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On the atmospheric fragmentation of small asteroids

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Abstract. It is known, from observational data recorded from airbursts, that small asteroids breakup at dynamical pressures lower than their mechanical strength. This means that actual theoretical models are inconsistent with observations. In this paper, we present a detailed discussion about data recorded from airbursts and about several theoretical models. We extend and improve a theory previously outlined for the fragmentation of small asteroids in the Earth atmosphere. The new condition for fragmentation is given by the shock wave–turbulence interaction, which results in sudden outburst of the dynamical pressure.

Key words. minor planets, asteroids – plasmas – shock waves

1. Introduction

Collisions between cosmic bodies are divided into two regimes: gravity– and strength–dominated regime. But the transition from one regime to the other is not well known (see, for example, Fig. 5 in Durda et al. 1998). In the case of collision with Earth, the presence of the atmosphere makes things more difficult. Observations of airburst of small asteroids (up to tens of metres) show that the fragmentation occurs when the dynamical pressure is lower than the mechanical strength and this conundrum has not a satisfactory explanation yet (see Ceplecha 1996b; Foschini 2000). It is worth noting that also the fracture itself has still many unknown features (for a review, see Fineberg & Marder 1999).

Studies on the fragmentation of small asteroids have also a great importance in impact hazard. Although the damage caused by Tunguska–like objects can be defined as "local", it is not negligible. The Tunguska event of 30 June 1908 resulted in the devastation of an area of 2150 ± 25 km^2 and the destruction of more than 80 million trees (for a review, see Trayner 1997; Vasilyev 1998). Still today there is a wide debate all over the world about the nature of the cosmic body which caused that disaster. Just on July 1999 an Italian scientific expedition, *Tunguska99*, went to Siberia to collect data and samples (Longo et al. 1999).

In this article, we discuss firstly the current models on atmospheric fragmentation of small asteroids and inconsistencies with observations (Sect. 2). In Sect. 3, we deal with the question of strength and aerodynamic load. In Sect. 4, we extend the approach previously outlined in Foschini (1999b), thereafter called Paper I. We study the hypersonic flow around a small asteroid, with particular reference to the definition of the pressure crushing on a cosmic body (Sect. 5). Therefore (Sect. 6), it is possible to obtain the condition for fragmentation under steady state conditions, that was already outlined in the Paper I. The Sect. 7 deals with the turbulence and massive ablation, putting the basis for the condition of fragmentation under unsteady conditions (Sect. 8). Some numerical examples conclude the paper (Sect. 9).

2. Problems with current models

Present models consider that the fragmentation begins when the condition:

$$\rho_\infty V^2 = S \tag{1}$$

is satisfied. In Eq. (1), ρ_∞ is the density of undisturbed stream, V is the speed of the body, S is the material mechanical strength. The term $\rho_\infty V^2$ is referred as the dynamical pressure in the front of the cosmic body.

However, observations of very bright bolides prove that large meteoroids or small asteroids breakup at dynamical pressures lower than their mechanical strength. Today there is still no explanation for this conundrum. This is a scientific problem of great interest, but it is also of paramount importance, because it allows us to know whether or not an asteroid might reach the Earth surface. In addition to this, the atmospheric breakup has also effect on the crater field formation (Passey & Melosh 1980) or on the area devastated by the airblast.

The interaction of a cosmic body in the Earth atmosphere in the strength–dominated regime can be divided into two parts, according to the body dimensions. For millimetre– to metre–sized bodies (meteoroids), the most

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Table 1. Meteoroid strength category. After Ceplecha et al. (1993)

Category	Range of $\rho_\infty V^2$ [MPa]	Mean $\rho_\infty V^2$ [MPa]
a	$\rho_\infty V^2 < 0.14$	0.08
b	$0.14 \leq \rho_\infty V^2 < 0.39$	0.25
c	$0.39 \leq \rho_\infty V^2 < 0.67$	0.53
d	$0.67 \leq \rho_\infty V^2 < 0.97$	0.80
e	$0.97 \leq \rho_\infty V^2 < 1.20$	1.10

useful theoretical model is the gross–fragmentation model developed by Ceplecha et al. (1993) and Ceplecha (1999). In this model, there are two basic fragmentation phenomena: *continuous fragmentation*, which is the main process of the meteoroid ablation, and *sudden fragmentation* or the discrete fragmentation at a certain point.

For small asteroids another model is used, where the ablation is contained in the form of explosive fragmentation, while at high atmospheric heights it is considered negligible. Several models have been developed: Baldwin & Shaeffer (1971), Grigoryan (1979), Chyba et al. (1993), Hills & Goda (1993), Lyne et al. (1996). A comparative study on models by Grigoryan, Hills & Goda, and Chyba–Thomas-Zahnle was carried out by Bronshten (1995). He notes that the model proposed by Chyba et al. does not take into account fragmentation: therefore, the destruction heights are overestimated (about 10–12 km). Bronshten also concludes that Grigoryan and Hills–Goda's models are equivalent.

Despite the particular features of each model, fragmentation is always considered to start when Eq. (1) is satisfied.

Although direct observations of asteroid impact are not available, it is possible to compare these models with observations of bodies with dimensions of several metres or tens of metres. Indeed, in this range, the gross–fragmentation model overlaps the explosive fragmentation models. As underlined several times by Ceplecha (1994, 1995, 1996b), observations clearly show that meteoroids breakup at dynamical pressures lower (10 times and more) than their mechanical strength. These data are obtained from photographic observation of bright bolides and the application of the gross–fragmentation model, that can be very precise.

