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IAU resolutions on reference systems and time scales in practice

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Abstract. To be consistent with IAU/IUGG (1991) resolutions ICRS and ITRS should be treated as four–dimensional reference systems with TCB and TCG time scales, respectively, interrelated by a four–dimensional general relativistic transformation. This two–way transformation is given in the form adapted for actual application. The use of TB and TT instead of TCB and TCG, respectively, involves scaling factors complicating the use of this transformation in practice. New IAU B1 (2000) resolution is commented taking in mind some points of possible confusion in its practical application. The problem of the relationship of the theory of reference systems with the parameters of common relevance to astronomy, geodesy and geodynamics is briefly outlined.

Key words. relativity – reference systems – time – astrometry – celestial mechanics

1. Introduction

First general–relativity based IAU resolutions on reference systems and time scales have been formulated in 1990 after intense discussion at IAU Colloquium 127 in Virginia Beach. One year later they were adopted by the General Assembly of IAU in Buenos–Aires in form of IAU Resolution A4 (1991). At the same time they were confirmed by IUGG Resolution 2 (1991). The main practical consequence of these resolutions is that ICRS and ITRS, two basic reference systems (RSs) maintained by the International Earth Rotation Service (IERS), are to be considered as four–dimensional relativistic RSs related by a four–dimensional relativistic transformation (generalized Lorentz transformation) with additional necessary three–dimensional rotation of the spatial axes of ITRS. The time scales of ICRS and ITRS are TCB and TCG, respectively. In fact, these resolutions had never been implemented in complete manner in practical work. ICRS and ITRS are usually treated as three–dimensional spatial RSs referred to TB and TT, respectively. IAU Resolution C7 (1994) did not remove confusion in using TCG and TT. Defining the epoch J2000.0 and the Julian century in terms of TT it recommends to develop new ephemerides in terms of TCB and TCG. IAU Resolution B6 (1997) recommended once again to use barycentric and geocentric RSs as defined by IAU Resolution A4 (1991) avoiding any scaling factors resulting from the replacement of TCB and TCG by TB and TT, respectively. IAU Colloquium 180 held in Washington in March 2000 formulated a much more advanced set of resolutions on RSs adopted in the same year by the 24th IAU General Assembly in form of IAU Resolution B1 (2000). At time being, it seems most likely that these new resolutions result in enlarging the difference between IAU theoretical concepts and their practical implementation in astronomy and geodesy.

This paper aims to consider the IAU resolutions taking in mind their practical use in astronomy and geodesy. In Sect. 2 we demonstrate how to use in practice the two–way relationship between ICRS and ITRS. Section 3 exposes the consequences of using TB and TT instead of TCB and TCG, respectively. In Sect. 4 we comment on the new IAU resolutions indicating the points of possible confusion in applying them in practice. In Sect. 5 we try to relate the theory of relativistic RSs with the numerical estimates of the parameters of common use in astronomy, geodesy, and geodynamics.

All expressions reproduced below are given within the post–Newtonian accuracy neglecting $O(c^{-4})$ and higher order terms in the expansions in powers of c^{-2} (c being the light velocity).

2. Relationships between ICRS and ITRS

For practical purposes it is sufficient to have only ICRS and ITRS. The first system referred to TCB is determined by VLBI observations of quasars. The second one referred to TCG is determined by positions of terrestrial ground stations. Both systems may be fixed experimentally without any relation to fundamental astronomy concepts. But these concepts such as equator, ecliptic, obliquity, Earth's rotation angular velocity, etc., are used in setting the

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mutual relationship between ICRS and ITRS. To have unambiguous relativistic interpretation of fundamental astronomy concepts and constants one has to use several RSs at the barycentric and geocentric level as described in (Brumberg et al. 1996). We will use below the same notation as in this paper, i.e. B – barycentric, G – geocentric, V – VLBI, C – ecliptical, Q – equatorial, D – dynamical, K – kinematical, $^+$ – rotating (C in front of abbreviation CRS means, of course, celestial reference system). At the barycentric level we have three systems BRSV, BRSC and BRSQ with time scale t = TCB and spatial coordinates $x = (x^i)$, $x_{\rm C} = (x^i_{\rm C})$ and $x_{\rm Q} = (x^i_{\rm Q})$, respectively. At the geocentric level each of these systems generates by means of the BRS→GRS transformation two geocentric RSs, dynamically or kinematically nonrotating ones with respect to the generating BRS. In result, there will be six geocentric RSs, i.e. GRSV, GRSC and GRSQ for D and K versions with time scale u = TCG and spatial coordinates $w = (w^i)$, $w_{\rm C} = (w^i_{\rm C})$ and $w_{\rm Q} = (w^i_{\rm Q})$, respectively. If necessary, we will distinguish D and K versions by writing $\underset{q}{w}^i$ with $q = 1$ for version D and $q = 0$ for version K. One more geocentric system is GRS$^+$ rotating with the Earth and having spatial coordinates $y = (y^i)$. By identifying BRSV and GRS$^+$ with ICRS and ITRS, respectively, the problem is to determine all relationships between these systems enabling one to get unambiguous relativistic interpretation of Earth's rotation parameters as well as related astronomical concepts.

