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Could dark matter interactions be an alternative to dark energy?

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ABSTRACT

We study the global dynamics of the universe within the framework of the interacting dark matter (IDM) scenario. Assuming that the dark matter obeys the collisional Boltzmann equation, we can derive analytical solutions of the global density evolution, that can accommodate an accelerated expansion, equivalent to either the *quintessence* or the standard Λ models, with the present time located after the inflection point. This is possible if there is a disequilibrium between the DM particle creation and annihilation processes with the former process dominating, which creates an effective source term with negative pressure. Comparing the predicted Hubble expansion of one of the IDM models (the simplest) with observational data, we find that the effective annihilation term is quite small, as suggested by various experiments.

Key words. cosmology: theory – methods: analytical

1. Introduction

The analysis of high quality cosmological data (e.g. supernovae type Ia, CMB, galaxy clustering) have suggested that we live in a flat, accelerating universe, that contains cold dark matter to explain clustering and an extra component with negative pressure, the vacuum energy (or more generally the *dark energy*), to explain the observed accelerated cosmic expansion (Spergel et al. 2007; Davis et al. 2007; Kowalski et al. 2008; Komatsu et al. 2009, and references therein). Because of the absence of a physically well-motivated fundamental theory, there have been many theoretical speculations about the nature of the exotic dark energy (DE) including a cosmological constant, or either scalar or vector fields (see Weinberg 1989; Wetterich 1995; Caldwell et al. 1998; Brax & Martin 1999; Peebles & Ratra 2003; Perivolaropoulos 2003; Brookfield et al. 2006; Boehmer & Harko 2007, and references therein).

Most papers in this type of study are based on the assumption that DE evolves independently of the dark matter (DM). The unknown nature of both DM and DE implies that we cannot preclude the possibility to find interactions in the dark sector. This is very important because interactions between the DM and *quintessence* could provide possible solutions to the cosmological coincidence problem (Grande et al. 2009). Several papers have been published in this area (e.g., Amendola et al. 2003; Cai & Wang 2005; Binder & Kremer 2006; Campo et al. 2006; Wang et al. 2006; Das et al. 2006; Olivares et al. 2008; He & Wang 2008, and references therein) proposing that the DE and DM could be coupled, assuming also that there is only one type of non-interacting DM.

However, there are other possibilities. It is plausible, for example, that the dark matter is self-interacting (IDM) (Spergel & Steinhardt 2000). This possibility was proposed in order to solve discrepancies between theoretical predictions and astrophysical observations, including less cuspy halo profiles, predicted by the IDM model, allowing for the observed gamma-ray and microwave emission from the center of our galaxy (Flores & Primack 1994; Moore et al. 1999; Hooper et al. 2007; Regis & Ullio 2008, and references therein) and the discrepancy between the predicted optical depth, τ, inferred from the Gunn-Peterson test in the spectra of high-z QSOs and the WMAP-based value (e.g., Mapelli et al. 2006; Belikov & Hooper 2009; Cirelli et al. 2009, and references therein). It has also been shown that some dark matter interactions could provide an accelerated expansion phase of the universe (Zimdahl et al. 2001; Balakin et al. 2003; Lima et al. 2008). In addition, DM could potentially contain more than one particle species, for example a mixture of cold, warm, or hot dark matter (Farrar & Peebles 2004; Gubser & Peebles 2004), with or without inter-component interactions.

In this work, we are not concerned with the viability of the different possibilities, nor with the properties of interacting DM models. The single aim of this work is to investigate whether there are repercussions of DM self-interactions on the global dynamics of the universe and specifically whether these models can yield an accelerated phase of the cosmic expansion, without the need for dark energy. We note that we do not "design" the fluid interactions to produce the desired accelerated cosmic evolution, as in some previous works (e.g., Balakin et al. 2003), but investigate the circumstances under which the analytical solution space of the collisional Boltzmann equation, in the expanding universe, allows for a late accelerated phase of the universe.

