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An analytical representation of the systematic differences HIPPARCOS–FK5

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Abstract. In the present paper we present an analytical representation of the relations between the systems of the HIPPARCOS catalogue and the Basic FK5 catalogue. These relations were used in the construction of the "Sixth Catalogue of Fundamental Stars (FK6)" for the reduction of the FK5 to the HIPPARCOS system. A comparison has been made between the present results and those derived by Mignard and Frœschlé recently. A computer program for evaluating analytically the systematic differences will be provided via the Internet facility.

Key words. catalogs – astrometry – reference systems

1. Introduction

At the General Assembly held in Kyoto 1997 the IAU has resolved (IAU, 1999, Resolution B2) that, from 1 January 1998, the official celestial reference system shall be the International Celestial Reference System (ICRS) with the HIPPARCOS catalogue (ESA, 1997) as its primary realization at optical wavelengths. The HIPPARCOS catalogue thus replaces the system defined by the FK5 catalogue (Fricke et al. 1988) which was constructed completely from ground based observations.

For the construction of the FK6 (Wielen et al. 1999), which is a combination of the data given in the FK5 and in the HIPPARCOS catalogue, we had to transform the FK5 to the HIPPARCOS system. Since there exists at present no generally adopted and practicable formulation for this transition we had to derive the relation between both systems.

In the present paper we briefly describe the procedure applied for the determination of these relations, give explicitly the numerical results with some supplementary graphical illustrations, and discuss the differences between our results and those derived recently by Mignard & Frœschlé (2000) (abbreviated sometimes as M&F in the following).

2. The epoch of comparison

We have made the comparison between the HIPPARCOS and FK5 positions at the mean epoch 1949.4 of the FK5, whereas Mignard and Frœschlé have chosen the mean HIPPARCOS epoch 1991.25 for their comparison. The reason for choosing the FK5 epoch is that the positions depend much less on the proper motions at this epoch, and the separation of the systematic differences in position from those in proper motion is much safer. If the comparison is made at the HIPPARCOS epoch 1991.25, then the differences in position contain the systematic differences in proper motion for nearly half a century and the positions and proper motions are highly correlated.

The dispersion of the residuals around the systematic trend is practically the same at 1949.4 (namely 52.26 mas in right ascension and 56.82 mas in declination, see Sect. 5) and at 1991.25 (46.22 mas and 62.36 mas in right ascension and declination, respectively. These values were obtained from a comparison at 1991.25, which is not presented here). This means that the systematic trend can be determined at both epochs with the same accuracy. However, because of the high correlation between the positions and proper motions at 1991.25, the comparison should be made near the mean FK5 epoch, 1949.4.

It is of some importance to mention, that also in the case of choosing the mean FK5 epoch 1949.4 for the epoch of comparison, proper motion effects are propagated into the mean positions to some extent. The reasons are as follows.

First we have chosen, for practical reasons, our epoch of comparison as the average epoch over all FK5 stars and over both coordinates. In reality the average epoch is about 1955 in right ascension and 1945 in declination, respectively. This brings proper motion effects over about ± 5 years into the systems of positions in right ascension and declination, respectively.

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Table 1. Systematic deviations of the individual mean epochs of the FK5 stars from the average mean FK5 epoch (average over all stars) as a function of the declination (ordinate) and the apparent magnitude (abscissa) in years. The average mean reference epoch is 1955 in right ascension (left hand part of the table) and 1945 in declination (right hand part). The units are years

	right ascension				declination			
$\delta\backslash$ mag	1	3	5	7^m	1	3	5	7^m
80°	–24	–20	–9	–3	–26	–18	–11	–3
60	–9	–6	0	2	–16	–8	0	8
40	–9	–5	2	7	–16	–6	3	12
20	–15	–10	0	6	–21	–10	0	11
0	–20	–14	0	7	–21	–10	0	10
–20	–19	–15	–3	3	–19	–10	0	8
–40	–3	0	6	9	–8	–1	5	12
–60	–4	–1	7	11	–5	0	3	8
–80	–12	–9	–2	1	–12	–9	–6	–3

Second, there is a range of about 50 years to the epochs of the individual FK5 stars in each coordinate, and, even more important, these variations depend systematically on the position on the sky and the apparent magnitude. An analysis of the differences between the stars' individual epochs (separately in right ascension and declination) and the mean epoch over all stars (1955 in right ascension and 1945 in declination) with the analytical method described below has shown, that the mean epochs are systematically smaller (i.e. shifted towards older epochs) in the declination zones about $-70° < \delta < -30°$ and $+25° < \delta < +50°$. In the other regions the epochs are on the average more modern than the average mean epoch. Moreover the bright stars have systematically older epochs than the faint stars, whose epochs are shifted towards more modern epochs. Some numerical results describing this effect are presented in Table 1, where the systematic deviation of the individual mean epochs from the average mean epoch (1955 in α and 1945 in δ) is given as a function of the declination δ and the apparent magnitude m. No systematic effect depending on the right ascension was found. As an example, the mean epochs in right ascension for stars of magnitude 3^m in the equatorial zone are systematically shifted 14 years to older epochs, i.e. their average mean epoch is $1955 - 14 = 1941$.

The proper motion differences are practically independent of the epoch, but they have, nevertheless, also been computed at the mean FK5 epoch. The transformation of the HIPPARCOS and FK5 positions and proper motions to 1949.4 has been made rigourously using a constant velocity in space and time. If no radial velocity was available for the star it was defined as zero. The parallaxes used were those from the HIPPARCOS catalogue, the radial velocities are those used in the construction of the FK6, part 1 (Wielen et al. 1999).

