The geometry of the universe from high resolution VLBI data of AGN shocks

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Abstract. We propose to use the linear diameters of the shocks in AGN jets as standard rods for estimating the geometry of the Universe. The unique feature of shocks is that we can directly estimate their linear diameters from total flux density monitoring data and light travel time arguments. We demonstrate this method by using a small sample of 14 22 GHz VLBI observations. The accuracy of the derived values ($q_0 \approx 0, \Omega_m \approx 0$) compares favorably with traditional methods using much larger samples.

Key words. cosmology: observations – galaxies: jets – galaxies: active

1. Introduction

In principle, the geometry of the Universe can be determined by measuring the apparent size of a constant linear size object at different redshifts. In an Euclidean universe the angular size of such an object decreases in direct proportion to the distance. However, in four-dimensional cosmological models based on the Friedmann-Robertson-Walker geometry, the angular diameter may have a minimum near $z = 1$ and can increase at higher redshifts. The exact behavior of the $\theta - z$ relation depends on the cosmological parameters, so it can therefore be used to determine the geometry of the universe.

Unfortunately, there are severe difficulties in finding a good “standard rod” which is not biased by cosmic epoch. All proposed objects, such as galaxy clusters or double radio sources, have similar diameters only on the average, and they evolve significantly with the cosmological epoch. In addition the results are confused by severe selection effects.

In short, the problem with all proposed standard rods is that we cannot measure the true linear size of an individual object at cosmological distances.

2. Traditional methods

Perhaps the best example of the problems in radio “standard rods” is the angular size – redshift relation of double radio sources (Kellermann 1972). Instead of having the expected signature of standard relativistic cosmology, a flattening of the angular size $\theta$ around $z \approx 1$, $\theta$ falls off monotonically. This is a clear evidence of evolution in the intrinsic arcsecond-scale overall sizes of radio sources with cosmic epoch (Nilsson et al. 1993) which effectively masks the signature of cosmological effects. To some extent it is possible to separate the effects of source evolution (e.g., Buchalter et al. 1998, but the cosmological parameters are only loosely constrained even when large samples are used.

In recent years many studies have been published on $\theta - z$ relation of milliarcsecond structures in AGN cores (Kellermann 1993; Gurvits 1994; Stelmach 1994; Stepanas & Saha 1995; Wilkinson et al. 1997; Gurvits et al. 1999). While the angular size of the compact radio sources seemed to show the expected turnover, there has been criticism of possible biases in these samples.

In these studies the standard rod has been defined as the jet length from peak flux down to 2% (Kellermann 1993) or 1% (Wilkinson et al. 1997) contour or as a size estimate from a single component model fit to uv-data (Gurvits 1994). The fact that the jet length, observed using these methods, can change rapidly due to moving shocks in the jet does not necessarily cause a bias, only random errors if it is assumed that on average all sources produce similar moving shocks.

Sources at different redshifts are observed at different intrinsic wavelengths because $\lambda_{\text{obs}} = \lambda_{\text{int}}(1 + z)$. This effect may bias the sizes of high-redshift sources downwards. Also if there is a correlation between intrinsic luminosity and jet size, $\theta - z$ relation will be biased. Because samples are flux-limited, Doppler-boosted sources at high redshifts will be favored and so the jets will be pointed closer to line of sight and appear smaller. The evolution in the intrinsic sizes of the sources with cosmic epoch is smaller because of small size and shorter lifetimes of jets in compact sources (Kellermann 1993), but even mild size evolution can...
either remove or produce a minimum in the $\theta - z$ diagram (Dabrowski et al. 1995; Krauss et al. 1993).

These studies have given the thus far best estimates of $q_0$, but the accuracy is hardly sufficient to differentiate between closed and open universes. Large samples are necessary both for minimizing the random errors and for trying to eliminate the various biases and selection effects. In addition, there are severe methodological problems in analyzing binned data. The most recent, and probably the most reliable, result is by Gurvits et al. (1999), who derived $q_0 = 0.21 \pm 0.30$ (assuming $\Lambda = 0$) after a careful analysis of 330 compact radio sources.