At the barycentric level all three RSs are mutually related by the constant rotation matrices

$$x_{\rm C} = P_{\rm C} x\,, \qquad x_{\rm Q} = P_{\rm Q} x\,. \tag{2.1}$$

Very approximately $P_{\rm Q} = E$ and $P_{\rm C} = D_1(\varepsilon)$ where E stands for the unit matrix, ε is the mean obliquity and D_1 is the first elementary rotation matrix. This constant rotation is conserved at the geocentric level in the relationships between V, C and Q GRSs of the same type (with respect to D or K versions). Matrix $P_{\rm C}$ may be determined from the comparison of the planetary theories VSOP87 (Bretagnon & Francou 1988) and VSOP2000 (Moisson 1999) with observations of the major planets within BRSV (ICRS) background. Matrix $P_{\rm Q}$ is to be determined at the geocentric level by comparison of the Earth's rotation theory like SMART97 (Bretagnon et al. 1998) with observations within GRS$^+$ (ITRS) background. The relationship with GRS$^+$ (ITRS) involves the Earth's rotation matrix $\hat{P}(u)$

$$y = \underset{q}{\hat{P}}(u)\,\underset{q}{w}_{\rm C} = \underset{q}{\hat{P}}(u) P_{\rm C}\,\underset{q}{w} = \underset{q}{\hat{P}}(u) P_{\rm C} P_{\rm Q}^{\rm T}\,\underset{q}{w}_{\rm Q}\,. \tag{2.2}$$

The relationships between D and K systems involve the geodetic rotation matrix F

$$\begin{aligned} \underset{0}{w} &= (E - c^{-2}F)\,\underset{1}{w}\,, \\ \underset{0}{w}_{\rm C} &= (E - c^{-2}F_{\rm C})\,\underset{1}{w}_{\rm C}\,, \\ \underset{0}{w}_{\rm Q} &= (E - c^{-2}F_{\rm Q})\,\underset{1}{w}_{\rm Q} \end{aligned} \tag{2.3}$$

and

$$\underset{1}{\hat{P}}(u) = \underset{0}{\hat{P}}(u)(E - c^{-2}F_{\rm C}) \tag{2.4}$$

with

$$F_{\rm C} = P_{\rm C} F P_{\rm C}^{\rm T}\,, \qquad F_{\rm Q} = P_{\rm Q} F P_{\rm Q}^{\rm T}\,. \tag{2.5}$$

Instead of rotation matrix F one may consider its equivalent vector representation

$$F^{ij} = \varepsilon_{ijk} F^k\,, \qquad \varepsilon_{ijk} = \tfrac{1}{2}(i-j)(j-k)(k-i) \tag{2.6}$$

so that the matrix product in (2.3) may be reduced to vector

$$(Fw)^i = F^{ik} w^k = -\varepsilon_{ijk} F^j w^k\,. \tag{2.7}$$

The components of $F^k_{\rm C}$ and $\dot{F}^k_{\rm C}$ have been evaluated in Brumberg et al. (1992) and more precisely in Brumberg & Bretagnon (2000b).

The relationships between barycentric and geocentric RSs are based on BRS→GRS transformation (Brumberg 1991 and references therein)

$$u = t - c^{-2}[A(t) + v^k_{\rm E} r^k_{\rm E}] + \ldots\,, \tag{2.8}$$

$$\begin{aligned} w^i \;=\; & r^i_{\rm E} + c^{-2}\Big\{\big[\tfrac{1}{2} v^i_{\rm E} v^k_{\rm E} + qF^{ik}(t) + D^{ik}(t)\big] r^k_{\rm E} \\ & + D^{ikm}(t) r^k_{\rm E} r^m_{\rm E}\Big\} + \ldots \end{aligned} \tag{2.9}$$

with

$$r^i_{\rm E} = x^i - x^i_{\rm E}(t), \qquad v^i_{\rm E} = \dot{x}^i_{\rm E}(t), \tag{2.10}$$

$$\dot{A}(t) = \tfrac{1}{2} v^2_{\rm E} + \bar{U}_{\rm E}(t, \boldsymbol{x}_{\rm E}) \tag{2.11}$$

and

$$\begin{aligned} D^{ik}(t) &= \delta_{ik} \bar{U}_{\rm E}(t, \boldsymbol{x}_{\rm E}), \\ D^{ijk}(t) &= \tfrac{1}{2}(\delta_{ij} a^k_{\rm E} + \delta_{ik} a^j_{\rm E} - \delta_{jk} a^i_{\rm E}) \end{aligned} \tag{2.12}$$

where $\bar{U}_{\rm E}(t, \boldsymbol{x})$ stands for the Newtonian potential of all solar system bodies excepting the Earth, $x^i_{\rm E}$, $v^i_{\rm E}$ and $a^i_{\rm E}$ being Earth's BRS position, velocity and acceleration, respectively. This transformation is written for V versions of RSs. For C or Q versions one should convert all V quantities to C or Q quantities, respectively.

