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Nonlinear resonant absorption of fast magnetoacoustic waves due to coupling into slow continua in the solar atmosphere

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Abstract. Nonlinear resonant absorption of fast magnetoacoustic (FMA) waves in inhomogeneous weakly dissipative, isotropic and anisotropic plasmas in static and steady equilibria is studied. Both isotropic and anisotropic plasmas are considered and for the background equilibrium state 1D planar static and steady models are used. The equilibrium configuration consists of three layers, where an inhomogeneous magnetised plasma slab is surrounded by two homogeneous magnetised semi-infinite plasma regions. The propagating FMA waves are partly absorbed due to coupling to local nonlinear slow magnetohydrodynamic (MHD) waves in the inhomogeneous layer, and are partly reflected. The coefficient of wave energy resonant absorption is derived using two simplifying assumptions (i) weak nonlinearity and (ii) the thickness of the inhomogeneous layer is small compared to the wavelength of the waves, i.e. the so-called long-wavelength approximation is used.

Key words. magnetohydrodynamics (MHD) – Sun: oscillations – Sun: atmosphere

1. Introduction

One of the most interesting processes in the solar atmosphere is the complicated interaction of the motions of the plasma with the magnetic fields. The solar atmosphere is a highly non-uniform and dynamic system and as a consequence it is a natural medium for magnetohydrodynamic (MHD) waves. Besides many other features of these waves, they are able to transport momentum and energy which can be dissipated. One of the many possibilities of the kinetic energy conversion into heat (i.e., dissipation) is via resonant absorption which owes its existence to coupling of global oscillations to local waves in a non-ideal plasma.

The ability to absorb wave energy in a weakly dissipative plasma, such as the upper parts of the solar atmosphere (e.g., magnetic canopy, transition region, corona) was intensively studied in recent decades. In controlled thermonuclear fusion research, resonant absorption of MHD waves is studied as a mechanism to provide supplementary heating to laboratory plasmas in order to bring them into the ignition regime of temperature (see, e.g., Grossman & Tataronis 1973; Chen & Hasegawa 1974; Hasegawa & Chen 1976; Goedbloed 1984; Poedts et al. 1989; Vaclavik & Appert 1991).

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Ionson (1978) proposed resonant absorption of MHD waves as a mechanism for heating magnetic loops in the solar corona. Since then, resonant absorption became not only a very popular but also a viable heating mechanism when trying to explain the high temperature of the corona (see, e.g., Kuperus et al. 1981; Ionson 1985; Davila 1987; Poedts et al. 1989; Poedts et al. 1990a,b,c; Hollweg 1990; Goossens 1991; Ofman & Davila 1995; Erdélyi & Goossens 1995, 1996). In addition, resonant absorption was proposed as a mechanism to explain the loss of energy of acoustic oscillations in the vicinity of sunspots (see, e.g., Hollweg 1988; Lou 1990; Sakurai et al. 1991; Goossens & Poedts 1992; Erdélyi & Goossens 1994, 1995).

The effect of resonant absorption was mostly studied in linear MHD. However, linear theory shows that in the vicinity of the resonant position, the amplitudes of the variables can be very large even when they are small when far away from this position. This observation implies that the linear theory can break down in this region. Recently, Ruderman et al. (1997b) developed a nonlinear theory of resonant slow waves in isotropic plasmas which has been extended to anisotropic plasmas by Ballai et al. (1998b).

Observations show that the solar atmosphere is not only an inhomogeneous and structured medium (i.e., ideal medium for plasma wave propagation and dissipation) but also highly dynamic. Solar magnetic structures (e.g., coronal loops, sunspots, prominences, etc.) exhibit large-scale motions. However, even recent high resolution satellite observations are not enough to derive detailed diagnostics of these motions. Theoretical studies, on the other side, have shown that these motions change the efficiency of MHD wave dissipation (see, e.g., Erdélyi & Goossens 1996; Erdélyi 1997, 1998). The effect of a steady (shear) flow on the resonant behaviour of nonlinear slow waves was discussed first by Ballai & Erdélyi (1998).

Many studies of resonant absorption considered only the sound (or slow) and Alfvén waves as excellent candidates for plasma heating. Alfvén waves can carry energy only along the magnetic field lines and slow waves are able to carry only 1–2% of energy under coronal (i.e., low plasma-β) conditions. However, fast magnetoacoustic waves might also have an important contribution in explaining the coronal temperatures, as has been shown, e.g., by Čadež et al. (1997), Csík et al. (1998).

FMA waves are magnetic waves which can propagate carrying energy *across* the magnetic field lines. They are compressive and therefore subject to dissipation by, e.g., viscosity, heat conduction, Landau and transit-time damping, etc. in the high-frequency limit where the Coulomb collisions are ineffective. In order to have an acceptable heating by fast MHD waves coming from lower regions (e.g., generated by convective motions in the photosphere), these waves should not be reflected by the steep rise of the Alfvén and/or slow wave speed with height and should not become evanescent. This is, however, difficult to prevent. In addition, only the high frequency waves of the full FMA spectrum can reach the corona (with periods of a few tens of seconds). The energy flux density of fast waves at the bottom of the corona required for significant heating is of the order of 10^5 erg cm^{-2} s^{-1}. This value is not inconsistent with the upper limit on acoustic waves of 10^4 erg cm^{-2} s^{-1} (Athay & White 1978), provided the coronal base magnetic field is sufficiently large ($\geq$10 G).

There is, however, another possibility. It is not necessary that waves have to travel up from photospheric regions and transport energy from the huge kinetic energy reservoir (convection motion) and then be dissipated in the corona. FMA waves (and slow or Alfvén waves as well!) may be generated locally in the corona by, e.g., magnetic reconnection. It has been suggested by Parker (1988) that shuffling the magnetic field in the solar atmosphere (by convective motion down in lower regions) builds up magnetic stresses which can be released through, e.g., reconnection providing energy to maintain the high temperatures in the solar corona. This mechanism was called nanoflareing.

There are, however, a few problems. The required amount of nanoflares is not (yet!) confirmed by observations because of their very localised nature. Extending the theory to larger observable scales (explosive events, micro-flares, blinkers, etc.) still does not help to prove this theory (Aschwanden 1999). Eventhough the released heating via reconnection may not provide enough energy, we still assume the amount of magnetic energy built up by the shuffles can provide enough energy to heat the corona. The necessary supplementary heating *may come from reconnection-driven, locally generated MHD waves.* Roussev et al. (2001) have carried out numerical simulations of explosive events where reconnection *has* excited and driven MHD (FMA) waves. They did not have comments, however, about their dissipation.

The aim of the present paper is to study the nonlinear resonant interaction of FMA waves coupled to slow continua in isotropic and anisotropic, static and steady state plasmas. We use a simplified slab geometry throughout the paper, where an inhomogeneous magnetic slab is sandwitched by two semi-infinite plasma regions. More realistic studies might be carried out using computer simulations similar to Roussev et al. (2001).

The paper is organized as follows. In Sect. 2, we specify the equilibrium model. The paper has three further sections which deal with three cases: isotropic and anisotropic plasmas and finally the effect of an equilibrium flow on resonant coupling. Analytical solutions to the nonlinear equations are found with the aid of a perturbation method. Finally, we apply the theoretical results to study the efficiency of wave energy resonant absorption in solar magnetic structures modelling, e.g., the magnetic canopy region.

2. The equilibrium

Bearing in mind solar applications, we study the nonlinear interaction of incident FMA waves in a one-dimensional plasma, i.e., the coupling of fast MHD waves with nonlinear slow MHD continuum waves. For similar studies in the *linear* regime see, e.g., Čadež et al. (1997), Csík et al. (1998). Monochromatic fast MHD waves are impinging and penetrating from the $x < 0$ homogeneously magnetised half-space (Region I, with magnetic induction B_e) into an inhomogeneous region, $0 < x < x_0$ (Region II, with $B_0(x)$). This inhomogeneous region is bounded on its right by another semi-infinite half-space containing a different homogeneously magnetised plasma (Region III, with B_i). The magnetic field is unidirectional and parallel to the z axis. In what follows we use the subscripts "e", "0" and "i" to indicate equilibrium quantities in the three regions (Regions I, II and III, respectively). All equilibrium quantities are continuous at the boundaries of Region II, and they satisfy the equation of total pressure balance,

$$p_e + \frac{B_e^2}{2\mu} = p_0(x) + \frac{B_0^2(x)}{2\mu} = p_i + \frac{B_i^2}{2\mu}, \tag{1}$$

which, in particular, leads to

$$\frac{\rho_i}{\rho_e} = \frac{2c_{\mathrm{S}e}^2 + \gamma v_{\mathrm{A}e}^2}{2c_{\mathrm{S}i}^2 + \gamma v_{\mathrm{A}i}^2}, \tag{2}$$

where $v_{\mathrm{A}}^2 = B_0^2/(\mu\rho_0)$ and $c_{\mathrm{S}}^2 = \gamma p_0/\rho_0$ are the squares of the Alfvén and sound speeds, respectively.