The basic data for the further investigation were therefore the differences

$$\begin{aligned}\Delta\alpha_* &= (\alpha_{\rm H} - \alpha_{\rm F})\cos\delta\\ \Delta\delta &= \delta_{\rm H} - \delta_{\rm F}\\ \Delta\mu_{\alpha_*} &= (\mu_{\alpha_{\rm H}} - \mu_{\alpha_{\rm F}})\cos\delta\\ \Delta\mu_\delta &= \mu_{\delta_{\rm H}} - \mu_{\delta_{\rm F}}\end{aligned}$$

between the HIPPARCOS positions and proper motions $\alpha_{\rm H}$, $\delta_{\rm H}$, $\mu_{\alpha_{\rm H}}$, $\mu_{\delta_{\rm H}}$, and the corresponding FK5 values $\alpha_{\rm F}$, $\delta_{\rm F}$, $\mu_{\alpha_{\rm F}}$, $\mu_{\delta_{\rm F}}$, all computed at the mean epoch 1949.4 of the FK5.

3. Elimination of discordant stars

We have eliminated a priori all stars for which we had information on duplicity, either from ground based observations or from the HIPPARCOS observations. In addition we have eliminated stars where the total proper motion difference $\Delta\mu_{\rm tot} = (\Delta\mu^2_{\alpha_*} + \Delta\mu^2_\delta)^{\frac{1}{2}}$ exceeded 10 mas/yr. These stars are so-called $\Delta\mu$ binaries, for which the "instantaneously" obtained HIPPARCOS proper motion (in reality determined from observations over about 3 years) deviates from the FK5 "center of mass" motion (obtained from averaging observations over about 200 years). The discordant HIPPARCOS proper motions of these stars produce also discordant differences in position at the FK5 epoch. Finally we have eliminated stars for which we have found discordant residuals in a preliminary determination of the systematic differences. For the determination of the final systematic differences, which are the result of this iteration process, 1151 stars have been left.

This procedure is in agreement with the IAU resolution on the optical frame passed at Manchester (IAU, 2001) which mandates that the HIPPARCOS frame is to be defined only by those stars that are indicated to be single in the HIPPARCOS catalogue.

4. The global orientation

In a first step the FK5 catalogue was rotated rigidly on the HIPPARCOS catalogue. The rotational parameters ϵ_x, ϵ_y, ϵ_z for the positions and ω_x, ω_y, ω_z for the proper motions were determined by the method of least squares using the equations describing an infinitesimal rotation:

$$\begin{aligned}\Delta\alpha_* &= +\epsilon_x\sin\delta\cos\alpha + \epsilon_y\sin\delta\sin\alpha - \epsilon_z\cos\delta\\ \Delta\delta &= -\epsilon_x\sin\alpha + \epsilon_y\cos\alpha\end{aligned}$$

for the differences in position and correspondingly for the proper motions:

$$\begin{aligned}\Delta\mu_{\alpha_*} &= +\omega_x\sin\delta\cos\alpha + \omega_y\sin\delta\sin\alpha - \omega_z\cos\delta\\ \Delta\mu_\delta &= -\omega_x\sin\alpha + \omega_y\cos\alpha.\end{aligned}$$

The signs in these equations are, for historical reasons, in the sense used by Lindegren in the NDAC reduction consortium. They are opposite to those used in the HIPPARCOS catalogue (ESA, 1977, Vol. 3, p. 390 and

391). Since Mignard and Frœschlé use differences FK5–HIPPARCOS, which are the negative of those used here, the signs are reversed twice and both results are directly comparable.

Table 2. Rotational parameters $\epsilon_x, \epsilon_y, \epsilon_z$ between the Hipparcos and FK5 positions at 1949.4 and $\omega_x, \omega_y, \omega_z$ between the proper motions

	Schwan	M&F
ϵ_x[mas]	– 3.01 ± 2.39	–4.75 ± 4.78
ϵ_y[mas]	–41.00 ± 2.39	–39.41 ± 4.78
ϵ_z[mas]	–19.12 ± 2.51	–12.50 ± 4.78
ω_x[mas/yr]	– 0.34 ± 0.08	– 0.30 ± 0.10
ω_y[mas/yr]	+ 0.74 ± 0.08	+ 0,60 ± 0.10
ω_z[mas/yr]	+ 0.89 ± 0.08	+ 0.70 ± 0.10

The derived rotational parameters are given in Table 2. In our transformation, which was made at the epoch 1949.4, we have determined and used the parameters at that epoch for the global rotation of the FK5 onto the HIPPARCOS frame. Merely for comparison we add the corresponding values as determined by Mignard and Frœschlé. Since their result was derived at the HIPPARCOS epoch 1991.25, we have transformed their positional rotation parmeters to 1949.4 with the aid of their proper motion parameters. The agreement between our results and that of M&F is satisfactory. The errors of the rotations in position given for M&F include the corresponding errors of the proper motion parameters which were used to transform the positions from 1991.25 to 1949.4. This explains the larger errors of the positional rotations for Mignard and Frœschlé.

5. Regional systematic errors

In order to understand the results presented below we give first a very short description of the method applied for the determination of the regional errors. The residuals $\Delta = \Delta\alpha_*$, $\Delta\delta$, $\Delta\mu_{\alpha_*}$ or $\Delta\mu_\delta$ which were obtained in Sect. 4 were analyzed on systematic errors depending on the right ascension, declination, and the apparent magnitude, using the analytical method as described by Bien et al. (1978). Let Δ_s and Δ_i denote the systematic and the accidental part of a difference, i.e.

$$\Delta = \Delta_\mathrm{s} + \Delta_\mathrm{i}.$$

The analytical method develops the systematic part into a series of statistically significant functions $Y_j(\alpha, \delta, m)$,

$$\Delta_\mathrm{s} = \sum_{j=1}^{g} Y_j.$$

Each Y_j is the product of a Legendre polynomial $L_n(x(\delta))$, a Fourier term $F_{ml}(\alpha)$ and a Hermite polynomial $H_p(y(m))$, where the subscripts n and p denote the degree of the Legendre and Hermite polynomial, respectively, and m is the frequency of the Fourier term:

$$Y_j = R_{pnm} L_n(x(\delta)) F_{ml}(\alpha) H_p(y(m)).$$

R_{pnm} is a normalizing factor and the subscript l differentiates between the sin and the cos terms. Please note the double use of the letter m as a subscript in F_{ml} and as the apparent magnitude in $H_p(y(m))$.