2. Collisional Boltzmann equation in an expanding universe

It is well established that the global dynamics of a homogeneous, isotropic, and flat universe is given by the Friedmann equation

$$\left(\frac{\dot{\alpha}}{\alpha}\right)^2 = \frac{8\pi G}{3}\rho, \tag{1}$$

where ρ is the total energy-density of the cosmic fluid, containing (in the matter-dominated epoch) dark matter, baryons, and any type of exotic energy. Differentiating Eq. (1), we derive the second Friedmann equation, given by:

$$\frac{\ddot{\alpha}}{\alpha} = -\frac{4\pi G}{3}\left(-2\rho - \frac{\dot{\rho}}{H}\right)\cdot \quad (2)$$

As we mentioned in the introduction, the dark matter is usually considered to contain only one type of particle that is stable and neutral. In this work, we investigate, using the Boltzmann formulation, the cosmological potential of a scenario in which the dominant "cosmic" fluid does not contain dark energy, is not perfect, and at the same time is not in equilibrium[1]. Although our approach is phenomenological, we briefly review a variety of physically motivated dark matter self-interaction models that have appeared in the literature.

The time evolution of the total density of the cosmic fluid is described by the collisional Boltzmann equation

$$\frac{d\rho}{dt} + 3H(t)\rho + \kappa\rho^2 - \Psi = 0, \quad (3)$$

where $H(t) \equiv \dot{\alpha}/\alpha$ is the Hubble function, Ψ is the rate of creation of DM particle pairs, and $\kappa(\geq 0)$ is given by:

$$\kappa = \frac{\langle\sigma u\rangle}{M_x}, \quad (4)$$

where σ is the cross-section for annihilation, u is the mean particle velocity, and M_x is the mass of the DM particle.

We note that, in the context of a spatially flat FLRW cosmology, for an effective pressure term of:

$$P = \frac{\kappa\rho^2 - \Psi}{3H}, \quad (5)$$

the collisional Boltzmann equation reduces to the usual fluid equation: $\dot{\rho} + 3H(\rho + P) = 0$. Inserting Eqs. (3) and (5) into Eq. (2), we obtain

$$\frac{\ddot{\alpha}}{\alpha} = -\frac{4\pi G}{3}\left(\rho + \frac{\kappa\rho^2 - \Psi}{H}\right) = -\frac{4\pi G}{3}(\rho + 3P)\,. \quad (6)$$

Obviously, a negative pressure (whatever its cause) can effectively act as a repulsive force possibly providing a cosmic acceleration.

We investigate the effects of DM self-interactions on the global dynamics of the universe and under which circumstances they can produce a negative pressure and thus provide an alternative to conventional dark energy. It is well known that negative pressure implies tension rather than compression, which is an impossibility for ideal gases but not for some physical systems that depart from thermodynamic equilibrium (Landau & Lifshitz 1985).

The particle annihilation regime was described by Weinberg (2008), using the collisional Boltzmann formulation, in which the physical properties of the DM interactions are related to massive particles (which are still present) that, if they carry a conserved additive or multiplicative quantum number, would imply that some particles must remain after all the antiparticles have been annihilated (Weinberg calls them L-particles). The L-particles may annihilate to form other particles, which during the period of annihilation they can be assumed to be in thermal and chemical equilibrium (see Weinberg 2008). This DM self-interacting model can affect the global dynamics of the universe (see our *Case 2* below).

The corresponding effects on the global dynamics of the particle creation regime, which provides an effective negative pressure, has also been investigated by a number of authors (e.g., Prigogine et al. 1989; Lima et al. 2008, and references therein).

In the framework of a Boltzmann formalism, a negative pressure could in general be the outcome of dark matter self-interactions, as suggested in Zimdahl et al. (2001) and Balakin et al. (2003), if an "anti-frictional" force is self-consistently exerted on the particles of the cosmic fluid. This possible alternative to dark energy has the caveat of its unknown exact nature, which is also however the case for all dark energy models. Other sources of negative pressure have been proposed, including gravitational matter "creation" processes (Zeldovich 1970), modeled by non-equilibrium thermodynamics (Prigogine et al. 1989) or even the C-field of Hoyle & Narlikar (1966). The effects of the former proposal (gravitational matter creation) on the global dynamics of the universe have been investigated, based on the assumption that the particles created are non-interacting (Lima et al. 2008). The merit of all these alternative models is that they unify the dark sector (dark energy and dark matter), since a single dark component (the dark matter) needs to be introduced into the cosmic fluid.

In a unified manner we present, the outcome for the global dynamics of the universe of different type of dark matter self-interactions, using the Boltzmann formulation in the matter-dominated era.