To have a uniquely determined equilibrium state, we adopt a simple profile for the local cusp speed, namely, being constant in the homogeneous regions and a monotonic function inside the inhomogeneous layer. It can be shown (e.g., in Čadež et al. 1997) that cusp (or slow) resonance may take place if the frequency of an incoming fast wave exceeds the cut-off frequency for the fast waves.

In order to make analytical progress we assume from the very beginning the inhomogeneous region is thin or that the wavelength of the impinging waves is long, i.e., $kx_0 \ll 1$.

In principle there may also exist an Alfvén resonant position at $x = x_{\rm A}$ in Region II. However the objective of the present paper is to study the effect of nonlinearity on the interaction of incoming *fast waves* with *resonant slow waves* in dissipative layers. The presence of an Alfvén resonance would complicate the analysis and would obscure results. To remove the Alfvén resonance we assume wave propagations and perturbations of all quantities to be independent of y and we consider the incoming fast wave to be entirely in the xz-plane, i.e., $k_y = 0$. Thus the dispersion relation for the impinging propagating fast waves takes the form

$$\omega^2 = \frac{1}{2} \mid \boldsymbol{k} \mid^2 \left\{(v_{\rm A}^2 + c_{\rm S}^2) + \left[(v_{\rm A}^2 + c_{\rm S}^2)^2 - 4 v_{\rm A}^2 c_{\rm S}^2 \cos^2\varphi\right]^{1/2}\right\}, \quad (3)$$

where $k = (k_x^2 + k_z^2)^{1/2}$, φ is the angle between the direction of propagation and the background magnetic field within the xz-plane. In higher coronal regions with strong magnetic fields, the sound speed is much smaller than the Alfvén speed, and the dispersion relation reduces to

$$\omega \approx k v_{\rm A}.$$

Let us write for simplicity k instead of k_z and we note with κ_e the ratio k_x/k_z. Bearing in mind that the equilibrium magnetic field is taken along the z axis, it follows that $\cos^2\varphi = 1/(\kappa_e^2 + 1)$. With these considerations the dispersion relation (3) becomes

$$\omega^2 = \frac{1}{2} k^2 (1 + \kappa_e^2) \times \left\{(v_{\rm A}^2 + c_{\rm S}^2) + \left[(v_{\rm A}^2 + c_{\rm S}^2)^2 - 4 v_{\rm A}^2 c_{\rm S}^2 \frac{1}{\kappa_e^2 + 1}\right]^{1/2}\right\}. \quad (4)$$

3. Isotropic plasma

Dissipation is one of the basic features of the solar plasma. The deviation from an ideal medium is reflected by the dimensionless Reynolds numbers (viscous or magnetic), which, under solar conditions, take large values (10^6 in the solar photosphere and 10^{12} in the solar corona). In the process of resonant absorption, this fact implies that dissipation is relevant only in a very narrow region around the ideal resonant position. Outside this narrow layer the plasma motion still can be described by the ideal MHD equations.

The governing equations for plasma motion inside and outside the dissipative layer have been intensively studied in recent years. Ruderman et al. (1997b) were the first to derive the equations that govern nonlinear resonant slow waves. They considered a static background in Cartesian geometry. In what follows, we give a short summary of their relevant results necessary for our paper.

The plasma motion outside the dissipative layer is described by a system of coupled first order PDE's for the total pressure perturbation, P, and the normal component of the velocity, u

$$\frac{\partial u}{\partial x} = \frac{V}{D}\frac{\partial P}{\partial \theta}, \qquad \frac{\partial P}{\partial x} = \frac{\rho_0 D_{\rm A}}{V}\frac{\partial u}{\partial \theta}, \quad (5)$$

where

$$D = \frac{\rho_0 D_{\rm T}}{(V^2 - c_{\rm S}^2)}, \quad (6)$$

$$D_{\rm A} = V^2 - v_{\rm A}^2, \qquad D_{\rm T} = (v_{\rm A}^2 + c_{\rm S}^2)(V^2 - c_{\rm T}^2). \quad (7)$$

Here $\theta = z - Vt$ is a running variable with $V = \omega/k$; the phase velocity and all other variables can be expressed in terms of u and P.

From a mathematical point of view the narrow dissipative layer can be considered as a surface of discontinuity (as in the case of shock waves) where the position of this surface coincides with the resonant position. In order to solve the system of governing equations for the whole domain of the outer region, we need boundary conditions for u and P at the surface of discontinuity. These boundary conditions provide a connection of the solutions at both sides of the resonant surface, just as the Rankine-Hugoniot relations connect solutions at both sides of a shock wave (see, e.g., Goossens et al. 1995).

One connection formula is given by

$$[P] = 0, \quad (8)$$

where square brackets indicate a jump in a quantity across the dissipative layer considered as a surface of the discontinuity. This formula coincides with its counterpart in linear theory. However, in contrast to the linear theory, the second connection formula can only be written in an implicit form. A second boundary condition is calculated for the normal component of the velocity u. Let us introduce the variables

$$\sigma = \delta_{\rm C}^{-1} x, \quad q = \frac{kV\delta_{\rm C}}{v_{\rm A}^2} v_{\parallel},$$

$$\delta_{\rm C} = \left[\frac{V}{k|\Delta|}\left(\nu + \frac{c_{\rm T}^2}{v_{\rm A}^2}\eta\right)\right]^{1/3}, \quad \Delta = -\frac{\rm d}{{\rm d}x} c_{\rm T}^2|_{x_{\rm C}}, \quad (9)$$

where $v_\parallel (= v_z = w)$ is the component of the velocity perturbation parallel to the magnetic field lines. Here $\delta_{\rm C}$ denotes the thickness of the dissipative layer and q can be considered as the dimensionless component of the perturbed velocity parallel to the equilibrium magnetic field lines.

Using the new variables introduced by (9), the governing equations for u and q are:

$$\frac{\partial u}{\partial \sigma} = -\frac{V}{k}\frac{\partial q}{\partial \theta}, \tag{10}$$

$$\sigma\frac{\partial q}{\partial \theta} + \Lambda q\frac{\partial q}{\partial \theta} - k\frac{\partial^2 q}{\partial \sigma^2} = -\frac{kV^4}{\rho_0 v_{\rm A}^4|\Delta|}\frac{{\rm d}P}{{\rm d}\theta}, \tag{11}$$

with

$$\Lambda = V^4\frac{(\gamma+1)v_{\rm A}^2 + 3c_{\rm S}^2}{k\delta_{\rm C}^2|\Delta|c_{\rm S}^4},$$

where all quantities are evaluated at the resonant position $x_{\rm C}$. For the jump in the normal component of the velocity we obtain

$$[u] = -\frac{V}{k}\mathcal{P}\int_{-\infty}^{\infty}\frac{\partial q}{\partial \theta}{\rm d}\sigma, \tag{12}$$

where we take the Cauchy principal part $\mathcal{P}$ because the integral is divergent at infinity. This equation is the non-linear connection formula obtained by assuming a static equilibrium state in the inhomogeneous region. In contrast to linear theory where the jump in u was obtained in terms of the total pressure P and some equilibrium quantities, the non-linear connection formula has an integral form of an unknown function q.

The system of Eq. (5) (obtained for the outer regions) and the two non-linear jump conditions (Eqs. (8) and (12), respectively) constitute a complete system of equations and boundary conditions. Note though, that we have to solve these *simultaneously* with Eq. (11).

3.1. Solutions for weak nonlinearity

In what follows we find a solution for the system of Eq. (5) in the three mentioned regions. The method how to find the desired solution has already been developed by Ruderman et al. (1997a). We do not repeat this derivation but rather give the key results essential for the current case.

Region I

The solution of Eq. (5) is given in the form of an incoming and outgoing wave of the form

$$P = \epsilon p_e\left[\cos[k(\theta+\kappa_e x)] + A(\theta-\kappa_e x)\right]. \tag{13}$$

With the aid of the system of equations governing the motion outside the dissipative layer, the normal component of the velocity is

$$u = \epsilon\frac{p_e\kappa_e V}{\rho_e(V^2 - v_{\rm Ae}^2)}\left[\cos[k(\theta+\kappa_e x)] - A(\theta-\kappa_e x)\right], \tag{14}$$

where ϵ, the dimensionless amplitude of the wave far away from the dissipative layer, is assumed to be small ($\epsilon \ll 1$). The frequency of the incoming wave is determined by Eq. (4). In addition to the incoming wave there is an outgoing wave which will be described in what follows.

Region III

To derive the governing equation for Region III we eliminate the normal component of the velocity from Eq. (5) and arrive at

$$\frac{\partial^2 P}{\partial x^2} + \kappa_i\frac{\partial^2 P}{\partial \theta^2} = 0, \tag{15}$$

with κ_i^2 defined as

$$\kappa_i^2 = -\frac{(V^2 - v_{{\rm A}i}^2)(V^2 - c_{{\rm S}i}^2)}{(c_{{\rm S}i}^2 + v_{{\rm A}i}^2)(V^2 - c_{{\rm T}i}^2)}. \tag{16}$$

Equation (15) is an elliptical differential equation. Its solution as a wave motion needs to be evanescent in Region III, because we do not want to allow wave leakage in the present study. The resonant condition ($V = c_{\rm T}(x_{\rm C})$) in Region II implies that $V < c_{{\rm T}i}$ and thus $\kappa_i^2 > 0$ which guarantees an evanescent solution.