The transformed declinations $x(\delta)$ and magnitudes $y(m)$ are defined according to

$$x(\delta) = b_0 + b_1 \sin(\delta),$$
$$y(m) = a_0 + a_1 m.$$

The parameters a_0, a_1, b_0, b_1 are determined such that $x(\delta)$ is transformed to the interval $[-1, +1]$, and $y(m)$ has an approximate normal distribution. With these transformed values the orthogonality of the Y_j holds approximately also in the practical application where the data are given merely for discrete values of α, δ, and m. For more details reference is made to Bien et al. (1978), where the explicit definitions of the various functions and the normalizing factors can be found.

A significance level of 5% has been chosen in our method which means, that 5% of the functions are included into the final development merely by chance.

The transformation parameters, which are needed in a numerical evaluation of the series developments, were in our case:

$$a_0 = -3.8256816038$$
$$a_1 = +0.8059786407$$
$$b_0 = -0.0000124009$$
$$b_1 = +1.0001534457.$$

The basic results are given numerically in the Tables 3 and 4 for the positions and proper motions in right ascension, and correspondingly in Tables 5 and 6 for the positions and proper motions in declination. The letter j is a running number, *p, n, m, l* describe the function Y_j, b_j is the coefficient of the function with its mean error $\sigma(b_j)$, and $z_j = x_1 10^{-x_2}$ is the significance of the function. Small values of z_j belong to highly significant functions. Two functions (running numbers $j = 9$ and $j = 27$) in the case of the right ascensions (Table 3) with a significane z_j less than 5 percent were included into the final development because they had shown up significantly in an analysis of the preliminary residuals.

No significant magnitude equation was detected (no significant function with $p \neq 0$).

The dispersion σ_res of the N residuals around the system is defined as

$$\sigma_\mathrm{res} = (\tfrac{1}{N-g} \textstyle\sum_{j=1}^{N} \Delta_i^2)^{\frac{1}{2}}.$$

In our application we have obtained a dispersion of 0.89 mas/yr and 0.57 mas/yr for the proper motions in right ascension and declination, respectively. If we exclude the region south of $-40°$ one obtains the values

Table 3. Analytical representation of the systematic differences HIPPARCOS–FK5 in right ascension for the epoch 1949.4. The coefficients are in [mas]

j	p	n	m	l	b_j	$\sigma(b_j)$	x_1	x_2
1	0	0	0	–1	–1.47182	1.572	2.69	2
2	0	1	0	–1	2.52241	1.499	2.77	3
3	0	2	0	–1	–2.59302	1.486	1.22	4
4	0	3	0	–1	12.15491	1.472	3.47	7
5	0	4	0	–1	–12.08197	1.458	9.17	8
6	0	5	0	–1	5.83088	1.478	3.09	3
7	0	6	0	–1	–12.59792	1.487	1.17	6
8	0	8	0	–1	2.40269	1.451	3.68	3
9	0	9	0	–1	4.19199	1.429	4.23	1
10	0	12	0	–1	1.24164	1.408	4.30	2
11	0	13	0	–1	–4.40693	1.409	2.99	5
12	0	19	0	–1	–2.92365	1.451	2.91	3
13	0	20	0	–1	4.35534	1.418	7.80	4
14	0	21	0	–1	–3.80601	1.487	2.89	2
15	0	23	0	–1	–4.84382	1.446	2.25	3
16	0	24	0	–1	–3.77238	1.446	3.38	2
17	0	30	0	–1	3.10153	1.422	3.29	2
18	0	0	1	–1	10.41052	1.569	4.47	6
19	0	0	1	1	5.92833	1.547	8.88	4
20	0	1	1	1	–4.64599	1.451	5.32	3
21	0	2	1	–1	–3.25977	1.469	1.35	2
22	0	3	1	–1	2.85461	1.459	5.08	3
23	0	4	1	–1	–3.49405	1.402	1.18	2
24	0	5	1	–1	3.06182	1.457	2.07	2
25	0	7	1	–1	3.20984	1.418	3.48	2
26	0	0	2	1	–7.93609	1.543	9.59	6
27	0	3	2	1	–4.34629	1.426	5.67	2
28	0	5	2	1	5.35261	1.423	6.12	4
29	0	0	3	–1	4.49004	1.550	1.44	2
30	0	1	3	–1	4.26997	1.448	3.42	3
31	0	1	3	1	3.73736	1.511	2.09	3
32	0	3	3	1	4.22523	1.440	5.89	3
33	0	0	4	–1	4.47191	1.551	4.08	3
34	0	1	4	–1	3.77471	1.473	1.77	2
35	0	1	4	1	–3.17723	1.480	4.92	2
36	0	5	4	1	4.21508	1.436	1.50	2
37	0	7	4	1	–3.48795	1.412	1.83	2
38	0	1	5	–1	–3.40530	1.454	2.17	2
39	0	0	7	1	–3.80774	1.541	1.27	2
40	0	0	8	–1	–5.87293	1.550	4.62	4
41	0	1	8	1	–3.62318	1.457	1.62	2
42	0	1	10	–1	3.09999	1.468	3.31	2

0.71 mas/yr and 0.47 mas/yr. For the positions we have 52.26 mas in right ascension and 56.82 mas in declination. These values are very near to those which were obtained in an additional comparison (not presented here) performed at the HIPPARCOS epoch 1991.25, namely 46.22 mas and 62.36 mas in declination.

With the coeffients given in Tables 3 through 6 and the transformation parmeters a_0, a_1, b_0, b_1 the systematic difference HIPPARCOS–FK5 in position and proper motion can be computed at the FK5 epoch 1949.4 for any α or δ. If necessary, the systematic differences in position at any other epoch can be obtained by applying the proper motion differences over the respective time interval.