3. The Cosmic density evolution for different DM interactions

We proceed to analytically solve Eq. (3). We change variables from t to α and thus Eq. (3) can be written

$$\frac{d\rho}{d\alpha} = f(\alpha)\rho^2 + g(\alpha)\rho + R(\alpha), \quad (7)$$

where

$$f(\alpha) = -\frac{\kappa}{\alpha H(\alpha)} \quad g(\alpha) = -\frac{3}{\alpha} \quad R(\alpha) = \frac{\Psi(\alpha)}{\alpha H(\alpha)}\cdot \quad (8)$$

Within this framework, based on Eqs. (5), (7) and (8), we can distinguish four possible DM self-interacting cases:

Case 1: $P = 0$: If the DM is collisionless or the collisional annihilation and pair creation processes are in equilibrium (i.e., $\Psi \equiv \kappa\rho^2$), the corresponding solution of the above differential equation is $\rho \propto \alpha^{-3}$ (where α is the scale factor of the universe), and thus we obtain, as we should, the dynamics of the Einstein de-Sitter model, with $H(t) = 2/3t$.

Case 2: $P = \kappa\rho^2/3H$: If we assume that in the matter-dominated era the particle creation term is negligible, $\Psi = 0$ [$R(\alpha) = 0$], (the case discussed in Weinberg 2008), then the corresponding pressure becomes positive. It is clear that Eq. (7)

[1] Initially, the total energy density is $\rho = \rho_{IDM} + \rho_r$. We consider that the self-interacting dark matter does not interact significantly with the background radiation, and thus in the matter-dominated epoch, radiation is irrelevant to the global dynamics (because of the well-known dependence: $\rho_r \propto a^{-4}$). Therefore, taking the above considerations into account and assuming that there are no residual radiation products of the DM interactions (otherwise see Appendix A), we conclude that in the matter-dominated era the total cosmic dark-matter density reduces to that of the IDM density ($\rho \simeq \rho_{IDM}$), which obeys the collisional Boltzmann equation (see Eq. (3)).

becomes a Bernoulli equation, the general solution of which provides the evolution of the global energy-density, which is that corresponding to the IDM ansanz:

$$\rho(\alpha) = \frac{\alpha^{-3}}{C_2 - \int_1^{\alpha} x^{-3} f(x)\mathrm{d}x} = \frac{\alpha^{-3}}{C_2 + \kappa \int_{t_0}^{t} \alpha^{-3}(t)\mathrm{d}t} \cdot \quad (9)$$

Prior to the present epoch ($\alpha \simeq 1$), we find that $\rho(\alpha) \propto \alpha^{-3}$, while at late enough times ($\alpha \gg 1$) the above integral converges, which implies that the corresponding global density tends to evolve again as the usual dark matter (see Weinberg 2008), with

$$\rho(\alpha) \rightarrow \frac{\alpha^{-3}}{C_2 + \kappa \int_{t_0}^{\infty} \alpha^{-3}(t)\mathrm{d}t} \propto \alpha^{-3}, \quad (10)$$

where t_0 is the present age of the universe. The latter analysis, relevant to the usual weakly interacting massive particle case – Weinberg (2008), leads to the conclusion that the annihilation term has no effect resembling that of dark energy, but does affect the evolution of the self-interacting DM component, with the integral in the denominator rapidly converging to a constant (which depends on the annihilation cross-section).

Case 3: $P = (\kappa\rho^2 - \Psi)/3H$: For the case of a non-perfect DM fluid (i.e., having up to the present time, a disequilibrium between the annihilation and particle pair creation processes), we can have either a positive or a negative effective pressure term. Although the latter situation may or may not appear plausible, even the remote such possibility, i.e., the case in which the DM particle creation term is larger than the annihilation term ($\kappa\rho^2 - \Psi < 0$), is of particular interest because of its effect on the global dynamics of the universe (see for example Zimdahl et al. 2001; Balakin et al. 2003).

It is interesting to note that this case can be viewed as a generalization of the gravitational matter creation model of Prigogine et al. (1989) (see also Lima et al. 2008, and references therein) in which annihilation processes are also included, although the matter-creation component dominates over annihilations. In this scenario, as in any interacting dark-matter model with a left-over residual radiation, a possible contribution from the radiation products to the global dynamics is negligible, as we show in Appendix A.