Region II

Since we are not able to solve analytically the governing equation in the dissipative layer, we consider the limit of weak nonlinearity and we use the regular perturbation method. Hence, we look for solutions in the form

$$f = \epsilon\sum_{n\geq 1}\lambda_i^{n-1}f_n, \tag{17}$$

where f represents any of the quantities P, u, and q. Here the nonlinear parameter, λ_i, is defined as a measure of the ratio of the second term in Eq. (11) (nonlinear term) to the third term (dissipative term), i.e.,

$$\lambda_i = \epsilon\Lambda \sim \epsilon(kx_0R)^{2/3}. \tag{18}$$

R is the total Reynolds number defined as

$$R^{-1} = R_{\rm v}^{-1} + R_{\rm m}^{-1},$$

where $R_{\rm v}$ and $R_{\rm m}$ are the viscous and magnetic Reynolds numbers, respectively. Analytical solutions and the absorption rate are derived under the assumptions that (i) $kx_0 \ll 1$, i.e., the long wavelength limit; and (ii) $kx_0 \gg \epsilon$, i.e., the limit of weak nonlinearity.

In the first order approximation we obtain from Eq. (11)

$$\sigma\frac{\partial q_1}{\partial \theta} - k\frac{\partial^2 q_1}{\partial \sigma^2} = -\frac{kV^4}{\rho_{0c}v_{\rm Ac}^4|\Delta|}\frac{{\rm d}P_1}{{\rm d}\theta}. \tag{19}$$

Because the total pressure, P, is continuous through the dissipative layer *and* is periodical with respect to θ, we can look for the solution in the form

$$f_1 = \Re(\hat{f}_1{\rm e}^{ik\theta}), \tag{20}$$

where f_1 represents u_1, P_1, or q_1, and $\Re$ indicates the real part of a quantity.

In Region I the solutions for the pressure and velocity exactly recover the results found in linear theory, i.e.,

$$\hat{P}_1 = p_e{\rm e}^{ik\kappa_e x} + A_1{\rm e}^{-ik\kappa_e x}, \tag{21}$$

$$\hat{u}_1 = \frac{\kappa_e V}{\rho_e(V^2 - v_{Ae}^2)}(p_e \mathrm{e}^{ik\kappa_e x} - A_1 \mathrm{e}^{-ik\kappa_e x}). \quad (22)$$

The first term in the RHS of $\hat{P}_1$ and $\hat{u}_1$ represents the incoming wave, while the second terms are the outgoing (reflected) waves. Following the procedure developed by Ruderman et al. (1997a) we finally obtain for the constant A_1

$$A_1 = -p_e \frac{\alpha_1 - \alpha_2 + i\alpha_3}{\alpha_1 + \alpha_2 + i\alpha_3} + \mathcal{O}(k^2 x_0^2), \quad (23)$$

where

$$\alpha_1 = \frac{\pi k V^5}{\rho_{0c} v_{Ac}^4 |\Delta|}, \quad \alpha_2 = \frac{\kappa_e V}{\rho_e(V^2 - v_{Ae}^2)},$$

$$\alpha_3 = \frac{\kappa_i V}{\rho_i(V^2 - v_{Ai}^2)} - kV\mathcal{P}\int_0^{x_0} \frac{\mathrm{d}x}{D(x)}. \quad (24)$$

Following the procedure of the implemented regular perturbation method one can derive the governing equations, and solutions in higher (e.g., second, third, etc.) order approximations.

Before we continue to derive the actual absorption rate, let us make a brief note. Nonlinearity, in general generates higher harmonics. In the second order approximation they appear of the order of $\mathcal{O}(k^2 x_0^2)$ which is the reason that they do not have a contribution. In the third order approximation, however, there a contribution appears in the reflected wave. Finally, in even higher order the contributions will no longer be monochromatic. For details see, e.g., Ballai & Erdélyi (1998).

3.2. Coefficient of wave energy absorption

The amount of absorbed energy is measured by

$$\mathcal{A} = 1 - \frac{\Phi_2}{\Phi_1}, \quad (25)$$

where Φ_1 and Φ_2 are the x-components of the energy fluxes of the incoming and outgoing waves, respectively, averaged over a period. It can be shown that the absorption coefficient can be approximated by

$$\mathcal{A} \approx \mathcal{A}_{\mathrm{L}} + \lambda_i^2 \mathcal{A}_{\mathrm{NL}}, \quad (26)$$

where $\mathcal{A}_{\mathrm{L}}$ and $\mathcal{A}_{\mathrm{NL}}$ are the linear and nonlinear coefficients of absorption, respectively, given by

$$\mathcal{A}_{\mathrm{L}} = \frac{4\alpha_1\alpha_2}{(\alpha_1 + \alpha_2)^2 + \alpha_3^2} + \mathcal{O}(k^2 x_0^2),$$

$$\mathcal{A}_{\mathrm{NL}} = \frac{4p_e^2 \alpha_1^3 \alpha_2^3 I}{\pi^2 V^2} \frac{(\alpha_1^2 - \alpha_2^2 - \alpha_3^2)}{[(\alpha_1 + \alpha_2)^2 + \alpha_3^2]^3} + \mathcal{O}(k^2 x_0^2). \quad (27)$$

Here I is an integral and its value is estimated to be of the order of one (Ruderman et al. 1997a). It can be shown that α_1 is of the order of $\sim kx_0$ while α_2 and α_3 are of the order of ~ 1. We consider in this paper the long wavelength approximation (i.e., $kx_0 \ll 1$) resulting in $\alpha_1^2 < \alpha_2^2 + \alpha_3^2$. This yields the nonlinear correction of the coefficient of energy absorption

$$\mathcal{A}_{\mathrm{NL}} = -\frac{4p_e^2 \alpha_1^3 \alpha_2^3 I}{\pi^2 V^2} \frac{1}{(\alpha_2^2 + \alpha_3^2)^2}. \quad (28)$$

The nonlinear correction of energy absorption is negative i.e., the nonlinearity in the dissipative layer *decreases* the value of wave energy absorption. A similar result was obtained by Ruderman et al. (1997a) for resonant interaction of sound waves with resonant slow waves in the limit of long wavelength approximation (i.e., $kx_0 \ll 1$). It was shown by Ruderman et al. (1997a) in Cartesian and by Ballai et al. (2001) in cylindrical geometry that in a nonlinear regime mean flows are generated outside the dissipative layer due to the nonlinear interaction of harmonics. In our view another main effect of nonlinearity is the generation of a mean flow as well as the modification of the amount of absorbed energy. The decrease in the amount of energy absorbed is due to the fact that part of the available energy is now used to drive this mean flow. This later statement is not proven yet, but will be addressed in a follow-up paper where particular attentions is paid to nonlinear resonant flow instabilities.

The situation is even more complicated when there is a background flow already present in the plasma. The generated mean flow can interact with the equilibrium bulk motion causing Kelvin-Helmholtz instability or resonant flow instability *below* the KHI threshold. These instabilities have a serious consequence as they may destroy the resonant layer leading to poor coupling between external global motions (i.e. external driver) and local resonant magnetic surfaces. Again, a detailed study is needed and will be carried out.

Let us come back to the static case and investigate further the absorption coefficient, $\mathcal{A}$. Taking into account that $\alpha_1/\alpha_2 = \mathcal{O}(kx_0)$, the coefficients of absorption finally becomes

$$\mathcal{A} = \frac{4\alpha_1\alpha_2}{\alpha_2^2 + \alpha_3^2} - \lambda_i^2 \frac{4p_e^2 \alpha_1^3 \alpha_2^3 I}{\pi^2 V^2} \frac{1}{(\alpha_2^2 + \alpha_3^2)^2}. \quad (29)$$

4. Anisotropic plasma

It is known that the plasma in the upper part of the solar corona is very anisotropic, its structures being dominated by the magnetic field. A key quantity in our discussion is the quantity $\omega_i\tau_i$, where ω_i is the ion cyclotron frequency and τ_i is the mean collision time of the ions. If $\omega_i\tau_i \ll 1$ the viscous tensor (Braginskii 1965) becomes isotropic and in this case the viscous force is usually given by the scalar kinematic viscosity, ν of the form $\rho\nu(\nabla^2 \boldsymbol{v} - \frac{1}{3}\nabla\nabla\cdot\boldsymbol{v})$, where the volume viscosity was neglected. These conditions hold well in the solar photosphere (as discussed earlier in the previous section).

On the other hand if $\omega_i\tau_i \gg 1$, the full expression of the viscosity tensor should be taken into account. This tensor contains five terms. These terms are proportional to the viscosity coefficients η_0, η_1, η_2, η_3, and η_4.