Table 4. Analytical representation of the systematic differences HIPPARCOS–FK5 for the proper motions in right ascension at the epoch 1949.4. The coefficients are in [mas/yr]

j	p	n	m	l	b_j	$\sigma(b_j)$	x_1	x_2
1	0	0	0	–1	.17179	.050	6.21	5
2	0	2	0	–1	.61486	.047	1.12	9
3	0	3	0	–1	–.77633	.046	1.20	9
4	0	4	0	–1	.79269	.046	4.02	10
5	0	5	0	–1	–.22279	.047	9.06	2
6	0	6	0	–1	.32411	.047	3.45	4
7	0	7	0	–1	.15044	.046	5.98	5
8	0	8	0	–1	–.17661	.046	2.37	5
9	0	11	0	–1	.19689	.046	5.26	6
10	0	12	0	–1	.01053	.045	4.71	2
11	0	15	0	–1	.00515	.046	4.04	3
12	0	16	0	–1	.04388	.046	1.90	2
13	0	17	0	–1	.21170	.045	3.94	5
14	0	18	0	–1	–.34898	.046	4.23	7
15	0	19	0	–1	.17332	.046	4.38	3
16	0	29	0	–1	.14754	.044	2.77	3
17	0	0	1	1	–.20905	.049	8.73	4
18	0	1	1	–1	.23271	.046	4.33	5
19	0	1	1	1	.28595	.046	1.25	5
20	0	2	1	–1	–.42140	.047	1.65	7
21	0	2	1	1	.07833	.046	2.68	2
22	0	4	1	–1	.15578	.045	6.04	4
23	0	6	1	–1	.09415	.045	2.83	2
24	0	6	1	1	.12027	.045	1.60	2
25	0	7	1	1	–.07923	.045	4.27	2
26	0	9	1	1	–.08704	.044	3.17	2
27	0	1	2	1	–.09044	.046	2.71	2
28	0	2	2	–1	–.10978	.045	2.73	2
29	0	2	2	1	.09769	.044	4.24	2
30	0	0	3	–1	–.26302	.049	1.85	5
31	0	0	3	1	–.10322	.049	4.01	2
32	0	1	3	–1	.12560	.045	1.06	2
33	0	3	3	1	.08008	.045	1.48	2
34	0	0	4	–1	.24088	.049	1.53	4
35	0	1	4	–1	–.17619	.047	4.33	3
36	0	1	4	1	–.16253	.046	3.20	3
37	0	3	4	–1	.12767	.045	4.56	3
38	0	4	4	–1	–.06535	.044	4.24	2
39	0	7	4	1	.08553	.043	4.42	2
40	0	8	4	–1	–.16043	.045	1.57	3
41	0	11	4	–1	–.11135	.044	1.72	2
42	0	3	5	1	.14551	.044	1.27	3
43	0	2	6	–1	–.12900	.045	4.07	3
44	0	0	8	–1	.15088	.049	3.03	3
45	0	3	8	–1	.08510	.044	4.90	2
46	0	6	8	1	–.11201	.044	1.11	2

We have used our results for the transformation of the “original” Basic FK5 onto the HIPPARCOS system. The residual differences between HIPPARCOS and this FK5 (on HIPPARCOS) were analyzed with our analytical method. No additonal systematic trend was detected.

Table 5. Analytical representation of the systematic differences HIPPARCOS–FK5 in declination for the epoch 1949.4. The coefficients are in mas

j	p	n	m	l	b_j	$\sigma(b_j)$	x_1	x_2
1	0	0	0	–1	37.94734	1.706	1.57	11
2	0	1	0	–1	–8.30112	1.631	1.80	4
3	0	2	0	–1	–20.71092	1.611	5.17	9
4	0	3	0	–1	10.43780	1.594	5.43	6
5	0	4	0	–1	5.63969	1.568	4.66	5
6	0	5	0	–1	–2.22992	1.603	1.51	2
7	0	6	0	–1	6.61965	1.565	6.49	5
8	0	7	0	–1	–3.63669	1.565	4.96	3
9	0	15	0	–1	–2.20583	1.501	3.09	2
10	0	16	0	–1	–3.67663	1.479	4.31	2
11	0	19	0	–1	–2.47205	1.525	4.00	2
12	0	22	0	–1	2.81729	1.522	3.98	2
13	0	0	1	1	–3.02465	1.679	5.00	2
14	0	2	1	–1	–7.58940	1.562	2.22	4
15	0	3	1	–1	–3.01592	1.505	3.95	2
16	0	8	1	–1	4.59738	1.471	2.12	3
17	0	0	2	–1	–3.40622	1.686	4.45	2
18	0	3	2	–1	3.71959	1.514	1.07	2
19	0	1	3	–1	3.26399	1.555	2.84	2
20	0	1	3	1	–3.19004	1.600	4.24	2
21	0	0	4	–1	3.69107	1.683	2.47	2
22	0	0	5	–1	4.26399	1.684	1.19	2
23	0	0	6	1	–3.91402	1.671	1.85	2

Table 6. Analytical representation of the systematic differences HIPPARCOS–FK5 for the proper motions in declination at the epoch 1949.4. The coefficients are in mas/yr

j	p	n	m	l	b_j	$\sigma(b_j)$	x_1	x_2
1	0	0	0	–1	.25909	.048	2.05	4
2	0	1	0	–1	–.54587	.046	2.04	8
3	0	4	0	–1	–.17570	.044	1.25	4
4	0	5	0	–1	.59082	.045	2.46	9
5	0	6	0	–1	–.15332	.045	1.85	2
6	0	7	0	–1	–.10474	.045	6.37	4
7	0	8	0	–1	.14863	.045	2.42	3
8	0	9	0	–1	–.18563	.044	1.09	4
9	0	10	0	–1	–.10955	.044	3.82	2
10	0	19	0	–1	–.07794	.041	3.01	2
11	0	0	1	–1	.17072	.048	2.39	3
12	0	0	1	1	–.16140	.048	7.59	3
13	0	2	1	1	.16726	.044	1.44	4
14	0	3	1	1	–.08194	.044	4.71	3
15	0	5	1	1	–.05052	.045	3.74	2
16	0	6	1	–1	–.06417	.043	3.66	2
17	0	7	1	1	–.07900	.044	2.31	2
18	0	8	1	–1	–.09188	.043	3.10	2
19	0	10	1	1	.09270	.041	1.58	2
20	0	11	1	1	–.10301	.043	1.78	2
21	0	1	2	–1	.17683	.046	1.31	3
22	0	1	2	1	.09980	.044	2.01	2
23	0	3	2	–1	–.11075	.044	1.11	2