For $\kappa \neq 0$ and $\Psi \neq 0$, it is not an easy task in general to solve analytically Eq. (7), because it is a non-linear differential equation (Riccati type). However, Eq. (7) could be fully solvable if (and only if) a particular solution is known. We indeed find that for some special functional forms of the interactive term, such as $\Psi = \Psi(\alpha, H)$, we can derive analytical solutions. We identified two functional forms for which we can solve the previous differential equation analytically, only one of these two is of interest because it provides a $\propto a^{-3}$ dependence of the scale factor (see Appendix B), which is:

$$\Psi(\alpha) = \alpha H(\alpha) R(\alpha) = C_1(m+3)\alpha^m H(\alpha) + \kappa C_1^2 \alpha^{2m}. \quad (11)$$

Although, the above functional form was not motivated by physical theory, but rather phenomenologically because it provides analytical solutions to the Boltzmann equation, its exact form can be justified a posteriori within the framework of IDM (see Appendix C).

The general solution of Eq. (7) for the total energy density, using Eq. (11), is:

$$\rho(\alpha) = C_1\alpha^m + \frac{\alpha^{-3}F(\alpha)}{\left[C_2 - \int_1^{\alpha} x^{-3} f(x) F(x)\mathrm{d}x\right]}, \quad (12)$$

where the kernel function $F(\alpha)$ has the form

$$F(\alpha) = \exp\left[-2\kappa C_1 \int_1^{\alpha} \frac{x^{m-1}}{H(x)}\mathrm{d}x\right] \cdot \quad (13)$$

We note that κC_1 has units of Gyr^{-1}, while m, C_1, and C_2 are the corresponding constants of the problem. Obviously, Eq. (12) can be rewritten as

$$\rho(\alpha) = \rho_c(\alpha) + \rho'(\alpha), \quad (14)$$

where $\rho_c = C_1\alpha^m$ is the density corresponding to the residual matter creation that results from a possible disequilibrium between the particle creation and annihilation processes, while ρ' can be viewed as the energy density of the self-interacting dark matter particles that are dominated by the annihilation processes. This can easily be understood if we define the constant C_1 to equal to zero, implying that the creation term is negligible and reducing the current solution (Eq. (14)) to that of Eq. (9). We note that close to the present epoch as well as at late enough times ($\alpha \gg 1$), as also in *Case 2*, the ρ' evolves in a similar way to the usual dark matter (see also Weinberg 2008). Finally, if both κ and Ψ tend to zero, the above cosmological model reduces to the usual Einstein-deSitter model (*Case 1*).

We note that, since $\rho' > 0$, the constant C_2 obeys the restriction

$$C_2 > G(\alpha) = \int_1^{\alpha} x^{-3} f(x) F(x)\mathrm{d}x \geq 0. \quad (15)$$

Evaluating now Eq. (12) at the present time ($\alpha = 1$, $F(\alpha) = 1$), we obtain the present-time total cosmic density, which is: $\rho_0 = C_1 + 1/C_2$, with $C_1 \geq 0$ and $C_2 > 0$.

Case 4: $P = -\Psi/3H$: In this scenario, we assume that the annihilation term is negligible [$\kappa = 0$ and $f(\alpha)$=0] and the particle creation term dominates. This situation is mathematically equivalent to the gravitational DM particle creation process within the context of non-equilibrium thermodynamics Prigogine et al. (1989), the important cosmological consequence of which were studied by Lima et al. (2008, and references therein). Using our nomenclature and $\kappa = 0$, Eq. (7) becomes a first order linear differential equation, a general solution of which is:

$$\rho(\alpha) = \alpha^{-3}\left[\int_1^{\alpha} x^3 R(x)\mathrm{d}x + C_2\right]. \quad (16)$$

The negative pressure can yield a late accelerated phase of the cosmic expansion (as in Lima et al. 2008), without the need for the required (in "classical" cosmological models) dark energy.

4. Case 3: $P = (\kappa\rho^2 - \Psi)/3H$

In this section, we investigate the conditions under which Eqs. (12) and (16) could provide accelerating solutions, similar to the usual dark energy case.

