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LETTER TO THE EDITOR

Clusters in the solar neighbourhood: how are they destroyed?

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ABSTRACT

We predict the survival time of initially bound star clusters in the solar neighbourhood taking into account: (1) stellar evolution, (2) tidal stripping, (3) shocking by spiral arms and (4) encounters with giant molecular clouds. We find that the predicted dissolution time is $t_{\rm dis} = 1.7(M_i/10^4\ M_\odot)^{0.67}$ Gyr for clusters in the mass range of $10^2 < M_i < 10^5\ M_\odot$. The resulting predicted shape of the logarithmic age distribution agrees very well with the empirical one, derived from a complete sample of clusters in the solar neighbourhood within 600 pc. The required scaling factor implies a star formation rate of $4 \times 10^2\ M_\odot$ Myr^{-1} within 600 pc from the Sun or a surface formation rate of $3.5 \times 10^{-10}\ M_\odot$ yr^{-1} pc^{-2} for stars in bound clusters with an initial mass in the range of 10^2 to $3 \times 10^4\ M_\odot$.

Key words. Galaxy: open clusters and associations: general – Galaxy: solar neighbourhood – Galaxy: disk – Galaxy: general – Galaxies: star clusters – Galaxy: kinematics and dynamics

1. Introduction

The first empirical determination of the lifetime of clusters in the solar neighbourhood is by Oort (1958), who noticed the lack of clusters older than a few Gyr in the solar neighbourhood. Later, Wielen (1971) derived a mean dissolution time of 0.2 Gyr from the age distribution of clusters. Since most of the observed clusters within about 1 kpc from the sun have a mass in the range of 10^2 to a few $10^3\ M_\odot$ the value derived by Wielen is for clusters in that mass range. Theory predicts that the dissolution time of clusters depends on their initial mass in that massive clusters survive longer than low mass clusters (e.g. Spitzer 1958; Wielen 1985; Chernoff & Weinberg 1990; Gnedin & Ostriker 1997, and references therein). Baumgardt & Makino (2003) (hereafter BM03) showed from N-body simulations that the dissolution time of clusters in the tidal field of the galaxy depends on their initial mass, M_i, as $t_{\rm dis} \sim M_i^{0.62}$. Independently, this same power-law dependence was also derived empirically in a study of cluster samples in four galaxies by Boutloukos & Lamers (2003).

The dissolution time of clusters in the solar neighbourhood was recently redetermined by Lamers et al. (2005) (hereafter L05), based on a new cluster sample of Kharchenko et al. (2005). They found a dissolution time of $t_{\rm dis} = t_4(M_i/10^4\ M_\odot)^{0.62}$ with $t_4 = 1.3 \pm 0.5$ Gyr for clusters with $10^2 < M < 10^4$. This is a factor 5 shorter than the $t_4 = 6.9$ Gyr that follows from the $t_{\rm dis}(M_i)$ relation derived by the N-body simulations of BM03 for clusters more massive than 4500 $M_\odot$ at a Galactocentric distance of 8.5 kpc. The simulations of BM03 include a realistic stellar mass function, stellar evolution, two-body relaxation, a detailed treatment of binary evolution and close encounters of stars. Part of this difference may be due to the fact that in low mass clusters, with lifetimes shorter than about 1 Gyr, the dynamical evolution is affected by the presence of massive stars during most of their lifetime (see Fig. 5 of BM03). It is doubtfull that this effect alone can fully explain the difference between the results of L05 and BM03. In fact, the large discrepancy suggests that other, probably external, disruptive effects must play an important role in destroying star clusters in the solar neighbourhood.

In this paper we explain the lifetime of clusters in the solar neighbourhood, by taking into account the combined effects of stellar evolution, tidal stripping, encounters with giant molecular clouds (GMCs) and spiral arm shocks. We use stellar population models to describe the stellar evolution and the results of BM03 for tidal stripping. For the effects of GMCs and spiral arms we adopt the new estimates from the recent studies by Gieles et al. (2006c) (hereafter GPZB06) and Gieles et al. (2006a) (hereafter GAPZ06), which are based on N-body simulations.