When $\omega_i\tau_i \gg 1$ the estimates $\eta_1/\eta_0 \approx \eta_2/\eta_0 \sim (\omega_i\tau_i)^{-2}$, $\eta_3/\eta_0 \approx \eta_4/\eta_0 \sim (\omega_i\tau_i)^{-1}$ are valid. Since in the solar upper atmosphere $\omega_i\tau_i \gtrsim 10^5$, we obtain $\eta_1/\eta_0 \approx \eta_2/\eta_0 \lesssim 10^{-10}$, $\eta_3/\eta_0 \approx \eta_4/\eta_0 \lesssim 10^{-5}$. Hence, it seems that it is a good approximation to describe viscosity by only the first term of Braginskii's tensorial expression (Braginskii 1965), which is proportional to η_0 and is given by

$$\hat{\pi} = \bar{\eta}_0 \left(\boldsymbol{b} \otimes \boldsymbol{b} - \tfrac{1}{3}\hat{I}\right) [3\boldsymbol{b}\cdot(\boldsymbol{b}\cdot\nabla)\boldsymbol{v} - \nabla\cdot\boldsymbol{v}], \tag{30}$$

where $\boldsymbol{b} = \boldsymbol{B}/B$ is the unit vector in the direction of the magnetic field, and $\bar{\eta}_0$ is the first Braginskii's coefficient of viscosity. $\hat{I}$ is the unit tensor, and the symbol $\otimes$ denotes the dyadic (tensorial) product of vectors.

Another dissipative process important in solar applications is the thermal conduction, as given by, e.g., Priest (1982),

$$\boldsymbol{q} = -\kappa_\parallel \boldsymbol{b}\left(\boldsymbol{b}\cdot\nabla\overline{T}\right), \tag{31}$$

where $\kappa_\parallel$ denotes the thermal conduction coefficient along the field lines.

The dynamics of nonlinear resonant MHD waves in anisotropic plasmas was studied by, e.g, Ballai et al. (1998b). They derived the governing equations and the connection formulae necessary to study the effect of resonant absorption in anisotropic plasmas. This theory was later applied to study the nonlinear resonant interaction of *sound waves* with slow waves (Ballai et al. 1998a). We have already argued in the Introduction why fast waves also could be potential candidates for plasma heating irrespective of whether the plasma is isotropic or anisotropic. Therefore a study of nonlinear resonant absorption of fast waves in anisotropic plasma will be carried out for the first time.

We apply the same equilibrium structure as in the isotropic case. Since the dissipation should be taken into account only in a thin layer embracing the ideal resonant position, the governing equations for the wave motion outside the dissipative layer are identical in isotropic and anisotropic plasmas. Therefore, the governing equations in the outer region are given by the system of Eqs. (5). Ballai et al. (1998b) have derived the governing equations *inside* the dissipative layer of an anisotropic plasma. Let us recall the key steps and necessary results found by Ballai et al. (1998b).

The efficiency of dissipation is given by the viscous Reynolds number and the Pechlet number, so we can define the total Reynolds number as

$$R^{-1} = R_{\rm v}^{-1} + P_e^{-1}.$$

The thickness of the dissipative layer is given by

$$\delta_{\rm c} = \frac{V^3 k\gamma}{(c_{\rm Sc}^2 + v_{\rm Ac}^2)|\Delta|}, \tag{32}$$

where λ is defined as

$$\gamma = \eta_0 \frac{(2v_{\rm Ac}^2 + 3c_{\rm Sc}^2)^2}{3v_{\rm Ac}^2 c_{\rm Sc}^2} + \kappa_\parallel \frac{(\gamma-1)^2(c_{\rm Sc}^2 + v_{\rm Ac}^2)}{2\gamma\rho_{0c}c_{\rm S}^2\tilde{R}}, \tag{33}$$

and $\tilde{R}$ is the gas constant.

The plasma motion inside the nonlinear resonant slow dissipative layer is governed by (see Ballai et al. 1998b)

$$\sigma\frac{\partial q}{\partial\theta} + \Lambda q\frac{\partial q}{\partial\theta} - \frac{1}{k}\frac{\partial^2 q}{\partial\theta^2} = -\frac{kV^4}{\rho_{0c}v_{\rm Ac}^4|\Delta|}\frac{{\rm d}P}{{\rm d}\theta}, \tag{34}$$

where

$$\Lambda = R^2\frac{v_{\rm Ac}^4|\Delta|[(\gamma+1)v_{\rm Ac}^2 + 3c_{\rm Sc}^2]}{kV^8}. \tag{35}$$

(Note the differences between the isotropic and anisotropic cases!) The connection formulae across the dissipative layer are similar to those found for isotropic plasma and are given by Eqs. (8) and (12).

The nonlinearity parameter in an anisotropic plasma may be introduced as

$$N = \epsilon^{1/2}R. \tag{36}$$

In what follows we assume that $N \ll 1$ which enables us to consider the nonlinear term in the governing equation as a perturbation and use the regular perturbation method.

Let us introduce some new variables

$$\bar{q} = \frac{q}{kx_0}, \quad \bar{\lambda}_a = \frac{N^2\Lambda kx_0}{R^2}, \tag{37}$$

where $\bar{\lambda}_a$ is of the order ϵR^2 and $\bar{q}$ of the order ϵ. In what follows we drop the bar. With these new variables the governing equation and the jump condition for the normal component of the velocity can be rewritten as

$$\sigma\frac{\partial q}{\partial\theta} + \epsilon^{-1}\lambda_a q\frac{\partial q}{\partial\theta} - k^{-1}\frac{\partial^2 q}{\partial\theta^2} = -\frac{V^4}{\rho_{0c}v_{\rm Ac}^4|\Delta|x_0}\frac{{\rm d}P_c}{{\rm d}\theta}, \tag{38}$$

$$[u] = -Vx_0\mathcal{P}\int_{-\infty}^{\infty}\frac{\partial q}{\partial\theta}\,{\rm d}\sigma. \tag{39}$$

Again, we use the regular perturbation method and look for the solution in the form of expansions given by Eq. (17). Similar to the isotropic case one has to derive the governing equations in the first, second, etc. approximation. For details we refer to Ballai et al. (1998a).

4.1. Coefficient of wave energy absorption

The calculation of the coefficient of absorbed energy is analogous to that for isotropic plasma. Taking into account the assumption of the long wavelength approximation, we obtain for the linear and nonlinear coefficient of wave energy absorption in anisotropic plasmas

$$\mathcal{A}_{\rm L} = \frac{4\alpha_1\alpha_2}{\alpha_2^2 + \alpha_3^2} + \mathcal{O}(k^2x_0^2), \tag{40}$$

$$\mathcal{A}_{\rm NL} = -\frac{p_e^2\alpha_1^3\alpha_2^3}{6\pi^2V^2k^2x_0^2(\alpha_2^2+\alpha_3^2)^2} + \mathcal{O}(k^2x_0^2). \tag{41}$$

Since $\mathcal{A}_{\rm NL} < 0$ it follows again that nonlinearity decreases the coefficient of energy absorption. Note, while expression (40) for $\mathcal{A}_{\rm L}$ is similar to its counterpart obtained in the isotropic case (and to the results obtained by Ruderman et al. 1997a), Eq. (41) differs from its isotropic counterpart. Hence, *though the particular type of dissipation operating in the dissipative layer is not important when calculating the coefficient of the energy absorption in linear theory, it becomes important when nonlinearity is taken into account.*

5. Effect of equilibrium flow

Recent satellite observations revealed that the solar plasma is characterized by a high degree of dynamics; the plasma is moving on a wide range of time scales. In order to have a more accurate and precise description of the wave heating in the solar atmosphere, one has to include the observed large scale motions in the analysis. As a first approximation, an equilibrium background motion is incorporated in the theory. In what follows, we point out the changes caused (if any at all) in the theory of resonant heating when a simple equilibrium flow is present.

Hollweg et al. (1990) showed that, for an incompressible linear plasma in planar geometry, the damping or excitation of *surface waves* can be strongly influenced by an equilibrium shear flow. Erdélyi & Goossens (1995) studied numerically the loss of acoustic power due to resonant absorption in cylindrical magnetic flux tubes in linear compressible dissipative MHD with *steady* equilibrium state. They showed that resonant absorption can be significantly influenced by velocity shears. They also found negative absorption of wave power which apparently can be attributed to the resonant instability found by Hollweg et al. (1990). Erdélyi (1998) studied the effect of bulk motion on resonant Alfvén waves in coronal loops. He found that a steady state can either efficiently enhance or decease the wave energy dissipation depending on the relative direction of the bulk and wave motions. The absorption of FMA waves in the presence of a field-aligned equilibrium flow in *linear* regime was studied by Csík et al. (1998) and they also found a smooth variation of the wave energy absorption coefficient relative to the equilibrium flow.