4.1. Conditions to have an inflection point and galaxy formation

To have an inflection point at $\alpha = \alpha_I$, we must have $\ddot{\alpha}_I = 0$ (see Eq. (6)). The latter equality implies that the expression $\rho + 3P = 0$ should contain a real root in the interval: $\alpha \in (0, 1)$. Therefore, with the aid of Eq. (12), (5) and (11), we derive the following condition:

$$\frac{\alpha^{-3}F(H + 2\kappa C_1\alpha^m)}{C_2 - G} + \frac{\kappa\alpha^{-6}F^2}{(C_2 - G)^2} - (m+2)C_1\alpha^m H = 0, \quad (17)$$

from which we obtain that $m > -2$ (where $C_1 > 0$, $\kappa \geq 0$, and $C_2 - G > 0$). Evidently, if we parametrize the constant m according to $m = -3(1 + w_{\rm IDM})$, we obtain the condition $w_{\rm IDM} < -1/3$, which implies that the current cosmological model can be viewed as a viable *quintessence* dark-energy look-alike, as far as the global dynamics is concerned. We remind the reader that the same restriction holds for the conventional dark energy model in which $P_Q = w\rho_Q$ (w = const.; for more details see Appendix D).

Furthermore, to ensure the growth of spatial density fluctuations, the effective DM should be capable of clustering and providing the formation of galaxies, while the effective dark energy term should be close to being homogeneous. In our case, the effective term that emulates dark energy is homogeneous in the same sense as in the classical quintessence, while the $\kappa\rho^2$ term slightly modifies the pure DM evolution. In any case, the interacting DM term after the inflection point tends to an evolution $\propto a^{-3}$. During the galaxy formation epoch at high-z's, we expect (due to the functional form of the DM term) that the slope of the interacting DM term is not far from that of the classical DM evolution (we will explore these issues further in a forthcoming paper).

4.2. Relation to the Standard Λ Cosmology

As an example, we show that for $m = 0$ (or $w_{\rm IDM} = -1$), the global dynamics, provided by Eq. (12), is equivalent to that of the traditional Λ cosmology. To this end, we use $dt = d\alpha/(\alpha H)$ and the basic kernel (Eq. (13)) becomes

$$F(\alpha) = \exp\left[-2\kappa C_1 \int_1^{\alpha} \frac{1}{xH(x)} dx\right] = e^{-2\kappa C_1 (t-t_0)}, \quad (18)$$

where t_0 is the present age of the universe. In addition, the integral in Eq. (12) (see also Eq. (15)) now takes the form $G(\alpha) = -\kappa Z(t)$ and $Z(t) = \int_{t_0}^{t} \alpha^{-3} e^{-2\kappa C_1 (t-t_0)}$. We note that at the present time we have $G(1) = 0$. Therefore, using the above formula, the global density evolution (Eq. (12)) can be written

$$\rho(\alpha) = C_1 + \alpha^{-3} \frac{e^{-2\kappa C_1 (t-t_0)}}{[C_2 - G(\alpha)]} \cdot \quad (19)$$

As expected, at early enough times ($t \to 0$) the overall density scales according to $\rho(\alpha) \propto a^{-3}$, while close to the present epoch the density evolves according to

$$\rho(\alpha) \simeq C_1 + \frac{\alpha^{-3}}{C_2}, \quad (20)$$

which is approximately equivalent to the corresponding evolution in the Λ cosmology in which the term C_1 resembles the constant-vacuum term (ρ_Λ) and the $1/C_2$ term resembles the density of matter (ρ_m). We note that the effective pressure term (Eq. (5)), for $\kappa \to 0$, becomes $\Psi \sim 3C_1 H$, which implies that: $P \sim -\Psi/3H = -C_1$. Therefore, this case relates to the traditional Λ cosmology, since C_1 corresponds to ρ_Λ (see Eq. (20)). We now investigate in detail the dynamics of the $m = 0$ model.

From Eq. (19), using the usual unit-less Ω-like parametrization, we derive after some algebra that

$$\left(\frac{H}{H_0}\right)^2 = \Omega_{1,0} + \frac{\Omega_{1,0}\Omega_{2,0}\alpha^{-3} e^{-2\kappa C_1 (t-t_0)}}{\Omega_{1,0} + \kappa C_1 \Omega_{2,0} Z(t)}, \quad (21)$$

where $\Omega_{1,0} = 8\pi G C_1/3H_0^2$ and $\Omega_{2,0} = 8\pi G/3H_0^2 C_2$, which in the usual Λ cosmology relates to Ω_Λ and Ω_m, respectively.