The structure of the paper is as follows. In Sect. 2 we discuss the predicted mass loss from star clusters by stellar evolution, tidal stripping, encounters with GMCs and spiral arm shocks. We calculate the mass evolution of clusters due to these four effects. In Sect. 3 we compare our results with the observed age distribution of clusters in the solar neighbourhood. The discussion and conclusions are given in Sect. 4.

2. The decreasing mass of star clusters

2.1. Mass loss by stellar evolution

The mass loss from clusters due to stellar evolution has been calculated for cluster evolution models by several groups. We adopt the *GALEV* models for single stellar populations with a Salpeter type mass function in the range of $0.15 < M/M_\odot < 85$ (Schulz et al. 2002; Anders & Fritze-v. Alvensleben 2003). These models are based on stellar evolution tracks from the Padova group, which include overshooting, mass loss due to stellar winds and supernovae. (Bertelli et al. 1994; Girardi et al. 2000). L05 have shown that the fraction of the initial cluster mass that is lost by stellar evolution, $q_{\rm ev}(t) = \Delta M/M_i$, can be approximated accurately by

$$\log q_{\rm ev}(t) = (\log t - a_{\rm ev})^{b_{\rm ev}} + c_{\rm ev} \quad \text{for} \quad t > 12.5\ \text{Myr}. \tag{1}$$

with t in yrs. For solar metallicity models with a Salpeter mass function $a_{\rm ev} = 7.00$, $b_{\rm ev} = 0.255$ and $c_{\rm ev} = -1.805$. This function describes the mass loss fraction of the models at $t > 12.5$ Myr with an accuracy of a few percent. The mass loss at younger ages is negligible because stars with $M_* > 30\ M_\odot$ hardly contribute to the mass of the cluster. The mass loss of a cluster by stellar evolution is

$$\left(\frac{{\rm d}M}{{\rm d}t}\right)_{\rm evol} = -M(t)\frac{{\rm d}q_{\rm ev}}{{\rm d}t}. \tag{2}$$

2.2. Mass loss by the galactic tidal field

BM03 have calculated a grid of N-body simulations of clusters in circular and elliptical orbits in the tidal field of a galaxy for different initial cluster masses, galactocentric distances R, and different cluster density profiles. The stars follow a Kroupa initial mass function and stellar evolution is taken into account during the evolution. Gieles et al. (2004) have shown that for all models with clusters with $M_i > 4500\ M_\odot$ of BM03 the dissolution time can be expressed as a function of the initial cluster mass as

$$t_{\rm dis} = t_4\ (M_i/10^4\ M_\odot)^{0.62}, \tag{3}$$

where t_4 is a constant that depends on the tidal field strength of the galaxy in which the cluster moves and on the ellipticity of its orbit. (If the tidal field were the only disruptive process then t_4 would be the lifetime of a cluster with $M_i = 10^4\ M_\odot$.) The mass loss due to the Galactic tidal field can then be written as

$$\left(\frac{{\rm d}M}{{\rm d}t}\right)_{\rm tidal} = \frac{-M(t)}{t_{\rm dis}} = \frac{-(M/10^4\ M_\odot)^{0.38}}{t_4/10^4}\ M_\odot\ {\rm Myr}^{-1}. \tag{4}$$

We adopt the value of $t_4 = 6.9$ Gyr from BM03 for clusters in circular orbits at $R_0 = 8.5$ kpc, although this may slightly overestimate the disruption time of the lower mass clusters by about a factor 2 or so (see Sect. 1).

