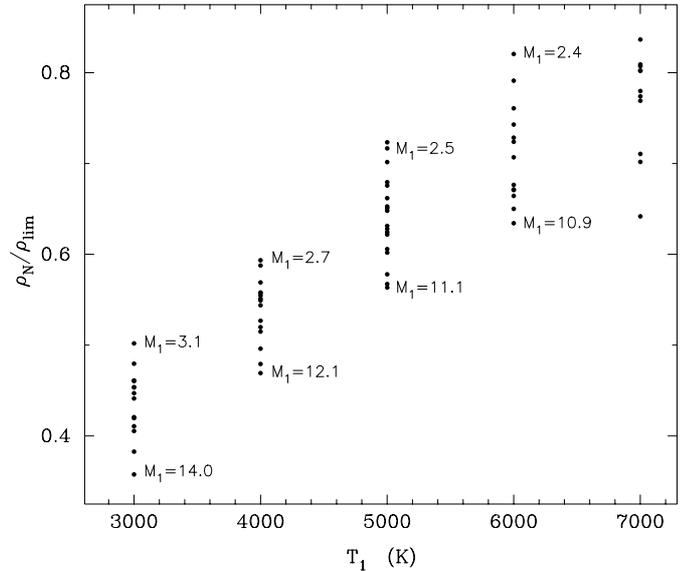
temperature though this dependence is not as prominent as that of  $\rho_{\text{lim}}/\rho_1$ .

In Fig. 4 we show the ratio  $\rho_N/\rho_{\text{lim}}$  for shock wave models with  $\rho_1 = 10^{-10}$  gm cm $^{-3}$ ,  $3000$  K  $\leq T_1 \leq 7000$  K,  $20$  km s $^{-1} \leq U_1 \leq 90$  km s $^{-1}$ . Each model on this plot is represented by a filled circle and for models with  $U_1 = 20$  km s $^{-1}$  and  $U_1 = 90$  km s $^{-1}$  we give the values of the upstream Mach number  $M_1$ .

At a fixed preshock gas temperature  $T_1$  the ratio  $\rho_N/\rho_{\text{lim}}$  decreases with increasing upstream velocity since the final compression ratio  $\rho_N/\rho_1$  grows with  $U_1$  more slowly than  $\rho_{\text{lim}}/\rho_1$  (compare dependencies of  $\rho_N/\rho_1$  and  $\rho_{\text{lim}}/\rho_1$  shown for  $T_1 = 3000$  K in Fig. 3). However the most interesting feature is that the increase in the preshock gas temperature  $T_1$  is accompanied by the gradual approach of the final compression ratio  $\rho_N/\rho_1$  to its upper limit  $\rho_{\text{lim}}/\rho_1$  corresponding to the isothermal shock wave. This is due to the decrease of the Mach number  $M_1$  with increasing  $T_1$ .

#### 4. Effects of the preshock density

The two upper panels of Fig. 5 show the postshock hydrogen ionization degree  $x_H$  and the postshock temperatures  $T_e$  and  $T_a$  as a function of distance from the discontinuous jump for shock wave models with  $10^{-12}$  gm cm $^{-3} \leq \rho_1 \leq 10^{-9}$  gm cm $^{-3}$ ,  $T_1 = 6000$  K and  $U_1 = 50$  km s $^{-1}$ . The two lower panels of Fig. 5 for the same models show the radiation flux  $F_R$  and the compression ratio  $\rho/\rho_1$ . Note that we use the logarithmic scale for the plot of radiative



**Fig. 4.** The final compression ratio  $\rho_N/\rho_{\text{lim}}$  at the postshock outer boundary in units of the compression ratio of the isothermal shock wave  $\rho_{\text{lim}}/\rho_1$  versus the preshock gas temperature  $T_1$  for models with  $\rho_1 = 10^{-10}$  gm cm $^{-3}$  and  $20$  km s $^{-1} \leq U_1 \leq 90$  km s $^{-1}$ . At fixed preshock temperature  $T_1$  the values of the Mach number  $M_1$  correspond to models with  $U_1 = 20$  km s $^{-1}$  and  $U_1 = 90$  km s $^{-1}$ , respectively