The effect of the equilibrium flow on nonlinear resonant MHD waves was studied recently by Ballai & Erdélyi (1998) where the generalized connection formulae and modified solutions due to the presence of a shear equilibrium background flow are determined in the *nonlinear regime* in isotropic plasmas. The effect of the equilibrium flow on the width of the dissipation layer was discussed. As an application, they studied the resonant nonlinear interaction of sound waves with local slow waves in the solar chromosphere in the limit of strong and weak nonlinearity (Erdélyi & Ballai 1999). They considered sound waves modelling the well-known five-minutes oscillations, and for the equilibrium flow they modelled the chromospheric down-flows observed very extensively in the past few years (see e.g., Doschek et al. 1976; Brekke et al. 1997; Warren et al. 1997). Their analysis was strongly limited by the applicability of the model since sound waves in the solar middle atmosphere need to have low frequencies to be able to tunnel into the chromospheric slow continuum. Another shortcoming of their model is that it is highly likely that even the middle atmosphere is magnetic, leading to MHD wave propagation (e.g., FMA waves) instead of pure sound waves.

Let us introduce the Doppler shifted phase velocity of the waves:

$$\mathcal{V} = V - v_0,$$

where $\boldsymbol{v}_0$ is the equilibrium flow speed and for the sake of simplicity it is taken to be parallel to the equilibrium magnetic field. In this case the slow resonant position is now determined by the condition $\mathcal{V}^2 = c_{\mathrm{T}}^2(x)$. In contrast to the static case, one of the main effect of the equilibrium flow is the introduction of a mode shifting due to the coupling between the localized slow MHD waves and the equilibrium flow. Let us consider a flow to be defined by

$$v_0 = \begin{cases} 0, & x < 0, \\ v_0(x), & x > 0. \end{cases} \tag{42}$$

The actual spatial dependence of the flow v_0 is specified later in an approximate form. We do not give all the details how to derive the modified absorption rate due to the presence of an equilibrium flow, as it is straightforward. The modified coefficient of wave energy absorption in isotropic plasma is finally given by

$$\mathcal{A} = -\frac{4\alpha_1\alpha_2}{\alpha_2^2+\alpha_3^2} + \lambda_i^2 \frac{4p_e^2\alpha_1^3\alpha_2^3 I}{\pi^2\mathcal{V}^2} \frac{1}{(\alpha_2^2+\alpha_3^2)^2}, \tag{43}$$

where now

$$\alpha_1 = \frac{\pi k \mathcal{V}^5}{\rho_{0c} v_{\mathrm{A}c}^4 \Delta}, \quad \alpha_2 = \frac{\kappa_e V}{\rho_e (V^2 - v_{\mathrm{A}e}^2)},$$

$$\alpha_3 = \frac{\kappa_i \mathcal{V}}{\rho_i(\mathcal{V}^2 - v_{\mathrm{A}i}^2)} - k\mathcal{V}\mathcal{P}\int_0^{x_0} \frac{\mathrm{d}x}{D(x)}. \tag{44}$$

For the anisotropic case the total coefficient of resonant absorption (linear plus nonlinear), in the presence of an equilibrium shear flow takes the form

$$\mathcal{A} = -\frac{4\alpha_1\alpha_2}{\alpha_2^2+\alpha_3^2} + \lambda_a^2 \frac{4p_e^2\alpha_1^3\alpha_2^3}{6\pi^2\mathcal{V}^2 k^2 x_0^2} \frac{1}{(\alpha_2^2+\alpha_3^2)^2}. \tag{45}$$

6. Numerical results

In spite of the many detailed observations of the dynamic solar atmosphere the spatial structure of its bulk plasma motion is not yet well-resolved. Here we suppose that the motion is inhomogeneous and approximately *proportional* to the local cusp speed. We are aware of the simplification and have considered different equilibrium models. Though the obtained quantitative results might not be directly applicable because of the poorly known spatial dependence of the large-scale equilibrium motions, the qualitative results still can tell us important information about and

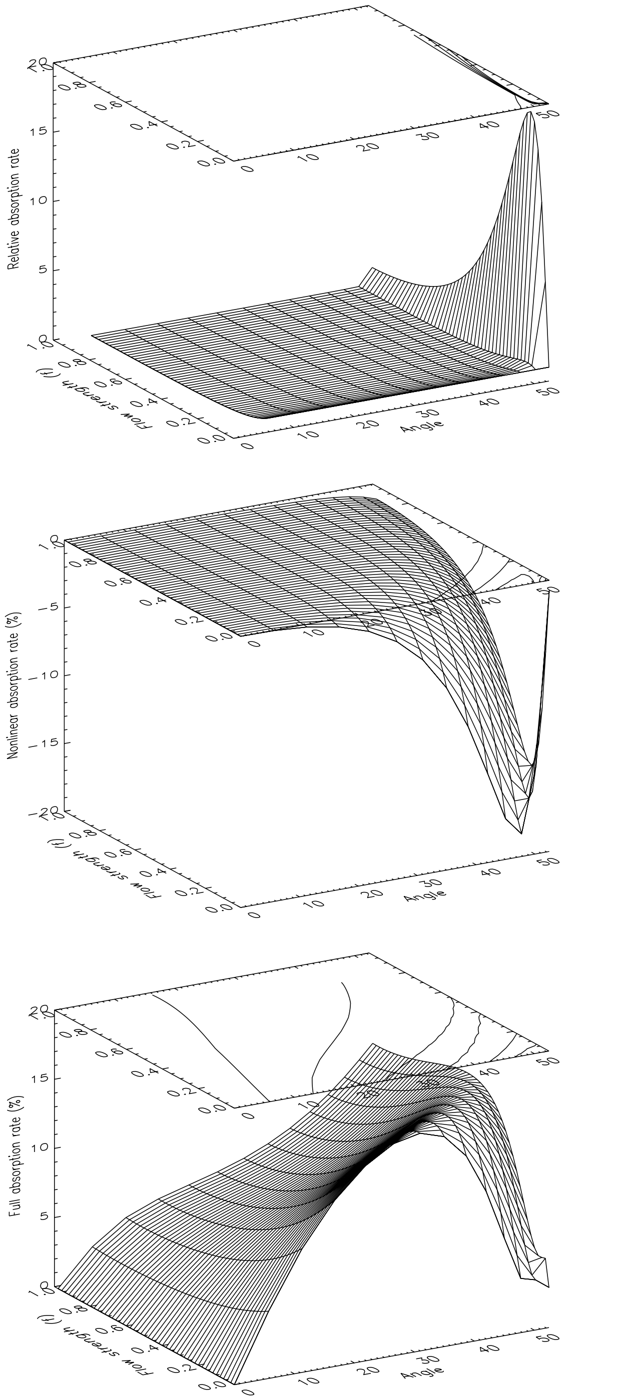

Fig. 1. The relative, nonlinear and full absorption coefficient in an isotropic plasma as a function of the field-aligned inhomogeneous plasma flow strength parameter, f, and the angle of incidence, φ

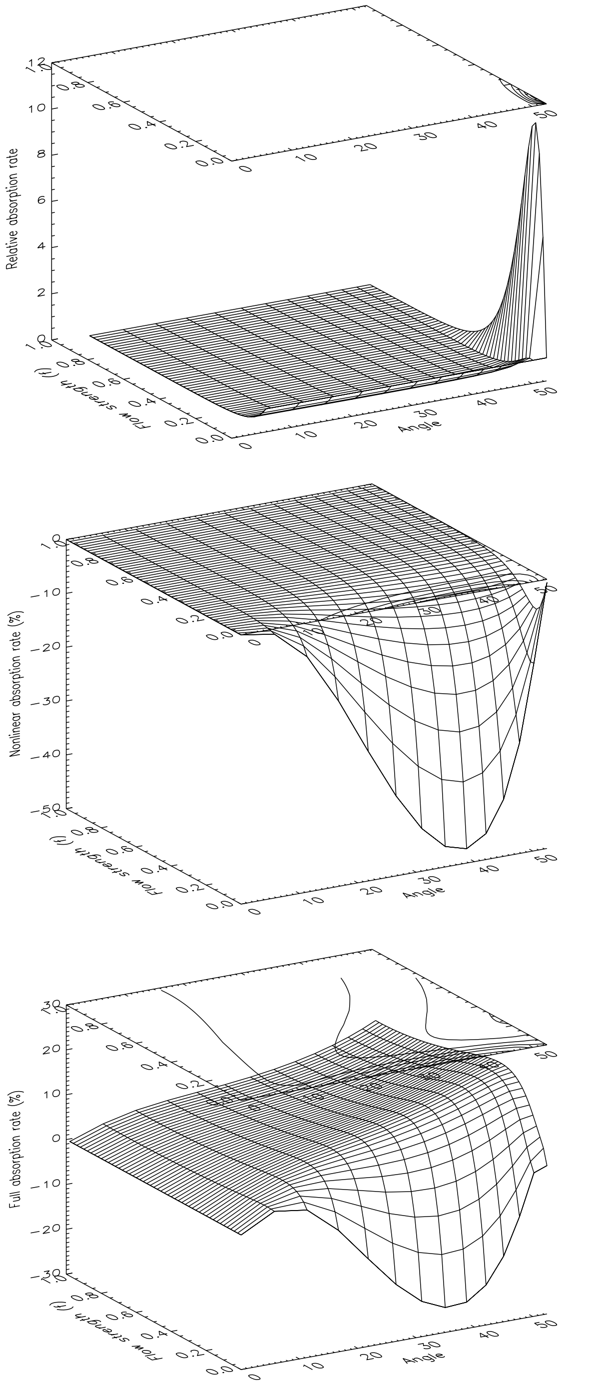

Fig. 2. The same as Fig. 1 but for an anisotropic plasma

give insight into the resonant dissipation mechanism in a steady state.