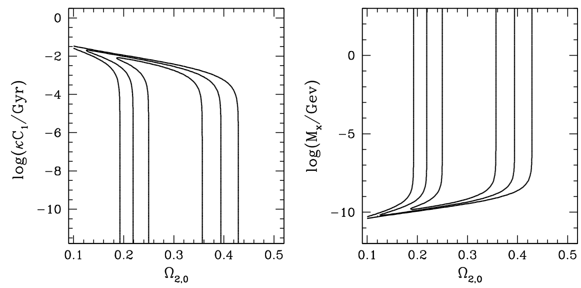

Fig. 1. *Left panel*: the $\Omega_{2,0} - \kappa C_1$ solution space provided by fitting our model to the early-type galaxy Hubble relation of Simon et al. (2005). *Right panel*: the corresponding $\Omega_{2,0} - M_x$ solution space.

We can now attempt to compare the Hubble function of Eq. (21) to that corresponding to the usual Λ model. To this end, we use a χ^2 minimization between the different models (our IDM Eq. (21) or the traditional ΛCDM model) and the Hubble relation derived directly from early-type galaxies at high redshifts (Simon et al. 2005). For the case of our IDM model, we simultaneously fit the two free parameters of the model, i.e., $\Omega_{2,0}$ and κC_1 for a flat background ($\Omega_{1,0} = 1 - \Omega_{2,0}$) with H_0 = 72 km s^{-1} Mpc^{-1} and $t_0 = H_0^{-1} \simeq 13.6$ Gyr which is roughly the age of the universe of the corresponding Λ cosmology. This procedure yields the best-fit model parameters $\Omega_{2,0} = 0.3^{+0.05}_{-0.08}$ and $\log(\kappa C_1) \simeq -9.3$ (with a stringent upper limit $\simeq -3$, but unconstrained towards lower values) where χ^2/d.f. = 1.29 (see left panel of Fig. 1). Using Eq. (4) we can now relate the range of values of κC_1 to the mass of the DM particle, from which we obtain that

$$M_x = \frac{1.205 \times 10^{-12}}{\kappa C_1} \frac{\langle\sigma u\rangle}{10^{-22}} \text{ GeV}, \quad (22)$$

(see also right panel of Fig. 1) and since κC_1 is unbound at small values, it is consistent with currently accepted lower bounds of M_x($\sim$ 10 GeV) (e.g., Cirelli et al. 2009, and references therein). The corresponding Hubble relation (Fig. 2), provided by the best-fit model free parameters, is indistinguishable from that of the traditional ΛCDM model, because of the very small value of $\kappa C_1 \simeq 10^{-9.3}$. For completeness, we also show, as the dashed line, the IDM solution provided by $M_x \sim 1$ eV ($\kappa C_1 \simeq 10^{-3}$), which is the stringent lower bound found by our analysis. In this case, the predicted Hubble expansion deviates significantly from the traditional Λ model at small α values indicating that it would probably create significant alterations to the standard BBN (e.g. Iocco et al. 2009, and references therein).

Although the present analysis does not provide any important constraints on M_x (within our model), we plan on the future to use a large amount of cosmologically relevant data to attempt to place stronger M_x constraints, also in the general case (see Eq. (12)).

5. Case 4: $P = -\Psi/3H$

We now prove that for $\kappa = 0$ (negligible annihilation), the global dynamics resembles that of the traditional quintessence cosmology (w = constant). Using again the phenomenologically selected form of Ψ, provided by Eq. (11), we obtain $R(\alpha) = C_1(m + 3)\alpha^{m-1}$. It is then straightforward to obtain the density evolution from Eq. (16), as:

$$\rho(\alpha) = \mathcal{D}\alpha^{-3} + C_1\alpha^m, \quad (23)$$

where $\mathcal{D} = C_2 - C_1$. The conditions in which the current model acts as a quintessence cosmology, are given by $\mathcal{D} > 0$, $C_1 > 0$,