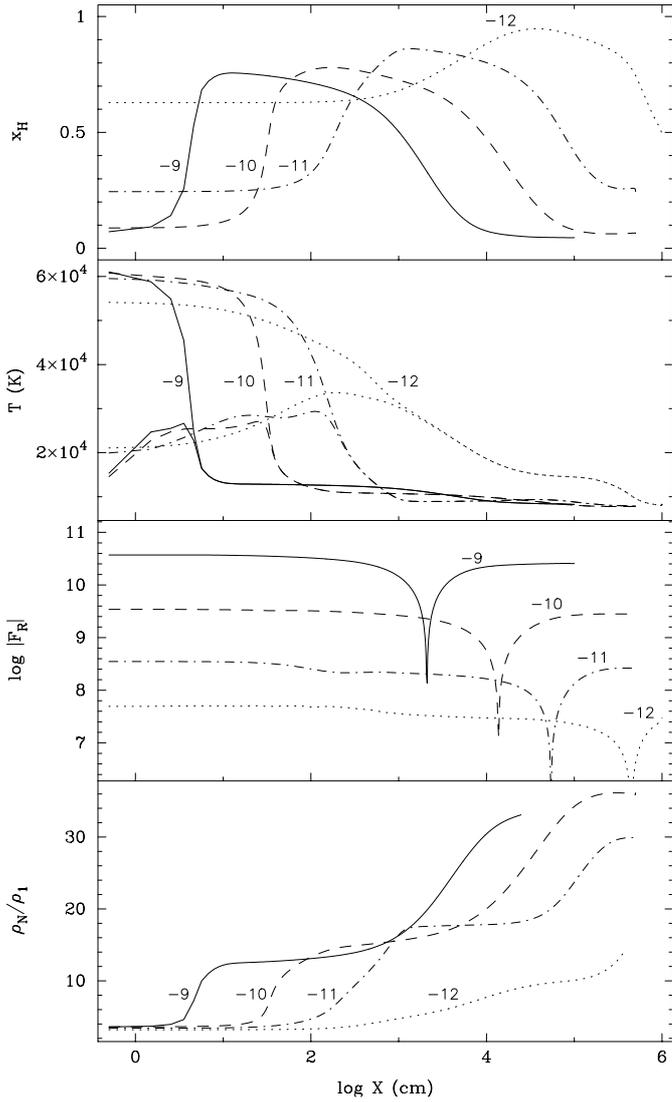
flux and the deep minima of  $\log |F_R|$  correspond to layers with  $F_R \approx 0$ .

The main results of these calculations are as follows. First, the width of the postshock relaxation zone increases with decreasing preshock gas density  $\rho_1$ . This is due to the fact that the mean collision time of particles is proportional to the gas density. Second, the radiative flux produced by the shock wave decreases with decreasing  $\rho_1$  because the kinetic energy of the gas flow at the preshock outer boundary is proportional to  $\rho_1$ . Third, more gradual ionization and recombination of hydrogen atoms at the lower preshock gas density  $\rho_1$  leads to smaller postshock compression ratios.

#### 5. Radiative flux of the shock wave

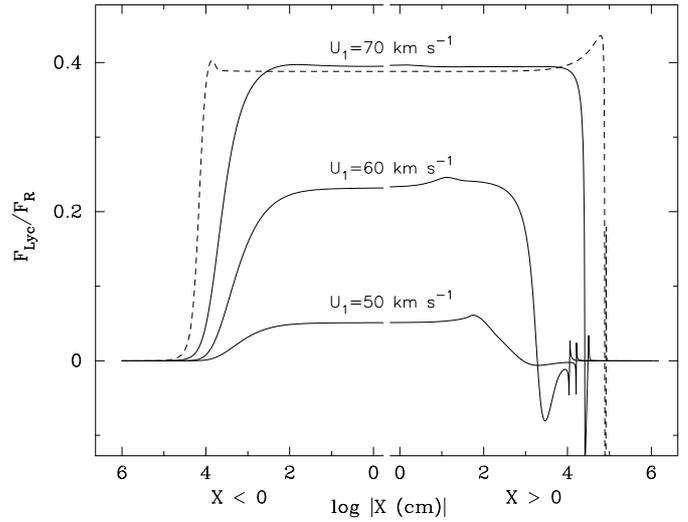
As seen from plots on the upper panel of Fig. 1, the upstream radiative flux  $F_{R1}$  emerging from the preshock outer boundary and the downstream radiation flux  $F_{RN}$  emerging in the opposite direction from the postshock outer boundary are not equal in absolute value. This difference increases with increasing upstream velocity. For example, at  $\rho_1 = 10^{-10}$  gm cm $^{-3}$  and  $T_1 = 6000$  K the ratio of these fluxes ranges from  $|F_{R1}|/F_{RN} \approx 1.01$  at  $U_1 = 20$  km s $^{-1}$  to  $|F_{R1}|/F_{RN} \approx 1.16$  at  $U_1 = 60$  km s $^{-1}$ .