Let t^* be the BRS instant of time corresponding to event with the GRS coordinates $(u, w^i = 0)$ (Klioner & Voinov 1993). Then u and t^* are related by the time equation

$$u = t^* - c^{-2} A(t^*) + \ldots \tag{2.13}$$

Hence,

$$t - t^* = c^{-2} v^k_{\rm E} w^k + \ldots \tag{2.14}$$

Expanding the right-hand member of (2.9) in the vicinity of t^* one gets the inverse GRS→BRS transformation

$$t = u + c^{-2}[A(u) + v^k_{\rm E} w^k] + \ldots\,, \tag{2.15}$$

$$x^i = w^i + z^i_{\rm E}(u) + c^{-2}\Big[\big(\tfrac{1}{2}v^i_{\rm E}v^k_{\rm E} - qF^{ik} - D^{ik}\big)w^k - D^{ikm}w^k w^m\Big] + \dots, \qquad (2.16)$$

with functions $z^i_{\rm E} = z^i_{\rm E}(u)$ characterizing the motion of the geocentre in terms of TCG and determined by

$$z^i_{\rm E}(u) = x^i_{\rm E}(t^*). \qquad (2.17)$$

In explicit form direct and inverse transformations (2.9) and (2.16) read

$$w^i = r^i_{\rm E} + c^{-2}\Big[\tfrac{1}{2}\boldsymbol{v}_{\rm E}\boldsymbol{r}_{\rm E}v^i_{\rm E} - q\varepsilon_{ijk}F^j r^k_{\rm E} + \bar{U}_{\rm E}(t,\boldsymbol{x}_{\rm E})r^i_{\rm E} + \boldsymbol{a}_{\rm E}\boldsymbol{r}_{\rm E}r^i_{\rm E} - \tfrac{1}{2}\boldsymbol{r}^2_{\rm E}a^i_{\rm E}\Big] + \dots \qquad (2.18)$$

and

$$x^i = w^i + z^i_{\rm E}(u) + c^{-2}\Big[\tfrac{1}{2}\boldsymbol{v}_{\rm E}\boldsymbol{w}v^i_{\rm E} + q\varepsilon_{ijk}F^j w^k - \bar{U}_{\rm E}(t,\boldsymbol{x}_{\rm E})w^i - \boldsymbol{a}_{\rm E}\boldsymbol{w}w^i + \tfrac{1}{2}\boldsymbol{w}^2 a^i_{\rm E}\Big] + \dots, \qquad (2.19)$$

respectively.

IAU Resolution B1.3 (2000) defines GCRS, geocentric celestial RS coinciding in our hierarchy with KGRSV. Therefore, the spatial part of the relationship between ICRS and GCRS is described by Eqs. (2.18) and (2.19) with $q = 0$. The relationship between GCRS and ITRS corresponds to the transformation between $\underset{0}{w}$ and y as indicated in (2.2). Matrix $\underset{0}{\hat{P}}(u)P_{\rm C}$ relating ITRS and GCRS is supposed to be determined from observations. The direct transformation BRS→GRS may be used to convert BRS space–time coordinates of quasars into their GRS space–time coordinates. On the contrary, the inverse transformation GRS→BRS may be used to transform GRS space–time coordinates of terrestrial ground stations into their BRS space–time coordinates.

For any TCB instant t coordinates $x^i_{\rm E}(t)$ of the centre of mass of the Earth may be determined from semi–analytical theories VSOP87 or VSOP2000, or else from numerical ephemerides such as JPL DE. Function $A(t)$ may be found from the solution of the time Eq. (2.11) as given, for example, by Fairhead & Bretagnon (1990). This solution is represented in the form

$$A(t) = A_0 + A_1 t + A_{\rm p}(t), \qquad (2.20)$$

A_0 and A_1 being constants and $A_{\rm p}(t)$ standing for the trigonometric, polynomial or mixed terms with respect to t (without a term linear in t). Very approximately $A_1 \approx \frac{3}{2}(GM_{\rm S}/a_{\rm ES})$, $a_{\rm ES}$ being the mean Earth–Sun distance, so that $c^{-2}A_1 \approx 1.5\ 10^{-8}$. More rigorously $c^{-2}A_1 = L_{\rm C}$, $L_{\rm C}$ being a constant indicated in IAU Resolution A4 (1991). Then ITRS space–time coordinates (t, x^i) are converted by (2.8) and (2.18) (with $q = 0$) into GCRS space–time coordinates (u, w^i).

The inverse transformation is a little bit more complicated. For any given TCG instant u one determines t^* by (2.13) resulting in

$$t^* = (1 + c^{-2}A_1)u + c^{-2}[A_0 + A_{\rm p}(u)] + \dots \qquad (2.21)$$

Then by solving Eq. (2.17) one finds $z^i_{\rm E}(u)$ either numerically, or analytically. One may use the expression

$$z^i_E(u) = x^i_{\rm E}(u) + c^{-2}A(u)v^i_{\rm E}(u) + \dots \qquad (2.22)$$

If $x^i_{\rm E}(t)$ and $A_{\rm p}(t)$ are restricted only by trigonometric terms then to retain such trigonometric form for $z^i_{\rm E}(u)$ one may use instead of (2.22) the trigonometric expansion for $x^i_{\rm E}(t^*)$ with the aid of Bessel functions. In any case, having found $z^i_{\rm E}(u)$ one easily convert by (2.15) and (2.19) (with $q = 0$) GCRS space–time coordinates (u, w^i) into ICRS space–time coordinates (t, x^i).