The mean errors $\sigma(b_j)$ allow one to approximately compute the mean error $\sigma(\Delta_\mathrm{s})$ of a systematic difference as a function of α and δ (no magnitude equation was found) according to

$$\sigma(\Delta_\mathrm{s}) = (\sum_{j=1}^{g} \sigma(b_j)^2 Y_j^2)^{\frac{1}{2}}.$$

This formula neglects the correlations between the functions Y_j. For most parts of the sky these correlations are small, however, since our transformed positions are approximately homogeneous distributed on the sphere. A comparison of the rather time-consuming correct computation with the above approximation has shown, that the approximate values differ from the correct ones by less than about 5 percent in the region $-70^\circ < \delta < +70^\circ$. For $|\delta| > 80^\circ$ differences significantly exceeding 10 percent may occure, however.

Typical values for the uncertainty of the systematic differences $\sigma(\Delta\alpha_\mathrm{s})$, $\sigma(\Delta\delta_\mathrm{s})$, $\sigma(\Delta\mu_{\alpha_\mathrm{s}})$ and $\sigma(\Delta\mu_{\delta_\mathrm{s}})$ are:

$$\begin{aligned} \sigma(\Delta\alpha_\mathrm{s}) &= 9.98 \text{ mas}, \\ \sigma(\Delta\delta_\mathrm{s}) &= 8.03 \text{ mas}, \\ \sigma(\Delta\mu_{\alpha_\mathrm{s}}) &= 0.33 \text{ mas/yr}, \\ \sigma(\Delta\mu_{\delta_\mathrm{s}}) &= 0.23 \text{ mas/yr}. \end{aligned}$$

These are averages over the whole sky. The errors increase with approaching the northern and the southern limiting declinations and they are smallest in the equatorial region.

In the Figs. 1 through 4, we present graphically the systematic differences in proper motion as functions of declination (Figs. 1 and 2), and right ascension in four declination zones (Figs. 3 and 4).

Since the mean positions are of lower interest we omit the corresponding figures for economical reasons. Each dot in the figures is the difference HIPPARCOS–FK5 for one FK5 star. The curves in Figs. 1 and 2 are obtained by merely evaluating those functions which are independent of α, i.e. functions with $m = 0$. The curves shown in Figs. 3 and 4 were obtained by evaluting those functions which depend also on α, i.e. functions with $m \neq 0$. Therefore the differences in Figs. 3 and 4 do not include the systematic effects depending exclusively on the declination (Figs. 1 and 2). It may also be worth mentioning that the scatter in Figs. 1 and 2 is partly produced by the systematic effects depending on the right ascension, which are still included in these figures. In Figs. 3 and 4, where the α and δ dependent terms are eliminated, the dispersion is much lower.

A computer program for the evaluation of the systematic differences as a function of α and δ will be provided via the Internet facility (URL: `http://www.ari.uni-heidelberg.de/`).

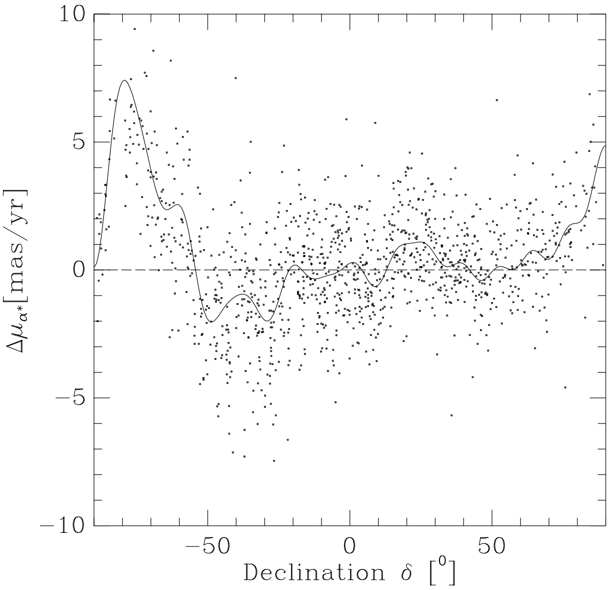

Fig. 1. Systematic differences $\Delta\mu_{\alpha*}$ [mas/yr] between the HIPPARCOS and the FK5 proper motions as a function of the declination. The global rotation is not included

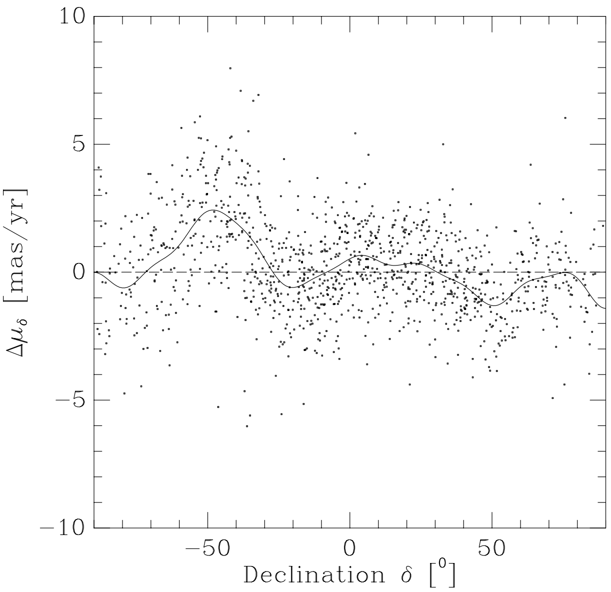

Fig. 2. Systematic differences $\Delta\mu_{\delta}$ [mas/yr] between the HIPPARCOS and the FK5 proper motions as a function of the declination. The global rotation is not included