According to Ceplecha et al. (1993) it is possible to distinguish five strength categories with an average dynamical pressure of fragmentation (Table 1). For continuous fragmentation the results obtained also indicate that the maximum dynamical pressure is below 1.2 MPa. Five exceptions were found: 4 bolides reached 1.5 MPa and one survived up to 5 MPa (Ceplecha et al. 1993).

It would be also very important to relate the ablation coefficient with the dynamical pressure $\rho_\infty V^2$ at the fragmentation point, in order to find a relationship between the meteoroid composition and its resistance to the air flow. To our knowledge, a detailed statistical analysis on this subject does not exist, but in the paper by Ceplecha et al. (1993) we can find a plot made by considering data on 30 bolides (we refer to Fig. 12 in that paper). We note that stony bodies (type I) have a wide range of $\rho_\infty V^2$ values at the fragmentation point. In the case of weak bodies, we can see that there is only one cometary bolide (type IIIA), but this is due to two factors: firstly, cometary bodies undergo continuous fragmentation, rather than a discrete breakup at certain points. Therefore, it is incorrect to speak about fragmentation pressure; we should use the maximum tolerable pressure. The second reason is that there is a selection effect. Indeed, from statistical studies, Ceplecha et al. (1997) found that a large part of bodies in the size range from 2 to 15 m are weak cometary bodies. However, a recent paper has shown that statistics from physical properties can lead to different results when compared with statistics from orbital evolution (Foschini et al. 2000). To be more precise, physical parameters prove that, as indicated above, a large part of small near Earth objects are weak cometary bodies, whilst the analysis of orbital evolution proves a strong asteroidal component.

In addition to data published in the paper by Ceplecha et al. (1993) and Ceplecha (1994) we can see in Table 2 some specific cases of bright bolides; for details, see the papers quoted or to Foschini (2000).

3. Stresses and strengths

For the sake of the simplicity, in the above section, we adopted commonly used values for mechanical strength, i.e. 1 MPa for cometary bodies, 10 MPa for carbonaceous chondrites, 50 MPa for stony bodies, and 200 MPa for iron bodies (see, for example, Hills & Goda 1993). Only Bronshten (2000) rejected the values for cometary bodies, proposing the range 0.02–0.4 MPa. He used values calculated by Öpik (1966) after observation of tidal disruption of Sun–grazing comets under the gravity of the Sun. However, it is worth noting that the strength obeys to scaling laws: therefore the larger is the body, the smaller is the strength. Öpik's calculations refer to the comet Ikeya–Seki (1965f), which has an estimated nucleus, before the breakup, of about 8.3 km. Small asteroids are in the range of several tens of metres, up to some hundreds of metres. The Tunguska Cosmic Body (TCB), to which Bronsthen apply his calculations, is in the range 50–100 m. We can calculate how the scaling law changes the value of the strength. The formula is shown in the paper by Tsvetkov & Skripnik (1991) and we recall it for simplicity:

$$S = S' \left(\frac{m'}{m}\right)^{\alpha} \quad (2)$$

where m and m' are the masses of the bodies and α is the scale factor: the more inhomogeneous is the material, the higher is α. Turcotte (1986) adopted a value of $\alpha = 0.12$ for glacial tills and we use this value. The comet mass can be calculated as $3\ 10^{14}$ kg, by using a density of 1000 kg/m^3, while the Tunguska Cosmic Body has an estimated value around 10^8 kg (Vasilyev 1998). With these

Table 2. Some special episodes of superbolides. Pressures are expressed in MPa

Name	Date	$\rho_\infty V^2$	S	Source
Přìbram	Apr. 7, 1959	9.2	50	ReVelle (1979)
Lost City	Jan. 3, 1970	1.5	50	Ceplecha (1996a)
Šumava	Dec. 4, 1974	0.14	1	Borovička & Spurný (1996)
Innisfree	Feb. 6, 1977	1.8	10	ReVelle (1979)
Space based obs.	Apr. 15, 1988	2.0	50	Nemtchinov et al. (1997)
Space based obs.	Oct. 1, 1990	1.5	50	Nemtchinov et al. (1997)
Benešov	May. 7, 1991	0.5	10	Borovička et al. (1998a,b)
Peekskill	Oct. 9, 1992	1.0	30	Ceplecha et al. (1996)
Marshall Isl.	Feb. 1, 1994	15	200	Nemtchinov et al. (1997)

values in Eq. (2) we obtain a value of the mechanical strength for a Tunguska–sized body, in the range 0.12–2.4 MPa, that is compatible with commonly used value of 1 MPa.

The scaling law for mechanical strength derives from the assumption that the fragmentation was a process of consecutive elimination of defects under increasing load. Baldwin & Sheaffer (1971) consider that the reason for the presence of cosmic bodies with very low fragmentation pressure can be explained by the assumption that additional flaws and cracks may be created by collisions in space, even though they do not completely destroy the cosmic body.

Tsvetkov & Skripnik (1991) made an interesting study on the fragmentation according to strength theory and scaling laws, but this kind of study is useful only for meteorites and cannot explain airbursts. They also show that the condition of fragmentation $S = \rho_\infty V^2$ is not valid even under the assumptions of scaling laws. Indeed, they showed that the aerodynamic loading never reach the ultimate strength. For this reason, they searched the cause of fragmentation in the particular structure and extreme inhomogeneity of cosmic bodies. With an appropriate selection of the scale factor, Tsvetkov & Skripnik obtained a good fit.

