A&A 500, 103–104 (2009)
DOI: 10.1051/0004-6361/200912150

Astronomy & Astrophysics
Special issue

COMMENTARY ON: CAVALIERE A. AND FUSCO-FEMIANO R., 1976, A&A, 49, 137

The β-model of the intracluster medium

M. Arnaud

Laboratoire AIM, IRFU/Service d'Astrophysique – CEA/DSM – CNRS – Université Paris Diderot, Bât. 709, CEA-Saclay,
91191 Gif-sur- Yvette Cedex, France
e-mail: `Monique.Arnaud@cea.fr`

The space between the galaxies in clusters of galaxies is filled with a hot tenous gas. Its high temperature (0.5–15 keV) is set by the the virialization process in the cluster's deep potential well, and the gas emits in X-rays predominantly through thermal Bremsstrahlung. This intracluster medium (ICM) is the main baryonic components of clusters and accounts for about 10% of the total dark matter dominated mass. While this is now well-established, it was not the case in the mid 70s, when the pioneering paper by Cavaliere & Fusco-Femiano (1976), which set the basis of the powerful β-model, was written.

The idea of an ICM was proposed as early as 1959, but it took 10 years after its first detection in 1966 to definitively prove the existence of the hot ICM (see also Sarazin 1986; Biviano 2000, for a historical review). Limber (1959) stated that it is "rather likely that there is an appreciable quantity of intracluster gas pervading (...) clusters", primordial gas left over from galaxy formation, gas swept out of galaxies during galaxy collisions, or both. However, "it (was) not at all clear at (that) time whether one should expect the intracluster gas to be hot or cold". The first observational clue was the detection of M 87 in the Virgo cluster with the Geiger counters onboard an Aerobee rocket (Byram et al. 1966), which was also the first detection of an X-ray source associated with a cluster. The next major advance was made with *Uhuru*, the first satellite dedicated to X-ray astronomy, launched in 1970. *Uhuru* performed an all-sky X-ray survey and permitted longer observations of individual sources than with rockets experiments. This established that clusters are X-ray luminous sources and that the X-ray emission is extended and not time variable (e.g. Gursky et al. 1972, online).

The detectors onboard *Uhuru* and rocket experiments were non-imaging proportional counters equipped with collimators to limit the field of view. Because of the crude spectral and spatial resolution (∼1°) of the detectors, the origin of the X-ray emission was ambiguous. Three possibilities were considered: 1) thermal bremsstrahlung from a hot tenuous gas; 2) inverse Compton (IC) scattering of CMB photons by relativistic electrons within the clusters; and 3) summed contributions of stellar sources (binaries or globular clusters). This was discussed very early by Felten et al. (1966) who correctly favored the thermal model on energetics arguments (unfortunately based on a spurious detection of the Coma cluster).

In their paper, Cavaliere and Fusco-Femiano outlined several problems with the IC interpretation and, in particular, with the low value of the magnetic field implied by the ratio of the X-ray and Synchrotron radio luminosity. On the other hand, they emphasized that the IC emission is expected to stand out at energies above the thermal emission's exponential cut-off, $E \gtrsim 10$ keV. The IC emission was indeed searched later on as an excess over the thermal emission in the hard X-ray band with *Beppo–SAX*, *RXTE*, and now *Suzaku*. Its level remains uncertain, and the high magnetic field value generally inferred is still an issue. Cavaliere and Fusco-Femiano then focused on the thermal interpretation and proposed a model for the gas distribution that they compared successfully with the *Uhuru* observations of Coma. The thermal origin of the X-ray emission from clusters was definitively demonstrated shortly afterwards with the detection of the 7 keV iron line, first in the spectra of Perseus and Coma measured with the *OSO-8* and *Ariel 5* satellites (Mitchell et al. 1976; Serlemitsos et al. 1977).

The hot ICM model proposed by Cavaliere & Fusco-Femiano (1976) was a major improvement over the model used previously by Lea et al. (1973) to fit the Coma, Perseus, and Virgo data. Lea et al. (1973) assumed that the gas was an isothermal self-gravitating plasma. This model was not consistent, since the derived gas mass was significantly lower than the bounding mass. Instead, Cavaliere and Fusco-Femiano assumed that the gas and the galaxies are in equilibrium in the same potential $\phi(r)$. (As mentioned in the note added in proof, a similar model was considered independently by Lea 1975 and Gull & Northover 1975.) A fundamental feature of the model is that the galaxy and gas radial distribution, $\rho_{\rm gal}$ and $n_{\rm gas}$, can then be directly related (their Eq. (1)) via the equilibrium equations:

$$\frac{1}{n_{\rm gas}}\frac{{\rm d}((kT/\mu\, m_{\rm p})n_{\rm gas})}{{\rm d}r} = -\frac{{\rm d}\phi}{{\rm d}r} = \frac{1}{\rho_{\rm gal}}\frac{{\rm d}(\sigma_{\rm r}^2\rho_{\rm gal})}{{\rm d}r} \qquad (1)$$

where T is the gas temperature and $\sigma_{\rm r}$ the galaxy velocity dispersion, assumed to be isotropic. Models for the galaxy distribution were available, issued from dynamical studies of self-gravitating collisionless systems (developed for globular clusters), assuming that the galaxy and the dark matter follow the same distribution. Cavaliere and Fusco-Femiano used the isothermal sphere model ($\sigma_{\rm r} = cst$) and the self-consistent truncated model of King (1966), which avoids the divergence of the total mass with r of the former model. They also considered both an isothermal and an adiabatic equation of state for the gas. For the simplest isothermal case, where both T and σ_r are spatially invariant, Eq. (1) reduces to their Eq. (2):

$$\frac{n_{\rm gas}(r)}{n_{\rm gas}(0)} = \left[\frac{\rho_{\rm gal}(r)}{\rho_{\rm gal}(0)}\right]^{\beta} \quad \text{with } \beta = \frac{\mu\, m_{\rm p}\, \sigma_{\rm r}^2}{kT}\,. \qquad (2)$$

However, the galaxy distribution derived in the isothermal model (or the truncated model) is not a simple analytical function. The full effectiveness of the model arose when Cavaliere & Fusco-Femiano (1978) proposed to use the empirical King approximation for this model, $\rho_{\rm gal}(r) = \rho_{\rm gal}(0)[1+r/r_{\rm c})^2]^{-3/2}$ where $r_{\rm c}$ is the core radius, yielding

$$n_{\rm gas}(r) = n_{\rm gas}(0)\left[1+\left(\frac{r}{r_{\rm c}}\right)^2\right]^{-3\beta/2}\,. \qquad (3)$$

This famous isothermal β-model[1] has the big advantage that it is simple, with a minimum of parameters. All the integrals needed to compute cluster quantities, such as the X-ray surface brightness, Sunayev-Zel'dovich decrement, gas mass, luminosity, and the total mass profile derived from the hydrostatic equilibrium equation, are also analytic. In particular, the surface brightness ($S_{\rm X}$) profile also follows a β-model with a slope of $-3\beta + 1/2$ (Gorenstein et al. 1978). The β-model can still be used for the density profile even if the temperature varies, although the model is no longer physically consistent. One simply fits the $S_{\rm X}$ profile extracted in the soft energy band, where the X-ray emissivity depends weakly on temperature.

The β-model was extensively used to fit cluster $S_{\rm X}$ profiles measured with *Einstein*, the first satellite with focusing telescopes (Jones & Forman 1984). This model was found to describe the data well, with $\beta \sim 2/3$. A central surface brightness (hence density) excess was detected in some clusters, though, which was explained as resulting from a cooling flow. It was also widely used to analyze *ROSAT* imagery data. When significant, the central excess was dealt with by excluding the central region in fitting the data or using an empirical double β-model (e.g. Mohr et al. 1999, online). Together with temperatures estimated from global spectra or temperature profiles for the brightest clusters that became available with the launch of *ASCA* (Markevitch et al. 1998), this modeling of the gas density profile was the basis of virtually all the statistical studies of cluster scaling and structural properties in the pre *XMM* and *Chandra* era. This includes the discovery of the "excess" gas entropy, the most direct evidence of the importance of non-gravitational processes, galaxy energy feedback, and cooling, in cluster evolution. The isothermal β-model is also commonly used for analyzing measurements of the Sunayev-Zel'dovich (SZ) effect.

From *XMM* and *Chandra* data, it is now well-established that the ICM is not isothermal, with decreasing temperature profiles on a large scale. Further deviations from the density β-model (Eq. (3)) were shown by the best *ROSAT* data and *XMM* and *Chandra* data. More centrally peaked density profiles are found, even for weakly cooling clusters, a consequence of the cusped nature of the dark matter profile not accounted for by King's model (Pratt & Arnaud 2002). The β-model also overestimates the true profiles at $r \gg r_{\rm c}$. They are steeper than the power-law behavior of the β-model (Vikhlinin et al. 1999), as is also found in numerical simulations (e.g. Rasia et al. 2004, online). New empirical parametric models of increasing complexity have been introduced (e.g. Vikhlinin et al. 2006), as well as new robust $S_{\rm X}$ deprojection methods (Croston et al. 2006). Such improved analyses are specially important when the physical quantity one derives from the data is particularly sensitive to the core and/or outskirt properties. This includes the hydrostatic mass profile in the center (critical for dark matter concentration estimates) and on a large scale (critical for precise cluster mass estimates), but also the integrated SZ decrement at large radii, for which more realistic models are beginning to be used (Mroczkowski et al. 2009).