Propagating FMA waves are impinging from the homogeneous magnetic (e.g., lower parts of the canopy) side (Region I) into the inhomogeneous magnetic part of the canopy (Regions II and III) where they are resonantly coupled to the slow resonant continuum waves. Physical parameters are in the dimensionless form. Length is measured in the thickness of the inhomogeneous layer (x_0) while all speeds are expressed in the unit of sound speed in Region I.

Figures 1–8 show the relative, nonlinear and full absorption coefficients as a function of the flow strength parameter and the angle of incidence for isotropic and anisotropic plasmas. The relative absorption rate is defined as the ratio of the nonlinear correction of absorption

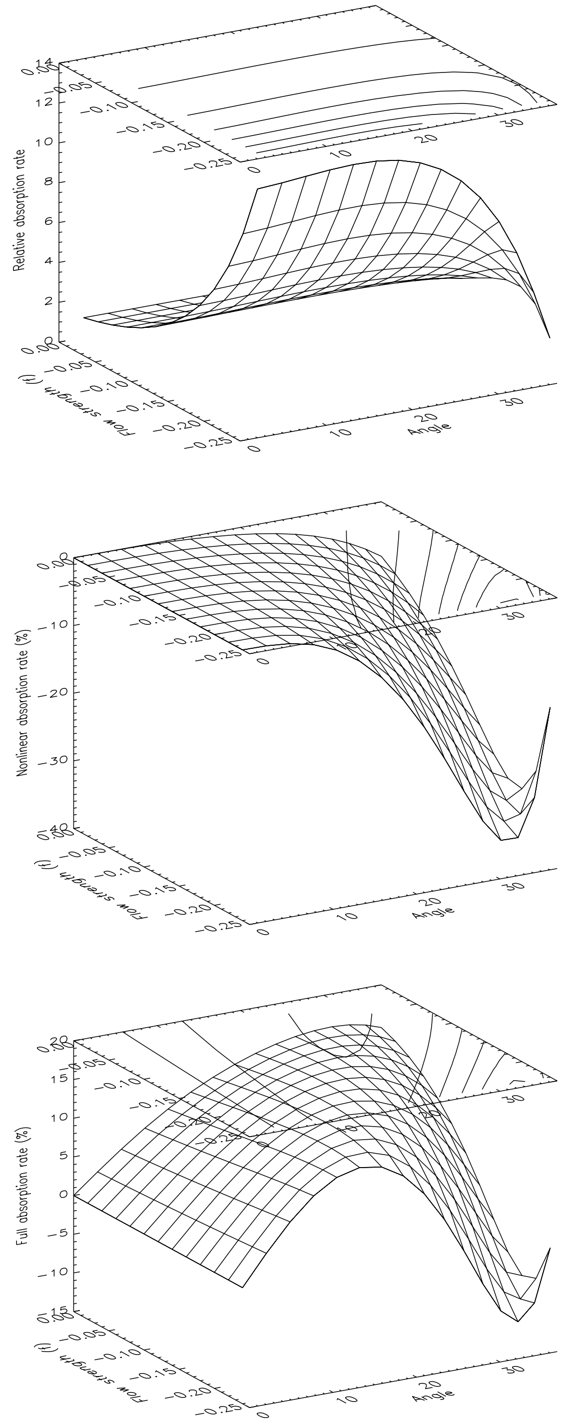

Fig. 3. The same as Figs. 1a-c but for an anti-parallel flow

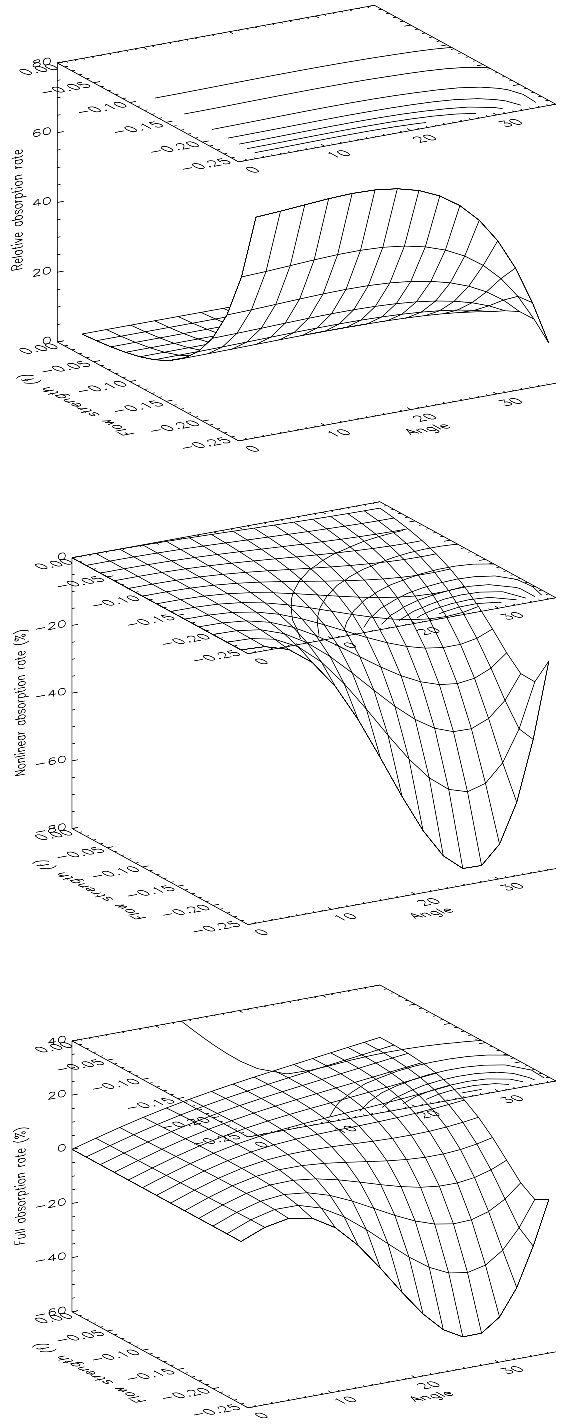

Fig. 4. The same as Figs. 2a-c but for an anti-parallel flow

for a given value of the flow strength parameter and angle of incidence and the nonlinear correction in the static case. The relative absorption is an appropriate measure to estimate the influence of flow. The nonlinear absorption is the correction due to purely nonlinear effects in the dissipative layer. Finally, the full absorption rate is defined as the sum of the linear and nonlinear correction (e.g., $\mathcal{A}$). All cases are investigated both in isotropic (Figs. 1, 3, 5, and 7) and anisotropic (Figs. 2, 4, 6, and 8) plasmas.

Since there is very little information available about the spatial structure of the equilibrium flows in the solar atmosphere we use two model approximations. In the first model the equilibrium flow is proportional to the local cusp speed, e.g., if the flow strength parameter, $f = 0.1$, this means there is an equilibrium flow 10% of the cusp speed at any given x-coordinate in Regions II and III.

Figures 1–4 show the relative, nonlinear and full absorption coefficient as a function of f and the angle

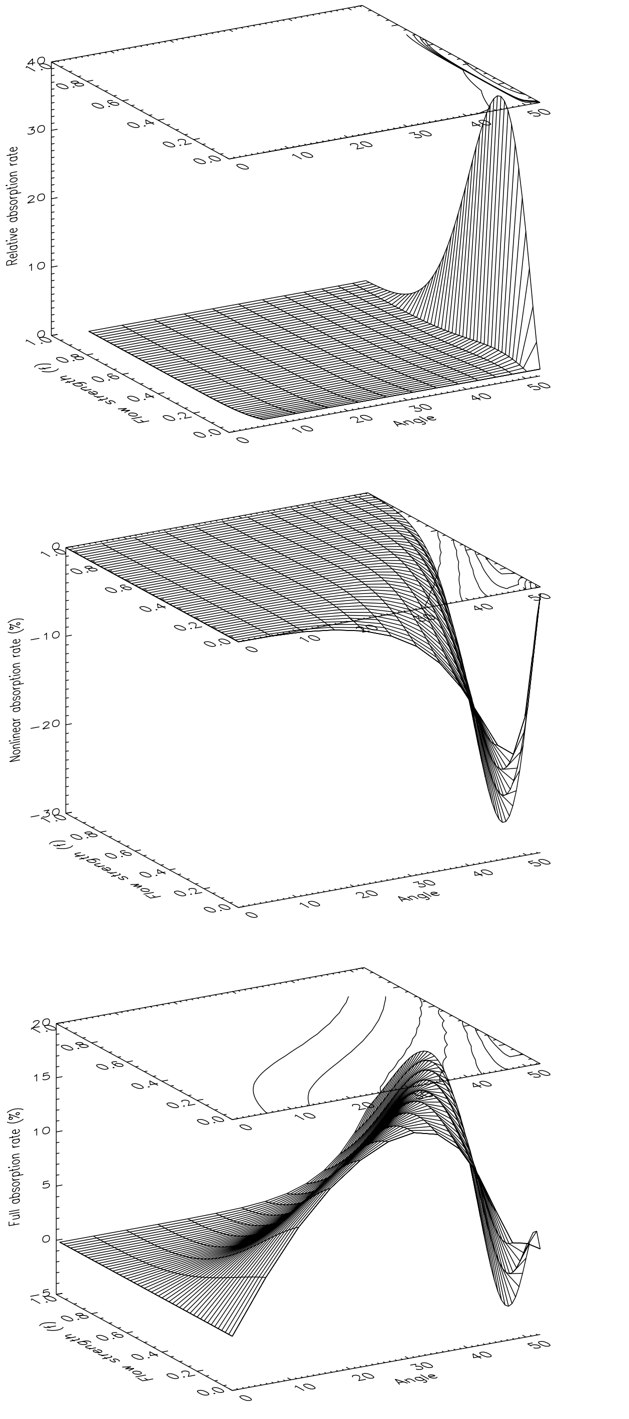

Fig. 5. The relative, nonlinear and full absorption coefficient in an isotropic plasma as a function of the field-aligned *homogeneous* plasma flow strength parameter and the angle of incidence, φ