3. A new standard rod: Normalized diameters of AGN shocks

Instead of using the whole milliarcsecond structures as standard rods, we propose to use the sizes of the individual shocks propagating along the jets.

As standard rods the VLBI shocks have several advantages over the previously used alternatives, although they do share many of the biases and selection effects. The shock size can be, and usually is, a function of frequency and therefore a function of redshift. The sizes can vary with cosmological epoch and physical environment of the AGN, and are presumably dependent on the AGN luminosity. However, some other familiar sources of error and bias are absent.

The sources in a flux limited sample are biased to have high Doppler boosting factors and thus small viewing angles. While this causes bias when using whole structures (Dabrowski et al. 1995), it works against the orientation bias when observing shock sizes. Because of relativistic aberration, the fast-moving shocks are always seen from the side. The quantity measured from the VLBI maps is therefore the transverse width of the shock. Furthermore, because the jet opening angles are small (Oppenheimer et al. 1994), the transverse size of an individual shock is nearly constant with time. The VLBI shock size may therefore be a better standard rod than the previously used total jet length. Large samples are still necessary for eliminating the various biases, selection effects and cosmological evolution.

However, the key advantage in using the shock sizes is that we have an independent method for estimating their true linear diameters. They are not just standard rods on the average; instead, each individual shock can be used as a calibrated rod.

A new shock becomes detectable both as an emerging VLBI component and as a flare in total flux density. Each flare has a characteristic variability timescale $\tau_{\text{var}}$ which can be estimated from flux monitoring data (Valtaoja et al. 1999). The transverse linear size $L$ of the shock component is proportional to the light travel time across the emitting region filling the jet. We therefore have

$$L = K \cdot c \cdot \tau_{\text{int}},$$

where $K$ is an unknown scaling factor that depends on the details of shock geometry and $\tau_{\text{int}}$ is the true intrinsic variability timescale, corrected for redshift and Doppler boosting. One can further define the variability angular size as $\theta_{\text{var}} = L/D_a$, where $D_a$ is the angular distance of the source. Since the derived $\theta_{\text{var}}$ depends on the geometry of the Universe while the directly observable $\theta_{\text{VLBI}}$ does not, a comparison of the two values for the same source can be used to reveal the geometry. This is the essence of our proposed new method: to use the relation $\theta_{\text{VLBI}}/\tau_{\text{int}}$ versus $z$.

In order to transfer the observed variability timescale $\tau_{\text{obs}}$ into the source frame of reference $\tau_{\text{int}}$, we must estimate the Doppler boosting factor $D_a$ of the source because

$$\tau_{\text{int}} = \frac{D}{1+z} \tau_{\text{obs}}.$$  

The traditional method for estimating the Doppler boosting factor is to use synchrotron-self-Compton arguments (e.g. Guerra & Daly 1997); however, as we have recently demonstrated, such values are highly unreliable and much better ones can be derived simply using total flux density variations (Lähteenmäki & Valtaoja 1999). Virtually all major total flux density outbursts in AGN have associated variability brightness temperatures far in excess of the equipartition limit $T_{b, \text{lim}}$ (Readhead 1994; Lähteenmäki et al. 1999), indicating significant Doppler boosting. The variability Doppler boosting $D_{a, \text{var}}$ is given by

$$D_{a, \text{var}} = \left( \frac{T_{\text{source}}}{T_{b, \text{lim}}} \right)^{1/3}. \quad (3)$$

Unfortunately $T_{\text{source}}/T_{b, \text{lim}}$ depends on $D_a$ and $z$:

$$T_{b, \text{var}} \propto \theta_{\text{var}}^{-2} (1+z)^3 \alpha (1+z)^3 D_a^2,$$

so we have to correct the original $\theta - z$ relation because the Doppler boosting factor derived from variability depends on the angular distance:

$$D_{a, \text{var}} \propto D_a^{2/3} (1+z). \quad (5)$$

The net result is that the “normalized rod length” $\theta_{\text{VLBI}}/\tau_{\text{int}}$ has a weaker dependence on the angular distance, $D_a^{-1/3}$ instead of the usual $\alpha \propto D_a^{-1}$.