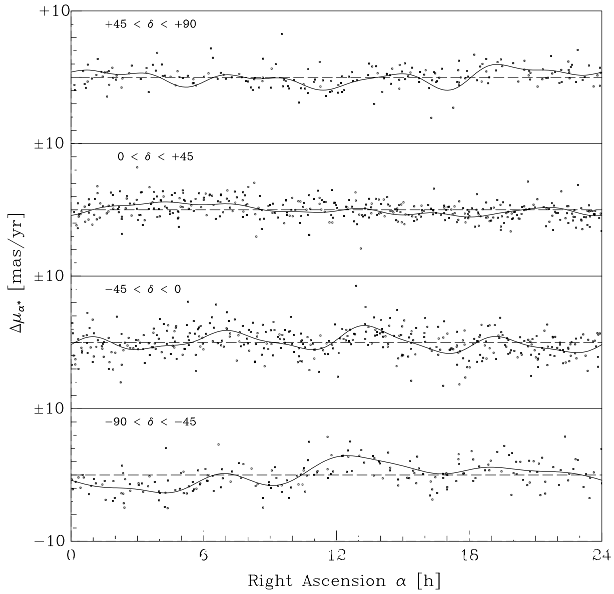

Fig. 3. Systematic differences $\Delta\mu_{\alpha*}$ [mas/yr] between the HIPPARCOS and the FK5 proper motions as a function of the right ascension in four declination zones

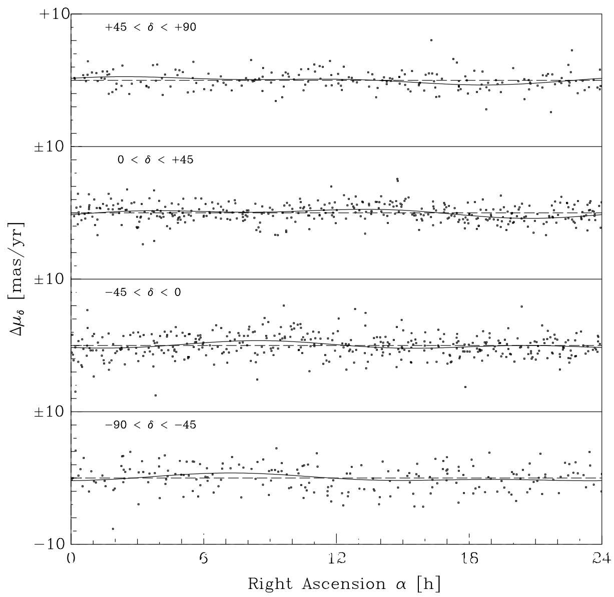

Fig. 4. Systematic differences $\Delta\mu_{\delta}$ [mas/yr] between the HIPPARCOS and the FK5 proper motions as a function of the right ascension in four declination zones

6. Comparison with the systematic relations given by Mignard and Frœschlé

Systematic differences between the FK5 and HIPPARCOS positions and proper motions have been provided by Mignard & Frœschlé (2000) in tabular form, giving these differences in steps of $\sin\delta = 0.2°$ in declination and 20° in α from $\sin\delta = -0.9$ ($\delta \approx -64.16°$) to $\sin\delta = +0.9$ and from $\alpha = 10°$ to $\alpha = 350°$. The global rotation is included in their systematic differences. The values were derived by smoothing the total differences over constant areas of 230 square degrees. Note that Mignard and Frœschlé use the differences FK5–HIPPARCOS which have the opposite sign of our values.

As explained in Sect. 2 we have preferred to compute the systematic differences at the mean epoch of the FK5 where the positions and proper motions are much less correlated than at other epochs. Since Mignard and Frœschlé have made their comparison at the HIPPARCOS epoch 1991.25 no direct comparison of the differences in position is possible. With the aid of the proper motion differences one could, in principle, transform the positional differences to other epochs and make the comparison between the present result and that by Mignard and Frœschlé at a

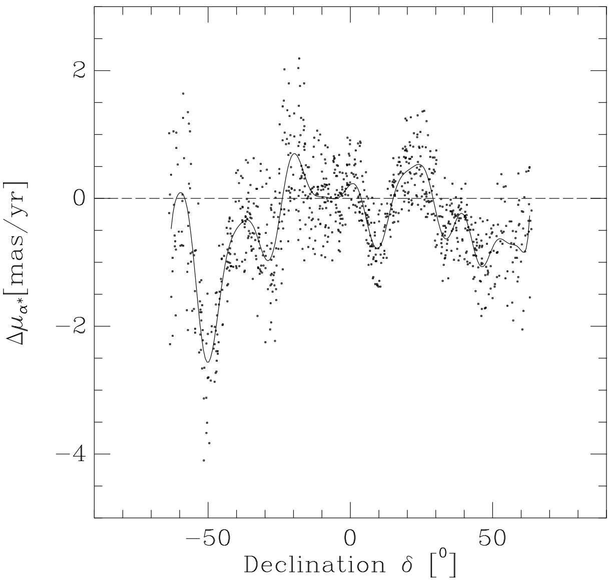

Fig. 5. Systematic differences $\Delta\mu_{\alpha*}$ [mas/yr] versus the declination between the systematic relations HIPPARCOS minus FK5 as determined by M&F and Schwan, respectively