However, the shock compression and heating during the atmospheric entry will result in an elimination of internal cracks, making the body more compact (see, for example, Davison & Graham 1979 or Zel'dovich & Raizer 1966). If internal voids are so large to survive to shock compression, they could give an explanation for some episodes, but not a general theory (Foschini 1998).

4. The hypersonic flow approach

Almost all models described in Sect. 2 deal with the motion of a cosmic body in the Earth atmosphere. However, it is worth noting that we cannot observe directly the cosmic body: we can only see the light emitted during its atmospheric entry. Therefore, we have to introduce in the equations several coefficients, that cannot be derived directly from observations.

If we turn our attention to the hypersonic flow around the body, we could have data from direct observations. Among models discussed above, only Nemtchinov et al. (1997) tried to investigate the hypersonic flow around the asteroid with a numerical model. Foschini (1999b) investigated the analytic approach: indeed, although the details of hypersonic flow are very difficult to calculate and there is need of numerical models, the pressure distribution can be evaluated with reasonable precision by means of approximate methods. In the limit of a strong shock ($M \gg 1$) several equations tend to asymptotic values and calculations become easier. For example, the ratio of densities across the shock is:

$$\frac{\rho_0}{\rho_\infty} \rightarrow \frac{\gamma+1}{\gamma-1} \tag{3}$$

where ρ_0 is the density in the stagnation point and γ is the specific heats ratio.

In the Paper I, it was showed the crucial role of the temperature, instead of the pressure, in the stagnation point. The gas is in local thermodynamic equilibrium (LTE), that is, matter is in equilibrium with itself, but not with radiation, which can escape. Particle densities depend on the temperature only and it is possible to use Boltzmann or Saha equations. The temperature is calculated directly from observations of spectra (e.g. Borovička & Spurný 1996; Borovička et al. 1998b) and, thus, it is a reliable starting point.

Particularly, as stated in the Paper I, the temperature at the stagnation point is very important. Changes in the stream properties are mainly due to changes in the stagnation temperature, which is a direct measure of the amount of the heat transfer. The enthalpy change Δh is:

$$\Delta h = c_{\rm P} \Delta T \tag{4}$$

where $c_{\rm P}$ is the specific heat with constant pressure. From Eq. (4), it is possible to relate the maximum speed of the stream (which is close to the cosmic body speed) to the stagnation temperature (see Paper I):

$$V_{\max} = \sqrt{\frac{2\gamma}{\gamma-1} R T_0} \tag{5}$$

where R is the constant of the specific gas. The Eq. (5) is valid for an adiabatic flow whether reversible or not.

Under certain approximations, the flow can be considered adiabatic: heat transfer in hypersonic motion is strongly reduced. The boundary layer, where convective heat transfer takes place, is very thin and can be considered negligible.

On the other hand, if we consider the Eq. (5), we see that it is satisfied for unrealistic high temperatures. For example, for a speed of $V = 12$ km s^{-1} in air we obtain $T_0 = 72\,000$ K. The introduction of ablated matter make γ higher and R lower, resulting in a further increase of T_0.

However, at high temperatures some effects of the gas become important: the main effects are the dissociation and ionization of air molecules. For $2000 < T < 4000$ K the oxygen molecules break down to single atoms; for $4000 < T < 9000$ K there is the dissociation of N_2; for $T > 9000$ K ionization occurs (see, for example, Oosthuizen & Carscallen 1997). In the Δh term of the Eq. (4), we should take into account also the heat absorbed by these processes. As a result, T_0 drops to more reasonable 10 000–15 000 K or so.

The treatment of real gas properties is probably one of the most difficult problem in aerothermodynamics and involves complex numerical modelling (for a review, see Tirsky 1993; Gnoffo 1999).

5. The equation of state

Before going on, it is necessary to evaluate the state of the shocked gas, in order to select the more appropriate equation of state.

From the point of view of pressure, we can see that the gas around an asteroid during the atmospheric entry, is still an ideal gas. Indeed, the limit of pressure to become a degenerate gas is around 10^{14} Pa (Eliezer 1991). A simple calculation shows that the asteroid reaches the maximum dynamical pressure at the sea level ($\rho_{\rm sl} = 1.293$ kg/m^3) and if it has the maximum geocentric speed ($V = 72$ km s^{-1}, although it is quite impossible to have an asteroid with such a speed). Therefore, the maximum $\rho_\infty V^2 \approx 7\ 10^9$ Pa, about four orders of magnitude below the limit of degeneracy.

From the point of view of the temperature, the question is a bit difficult to handle. From spectroscopic observations of bright bolides (Borovička & Spurný 1996; Borovička et al. 1998b), we can see that the spectrum is composed by two temperatures emission: the first value lies in the range between 4000–6000 K and the second value is about 10 000 K. The temperature of 10 000 K is about the limit between normal and partially ionized gas. Therefore, we shall adopt a more general point of view, based on microscopic physics.

From the kinetic theory, we know that the pressure is additive, therefore we can separate it in contributions from different species composing the gas: electrons, photons, ions, and atoms. We can write:

$$P = P_{\rm i} + P_{\rm e} + P_{\rm r} = \sum_j N_j k T_j + N_{\rm e} k T_{\rm e} + \frac{aT^4}{3} \qquad (6)$$

where k is the Boltzmann constant and a is the radiation constant, related to the Stefan–Boltzmann constant ($a = 4\sigma/c = 7.56591\ 10^{-16}$ [Pa K^{-4}]); N is the volume density of species (the index e is for electrons); the sum over the index j concerns all heavy particles (ions and neutral atoms).