However the much stronger asymmetry of the radiation field is revealed at frequencies  $\nu > \nu_1$  because the Lyman continuum flux is directed mostly upstream. The region of the effective transport of the Lyman continuum radiation encompasses the near vicinity of the discontinuous jump. Ahead of the discontinuity, this is the radiative precursor



**Fig. 5.** The hydrogen ionization degree  $x_H$ , the electron temperature  $T_e$  and the temperature of heavy particles  $T_a$ , the radiation flux  $F_R$  and the compression ratio  $\rho/\rho_1$  as a function of distance from the discontinuous jump in the postshock region at  $T_1 = 6000$  K and  $U_1 = 50$  km s<sup>-1</sup>. In solid, dashed, dot-dashed and dotted lines are shown the dependencies corresponding to the preshock gas densities  $\rho_1 = 10^{-9}$ ,  $10^{-10}$ ,  $10^{-11}$  and  $10^{-12}$  gm cm<sup>-3</sup>, respectively. Each curve is labeled with  $\log \rho_1$ . In temperature plots, the lower and upper converging curves represent the electron temperature  $T_e$  and the heavy particle temperature  $T_a$ , respectively

and behind the discontinuity the Lyman continuum flux becomes negligible where the total radiation flux changes its sign (i.e. in the layers with  $F_R \approx 0$ ). The contribution of the Lyman continuum to the total radiation flux within this region rapidly increases with increasing upstream velocity. For example, for shock waves propagating in a gas with  $\rho_1 = 10^{-10}$  gm cm<sup>-3</sup> and  $T_1 = 3000$  K, the ratio of the Lyman continuum flux to the total radiation flux increases from  $F_{\text{Lyc}}/F_R \approx 10^{-2}$  at  $U_1 = 40$  km s<sup>-1</sup> to  $F_{\text{Lyc}}/F_R \approx 0.2$  at  $U_1 = 60$  km s<sup>-1</sup>. The growth of the ratio  $F_{\text{Lyc}}/F_R$  ceases at an upstream velocity  $U_1 \approx 75$  km s<sup>-1</sup>,



**Fig. 6.** The ratio of the radiation flux in the Lyman continuum to the total radiation flux  $F_{\text{Lyc}}/F_R$  in the vicinity of the discontinuous jump of shock waves with an upstream velocity  $U_1 = 50, 60, 70$  (solid lines) and  $80$  km s<sup>-1</sup> (dashed line)

corresponding to almost full hydrogen ionization in the radiative precursor. At larger upstream velocities the fraction of the Lyman continuum radiation does not exceed  $\approx 40\%$  of the total radiation flux, but the region of the effective transport of the Lyman continuum radiation becomes wider, both ahead of and behind the discontinuity.

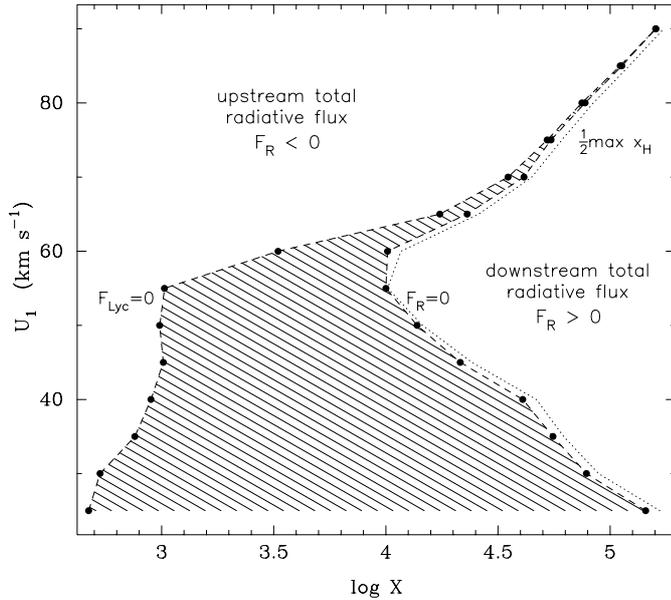
The plots of the ratio  $F_{\text{Lyc}}/F_R$  for shock wave models with upstream velocities  $50$  km s<sup>-1</sup>  $\leq U_1 \leq 80$  km s<sup>-1</sup> are shown in Fig. 6. It should be noted that the positive value of the ratio  $F_{\text{Lyc}}/F_R$  implies that the total radiation flux and the Lyman continuum flux are both negative and are directed upstream. Negative values of this ratio imply that the Lyman continuum flux is directed downstream, whereas the total radiation flux is still upstream. Oscillations of the ratio  $F_{\text{Lyc}}/F_R$  in the postshock region are due to vanishing values of  $F_R$ .