2.3. Mass loss by spiral arm shocking

GAPZ06 studied the dissolution of star clusters by spiral arms by means of N-body simulations. They used and adjusted the analytical expression of Ostriker et al. (1972) for the dissolution time of star clusters due to disk shock, to derive an expression for the dissolution time of star clusters by spiral arms ($t_{\rm sp}$). Mass loss by spiral arm shocks will occur just at the moment the cluster crosses the spiral arm. Assuming that spiral arms move with a constant pattern speed ($\Omega_{\rm p}$) and that the matter in the disk has a constant circular velocity ($V_{\rm disk}$), the relative velocity between the two ($V_{\rm drift}$) depends on the location in the galaxy (R). Density waves that pass with a low velocity have a large effect on the star clusters (e.g. Ostriker et al. 1972). Therefore, the disruptive effect of spiral arm shocks is most important close to the corotation radius ($R_{\rm CR}$), i.e. the point where the disk and the spiral arms have the same rotational velocity. We adopt the "average" spiral arm model of GAPZ06, which is based on the study of Elmegreen et al. (1989) of the spiral galaxies M81 and M100, to derive the density contrast of the spiral arm. Dias & Lépine (2005) found $\Omega_{\rm p} = 25.9$ km s^{-1} kpc^{-1} for the spiral arms in the Galaxy, from a study of the nearby star clusters, and a corotation radius ($R_{\rm CR}$) almost coinciding with the solar radius $R_{\rm CR}/R_0 = 1.06 \pm 0.08$. Based on the adopted values of $R_0 = 8.5$ kpc, $V_{\rm disk} = 220$ km s^{-1} and the assumption that our Galaxy has 4 spiral arms (Vallée 2005), GAPZ06 used $V_{\rm drift} = 12.5$ km s^{-1} = 12.7 pc Myr^{-1} and $t_{\rm drift} = 1.05$ Gyr. Taking into account the ratio $f = (\Delta M/M)/\Delta E/E) = 0.3$ between the energy gain and the mass loss, predicted by GAPZ06 we find for the solar neighbourhood that

$$\begin{aligned} t_{\rm sp} &= 20\left(\frac{M}{10^4\ M_\odot}\right)\left(\frac{3.75\ {\rm pc}}{r_{\rm h}}\right)^3\ {\rm Gyr} \\ &= 20\left(M/10^4\ M_\odot\right)^{1-3\lambda}\ {\rm Gyr}, \end{aligned} \tag{5}$$

where we have substituted the observed mass-radius relation of clusters in nearby spiral galaxies of Larsen (2004): $r_{\rm h} = 3.75\ (M/10^4\ M_\odot)^\lambda$, with $\lambda = 0.10 \pm 0.03$. The mass loss of clusters due to spiral arm shocks is then

$$\left(\frac{{\rm d}M}{{\rm d}t}\right)_{\rm sp} = \frac{-M(t)}{t_{\rm ds}} = -0.5\left(\frac{M(t)}{10^4\ M_\odot}\right)^{3\lambda}\ M_\odot\ {\rm Myr}^{-1}. \tag{6}$$

Notice that for $\lambda = 0.1$ the mass loss due to shocking by spiral arms has almost the same mass dependence, i.e. $\propto M^{0.3}$, as the mass loss by the tidal field (viz. $M^{0.38}$). We will use the mass loss relation Eq. (6) as a statistical mean.

2.4. Mass loss by giant molecular cloud encounters

GPZB06 studied the encounters between GMCs and clusters with N-body simulations. They derived an expression for the energy gain and the resulting mass loss for the full range of encounter distances, from head-on to distant encounters. Adopting a mean GMC density in the galactic plane near the sun of $\rho_n = 0.03\ M_\odot$ pc^{-3}, a surface density of GMCs $\Sigma_n = 170\ M_\odot$ pc^{-2} (Solomon et al. 1987) and a mean velocity dispersion of clusters and GMCs of $\sigma_v \simeq 10$ km s^{-1}, they derived a dissolution time ($t_{\rm GMC}$) for clusters by GMC encounters in the solar neighbourhood of

$$t_{\rm GMC} = 2.0\left(\frac{M}{10^4\ M_\odot}\right)\left(\frac{3.75\ {\rm pc}}{r_{\rm h}}\right)^3\ {\rm Gyr}. \tag{7}$$

If we assume the same mass-radius relation as before we find that the mass loss rate due to encounters with GMCs is

$$\left(\frac{{\rm d}M}{{\rm d}t}\right)_{\rm GMC} = \frac{-M(t)}{t_{\rm GMC}} = -5.0\left(\frac{M(t)}{10^4\ M_\odot}\right)^{3\lambda}\ M_\odot\ {\rm Myr}^{-1}. \tag{8}$$

Notice that the mass dependence is the same as for dissolution by spiral arm shocking, but that the effect is ten times stronger.