Equation (6) has been obtained by means of quite general assumptions, without any particular restrictions (more details can be found in many places; for example, in Lang 1999). We can apply Eq. (6) to hypersonic flow around an asteroid in the Earth atmosphere. The only restrictions are determined by the momentum distributions of particles: ions, atoms, and electrons obey generally to a Maxwellian distribution, photons obey to the Bose–Einstein statistics. These conditions are satisfied in processes occurring during the atmospheric entry of a cosmic body.

6. The pressure of fragmentation

We can note that, for typical temperatures in airbursts, the radiation pressure is negligible. For $T = 10\,000$ K, we have that $P_{\rm r} \approx 2.5$ Pa, which is negligible when compared to $\rho_\infty V^2$ at the fragmentation point (see Sect. 2). Therefore, for airbursts and meteors, the Eq. (6) becomes:

$$P \approx P_{\rm i} + P_{\rm e} = \sum_j N_j k T_j + N_{\rm e} k T_{\rm e}. \qquad (7)$$

In order to calculate the densities of species, we have to take into account that the fluid around the asteroid is in local thermodynamic equilibrium (LTE) and we can use the Saha relation:

$$\frac{N_{\rm e} N_{\rm i}}{N_{\rm a}} = \frac{(2\pi m k T)^{3/2}}{h^3} \frac{2 g_{\rm i}}{g_{\rm a}} \exp\left(-\frac{eE_{\rm i}}{kT}\right) \qquad (8)$$

where $N_{\rm a}$ is the density of neutral atoms, $E_{\rm i}$ is the ionization potential, $g_{\rm a}$ and $g_{\rm i}$ are respectively the statistical weight of the ground state of the neutral atom and of the ion. Some of these values are listed in Table 3.

Table 3. Values of $E_{\rm i}$, $g_{\rm a}$, and $g_{\rm i}$ for some species in the flow

Species	$E_{\rm i}$ [eV]	$g_{\rm a}$	$g_{\rm i}$
Na	5.14	2	1
K	4.34	2	1
O_2	12.05	3	4
N_2	15.6	1	2

For each airburst, with a given composition, we can calculate densities of all species and obtain the effective pressure. But this is not the purpose of this work. We will introduce some hypoteses for the sake of the simplicity, in order to do some calculations of order of magnitude.

The Saha Eq. (8) can be rewritten in terms of degree of ionization α:

$$\frac{N_{\rm tot}\alpha^2}{(1-\alpha)} = \frac{(2\pi m k T)^{3/2}}{h^3} \frac{2 g_{\rm i}}{g_{\rm a}} \exp\left(-\frac{eE_{\rm i}}{kT}\right) \qquad (9)$$

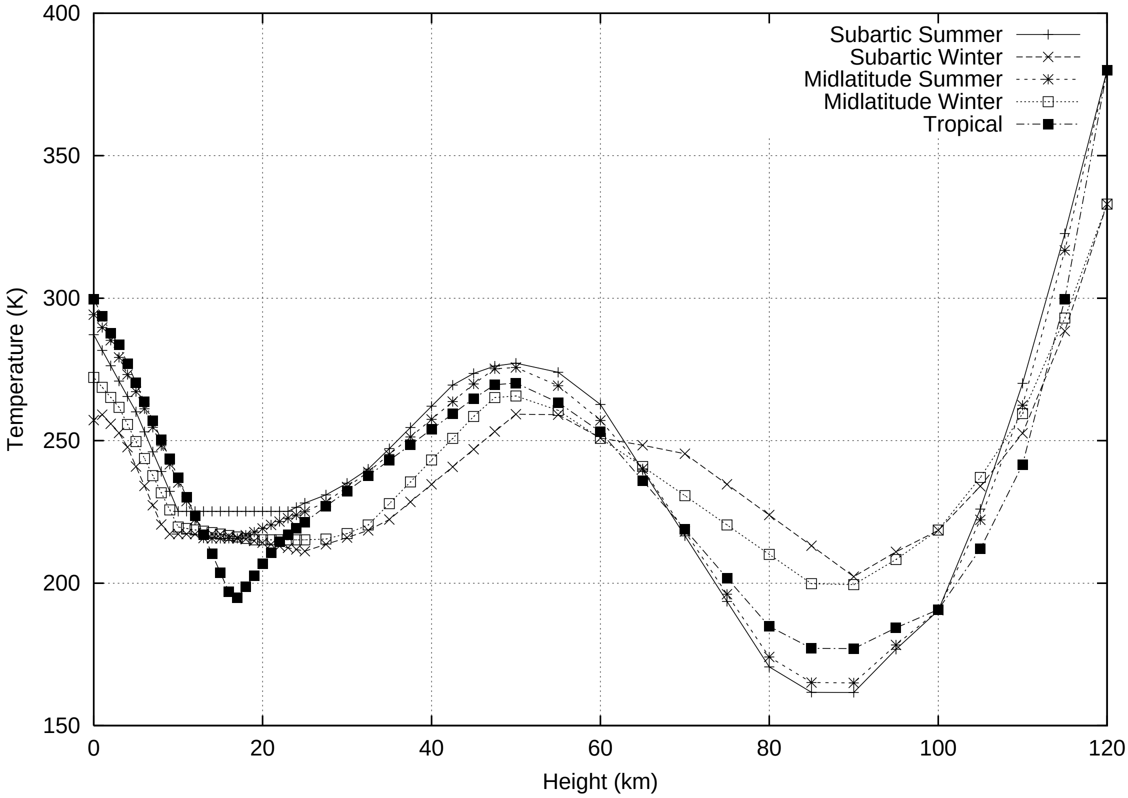

Fig. 1. Temperature as a function of the height, for different latitudes and seasons. Data from US Standard Atmosphere 1976

where $N_{\rm tot}$ is the total number density of particles; therefore, $N_{\rm e} = \alpha N_{\rm tot}$ and so on. The Eq. (7) can be rewritten:

$$P = (1+\alpha)N_{\rm tot}kT = (1+\alpha)\rho RT \qquad (10)$$

that is the well known equation of state for a gas with a degree of ionization α.