3. Scaling factors

In spite of the fact that IAU Resolution A4 (1991), IAU Resolution B6 (1997) and IUGG Resolution 2 (1991) imply the use of ICRS and ITRS with TCB and TCG, respectively, without introducing any scaling factors the actual situation is more complex than described in the previous section.

In accordance with IERS Standards (1992) and IERS Conventions (1996) language, ICRS represents mathematically a BRS based on VLBI observations (BRSV in terms of Brumberg et al. 1996). Its approximate metric as given by IAU (1991) resolutions is (up to the sign conversion)

$$\mathrm{d}s^2 = \big[1 - 2c^{-2}U(t,\boldsymbol{x})\big]c^2\mathrm{d}t^2 - \big[1 + 2c^{-2}U(t,\boldsymbol{x})\big]\left(\mathrm{d}{x^1}^2 + \mathrm{d}{x^2}^2 + \mathrm{d}{x^3}^2\right), \quad (3.1)$$

$U(t, \boldsymbol{x})$ being the Newtonian potential of all solar system bodies. Instead of t = TCB and spatial coordinates $\boldsymbol{x}$ JPL DE/LE, IERS Standards (1992) and IERS Conventions (1996) use (rather implicitly)

$$\begin{aligned} \mathrm{TB} &= (1 - L_{\rm B})\,\mathrm{TCB}, \\ (\boldsymbol{x})_{\rm TB} &= (1 - L_{\rm B})\,\boldsymbol{x}, \\ (GM)_{\rm TB} &= (1 - L_{\rm B})\,GM\,, \end{aligned} \qquad (3.2)$$

with $L_{\rm B} \approx 1.550505\ 10^{-8}$ given in IAU (1991) resolutions for the relationship between TCB and TB of JPL DE/LE ephemerides. Actually each version of JPL DE/LE provides its own $L_{\rm B}$. Scaling of spatial coordinates and mass factors is produced to keep invariance of the light velocity and the equations of motion of the solar system bodies under transformation from TCB to TB. This transformation involves also the relationship

$$(\mathrm{d}s^2)_{\rm TB} = (1 - L_{\rm B})^2\mathrm{d}s^2\,, \qquad (3.3)$$

where $(\mathrm{d}s^2)_{\rm TB}$ keeps the same form in terms of TB, $(\boldsymbol{x})_{\rm TB}$ and $(GM)_{\rm TB}$ as expression (3.1) in terms of t, $\boldsymbol{x}$ and GM. Hence, BRS of IAU (1991) resolutions and ICRS differ by introducing $L_{\rm B}$ factor.

From the theoretical point of view, ITRS maintained by IERS is supposed to be a realization of GRS$^+$, a geocentric system rotating with the Earth and resulting from application of Newtonian rotation of the spatial axes of

GRS envisaged by IAU/IUGG (1991) resolutions. The latter system (GRSV in terms of Brumberg et al. 1996) is described by the metric of the form

$$\begin{aligned} ds^2 = & \Big\{1 - 2c^{-2}\big[\hat{U}_E + Q_j w^j + T(u, \boldsymbol{w})\big]\Big\}c^2 du^2 \\ & +2c^{-3}\big[(1-q)\varepsilon_{ijk}\dot{F}^j w^k + \ldots\big] c du dw^i \\ & -\Big\{1 + 2c^{-2}\big[\hat{U}_E + Q_j w^j + T(u, \boldsymbol{w})\big]\Big\}\Big(dw^{1^2} \\ & +dw^{2^2} + dw^{3^2}\Big). \end{aligned} \quad (3.4)$$

Here $\hat{U}_E$ is the GRS geopotential, $T(u, \boldsymbol{w})$ is the tidal potential vanishing at the GRS origin (only these two quantities are indicated in IAU 1991 resolutions), Q_j is the non–geodesic acceleration in the BRS motion of the Earth (caused by the distinction of the actual Earth from the point-mass model), F^j is the geodetic rotation vector of the Earth. Dots stand for the additive c^{-3}-order terms in the mixed components of the metric. For $q = 0$ this metric corresponds to GCRS envisaged by IAU Resolution B1.3 (2000).