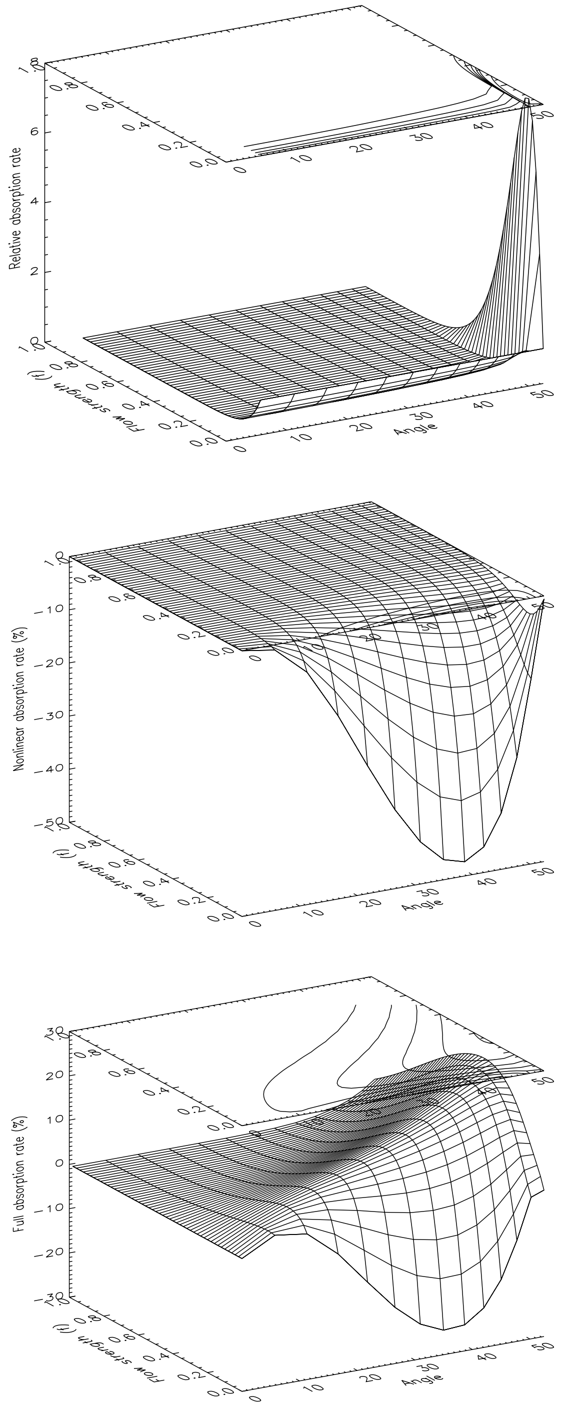

Fig. 6. The same as Figs. 5a-c but for an anisotropic plasma

of incidence. The general conclusion is in agreement with the previous studies, so that we can generally conclude the flow has a strong influence on the absorption rate. When the plasma is anisotropic this effect is even more pronounced (compare Figs. 1 and 2 or Figs. 3 and 4). Nonlinearity indeed decreases the absorption rate (Figs. 1b, 2b, 3b, and 4b) resulting in negative absorption (may be an over-reflection) in the anisotropic case (Figs. 2c and 4c). When the absorption rate becomes less than zero for a given flow this means the outgoing wave has larger amplitude than its incoming counterpart. Energy is transferred from the equilibrium flow to the waves. Figures 3–4 also show that in the case of anti-parallel propagation only a fairly small amount of flow may have an effect on the resonant heating since at around $f < -0.2$ there is no coupling of the FMA waves into the

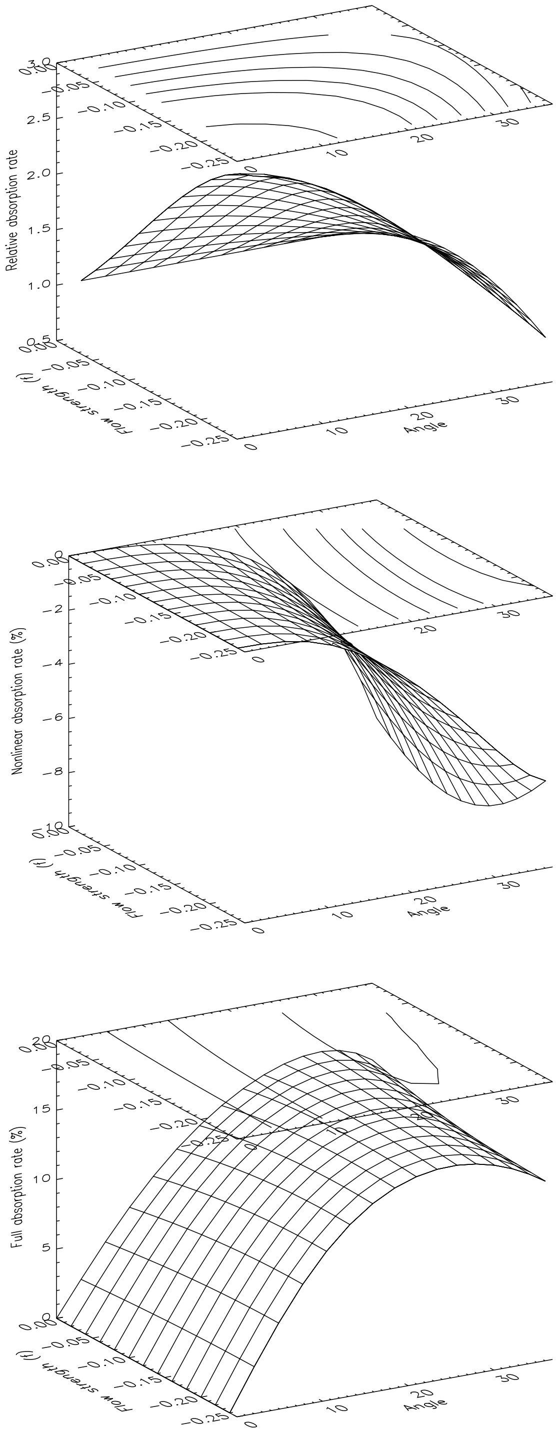

Fig. 7. The same as Figs. 5a-c but for an anti-parallel flow

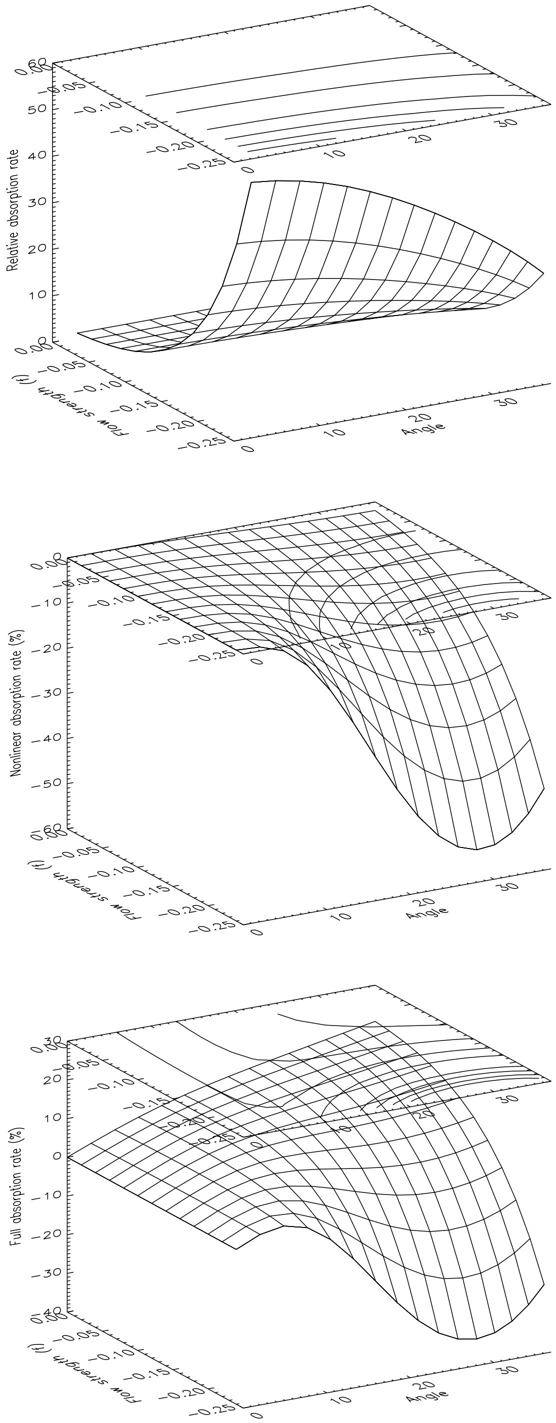

Fig. 8. The same as Figs. 6a-c but for an anti-parallel flow

slow continuum because the slow continuum is Doppler-shifted too much.