4. Testing the new method

As a demonstration of the method we calculated the normalization $\theta_{\text{VLBI}}/\tau_{\text{int}}$ (mas/light-day) for 14 shocks from Bloom et al. (1999) and Wiik et al. (1998). From the two 22 GHz global VLBI surveys we first picked the sources also monitored at 22 GHz in Metsähovi (Teräsranta et al. 1998), and then selected all the 14 shocks (in 13 different sources) which were resolved in the VLBI maps and could also be identified with corresponding flares in the total flux. The required Doppler boosting factors were taken from Lähteenmäki & Valtaoja (1999). In accordance with current cosmological thinking, the normalized shock sizes
were fitted to two models, one without and one with a cosmological constant term $\Lambda$. Alternative cosmologies, e.g., steady or quasi-steady state models or models with $\Omega \neq 1$ may of course be fitted to the data. However, with our present small data set we wish just to demonstrate the new method, not to attempt one more exploration of the whole cosmological parameter space.

The first model assumes $\Lambda = 0$, and so $q_0 = \Omega_m/2$ (Eq. (6)). This model has been used in previous studies because it can be expressed in a simple closed form.

$$\frac{\theta_{\text{VLBI}}}{\tau_{\text{int}}} = \frac{KH_0}{2^{2/3}} \cdot \frac{(1+z)^2}{(1+z-\sqrt{1+z})^{2/3}} \cdot \left[\frac{q_0^2}{q_0 z + (q_0 - 1)(\sqrt{2q_0 z + 1} - 1)}\right]^{1/3}.$$  \hspace{1cm} (6)

When the cosmological constant is not assumed to be zero ($\Lambda \neq 0, k = 0$), but $\Omega = \Omega_m + \Omega_A = 1$, to keep the amount of free parameters constant, it is not possible to express the model in simple analytical form. The integral in Eq. (7) must be evaluated numerically in the fitting algorithm.

$$\frac{\theta_{\text{VLBI}}}{\tau_{\text{int}}} = \frac{KH_0}{2^{2/3}} \cdot \frac{(1+z)^2}{(1+z-\sqrt{1+z})^{2/3}} \cdot \left[\int_1^{1+z} \frac{d\mu}{\sqrt{\Omega_m(\mu^3 - 1) + 1}}\right]^{-1/3}.$$  \hspace{1cm} (7)

To enable numerical integration in the cost function, differential evolution algorithm DE (Storn & Price 1997) was used instead of the more traditional gradient-expansion (Levenberg-Marquardt) method. DE-algorithm alleviated also the problems caused by the constrained regions ($q_0, \Omega_m \geq 0$). This optimization method is also robust in the sense that it does not have a single starting point but a population of randomly selected points around the starting value.

We tried to use the bootstrap algorithm to estimate errors in the fitted parameters, but this lead to unrealistically low error estimates because the distributions are cut by the constrained regions. By assuming equal standard deviation in the datapoints, and using the standard deviation of the residual, an estimate for the one sigma error of $q_0$ is $\pm 0.1$ for the model 6. However, such an assumption may not be very good, leading to an underestimated error.

To study the behavior of $\chi^2(q_0, \Omega_m)$, we made fits with $2^{-2/3}KH_0$ as a free parameter and $q_0$ or $\Omega_m$ set to fixed discrete values. The normalized data with two values of $q_0$ and $\Omega_m$ are plotted in Figs. 1 and 2. Plots of the (not reduced) $\chi^2$, Fig. 3, confirm that on the basis of our small data set, the models favor small values of $q_0$ and $\Omega_m$, perhaps model in Eq. (7) is giving a better fit.