Thus, behind the discontinuous jump, the Lyman continuum flux  $F_{\text{Lyc}}$  changes its sign at smaller distance than the radiative flux

$$F(\nu < \nu_1) = \int_0^{\nu_1} F_\nu d\nu, \quad (10)$$

transported at frequencies below the Lyman continuum edge  $\nu_1$ . In Fig. 7 we depict the zones of the postshock region with upstream and downstream total radiative flux  $F_R$  in shock wave models with  $\rho_1 = 10^{-10}$  gm cm<sup>-3</sup> and  $T_1 = 6000$  K. The zone with downstream Lyman continuum flux ( $F_{\text{Lyc}} > 0$ ) and upstream flux  $F(\nu < \nu_1)$  is shown on this diagram by the shaded area. Space coordinates  $X[F(\nu < \nu_1) = 0]$  and  $X(F_R = 0)$  coincide because most of the radiation is transported in these layers at frequencies below the Lyman continuum edge.

In Fig. 7 we also show the space coordinates of the layers where half of the ionized hydrogen atoms are



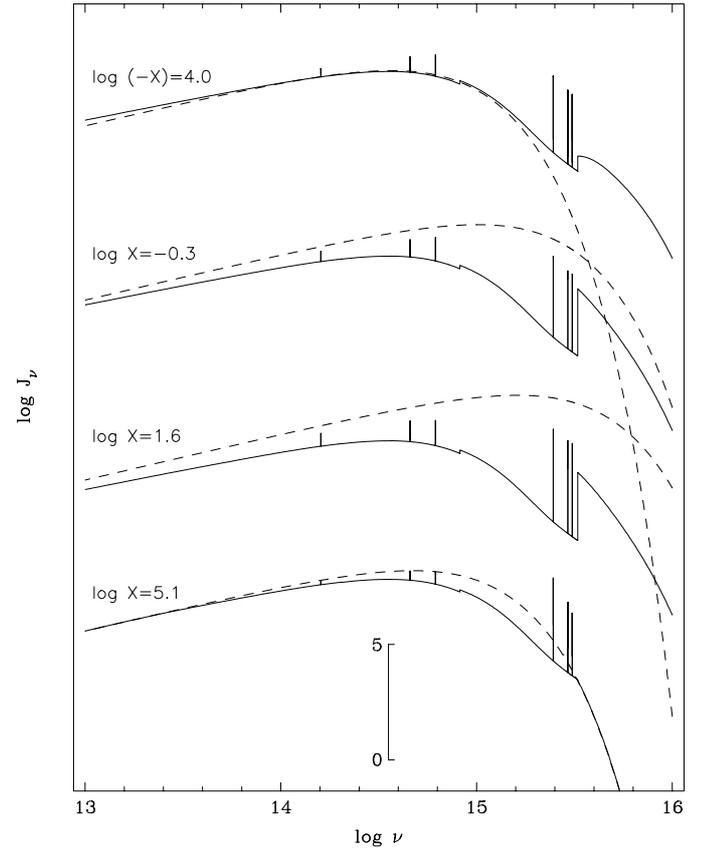
**Fig. 7.** The zones of upstream ( $F_R < 0$ ) and downstream ( $F_R > 0$ ) total radiative flux in the postshock region at  $\rho_1 = 10^{-10} \text{ gm cm}^{-3}$  and  $T_1 = 6000 \text{ K}$ . The shaded area represents the zone with downstream Lyman continuum flux and upstream flux  $F(\nu < \nu_1)$ . The dotted curve marked  $\frac{1}{2} \max x_H$  gives the locus where the hydrogen ionization degree is a half of its maximum postshock value

recombined. The close values of  $X(F_R = 0)$  and  $X(\frac{1}{2} \max x_H)$  clearly illustrate that the main mechanism of the shock wave energy dissipation is the ionization of hydrogen atoms.