Our second model has a constant and field-aligned equilibrium flow in the entire Regions II and III. In this case the flow strength parameter, f, simply measures the equilibrium dimensionless flow $\boldsymbol{v}_0$. Results are shown in Figs. 5–8. Most of the conclusions for the studies of the effect of inhomogeneous flow also may apply here. This gives an assurance that the actual structure of flow is less important to obtain a qualitative picture of the effect of equilibrium motion on resonant FMA waves. There is however one difference: in the latter case, over-reflection may occur both in isotropic or anisotropic plasmas (Figs. 5c, 6c and 8c).

Finally, Figs. 9–10 demonstrate that indeed the *linear* absorption rate is *independent* of the actual dissipation mechanisms (e.g., isotropic or anisotropic viscosity, etc.). This result is in full agreement with previous analytical

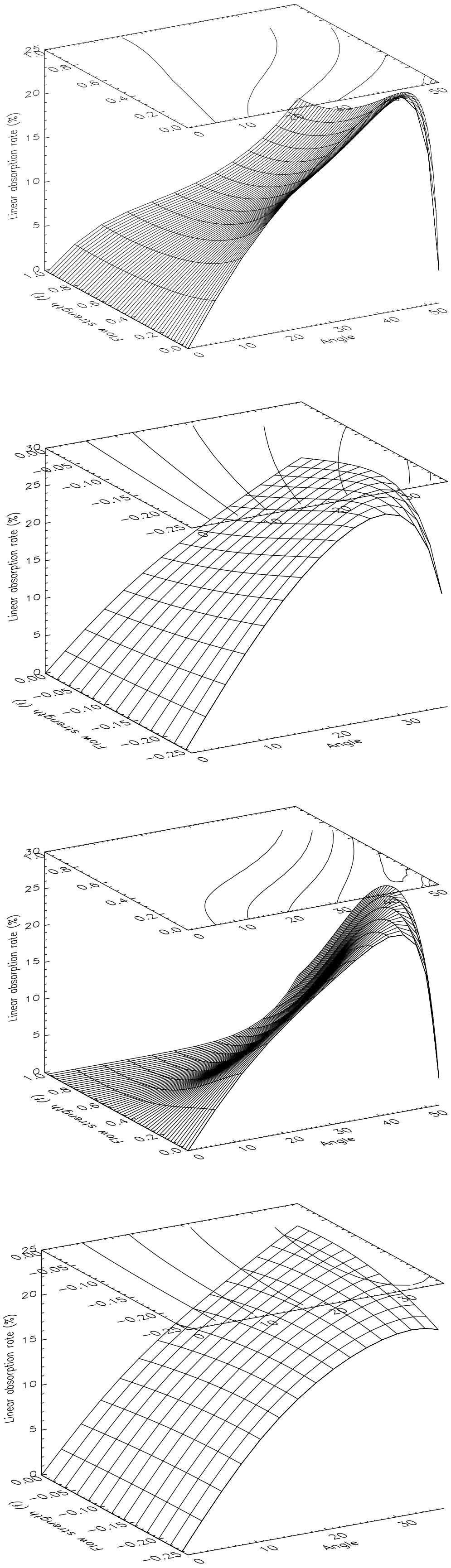

Fig. 9. The linear coefficient of absorption for an isotropic plasma for parallel and anti-parallel inhomogeneous flows (Figs. 9a-b) and homogeneous flows (Figs. 9c-d), respectively

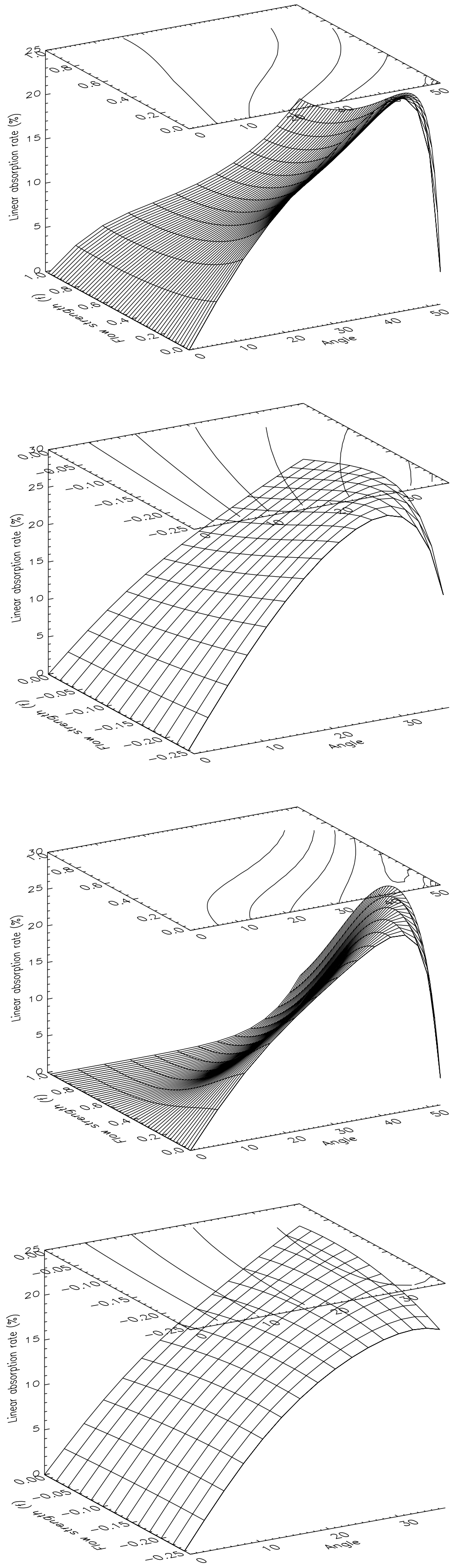

Fig. 10. The same as Fig. 9 but for an anisotropic plasma

predictions (e.g., Sakurai et al. 1991; Goossens et al. 1995; Ruderman & Goossens 1996; Erdéyi 1997).

7. Conclusion

The present paper applies the nonlinear theory of resonant coupling of FMA waves into slow MHD continuum in isotropic and anisotropic plasmas. The theory was originally derived by Ruderman et al. (1997b) and Ballai et al. (1998b).

The modifications due to an equilibrium flow on the resonant absorption coefficient is derived both for isotropic and anisotropic plasmas. The effect of an equilibrium flow is not easy to predict in general terms. By means of analytical and simple numerical investigations we found how a steady flow can strongly affect the efficiency of the resonant coupling and absorption leading to a complex picture of interaction between the incoming FMA waves and the local inhomogeneous plasma.

We applied the obtained results to cases appropriate to solar physics. The model studied in the present paper can be considered as an approximate model of interaction of FMA waves within the solar chromosphere-magnetic canopy and/or low corona.

We have assumed that (i) the thickness of the slab containing the inhomogeneous plasma (Region II) is small in comparison with the wavelength of the incoming fast wave (i.e., $kx_0 \ll 1$); and (ii) the nonlinearity in the dissipative layer is weak; the nonlinear term in the equation describing the plasma motion in the dissipative layer can be considered as a perturbation and nonlinearity gives only a correction to the linear results.

Applying a regular perturbation method, analytical solutions are obtained in the form of power expansions with respect to the nonlinearity parameters $\lambda_{i,a}$. We found that:

(i) An equilibrium flow in the slow dissipative layer can *either* increase or decrease the coefficient of the wave energy absorption. Thus, a field-aligned flow has an important effect on the resonant interaction of fast waves and nonlinear slow resonances and energy transfer;
(ii) Negative absorption rate (i.e., resonant flow instability or over-reflection) has been found for a wide range of parameters. A more accurate quantitative analysis would require observations and diagnostics of large-scale flow fine structures in the low and middle magnetic atmosphere.

The present paper deals with weak nonlinearity. In a forthcoming study we address the question of strong nonlinearity and will discuss the nonlinear aspects of resonant instability and their applicability with some observation consequences to solar physics.

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