Foschini (1999a) underlined the importance of alkaline metals in the production of electrons in meteors. In addition, von Zahn et al. (1999) recently showed the particular role of the potassium in Leonid meteors. Indeed, alkaline metals, owing to their low ionization potential easily ionize even at low temperatures. For example, we assume that $N_{\rm tot} = 6.022\ 10^{23}\ {\rm m}^{-3}$ (it is the Avogadro number) and we calculate the degree of ionization of a gas composed by single species listed in Table 3. For a temperature $T = 10\,000$ K, we have: for sodium, $\alpha = 0.92$; for potassium, $\alpha = 0.96$; for oxygen, $\alpha = 0.1$ and for nitrogen, $\alpha = 0.01$.

It is known that, in meteors, the impact ionization from impinging molecules is very important. A nitrogen molecule with a speed of 18 $\rm km\,s^{-1}$ has an energy of about 45 eV. However, as the asteroid penetrates deeper in the atmosphere, the flow generates a shield that protects the asteroid from direct impacts (for a detailed description see Ceplecha et al. 1998). Therefore, the thermal ionization become important particularly in small asteroids/comets, when impact ionization is strongly reduced. So, alkaline metals play an important role, because they provide a huge amount of electrons.

We have also to take into account that we are dealing with particular condition in the stagnation point, where there is the maximum thermo–mechanical stress. In this point, we can consider the fluid as highly ionized: the ablation of alkaline metals provide a source of ions for the flow. In the stagnation point – not in the whole flow – we can consider the gas with a degree of ionization $\alpha \to 1$. Therefore, by substituting Eq. (10) in Eq. (5), we obtain the condition for fragmentation:

$$V_{\rm max} = \sqrt{\frac{2\gamma}{\gamma-1}\frac{P}{(1+\alpha)\rho}}\,. \qquad (11)$$

Equation (11) was already outlined in the Paper I, where it was considered $\alpha = 1$.

It is worth noting that Eq. (11) is the condition of fragmentation, not the condition for the airburst, as deduced by Bronshten (2000). The airburst generally occurs after a scale height, as shown by several studies on superbolides. In the Paper I, there was an unfortunate error in considering the explosion height, instead of the fragmentation height (although it has negligible effect, as we shall see in the Sect. 9). Therefore, Eq. (11) must be revised as follows:

$$V_{\rm max} = \sqrt{\frac{2\gamma}{\gamma-1}\frac{P}{(1+\alpha)\rho_{\rm sl}}\exp\left(\frac{h+H}{H}\right)} \qquad (12)$$

where $\rho_{\rm sl}$ is the air density at sea level, h is the airburst height, and H is the scale height.

Before to analyse the Eq. (12), it is necessary to look at the value for γ. In the Paper I, $\gamma = 1.7$ was used and it was taken from experimental measurement in hypervelocity impacts. Bronshten (2000) does not agree with this value and invokes argument to support a value of $\gamma = 1.15$, that is compatible with air at high temperatures and under shock loading. Bronshten's assumption are reasonable when we have a diatomic gas at high temperature: the rotation and vibration of air molecules change the values for γ. But, if we have a monatomic gas or metal vapors, we have to consider a $\gamma = 5/3$, unless the temperature is so high to change appreciably the electron energy (see Zel'dovich & Raizier 1966); however, this is not our case. This is confirmed also by meteor spectra: they show that the surrounding gas is monatomic, although molecules bands are sometime recorded (Ceplecha et al. 1998).

It is worth noting that the question about the value of γ is not simple. Perhaps, it will be solved when experimental data will be available.

7. Turbulence and massive ablation

Equation (12) shows that it is necessary to have higher speeds in order to reach the value of the mechanical strength. It is worth noting that Eq. (12) is valid under steady state condition. In this case, we can neglect the contribution of turbulence, because of compressibility effects at high Mach number (see Andreopulos et al. 2000).

The influence of the turbulence in the large meteoroid entry was first analyzed by ReVelle (1979). In his approach based on the motion of a single body in the atmosphere, ReVelle studied how the convective heat transfer depends on turbulence. He found that the turbulent convective layer is negligible, except for speed lower than about 20 $\mathrm{km\,s^{-1}}$.

However, the presence of massive ablation changes the flow. Gupta (1983) noted that there are few experimental data available and, most important, they are obtained with freestream Mach number in the range 3–7 and with negligible ablation. There are less data about experiments with moderate ablation, but with lower Mach number (up to 2.6). Starting from these data, models predict that convective heating is negligible with almost any turbulence model. This was the situation in 1983, but more recent reviews (Gnoffo 1999) show that there are still several things unknown. The only news is the entry of the Galileo probe in the Jovian atmosphere, which gave us useful information about hypersonic motion with massive ablation. The probe entered in the atmosphere at a relative speed of 47.5 $\mathrm{km\,s^{-1}}$ and experienced an ablation rate of 7.4 $\mathrm{km\,s^{-1}}$, with a total mass loss of about 79 kg (Gnoffo 1999).