At upstream velocity  $U_1 < 60 \text{ km s}^{-1}$ , the postshock gas remains partially ionized and the maximum ionization degree increases with increasing  $U_1$ , while the space coordinate of the ionization maximum decreases. Correspondingly, the hydrogen recombination also occurs at a smaller distance from the discontinuous jump.

At upstream velocities of  $U_1 > 60 \text{ km s}^{-1}$ , the hydrogen is fully ionized and the width of the zone with  $x_H \approx 1$  increases with increasing  $U_1$ . Correspondingly, the hydrogen recombines at larger distances from the discontinuous jump and the coordinate  $X(F_R = 0)$  increases with increasing  $U_1$ . At full hydrogen ionization, the zone of the effective energy transport in the Lyman continuum spreads both upstream and downstream with increasing upstream velocity and at  $U_1 > 80 \text{ km s}^{-1}$  both the Lyman continuum flux and the total radiation flux change sign nearly in the same layers.

Another interesting conclusion which follows from the diagram in Fig. 7 is that the width of shock waves with partial postshock hydrogen ionization decreases with increasing upstream velocity  $U_1$ , whereas the width of shock waves with full postshock hydrogen ionization increases with increasing upstream velocity.



**Fig. 8.** Monochromatic mean intensity  $J_\nu$  (solid lines) and the local Planck function  $B_\nu(T_e)$  (dashed lines) as a function of frequency  $\nu$  for four distinct layers of the shock wave with  $\rho = 10^{-10} \text{ gm cm}^{-3}$ ,  $T_1 = 6000 \text{ K}$  and  $U_1 = 60 \text{ km s}^{-1}$ . The upper pair of plots represent the preshock region, whereas other plots represent the postshock region

## 6. Frequency dependent radiation field

The radiation field produced by the shock wave is remarkably non-equilibrium and any attempts to use an assumption of thermal equilibrium inevitably lead to large errors. In order to evaluate the degree of departure from thermal equilibrium, it is instructive to compare the monochromatic mean intensity  $J_\nu$  with the local Planck function  $B_\nu(T_e)$ . In Fig. 8 the plots of  $J_\nu$  and  $B_\nu(T_e)$  are shown as a function of frequency  $\nu$  for four distinct layers of the shock wave model with  $\rho = 10^{-10} \text{ gm cm}^{-3}$ ,  $T_1 = 6000 \text{ K}$  and  $U_1 = 60 \text{ km s}^{-1}$ . For the sake of convenience each pair of plots is shifted with respect to others.

The upper pair of plots represents  $J_\nu$  and  $B_\nu(T_e)$  at  $X \approx 10^4 \text{ cm}$ . As is seen from Fig. 1, the photoionization of hydrogen atoms in these layers due to absorption of the Lyman continuum radiation becomes perceptible and this layer can roughly be considered as the boundary of the radiative precursor. Excess radiation in comparison with  $B_\nu(T_e)$  at frequencies  $\nu > \nu_1$  is due to the fact that the optical depth of the radiative precursor decreases with increasing frequency  $\nu$ .

The second and third pairs of plots represent the cell just behind the discontinuous jump ( $X = 0.5 \text{ cm}$ ) and

the layer where the hydrogen ionization degree reaches its maximum ( $X \approx 40$  cm). Between these layers, various relaxation processes redistribute the energy of heavy particles among other degrees of freedom and the radiation field is furthest from equilibrium.

The lower pair of plots represent the layers ( $X \approx 1.2 \cdot 10^5$  cm) where most of the hydrogen atoms are recombined and the gas temperature gradually approaches its unperturbed value  $T_1$ . The optical depth at frequencies of the Lyman continuum is so large that  $J_\nu$  and  $B_\nu(T_e)$  coincide at  $\nu > \nu_1$ .