Instead of coordinate time u = TCG, spatial coordinates $\boldsymbol{w}$ and mass factors $G\hat{M}$ related to GRS, IERS Standards (1992) use actually

$$\begin{aligned} TT &= (1 - L_G)\,TCG, \\ (\boldsymbol{w})_{TT} &= (1 - L_G)\,\boldsymbol{w}, \\ (G\hat{M})_{TT} &= (1 - L_G)\,(G\hat{M}) \end{aligned} \quad (3.5)$$

with $L_G \approx 6.969291\ 10^{-10}$ as given by IAU (1991) resolutions. In the spirit of IAU (1991) resolutions L_G was geoid–model dependent but it is not true anymore according to IAU Resolution B1.9 (2000) (see the next section). Again, the scaling of the spatial coordinates and mass factors is aimed to keep invariance of the light velocity and the GRS equations of motion (of the Moon or Earth's artificial satellites) under the transformation from TCG to TT. This transformation accompanied by the scaling of spatial coordinates and mass factors involves the invariance of the metric (up to the constant factor)

$$(ds^2)_{TT} = (1 - L_G)^2 ds^2 \quad (3.6)$$

where $(ds^2)_{TT}$ has the same form in terms of TT, $(\boldsymbol{w})_{TT}$ and $(G\hat{M})_{TT}$ as expression (3.4) in terms of u, $\boldsymbol{w}$ and $G\hat{M}$. Both transformations (3.2) and (3.5) are often considered as the change of unit of time from SI unit to TB or TT units, respectively.

Hence, the GRS$^+$ system corresponding to IAU/IUGG (1991) resolutions differs from ITRS as given by IERS Standards (1992) by introducing L_G factor. These L_B and L_G scaling factors should be taken into account when applying the transformations of Sect. 2 to ICRS and GCRS constructed with TB and TT instead of TCB and TCG, respectively.

It seems that the procedures of IERS Conventions (1996) for LLR, SLR and GPS observations just use these scaling factors. But with respect to VLBI observations IERS Conventions (1996) did not stop on that. ITRS realized by a new VLBI formula of Conventions (1996) contains some other scaling factors. First of all, one should decide what kind of distances are demanded for ITRS and then make choice between two basic options:

(1) calculated coordinate distances associated to TCG, or

(2) locally measurable distances in the infinitesimal vicinity of an observer on the surface of the Earth.

The first option corresponds to the GRS metric adopted by IAU/IUGG (1991) resolutions. The second option corresponds to the new VLBI formula of IERS Conventions (1996) although this is not stated explicitly and may be demonstrated only by analysing the new VLBI formula. In principle, one may realize the second option in two different ways. The first one is based on a local transformation of any general–relativity metric to the flat (Minkowski) metric with proper time and proper distances. Indeed, any metric

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu\,, \qquad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad (3.7)$$

with Einstein summation from 0 to 3 for any repeated Greek index and Minkowski tensor $\eta_{\mu\nu}$ ($\eta_{00} = 1$, $\eta_{0i} = 0$, $\eta_{ij} = -\delta_{ij}$, δ_{ij} being the Kronecker symbol) may be locally transformed to the Minkowski metric (see, e.g., Brumberg 1991)

$$ds^2 = c^2 d\tau^2 - dl^2, \qquad dl^2 = \delta_{ik} dx^{(i)} dx^{(k)} \quad (3.8)$$

with elementary proper time

$$c d\tau = \left(1 + \frac{1}{2}h_{00} - \frac{1}{8}h_{00}^2\right) c dt + \left(h_{0i} - \frac{1}{2}h_{00}h_{0i}\right) dx^i \quad (3.9)$$

and elementary proper distances

$$dx^{(i)} = dx^i + \frac{1}{2}(-h_{ik} + h_{0i}h_{0k}) dx^k. \quad (3.10)$$

These formulas are accurate up to the third order with respect to c^{-1} provided that h_{00} and h_{ik} are of the second order and h_{0i} are of the first order.

Now let us apply these formulas to GRS$^+(u, \boldsymbol{y})$. This metric resulted from (3.4) by three-dimensional rotation of the spatial axes is as follows:

$$\begin{aligned} ds^2 = & \Big\{1 - 2c^{-2}\Big[\hat{U}_E^+ + \tfrac{1}{2}(\boldsymbol{\omega}_E \times \boldsymbol{y})^2 + Q_k^+ y^k + \ldots\Big]\Big\} \\ & \times c^2 du^2 - 2c^{-1}(\boldsymbol{\omega}_E \times \boldsymbol{y})^i c du dy^i - \Big\{1 + 2c^{-2}\Big[\hat{U}_E^+ \\ & + Q_k^+ y^k + \ldots\Big]\Big\}\Big(dy^{1^2} + dy^{2^2} + dy^{3^2}\Big). \end{aligned} \quad (3.11)$$

Here $\boldsymbol{\omega}_E$ is the Earth' rotation angular velocity and $\hat{U}_E^+$ and Q_k^+ have the same meaning as $\hat{U}_E$ and Q_k, respectively, but expressed in rotating coordinates. Hence, one has for the infinitesimal intervals of proper time and proper distances

$$c d\tau = (1 - c^{-2} W_E) c du - c^{-1}(\boldsymbol{\omega}_E \times \boldsymbol{y})^i dy^i \quad (3.12)$$

and

$$\begin{aligned} dy^{(i)} &= (1 + c^{-2}\hat{U}_{\rm E}^{+})dy^i \\ &+\tfrac{1}{2}c^{-2}(\boldsymbol{\omega}_{\rm E} \times \boldsymbol{y})^i(\boldsymbol{\omega}_{\rm E} \times \boldsymbol{y})^k dy^k \end{aligned} \quad (3.13)$$

with the potential of the force of gravity

$$W_{\rm E} = \hat{U}_{\rm E}^{+} + \frac{1}{2}(\boldsymbol{\omega}_{\rm E} \times \boldsymbol{y})^2. \quad (3.14)$$

Only the main terms are indicated explicitly in (3.11)–(3.14). Remembering that $L_G = c^{-2}W_0$, W_0 being the value of W_E on the geoid, and neglecting all the terms with the Earth's rotation velocity one may see in (3.12) and (3.13) the factors $1 - L_G$ and $1 + L_G$ used implicitly in the new VLBI model of IERS Conventions (1996). Even if it is good for a local network it cannot be applied successfully for the whole Earth taking into account the variability of $\hat{U}_E$ on the surface of the Earth.