Unfortunately, it is not possible to estimate the true error of each data point accurately. The intrinsic variability timescale $\tau_{\text{int}}$ (and the variability Doppler boosting factor $D$) is calculated using just two parameters, the observed variability timescale and the maximum flux of the total flux density flare. These can be derived with good accuracy from exponential model fits to the observed total flux density variations (Valtaoja et al. 1999). By comparing different methods for estimating the amount of Doppler boosting, Lähteenmäki & Valtaoja (1999) argued that the errors in $D_{\text{var}}$-values (and, in consequence, also in $\tau_{\text{int}}$) are very small in comparison with the typical factor of 2 accuracy in SSC-derived values. The main error in the normalized data points thus comes from $\theta_{\text{VLBI}}$. The true errors in the VLBI angular sizes are difficult to estimate, and the standard practice seems to be not to present them at all. However, Bloom et al. state that their size estimates, which we have used here, have accuracies between 13% and “several times larger”. To give at least some indication of the accuracy in the normalized datapoints we show in Fig. 2 two one sigma error bars, the larger corresponding to a 50% error in $\theta_{\text{VLBI}}$ and a 20% error in $\tau_{\text{int}}$, and the smaller to 13% and 20%, respectively. We note that the errors in the individual datapoints are comparable to the errors in the binned data of the traditional methods (e.g., Fig. 5 in Gurvits et al. 1999).

5. Conclusions

The angular sizes of the AGN shocks are generally of the order of 0.1 mas or even less. The VLBI components must also be identified with the corresponding total flux density flaring events. This requires millimeter VLBI data as

Fig. 1. Normalized datapoints and fitted models with $\Lambda = 0$ (Eq. (6)). $q_0$ was fixed and best fit was found with $2^{-2/3}KH_0$ as the free parameter. The two error bars illustrate the range of one sigma errors in the normalized angular sizes (see text for a detailed explanation).
Fig. 2. Normalized datapoints and fitted models with $\Lambda \neq 0$ (Eq. (7)). $\Omega_m$ was fixed and best fit was found with $2^{-2/3}KH_0$ as the free parameter.

Fig. 3. $\chi^2$ as a function of $\Omega_m$ and $q_0$. The fits have been made with $2^{-2/3}KH_0$ as the free parameter for each $\chi^2$ point.

well as simultaneous flux density monitoring at the VLBI observing wavelengths. Such data is only now starting to become available (cf. Valtaoja et al. 1999; Lähteenmäki et al. 1999). In this paper we have used one of the first published 22 GHz global VLBI samples (Bloom et al. 1999) as well as our own similar data (Wiik et al. 1998 and in preparation) to demonstrate the feasibility of using the normalized sizes of AGN shocks as standard rods.

By measuring both the angular and the linear sizes of our standard rods, we avoid most of the problems inherent in previous approaches. The main remaining source of uncertainty is the geometrical scaling factor $K$ (Eq. (1)), which plausibly could be dependent on source properties, for example if luminous AGN have different shock shapes than the weaker ones. The price to be paid is that we must be able to determine the Doppler boosting factors $D$, and that the resulting $\theta_{VLBI}/\theta_{int}$ vs. $z$-dependence is weaker than in the usual case if variability Doppler boosting factors are used. This can be avoided if one uses boosting factors derived by other methods; however, such values tend to be so inaccurate that the method does not work any more (cf. Lähteenmäki & Valtaoja 1999).

Using a very small set of test data, we find that the best fit is given by $q_0 \simeq 0, \Omega_m \simeq 0$. In general, models with a dominant cosmological constant are favored, if one assumes that $\Omega = \Omega_m + \Omega_\Lambda = 1$. Although our analysis is meant to be mainly a demonstration of our new method, not a new determination of $q_0$ or $\Omega_m$, we note that the accuracy obtainable using just a handful of normalized shock sizes and unbinned data is comparable to that of traditional methods with hundreds of sources. The accuracy can be further improved with careful measurements of the shock sizes and their errors. With constantly growing amounts of millimeter VLBI data becoming available, AGN shocks may prove to be the long sought after accurate and useful standard rods.

References
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