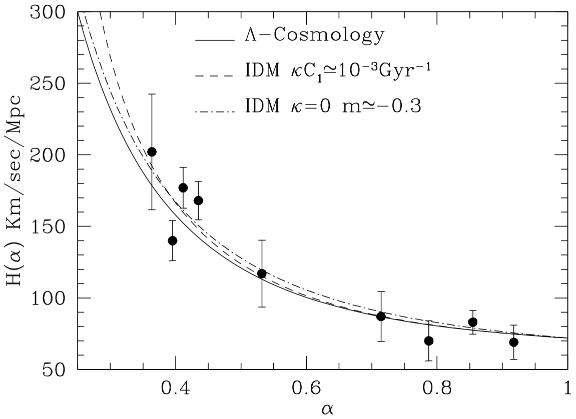

Fig. 2. Comparison of the Hubble function provided by the traditional ΛCDM model, which coincides with our $m = 0$ model (for the best-fit model of the two free parameters - see text). The dashed line corresponds to our $m = 0$ IDM model for the highest κC_1 bound, provided by our fitting procedure ($\sim 10^{-3}$). The dot-dashed line corresponds to our $\kappa = 0$ IDM model (*Case 4*) for the best-fit model parameters ($m \simeq -0.3$ and $\Omega_{2,0} \simeq 0.28$). Finally, the points correspond to the observational data (Simon et al. 2005).

and $w_{\rm IDM} = -1 - m/3$, which implies that to have an inflection point, the following should be satisfied: $w_{\rm IDM} < -1/3$ or $m > -2$ (see Appendix D). We note, that the Hubble flow is now given by

$$\left(\frac{H}{H_0}\right)^2 = \Omega_{2,0}\alpha^{-3} + \Omega_{1,0}\alpha^m, \quad (24)$$

where $\Omega_{2,0} = 8\pi G\mathcal{D}/3H_0^2$ and $\Omega_{1,0} = 8\pi GC_1/3H_0^2$. Finally, by minimizing the corresponding χ^2, we find that the best-fit model values are $\Omega_{2,0} \simeq 0.28$ and $m \simeq -0.30$ ($w_{\rm IDM} \simeq -0.90$) with χ^2/d.f. $= 1.29$. The corresponding Hubble flow curve is shown in Fig. 2 as the dot-dashed line. We note that this solution is mathematically equivalent to that of the gravitational matter creation model of Lima et al. (2008).

6. Conclusions

We have investigated the evolution of the global density of the universe in the framework of an interacting DM scenario by solving analytically the collisional Boltzmann equation in an expanding universe. A disequilibrium between the DM particle creation and annihilation processes, regardless of its cause and in which the particle creation term dominates, can create an effective source term with negative pressure, which acting like dark energy, provides an accelerated expansion phase of the universe. There are also solutions for which the present time is located after the inflection point. Finally, comparing the observed Hubble function of a few high-redshift elliptical galaxies with that predicted by our simplest IDM model ($m = 0$), we find that the effective annihilation term is quite small. In a forthcoming paper, we propose to use a multitude of cosmologically relevant observations to jointly fit the predicted, by our generic IDM model, Hubble relation and thus possibly provide more stringent constraints on the free parameters of the models. We also plan to derive the perturbation growth factor to study structure formation within the IDM model.

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Appendix A:The effect of the decay products

Here we attempt to investigate in the matter-dominated era, whether the possible radiation products related to dark matter interactions can affect the global dynamics. A general coupling can be viewed by the continuity equations of interacting dark matter $\rho_{\rm IDM}$ and residual radiation $\delta\rho_{\rm r}$,

$$\frac{{\rm d}\rho_{\rm IDM}}{{\rm d}t} + 3H(t)\rho_{\rm IDM} + \kappa\rho_{\rm IDM}^2 - \Psi = Q, \quad (25)$$

$$\frac{{\rm d}\delta\rho_{\rm r}}{{\rm d}t} + 4H(t)\delta\rho_{\rm r} = -Q, \quad (26)$$

where Q is the rate of energy density transfer. If $Q < 0$, then the IDM fluid transfers to residual radiation. As an example, we can use a generic model with $Q = -\epsilon\delta\rho_{\rm r}$, where $\epsilon > 0$. Thus, Eq. (26) has an exact solution

$$\delta\rho_{\rm r} = \delta\rho_{r0}\alpha^{-4}{\rm e}^{\epsilon(t-t_0)}, \quad (27)$$

where t_0 is the present age of the universe. This shows that the contribution of the residual radiation to the global dynamics was negligible in the past, since there is not only the usual $\propto a^{-4}$ dependence of the background radiation but also a further exponential drop, and thus $Q \simeq 0$. We therefore conclude that we can approximate the total energy-density with that of the interacting dark-matter density ($\rho \simeq \rho_{\rm IDM}$). Note, that $1/\epsilon$ can be viewed as the mean lifetime of the residual radiation particles.