The same VLBI model of IERS Conventions (1996) may be constructed in describing the VLBI procedures in a topocentric reference system (TRS). Indeed, at the spatial origin of any TRS its metric takes the Minkowski form enabling one again to operate with the infinitesimal intervals of proper time and proper distances. In such a way this model was obtained as formula (6.4.37) in (Brumberg 1991) to illustrate the possibility to construct a local network in any topocentric RS.

Hence, GRS^+ envisaged by IAU/IUGG (1991) resolutions and ITRS of VLBI model of IERS Conventions (1996) differ not only by TCG/TT dualism but also by different types of the employed distances (coordinate/proper distances dualism).

The problem of scaling factors has been widely discussed within the IAU Sub–Working Group "Relativity in Celestial Mechanics and Astrometry" during the period of 1995–1997. The final report of this discussion (RCMA 1998) demonstrates that the discrepancy between different existing models of constructing ITRS may achieve 5 mm in the ground–station coordinates.

4. Comments on IAU resolution B1 (2000)

It is evident that IAU/IUGG (1991) resolutions on RSs and time scales still are not completely used in practice. Meanwhile a much more advanced set of resolutions B1 was adopted at the 24th IAU General Assembly (2000). It seems reasonable to comment these new resolutions from the point of view of their actual application.

First of all, one may formulate some evident general comments to be taken into account in considering any IAU resolutions:

(1) Any quantity or a concept mentioned in the IAU resolutions should be clearly defined and well documented in the astronomical literature;
(2) In principle, IAU resolutions are aimed to ensure the use of unambiguous concepts (reference systems, time scales, astronomical constants, etc.) in astronomy and geodesy. The IAU resolutions should not be used for dictating the manner to solve a specific astronomical problem (according to the preference of the authors of resolutions);
(3) It would be reasonable to formulate first at some IAU General Assembly any IAU recommendations only as draft resolutions to be adopted as actual resolutions at the next General Assembly making it possible to discuss and even to test draft resolutions by more wide scientific community during the time span between two consequent assemblies.

From this point of view it is unclear to whom IAU Resolutions B1.3–B1.5 are addressed. They are formulated not too accurately for specialists in relativity (not saying that no resolutions are needed for scientific research at all). On the other hand, they are too complicated to be correctly understood and applied by non–specialists. Actually, they demand the knowledge of DSX papers (Damour et al. 1991–1994) designed for relativity specialists. The comments below are aimed to elucidate and to clarify some points of possible confusion.

Resolution B1.3. Definition of BCRS and GCRS

3.1 Harmonic gauge.

Indeed, the harmonic gauge is "a useful and simplifying gauge for many kinds of applications". However, just in case of the generalized potential w its advantage is doubtful. This function satisfies the wave equation when using harmonic coordinates and Poisson equation when using standard PPN coordinates. One may argue that this property is not valid for higher approximations but Resolution B1.3 describes only the post-Newtonian approximation.

3.2 Integral expressions for potentials

It is not clear what astronomers are supposed to do with these expressions. These expressions may be omitted or, at least, be replaced by the partial differential equations for the potentials (wave equation for w and Poisson equation for w^i).

3.3 TCB→TCG transformation

Equation for $\dot{A}(t)$ is written and should be solved in the post-Newtonian approximation (potential w differs from Newtonian potential by terms $O(c^{-2})$). Equation for $\dot{B}(t)$ is written and should be solved in Newtonian approximation. Therefore, one may replace these two equations just by one equation for $(\dot{A}(t) + c^{-2}\dot{B}(t))$. On the other hand, none of these two versions enables one to use directly the results already obtained in solving the first equation in Newtonian approximation in replacing w by Newtonian potential (needless to say, the second equation changes therewith). To avoid any possible misunderstanding it would be better to mention it in Notes.

3.4 References

There are 5 references. In fact resolution B1.3 is based only on one single reference (Damour et al. 1991–1994).

Moreover, the reader cannot find there the formulas in the form they appear in resolution B1.3.

Resolution B1.4. Post–Newtonian Potential Coefficients

4.1 Potential coefficients

The resolution states "that physically meaningful post-Newtonian potential coefficients can be derived from the literature" and recommends to expand the post-Newtonian Earth's potential in GCRS to "sufficient accuracy" (sufficient for what?) just as Newtonian potential. Let us remember again that the post-Newtonian potential satisfies the wave equation in harmonic coordinates and Poisson equation within the standard PPN gauge so that "physically meaningful coefficients" depend, of course, on the coordinate gauge and the Newtonian form of the expansion given in the Resolution B1.4 is more adequate to the standard PPN gauge. This recommendation seems to be of little help for astronomical practice.