2.5. The predicted mass evolution of clusters in the solar neighbourhood

The decrease of mass due to the combined effects of stellar evolution, tidal stripping, spiral arm shocks and GMC encounters can then be described as

$$\frac{{\rm d}M}{{\rm d}t} = \left(\frac{{\rm d}M}{{\rm d}t}\right)_{\rm ev} + \left(\frac{{\rm d}M}{{\rm d}t}\right)_{\rm tidal} + \left(\frac{{\rm d}M}{{\rm d}t}\right)_{\rm sp} + \left(\frac{{\rm d}M}{{\rm d}t}\right)_{\rm GMC}, \tag{9}$$

with the terms given by Eqs. (2), (4), (6) and (8). We have solved this equation numerically for clusters of different masses. The results are shown in Fig. 1 for a cluster with an initial mass of

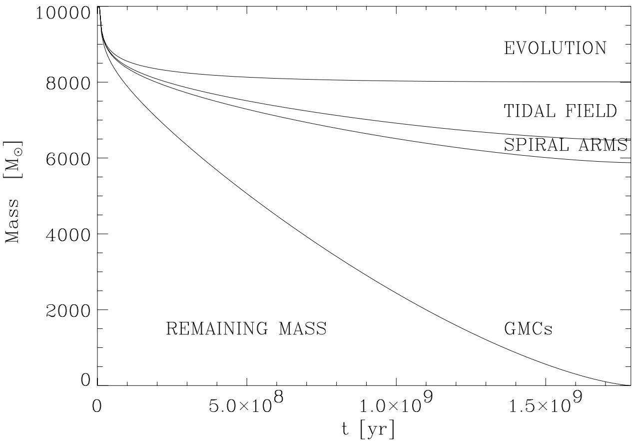

Fig. 1. The mass evolution of a cluster with an initial mass of 10^4 $M_\odot$ in the solar neighbourhood. The mass loss due to the four separate effects is indicated. Encounters with GMCs are the dominant dissolution effect in the solar neighbourhood.

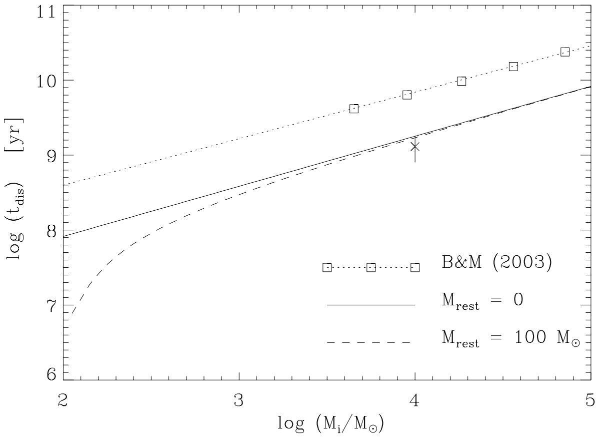

Fig. 2. The predicted dissolution times of clusters in the solar neighbourhood due to the combined effects of stellar evolution, tidal field, spiral arm shocks and encounters with GMCs, as a function of the initial mass. Full line: total dissolution time. Dashed line: time when the remaining mass is 100 $M_\odot$. Squares and dotted line: dissolution time due to stellar evolution and the Galactic tidal field only, predicted by BM03. Cross with error bar: the value of t_4 empirically derived by L05.