The probe did not suffer any fragmentation, despite of massive ablation, although it is still not clear the role of all processes during the entry. The layer between the shock and the probe is expected to be turbulent over almost the entire length of the Galileo probe, owing to the massive ablation and the large Reynolds number (Gupta 1983). The interaction of shock waves with turbulence leads to amplification of speed fluctuations and changes in length scales. It would be better to say that this occur when the flux is *unsteady* and the shock wave is subject to strong distortions. At high Mach number, but steady state motion, the effect of compressibility takes place and there is no amplification; we can adopt Eq. (12).

But the atmospheric motion is generally unsteady. Figure 1 shows the air temperature from tables of the US Standard Atmosphere 1976. The air temperature is instrumental to derive the local sound speed ($a = \sqrt{\gamma RT}$), as a function of the height, for different latitude and seasons. As the temperature changes, the sound speed changes. The US Standard Atmosphere gives also a reference atmosphere – and therefore a reference temperature – from which we can calculate a reference sound speed (Fig. 2).

It is known that, for a small asteroid, the deceleration due to ablation is negligible; therefore we can consider that the body speed does not change until the fragmentation begins. For a given asteroid, the Mach number depends only on the local sound speed. In Fig. 3 we can see the Mach number for different values of the speed of the cosmic body. It results that the Mach number change substantially for higher asteroid speed, while for lower speed, changes of M become smooth. We note also strong variations when the body crosses the mesopause, the stratopause, and the tropopause.

Ceplecha (1994) gave the mean values of end heights for large meteoroids (up to about 7 m): he found 32 km for type I; 43 km for type II; 57 km for type IIIA; and 69 km for type IIIB. These are end heights and if we consider that the fragmentation begins about a scale height above, we infer that the mean fragmentation height is: 39 km for type I; 50 km for type II; 64 km for type IIIA; and 77 km for type IIIB. That is, we can divide these bodies into two categories: a first category, made with type II and type IIIB, which contains bodies that breakup during the crossing of the stratopause and the mesopause, respectively. The second category, made with type I and type IIIA, which contains bodies that breakup with a certain delay after the crossing of atmospheric pauses. The delay can be explained with the different ability to ablate, and therefore to produce the turbulent boundary layer. Indeed, for bodies that breakup at the mesosphere (cometary bodies), the type IIIB has an average ablation coefficient higher than the type IIIA. For asteroidal bodies, that breakup at the stratosphere, the type II has a higher ablation coefficient than the type I. We should also take into account that the transition from laminar to turbulent flow depends on the body dimensions and speed. Large bodies at high speed reach the transition at higher heights (ReVelle 1979).

It is worth noting that there are also other factors that can change the fragmentation height, such as the rotation (Adolfsson & Gustafson 1994; Ceplecha 1996a), but they are independent from the body type and can explain only deviations from the average values.

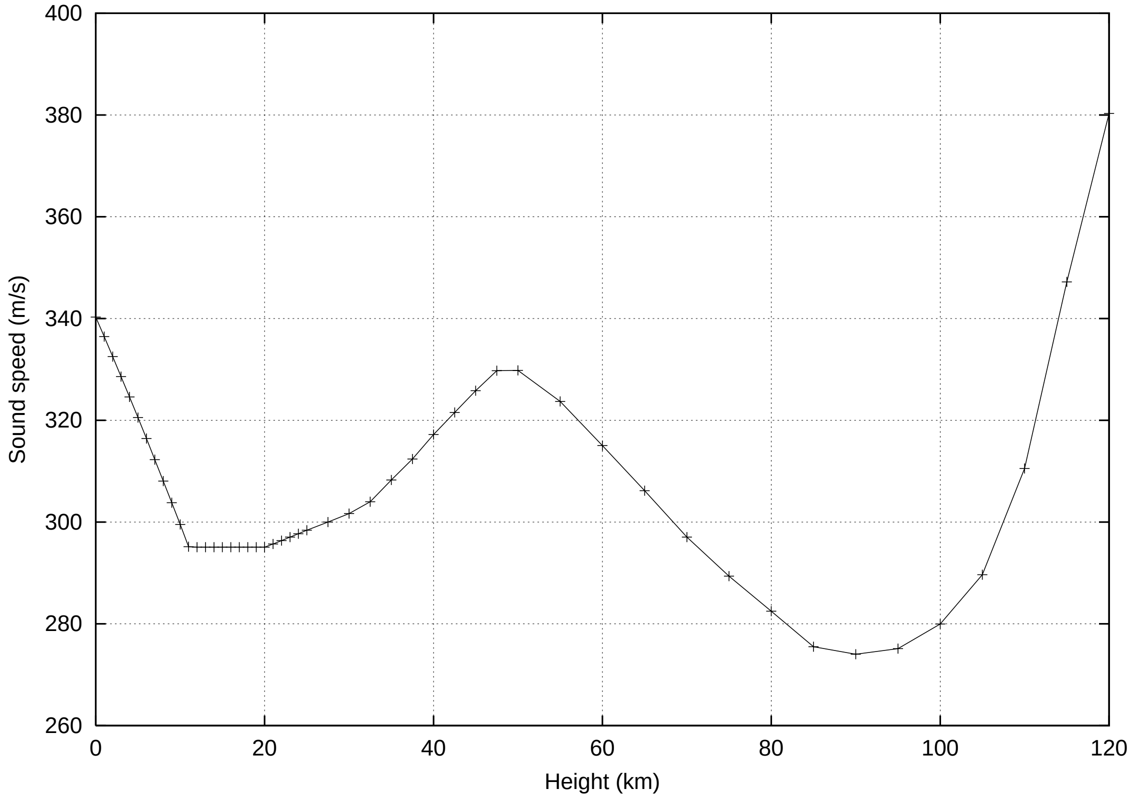

Fig. 2. Local sound speed from the reference temperature of the US Standard Atmosphere 1976