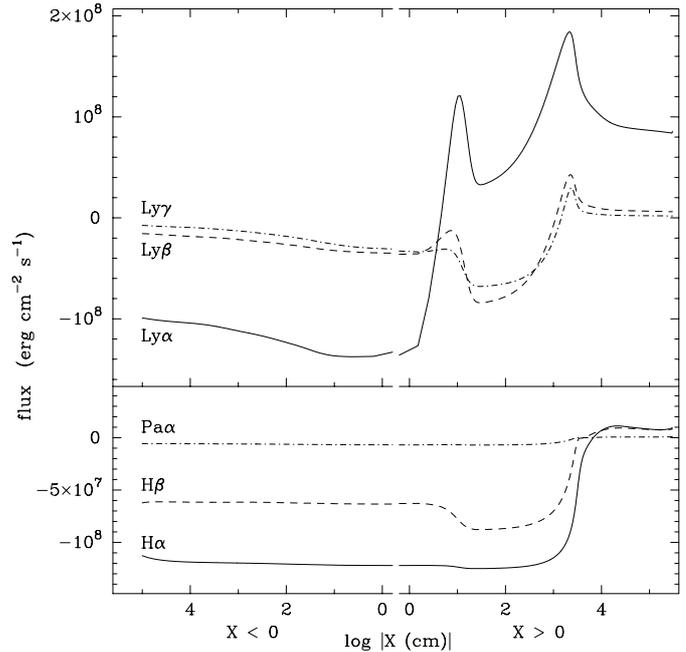
All shock wave models demonstrate two common features. First, we see an excess of Lyman continuum radiation in the preshock region, due to the existence of the radiative precursor. Second, there is a lower mean intensity of  $J_\nu$  in comparison with  $B_\nu(T_e)$ , due to the small optical depth of the shock wave at frequencies  $\nu < \nu_1$ .

The plots of the mean intensity  $J_\nu$  displayed in Fig. 8 reveal the presence of six spectral line features. These are  $\text{Ly}\alpha$ ,  $\text{Ly}\beta$ ,  $\text{Ly}\gamma$ ,  $\text{H}\alpha$ ,  $\text{H}\beta$  and  $\text{Pa}\alpha$ . In comparison with our previous work described in Paper II, we improved the treatment of the spectral line radiation transfer and considered each line of frequency  $\nu_0$  within the frequency interval as wide as  $[\nu_0(1 - \delta), \nu_0(1 + \delta)]$ , where  $\delta = 5 \cdot 10^{-4}$ . The use of wide frequency intervals is necessary because of the strong Doppler broadening of line profiles behind the discontinuous jump. In particular, the radiation flux transported in spectral lines is not negligible and contribution of the spectral line radiation to the total radiation flux increases with increasing upstream velocity. The most important are the  $\text{Ly}\alpha$  and  $\text{H}\alpha$  lines. For example, in the shock wave model with  $\rho_1 = 10^{-10} \text{ gm cm}^{-3}$ ,  $T_1 = 6000 \text{ K}$  and  $U_1 = 60 \text{ km s}^{-1}$  the upstream radiation flux in  $\text{Ly}\alpha$  and  $\text{H}\alpha$  lines emerging from the preshock outer boundary is  $\approx 2\%$  of the total radiation flux.

In Fig. 9 we show the plots of the radiation flux integrated within frequency intervals of the  $\text{Ly}\alpha$ ,  $\text{Ly}\beta$ ,  $\text{Ly}\gamma$  (upper panel),  $\text{H}\alpha$ ,  $\text{H}\beta$  and  $\text{Pa}\alpha$  (lower panel) lines. Comparing with plots of  $n_2/n_{\text{H}}$  and  $x_{\text{H}}$  displayed in Fig. 1 we can conclude that the spectral line radiation is produced mostly in the layers of hydrogen recombination. Another interesting feature is that the upstream and downstream  $\text{Ly}\alpha$  radiation fluxes are nearly equal in absolute value, whereas the radiation flux in Balmer lines emerges mostly upstream.