Appendix B: Solutions of the Riccati equation

With the aid of differential equation theory we present solutions that are relevant to our Eq. (7). In general, a Riccati differential equation is given by

$$y' = f(x)y^2 + g(x)y + R(x) \quad (28)$$

and it is fully solvable only when a particular solution is known. Below, we present two cases in which analytical solutions are possible:

– *Case 1*: for the case where

$$R(x) = C_1 m x^{m-1} - C_1^2 x^{2m} f(x) - C_1 x^m g(x) \quad (29)$$

the particular solution is x^m and thus the corresponding general solution can be written as

$$y(x) = C_1 x^m + \Phi(x)\left[C_2 - \int_1^x f(u)\Phi(u){\rm d}u\right]^{-1}, \quad (30)$$

where

$$\Phi(x) = \exp\left[\int_1^x (2C_1 u^m f(u) + g(u))\,{\rm d}u\right] \quad (31)$$

and C_1, C_2 are the integration constants. Using now Eq. (8), we obtain $\Psi(x) = xH(x)R(x) = C_1(m+3)x^m H(x) + \kappa C_1^2 x^{2m}$.

- *Case 2*: for the case where

$$R(x) = h'(x) \quad \text{with} \quad g(x) = -f(x)h(x), \tag{32}$$

the particular solution is $h(x)$ (in our case we have $h(x) = -3\kappa^{-1}H(x)$). The general solution now becomes

$$y(x) = h(x) + \Phi(x)\left[C_2 - \int_1^x f(u)\Phi(u)\mathrm{d}u\right]^{-1}, \tag{33}$$

where

$$\Phi(x) = \exp\left[\int_1^x f(u)h(u)\mathrm{d}u\right]. \tag{34}$$

In this framework, using Eq. (8) we finally obtain $\Psi(x) = xH(x)R(x) = -3\kappa^{-1}xH(x)H'(x)$.

Note that the solution to *Case 1* is the only one providing a $\propto\alpha^{-3}$ dependence of the scale factor (see Eqs. (12), (19) and (20)).

Appendix C: Justification of the functional form of Ψ

We assume that we have a non-perfect cosmic fluid in a disequilibrium phase with energy density ρ then from the collisional Boltzmann equation, we have that

$$\Psi = \dot{\rho} + 3H\rho + \kappa\rho^2 = \frac{\mathrm{d}\rho}{\mathrm{d}a}aH + 3H\rho + \kappa\rho^2. \tag{35}$$

Furthermore, we assume that for a convenient period of time, the cosmic fluid, in an expanding Universe, is slowly diluted according to $\rho \sim C_1\alpha^m$ ($m \leq 0$). From a mathematical point of view, the latter assumption simply means that a solution of the form $\propto\alpha^m$ is a particular solution of the Boltzmann equation. Therefore, we have finally that:

$$\Psi \simeq C_1(m+3)a^m H + \kappa C_1^2 a^{2m}. \tag{36}$$

Appendix D: Correspondence between our model and conventional dark energy models

We remind the reader that for homogeneous and isotropic flat cosmologies ($\Omega_\mathrm{m} + \Omega_Q = 1$), controlled by non-relativistic DM and a DE with a constant equation of state parameter (w), the density evolution of the cosmic fluid can be written as

$$\rho(\alpha) = \rho_{\mathrm{m},0}\alpha^{-3} + \rho_{Q,0}\alpha^{-3(1+w)}, \tag{37}$$

where $\rho_{\mathrm{m},0}$ and $\rho_{Q,0}$ are the present-day DM and DE densities, respectively.

The necessary criteria for cosmic acceleration and an inflection point in our past ($t_i < t_0$), are (a) $P < 0$ and (b) $\ddot{\alpha} = 0$, which leads to the conditions

- **Dark Energy models**: $P = P_m + P_Q = w\rho_Q < 0$, $P_m = 0$ with $w < -1/3$.
- **IDM models**: $P = \kappa\rho^2 - \Psi < 0$ and $m > -2$ (or $w_\mathrm{IDM} < -1/3$).

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