On the theory side, N-body/hydrodynamical cosmological simulations and analytical models of cluster formation, both of increasing sophistication, have been developed (see, e.g., the last paper by Cavaliere et al. 2009, online). One might conclude that the isothermal β-model is now obsolete, both from a theoretical and observational point of view. This is not true. This model is still fruitful. I believe that its longstanding success stems from its being a simple model that is easy to use and was adapted to the statistical quality of X-ray data for a long while, along with capturing the main characteristics of the ICM in a first approximation. When used with caution, it remains a working model, keeping in mind its limitations. It is useful both for observers when the data do not allow or require more sophisticated analysis (e.g. high z clusters) and for theoreticians when crude modeling of the ICM structure is sufficient (see Bower et al. 2008, for a recent example).

References

Biviano, A., in Constructing the Universe with Clusters of Galaxies, IAP 2000 meeting, Paris, ed. F. Durret, & D. Gerbal [`arXiv:astro-ph/0010409`]
Bower, R., McCarthy, I., & Benson, A. 2008, MNRAS, 390, 1399
Byram, E., Chubb, T., & Friedman, H. 1966, Science, 152, 66
Cavaliere, A., & Fusco-Femiano, R. 1976, A&A, 49, 137
Cavaliere, A., & Fusco-Femiano, R. 1978, A&A, 70, 677
Cavaliere, A., Lapi, A., & Fusco-Femiano, R. 2009, ApJ, in press [`arXiv:0903.5224`]
Croston, J., Arnaud, M., Pointecouteau, E., & Pratt, G. 2006, A&A, 459, 1007
Felten, J., Gould, R., Stein, W., & Woolf, N., 1966, ApJ, 146, 955
Gorenstein, P., Fabricant, D., Topka, K., Harnden, F., & Tucker, W. 1978, ApJ, 224, 718
Gull, S., & Northover, K. 1975, MNRAS, 173, 583
Gursky, H., Solinger, A., Kellogg, M., et al. 1972, ApJ, 173, L99
Henriksen, M., & Mushotzky, R. 1985, ApJ, 292, 441
Jones, C., & Forman, W. 1984, ApJ, 276, 38
Kellogg, E. 1973, in X- and Gamma-Ray Astronomy, ed. H. Bradt, & R. Giacconi, IAU Symp. (Dordrecht: Reidel), 55, 171
King, I. R. 1966, AJ, 71, 64
Lea, S., Silk, J., Kellogg, E., & Murray, S. 1973, ApJ, 184, L105
Lea, S. 1975, Astrophys. Lett., 16, L141
Limber, D. 1959, ApJ, 130, 414
Markevitch, M., Forman, W., Sarazin, C., & Vikhlinin, A. 1998, ApJ, 503, 77
Mohr, J., Mathiesen, B., & Evrard, A. 1999, ApJ, 517, 627
Mitchell, R. J., Culhane, J. L., Davison, P. J. N., & Ives, J. C. 1976, MNRAS, 175, 29
Mroczkowski, T., Bonamente, M., Carlstrom, J. 2009, ApJ, 694, 1034
Pratt, G. W., & Arnaud, M. 2002, A&A, 394, 375
Rasia, E., Tormen, G., & Moscardini, L. 2004, MNRAS, 351, 237
Serlemitsos, P. J., Smith, B. W., Boldt, E. A., Holt, S. S., & Swank, J. H. 1977, ApJ, 211, L63
Sarazin, C. 1986, Rev. Mod. Phys., 58, 1
Vikhlinin, A., Forman, W., & Jones, C. 1999, ApJ, 525, 47
Vikhlinin, A., Kravtsov, A., Forman, W., et al.2006, ApJ, 640, 691

---

[1] It is amusing to note that Cavaliere & Fusco-Femiano (1978, 1976) originally used the symbol τ for the β parameter, a notation introduced by Gorenstein et al. (1978), and that the now universal denomination of this model, as the "β-model", was introduced rather late by Henriksen & Mushotzky (1985), to my knowledge.