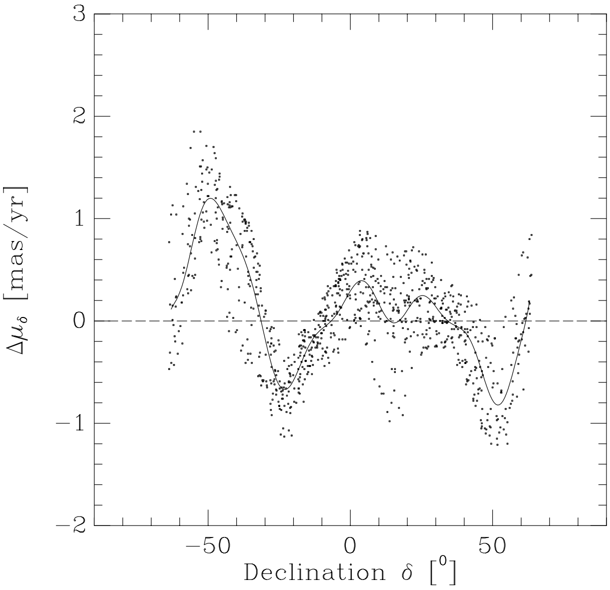

Fig. 6. Systematic differences $\Delta\mu_{\delta}$ [mas/yr] versus the declination between the systematic relations HIPPARCOS minus FK5 as determined by M&F and Schwan, respectively

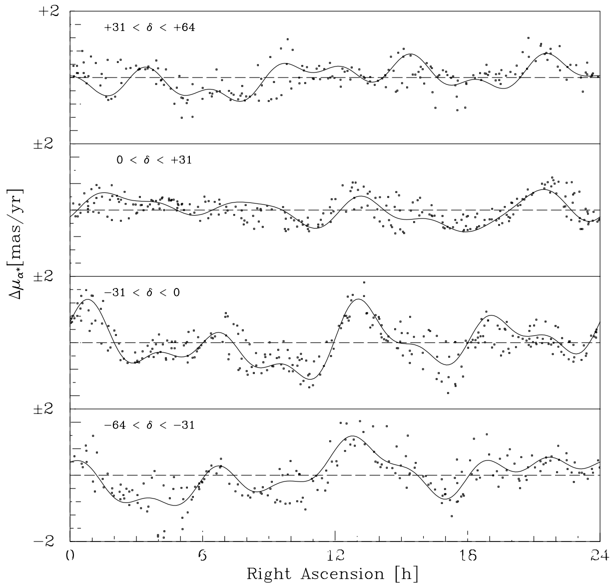

Fig. 7. Systematic differences $\Delta\mu_{\alpha*}$ [mas/yr] versus the right ascension between the systematic relations HIPPARCOS minus FK5 as determined by M&F and Schwan, respectively, given in four declination zones

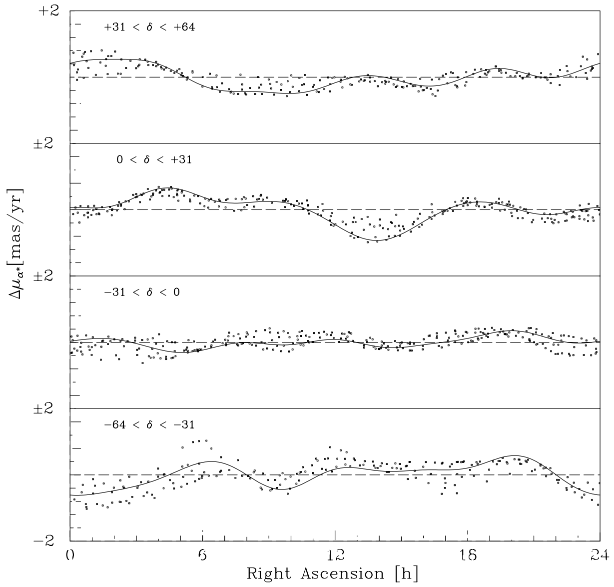

Fig. 8. Systematic differences $\Delta\mu_{\delta}$ [mas/yr] versus the right ascension between the systematic relations HIPPARCOS minus FK5 as determined by M&F and Schwan, respectively, given in four declination zones

common epoch. Since the positions are in general of less interest, and since the conclusions to be drawn in the following are independent from that comparison, we restrict our further analysis to the proper motions. Another difference between both representations is the inclusion of the global rotation into the regional errors in the case of Mignard and Frœschlé, whereas the global rotation has first been determined and eliminated in the present investigation. This procedure was also originally adopted in the HIPPARCOS catalogue (vol. 3, p. 418).

A comparison of tabular values (not shown) computed from the analytical result with those given in the tables by Mignard and Frœschlé reveals larger amplitudes and more fluctuations in the analytical result. This holds in particular for the proper motions in right ascension. The reason for this is the stronger smoothing made by Mignard and Frœschlé. With our significance level of 5 percent we have found significant functions up to a degree of $n = 30$ and $m = 8$. Such rather complicated functions model systematic trends in comparatively small areas on the sky.

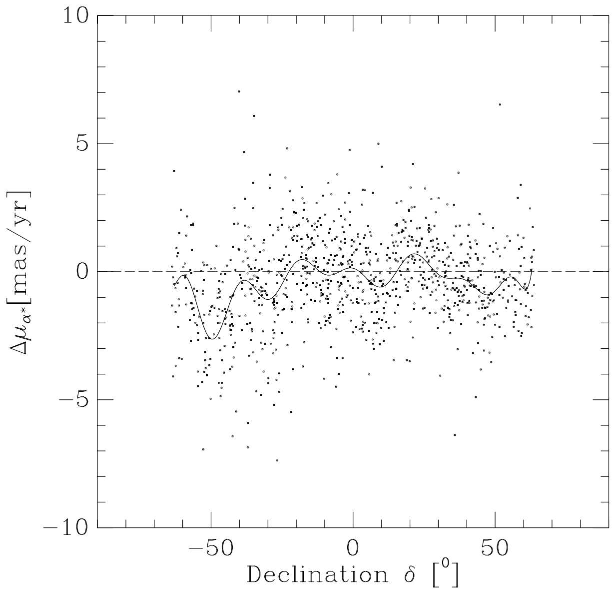

Fig. 9. Systematic differences $\Delta\mu_{\alpha_*}$ [mas/yr] versus the declination between the two systematic relations HIPPARCOS minus FK5 as determined by M&F and Schwan, respectively