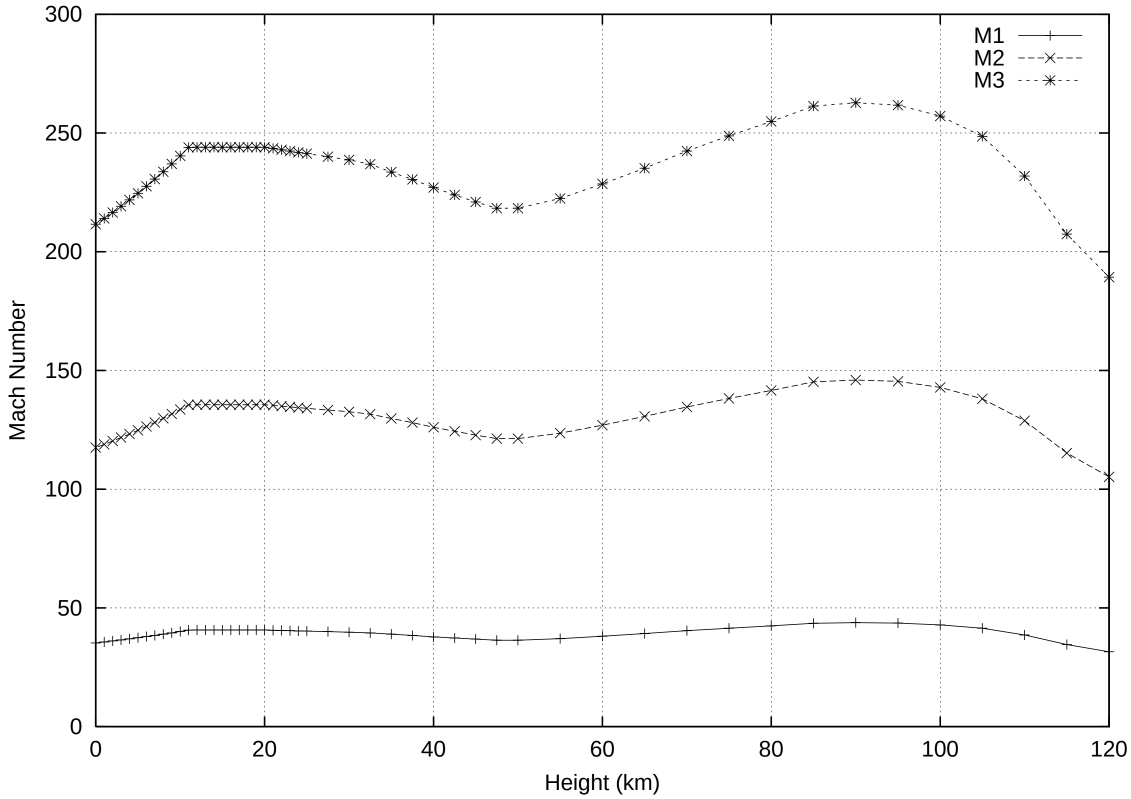

Fig. 3. Mach number as a function of the height for different asteroid speed. $M1 = 12\ \mathrm{km\,s^{-1}}$; $M2 = 40\ \mathrm{km\,s^{-1}}$; $M3 = 72\ \mathrm{km\,s^{-1}}$

8. The condition for fragmentation

To find a quantitative condition for fragmentation by taking into account the turbulence, at least for first order calculations, is very difficult. Problems derive from the lack of experimental data about the shock wave–turbulence interaction at very high Mach number. Moreover, the few feasible experiments at moderate Mach number are strongly dependent on measurement systems and available data are insufficient to fully describe the mutual interaction of shock waves and turbulence. Last, but not least, the turbulence itself is one of the oldest unsolved problems in the history of science. In recent years, numerical models allowed detailed investigation, but the problem is complicated by the fact that, when dealing with turbulence, the averaging of governing equations introduces new unknowns. Therefore, the number of available equations is not sufficient and it is necessary to assume a closure condition. The lacking of experimental data make it hard to make hypotheses on the closure condition and therefore numerical models are often contradictory. For reviews of these problems see Andreopulos et al. (2000), Lele (1994), Adamson & Messiter (1980), and references therein.

Despite of differences, it is clear that the shock wave–turbulence interaction produce an amplification of fluctuations. The amplification depends on the shock strength, the state of the turbulence, and its level of compressibility. The most important outcomes are the amplification of velocity fluctuations and changes in the length scales (Andreopulos et al. 2000). This leads to changes in the dynamical pressure in the front of the asteroid, but also changes in pressure along the flank of the cosmic body; these changes can be further amplified by local irregularity, such as small "hills".