## 7. Radiative losses of the shock wave

In order to estimate the irreversible loss occurring in the shock wave energy it is instructive to compare the values of each term of the left-hand side of relation (3) at both boundaries of the slab. In Fig. 10 these quantities are shown for shock wave models with  $\rho_1 = 10^{-10} \text{ gm cm}^{-3}$ ,  $T_1 = 6000 \text{ K}$  and  $20 \text{ km s}^{-1} \leq U_1 \leq 80 \text{ km s}^{-1}$ . As is seen by comparing the upper and lower panels representing the preshock and postshock outer boundaries, respectively, we see that most of the kinetic energy of the gas flow is irreversibly lost, since the ratio of the fluxes of ki-



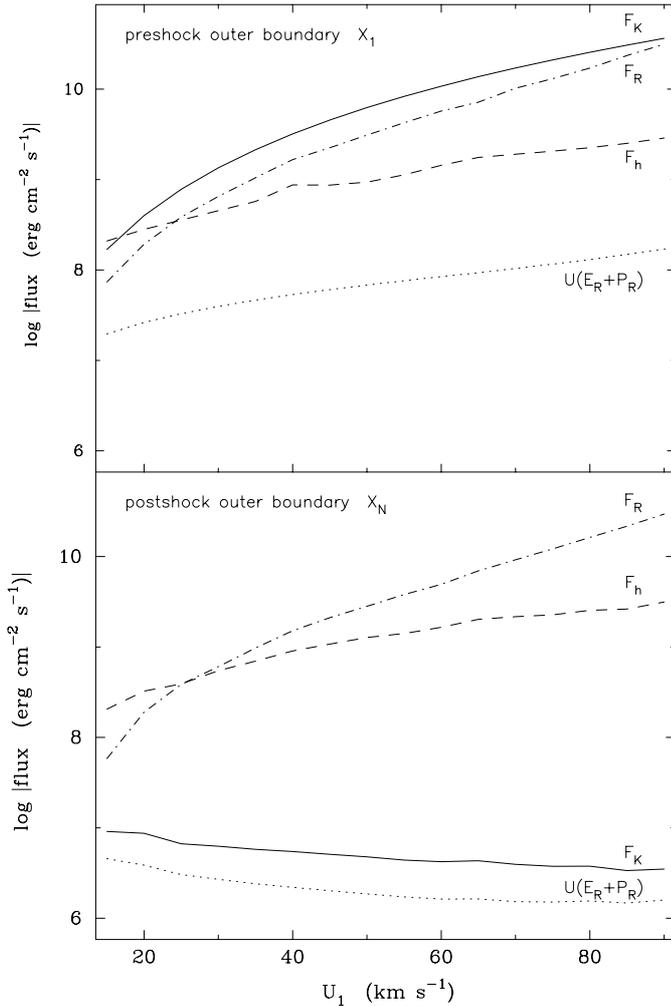
**Fig. 9.** The radiation flux integrated over spectral line frequency intervals as a function of space coordinate  $X$  for the shock wave model with  $\rho_1 = 10^{-10} \text{ gm cm}^{-3}$ ,  $T_1 = 6000 \text{ K}$  and  $U_1 = 60 \text{ km s}^{-1}$

netic energy at both boundaries of the slab ranges within  $10^2 \lesssim F_{\text{K1}}/F_{\text{KN}} \lesssim 7 \cdot 10^3$ . At the same time, the increase of the enthalpy flux after the passage of the parcel of gas through the slab does not exceed 20%. Thus, most of the energy of the shock wave is lost due to radiation.

The role of radiation in irreversible energy losses of the shock wave can be evaluated from comparison of the radiation flux emerging from the boundary of the slab with the total energy flux  $\mathcal{C}_2$ . In Fig. 11 the plots of the ratio  $F_{\text{RN}}/\mathcal{C}_2$  at the postshock outer boundary are shown for two sequences of shock wave models with  $T_1 = 3000 \text{ K}$  and  $T_1 = 6000 \text{ K}$ . As is seen, the radiative losses increase very rapidly with increasing upstream velocity  $U_1$ , though there is also the dependence on the preshock gas temperature  $T_1$ .

## 8. Conclusion

In this work we improved the treatment of radiation transfer and considered the structure of shock waves propagating through partially ionized hydrogen gas with densities from  $10^{-12} \text{ gm cm}^{-3}$  to  $10^{-9} \text{ gm cm}^{-3}$ . Our calculations showed that while the unperturbed hydrogen gas is partially ionized, the preshock gas temperature affects the shock wave structure, mostly due to the preshock hydrogen ionization because the postshock equilibration of translational degrees of freedom is very sensitive to the number density of hydrogen ions. In general, a higher temperature in the preshock gas will lead to larger postshock ionization and to a stronger radiation field produced by the shock wave. At the same time, the increase in the