4.2 Vector potential representation

Resolution B1.4 recommends also to express the vector potential W^i in terms of the total angular momentum vector $\boldsymbol{S}$ in the form

$$W^i = -\frac{G}{2}\frac{(\boldsymbol{x}\times\boldsymbol{S})^i}{r^3}\,. \qquad (4.1)$$

There is no indication on the approximation level of this expression mentioning only that it leads to the well–known Lense–Thirring effect. In fact this approximate formula is not sufficient for the complete study of the Lense–Thirring problem (motion of a test particle in the field of the rotating spherical body). The more accurate value of W^i resulting from the DSX theory (Damour et al. 1991–1994) includes not only spin moments S^m but also the time–derivatives of the mass moments M_{ij}. This approximate expression is of the form (Klioner 2000)

$$W^i = -\frac{1}{2}G\left(\frac{1}{r}\right)_{,j}(\dot{M}_{ij} - \varepsilon_{ijm}S^m) + \ldots \qquad (4.2)$$

with Levi–Civita symbol ε_{ijm} as in (2.6). Only the last term with spin S^m is indicated in Resolution B1.4 . In the approximation of the rigid–body rotation one has

$$S^m = \omega^m I^{ss} - I^{ms}\omega^s\,, \qquad M_{ij} = I^{ij} - \tfrac{1}{3}\delta_{ij}I^{ss} \qquad (4.3)$$

with standard (Fock) inertia moments I^{ij}. By substituting the expressions for the spin and time–derivatives

$$\dot{M}_{ij} = (\varepsilon_{ims}I^{js} + \varepsilon_{jms}I^{is})\omega^m \qquad (\dot{I}^{ss} = 0) \qquad (4.4)$$

one gets

$$W^i = -\frac{1}{2}G\left(\frac{1}{r}\right)_{,j}\omega^m(\varepsilon_{ims}I^{js} + \varepsilon_{jms}I^{is} + \varepsilon_{ijs}I^{ms} - \varepsilon_{ijm}I^{ss}). \qquad (4.5)$$

The first two terms in parentheses come from the mass moments whereas the last two terms come from the spin moments. All terms are of the same analytical order of smallness and they all are needed for the complete study of the Lense–Thirring problem. In virtue of the identity

$$\varepsilon_{ijs}I^{ms} + \varepsilon_{mis}I^{js} + \varepsilon_{jms}I^{is} = \varepsilon_{ijm}I^{ss} \qquad (4.6)$$

the last three terms may be reduced to the first one resulting in the well–known expression for the vector potential given, for example, by Fock (1955)

$$W^i = -G\left(\frac{1}{r}\right)_{,j}\omega^m\varepsilon_{ims}I^{js}\,. \qquad (4.7)$$

It is absolutely not evident why this expression in terms of angular velocity and inertia moments should be abandoned in favour of mass-moment and spin–moment representation. Each representation has its own merits and disadvantages. There is no need to specify the form of the vector potential. It is quite sufficient to indicate the Poisson equation related to it. Let us add that formula (4.2) is given explicitly in (Kopejkin 1991) in the equivalent form

$$W^i = -\frac{1}{2}G\left(\frac{1}{r}\right)_{,j}(\dot{I}_{ij} - \varepsilon_{ijm}S^m) + \ldots \qquad (4.8)$$

resulted from the detailed expansions derived in (Kopejkin 1988) for the functions entering into the coefficients of the barycentric and geocenric metric forms.

Resolution B1.4 as a whole cannot be considered as necessary for practical application of relativity in positional astronomy.

Resolution B1.5. Extended Relativistic Framework for Time and Frequency Comparisons and Realization of Coordinate Times in the Solar System

5.1 B1.3/B1.5 comparison

Resolutions B1.3 and B1.5 are in some distinction from each other because the generalized potential w of Resolution B1.3 is replaced in Resolution B1.5 by

$$w = w_0 + w_{\mathrm{L}} - c^{-2}\Delta\,, \qquad (4.9)$$

where $w_0 + w_{\mathrm{L}}$ represents Newtonian potential (separated into spherical and non–spherical parts) and Δ describes the relativity contribution to it. Since Resolution B1.5 is a "practical" resolution (in distinction to "theoretical" Resolution B1.3) it may represent difficulties for non–specialists in choosing one of these two equal possibilities.

5.2 Analytical or numerical order of smallness

In contrast to the formulations of Resolution B1.3 valid within the post-Newtonian approximation the formulations of Resolution B1.5 (in particular, TCB→TCG transformation) are given without some post-Newtonian terms evaluated as being negligible. It is better to reproduce all post-Newtonian terms indicating explicitly the terms being negligible in some cases.