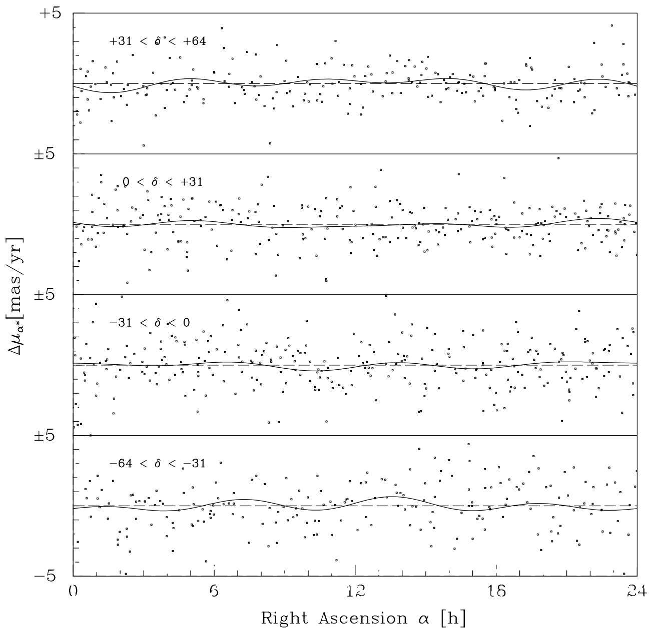

Fig. 10. Systematic differences $\Delta\mu_{\alpha_*}$ [mas/yr] versus the right ascension between the two systematic relations HIPPARCOS minus FK5 as determined by M&F and Schwan, respectively, given in four declination zones

In order to make a more quantitative comparison between both results we have transformed the FK5 to the HIPPARCOS system by applying the systematic corrections according to Mignard and Frœschlé. These corrections were computed by linear, two-dimensional interpolation, using the three tabulated values closest to a star.

By forming the difference between this "CERGA" version and the "ARI" version of the FK5 (also on HIPPARCOS) we get the difference between the two systems with some residual dispersion. From an analysis of these differences with our analytical method we have found systematic deviations between both results, particularly in $\Delta\mu_{\alpha_*}$, as expected already from the visual inspection. In Figs. 5 and 6 we give these differences between the ARI and CERGA proper motion systems as a function of declination, and in Figs. 7 and 8 as a function of right ascension in four declination zones. Each dot is the difference between the proper motions $\Delta\mu_{\alpha_*}$ of an FK5 star transformed to the HIPPARCOS system a) with the analytical series development and b) with the tables by Mignard and Frœschlé.

It is obvious that the differences between both method are largest in $\Delta\mu_{\alpha_*}$, in particular for the region south of $-40°$. As mentioned before we had found no systematic effects in the residuals obtained from our analytical method. It is therefore of interest to see, whether one can directly detect systematic trends in the residuals obtained from the total differences HIPPARCOS–FK5 after applying the systematic corrections according to the tabular values by Mignard and Frœschlé. We have therefore analyzed the residuals HIPPARCOS–FK5 (on HIPPARCOS with Mignard and Frœschlé) with the aid of our analytical method. With a significance level of 5 percent many significant functions were found, in particular for $\Delta\mu_{\alpha_*}$. Most significant were the functions depending exclusively on the declination. A graphical representation of the results is given for $\Delta\mu_{\alpha_*}$ in the Figs. 9 and 10. Figure 9 shows the δ-dependent systematic effects and Fig. 10 shows the α-dependent parts in four declination zones.

With some experience in judging systematic behaviour one can already visually identify the more striking features near the declinations at $-50°, -30°$, $+10°$, $+20°$ and $+50°$. Except from these features there is a remarkable global offset in $\Delta\mu_\alpha$ of -0.33 ± 0.06 [mas/yr]. The α-dependent effects are not so clearly pronounced and they were practically insignificant with the higher significance level of one percent.

We have also made an analytical comparison between HIPPARCOS and the FK5 adopting a significance level of one instead of five percent. Some of the minor features are thus suppressed. The more prominent differences between both results remain, however.

7. Conclusion

We have presented the analytical representation of the systematic relations between the HIPPARCOS and the FK5 systems. These relations were used in the construction of the FK6 for transforming the FK5 catalogue to the HIPPARCOS system. A comparison with the tabular values provided by Mignard and Frœschlé reveals some significant differences, mainly for the proper motions in right ascension in the the southern sky.

It is noted that the differences determined by Mignard and Frœschlé are much smoother than those obtained by the analytical method. Many of the systematic differences between both methods are of lower significance, but a few seem to be substantial.

The systematic differences, in particular for $\Delta\mu_{\alpha*}$, have significant variations over comparatively small intervals in declination (see Fig. 1). It seems to be doubtful, whether tabular values given, for instance, merely at $\delta = -64.2°$ and then at $\delta = -44.4°$ are sufficiently dense for describing such structures.

No tabular values are given by Mignard and Frœschlé north of about $\delta = +64.4°$ and south of $\delta = -64.4°$. It is therefore not clear how to transform from the FK5 to the HIPPARCOS system in these polar regions.

Mignard and Frœschlé have determined the systematic differences at the mean HIPPARCOS epoch, 1991.25. The proper motion differences for nearly half a century are thus included in the positions. The positions and poper motions are therefore highly correlated and the separation of the positional system from the proper motion system is rather uncertain at that epoch. We recommend therefore to compare at an epoch near to the mean FK5 epoch, since the HIPPARCOS system can be supposed to be essentially free of systematic errors and the effect of the FK5 proper motions on the positional differences are small.

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