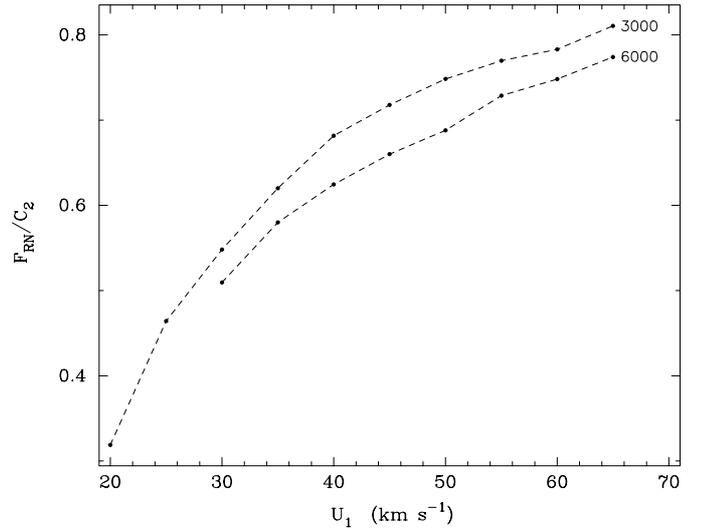


**Fig. 10.** The flux of kinetic energy  $F_K$  (solid lines), the flux of enthalpy  $F_h$  (dashed lines), the radiation flux  $F_R$  (dot-dashed lines), and the flux  $U(E_R + P_R)$  (dotted lines) at the preshock (upper panel) and postshock (lower panel) outer boundaries of the slab as a function of upstream velocity  $U_1$  in the models with  $\rho_1 = 10^{-10} \text{ gm cm}^{-3}$  and  $T_1 = 6000 \text{ K}$

preshock gas temperature is accompanied by the slight decrease of the postshock gas compression.

The preshock gas density  $\rho_1$  affects the shock wave structure mostly due to the fact that the preshock flux of kinetic energy as well as the radiation flux emerging from the shock wave are proportional to  $\rho_1$ . Therefore, in the gas with lower density, the shock wave produces weaker radiation flux and less compression of the postshock gas. Moreover, at lower gas density, the width of the shock wave increases. The width of the radiative precursor increases due to the smaller absorption in the Lyman continuum, whereas the width of the postshock relaxation zone increases because of the proportionality of the rates of relaxation processes to the gas density.

The increase in the shock wave velocity leads to higher rates of relaxation processes in the compressed gas behind the discontinuous jump and to higher postshock ionization. While the maximum ionization degree of the post-



**Fig. 11.** The ratio of the downstream radiation flux to the total energy flux  $F_{RN}/C_2$  at the postshock outer boundary as a function of upstream velocity  $U_1$  for shock wave models with  $\rho_1 = 10^{-10} \text{ gm cm}^{-3}$ . Each curve is labeled with  $T_1$

shock gas is less than unity, the width of the shock wave decreases with increasing shock wave velocity. However, when the postshock gas is fully ionized, the increase in the shock wave velocity leads to the extension of the postshock zone of hydrogen ionization and, therefore, to an increase in the shock wave width.

Strong radiative cooling of the postshock gas leads to compression ratios as high as  $\rho/\rho_1 \sim 10^2$ . Such large increases in the postshock gas density obviously can favour the condensation of dust grains in outer atmospheres of radially pulsating late-type giants.

The isothermal approximation undoubtedly overestimates the radiative losses of the shock wave energy by at least a factor of two for shocks with an upstream velocity of  $U_1 \lesssim 30 \text{ km s}^{-1}$  and can provide good accuracy at upstream velocities of  $U_1 > 80 \text{ km s}^{-1}$ , that is at Mach numbers  $M_1 \gtrsim 10$ .

Our calculations demonstrated that the monochromatic radiation flux at frequencies of hydrogen lines significantly exceeds the flux of the background continuum, so that our models reproduce the emission spectrum observed in radially pulsating late-type supergiants. Moreover, the fraction of the energy transported in hydrogen lines is not negligible and can be as large as one or two percent of the total radiation flux because of the strong broadening of the line profiles behind the discontinuous jump.

In this paper, we did not discuss the details of the emission line spectra because current results are insufficient for comparison with observations. The change in the velocity of the gas during its passage through the shock wave must inevitably influence the line profiles. Calculations of shock wave structure based on the solution of the transfer equation which takes into account effects of the gas flow

velocity gradient were beyond the scope of the present work and will be presented in the forthcoming paper.

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