5.3 Angular momentum (spin) or angular velocity?

The vector potential is expressed here within some numerical accuracy in terms of angular momentum vector (spin). In more detail this question has been discussed in analysing Resolution B1.4. In present astronomical practice (IERS activity) one deals with Earth's rotation parameters including Earth's angular velocity. The calculated and measurable quantities of this problem are related to the rotation matrix connecting ITRS and GCRS (precession, nutation, diurnal rotation, polar motion). Just these parameters are reproduced in IERS documents and in the system of astronomical constants. An alternative approach is based on the use of spin but in this case one should indicate where one can find its observed value, its relation to common Earth's rotation parameters, etc. Now one may find the value of the Earth's angular momentum but rather as a derived constant (based on the value of the Earth's angular velocity and some Earth's model). Perhaps, in the future the Earth's rotation problem will be investigated as purely physical problem with no relation to astronomical reference systems but now it is too early to replace Earth's angular velocity by the spin of the Earth.

5.4 Inadequate notes statement

The statement in the Notes that "the present Recommendation provides an extension of the IAU 1991 recommendations at the full post-Newtonian level" is not correct due to the neglection of some post-Newtonian terms.
Incompatibility of Resolution B1.3 with Resolutions B1.7 (Definition of Celestial Intermediate Pole) and B1.8 (Definition and Use of Celestial and Terrestrial Ephemeris Origin).

In accordance with the spirit of Resolution B1.3 BCRS is just ICRS and GCRS results from it by means of the direct four–dimensional transformation BCRS→GCRS. On the other hand, "non–relativistic" Resolutions B1.7 and B1.8 determine implicitly GCRS by its relationship to ITRS. Two versions of GCRS constructed by these two ways may not be identical. One may apply to this ITRS–induced GCRS the inverse GCRS→BCRS transformation of Sect. 2 to check if one returns back to ICRS. In fact, this situation might be clarified only after the corresponding IERS (2000) procedures will be available.

Resolution B1.8

The usefulness of this resolution stating a new longitude origin (NRO, non–rotating origin) for GCRS and ITRS is not so evident even within the Newtonian framework (see Fukushima 2000). ICRS and GCRS as four–dimensional relativistic RSs have advantage of being independent of any notion of equator. Introduction of NRO within the general–relativity framework demands again the re–construction of some Newtonian fundamental astronomy concepts including the notions of fixed equator, moving equator, etc. Without such re–construction the use of NRO will involve some additional relativistic ambiguities.

Resolution B1.9 (Re–definition of Terrestrial Time TT)

Actually, this resolution introduces L_G just as a defining constant ($L_{\mathrm{G}} = 6.969290134\ 10^{-10}$) and, hence, TAI cannot be regarded anymore as physical realization of TT. One may argue if TT is still necessary at all. Indeed, theoretically it is possible to derive from the readings of atomic clocks distributed anywhere on the Earth's surface and in its vicinity just TCG based on the equation relating proper time of an observer with TCG (Brumberg 1992). But just as Keplerian motion and Newtonian potential are still of great benefit for astronomy it might be too premature to forget the notion of geoid in geodesy as implied by this resolution.

5. Reference systems and astronomical constants

The preliminary analysis of the system of astronomical constants within the framework of the theory of relativistic RSs was given in (Brumberg et al. 1996). The main idea is to specify for any given constant a "proper" RS, i.e. the reference system adequate for the determination procedure of the constant under consideration, and to indicate the corresponding realization of such RS (scaling factors for the first instance). In such a way one may avoid relativistic ambiguities when treating different astronomical constants.

At the barycentric level using the BRS equations of the solar system bodies one can obtain the value of $GM_{\mathrm{S}} = GM_{\mathrm{sun}}$ and dimensionless ratios M_{S}/M_i for all other bodies. Theoretically, the GRS value $G\hat{M}_{\mathrm{E}}$ of the geocentric constant differs from its BRS value GM_{E} but this distinction may be presently ignored ($M_{\mathrm{E}} = \hat{M}_{\mathrm{E}}$). On the other hand, due to the scaling factors one often uses the value $(GM_{\mathrm{E}})_{\mathrm{TB}}$ in the BRS equations and the value $(GM_{\mathrm{E}})_{\mathrm{TT}}$ in the GRS equations. These two values are different in accordance with (3.2) and (3.5).

The constants characterizing the figure of the Earth and its gravitational field as reproduced in (Groten 2000) are supposed to be given in ITRS. One should strongly distinguish between its TCG and TT realizations. In any case all temporal variations of the terrestrial quantities should be described in TCG (but not in TCB).

The problem of Earth's rotation parameters is of particular interest. The theory of the Earth's rotation such as SMART97 (Bretagnon et al. 1998) is supposed to be constructed in DGRSC. The kinematical relativistic differences between Euler angles relating ITRS with DGRSC and KGRSC, respectively, are estimated in (Brumberg & Bretagnon 2000a). The Earth's rotation angular velocity should be always given with indication of the corresponding RS (for astrometric purposes one should have this value referred to GCRS = KGRSV).

6. Conclusion

Nobody can deny that if general relativity became nowadays a working theory to produce high–accurate theories of motion of celestial bodies and to analyse high–precision observations. At the same time much remains to be done to avoid confusion and ambiguities in applying general relativity in practical astronomy and geodesy. One of the effective tools in this may is provided by the theory of reference systems as recommended by IAU/IUGG.

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