However, for the sake of simplicity, we make the assumption that the zone of greatest stress is the stagnation point. Or, it would be better to say, that we will continue to hold this hypotesis. Being under unsteady conditions we cannot apply isentropic relations, i.e. Eq. (12). We start from the value of the pressure in the shock layer derived from the Rankine–Hugoniot relations in the hypersonic limit ($P \approx \rho_\infty V^2$). Being the gas cap around the asteroid a plasma, we have to consider the Eq. (10), so we have:

$$P \approx (1+\alpha)\rho_\infty V^2. \qquad (13)$$

For the sake of the simplicity, we assume that turbulence does not affect density; so we can consider that shock wave–turbulence interaction does affect only velocity. The problem is to set up the value of this amplification: we then introduce an amplification factor κ to evaluate:

$$P \approx (1+\alpha)\rho_\infty \kappa V^2. \qquad (14)$$

As written above, we have no experimental data and numerical models are often contradictory. We can try to set up lower and upper limits. Rotman (1991) reports amplification of the kinetic energy of about 2–2.15. Jacquin et al. (1993) found that the amplification of the kinetic energy depends on the density ratio; the factor can be up to 12.7 for diatomic gases. For monatomic gases and plasmas the upper limit of the amplification is 6.

Table 4. Special episodes of superbolides: the new pressure of fragmentation calculated according to Eq. (14). See Table 2 for other details. Pressures are expressed in MPa

Name	Min P	Max P	S
Přibram	37	110	50
Lost City	6	18	50
Šumava	0.6	1.7	1
Innisfree	7	22	10
Space based obs.	8	24	50
Space based obs.	6	18	50
Benešov	2	6	10
Peekskill	4	12	30
Marshall Isl.	60	180	200

In conclusion, we can assume for kinetic energy $2 \leq \kappa \leq 6$ and that this amplification value is valid also for pressure. Therefore, under distortion of the shock wave, the turbulence can leads to amplification of dynamical pressure up to 12 times the nominal value for a neutral gas (we have taken into account also the multiplicative factor $1+\alpha$). Comparing with experimental data of the fragmentation of asteroids showed in Sect. 2, we can see that they are in better agreement (see Table 4).

Therefore, the new condition for fragmentation under unsteady regime is:

$$V = \sqrt{\frac{S}{\kappa(1+\alpha)\rho_{\rm sl}} \exp\left(\frac{h+H}{H}\right)}. \qquad (15)$$

In addition, we have to consider that the distortion of the shock wave leads to the partial removal of the gas cap around the cosmic body, so that the ablation increases strongly. This enhances impact ionization, so that $\alpha \to 1$ also during unsteady conditions, where thermal ionization is negligible.

9. Examples

Let us to consider two episodes in order to show how this "embryo" of theory works. Firstly, we can consider the Lugo bolide of 19 January 1993 (Cevolani et al. 1993; Foschini 1998). It was a very bright bolide, which reached a peak magnitude of about −23 and released an estimated energy of about 14 kton, when exploded at about 30 km over the city of Lugo, in northern Italy. In previous analyses, it was considered that the fragmentation occurred when the dynamical pressure reached the mechanical strength.

If we now apply the condition given from Eq. (15), taking into account $\alpha \to 1$, $\rho_{\rm sl} = 1.293$ kg/m^3, and that the scale height $H = 6.8$ km at 30 km height, we obtain the values in Table 5.

We have three reasonable solutions, which are underlined. If we consider that the final airburst occurred

Table 5. Values of the speed of the Lugo bolide at the moment of fragmentation [$\mathrm{km\,s^{-1}}$]

S [MPa]	$\kappa = 2$	$\kappa = 6$
1	6.6	3.8
10	20.8	12.0
50	46.5	26.9
200	93.1	53.7

at about 30 km height, typical for type I bodies (see the discussion at the end of Sect. 7), we can consider 26.9 $\mathrm{km\,s^{-1}}$ as the most probable speed. This value is in agreement with first estimation from eyewitnesses (Cevolani et al. 1993).

For the Tunguska event, a more detailed analysis is in preparation in collaboration with the members of the Tunguska99 Scientific Expedition. Here we want to underline only one thing: from Fig. 1 we can see that, in subartic summer, the temperature does not change in the height interval crossing the troposphere. Therefore, the Mach number does not change and we can apply the Eq. (12), for steady state conditions, as shown in the Paper I. The error, noted by Bronshten (2000), introduced in considering the explosion height, instead of the fragmentation height is negligible: the new value is 16 $\mathrm{km\,s^{-1}}$ to be compared with the old one of 16.5 $\mathrm{km\,s^{-1}}$.

10. Conclusion

In this paper, we have done a further step toward the construction of a theory for the fragmentation of a small asteroid during the atmospheric entry. In the Paper I, we showed a specific part of the theory applied to a particular episode, the Tunguska event of 30 June 1908.

Here we showed more details, both explaining better some assumptions in the theory of steady state motion outlined in the Paper I, and extending the theory to unsteady motion. We have taken into account the effect of turbulence and its interaction with shock waves. Some examples are discussed and we have found a reasonable agreement with available experimental data.

We have found two conditions for the fragmentation, according to steadiness of the motion.

1. Steady state motion: in this case, the compressibility suppresses the turbulence and, therefore, we can use Eq. (12);
2. Unsteady motion: we have a strong interaction between the shock wave and the turbulence, which give rise to sudden pressure outburst. We have to use Eq. (15).

On the other hand, it is necessary to remember that we are speculating, because of the scarce experimental data and the large uncertainties affecting records. Therefore, these researches must be taken *cum grano salis.*

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