10^4 $M_\odot$. The figure shows the total mass loss as well as the mass lost by each mechanism independently. Encounters with GMCs are the dominant dissolution effect in the solar neighbourhood, contributing about as much as the three other effects combined.

Figure 2 shows the ages of clusters when their remaining mass is 0 and 100 $M_\odot$ as a function of the initial mass. The almost linear part from $\log(M_i/M_\odot) = 3.5$ to 5 has a slope of about 0.67. The figure also shows the dissolution times by the Galactic tidal field, predicted by BM03 for clusters with an initial concentration factor $W_0 = 5$ in a circular orbit at $R_0 = 8.5$ kpc. Our predicted timescales are about a factor 5 smaller, which agrees with the empirically determined dissolution time (L05).

3. Comparison with observed age distribution of clusters in the solar neighbourhood

Given the initial mass distribution of the clusters, their formation rate, CFR(t), and the time it takes for a dissolving cluster to fade below the detection limit, we can predict the distribution of observable clusters as a function of age or mass. L05 have derived an expression for the general case of a cluster sample that is set by a magnitude limit. Here we are interested in the prediction for a cluster sample that is complete down to a mass limit of 100 $M_\odot$, because this is the mass limit of the unbiased sample of clusters within 600 pc of Kharchenko et al. (2005) (see L05).

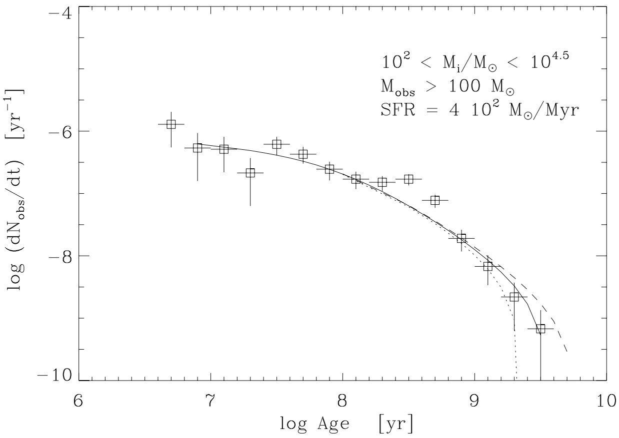

Fig. 3. The observed age distribution of an unbiased sample of clusters with $M > 100$ $M_\odot$ in the solar neighbourhood within 600 pc (Karchenko et al. 2005; L05) in units of nr yr^{-1} is given by squares with the Poisson error bars. The full line shows the predicted distribution for a cluster sample with a maximum mass of $M_{\rm max} = 3 \times 10^4$ $M_\odot$ and a SFR of 4×10^2 $M_\odot$ Myr^{-1}. The dotted line is for $M_{\rm max} = 1.5 \times 10^4$ $M_\odot$ and the dashed line is for $M_{\rm max} = 6 \times 10^4$ $M_\odot$.

For a constant CFR and a power law cluster IMF with a slope of $-\alpha = -2$ (Lada & Lada 2003) the number of clusters with $M > 100$ $M_\odot$ as a function of age is

$$N_{>100}(t) = C\,(M_{\rm lim}(t)^{-1} - M_{\rm max}^{-1}), \tag{10}$$

where $M_{\rm lim}(t)$ is the *initial* mass of clusters that reach $M(t) = 100$ $M_\odot$ at age t. (Clusters of age t with a smaller initial mass have $M < 100$ $M_\odot$ by now.) $M_{\rm max}$ is the maximum *initial* mass of the clusters that are formed. The constant C is related to the star formation rate (SFR) in bound clusters as SFR $= C \ln(M_{\rm max}/M_{\rm lim})$ for $-\alpha = -2$.

Figure 3 shows a comparison between the observed age distribution of clusters with $M > 100$ $M_\odot$ within 600 pc (from L05) with the predicted distribution for $M_{\rm max} = 3 \times 10^4$ $M_\odot$. This value of $M_{\rm max}$ is adopted because the observed distribution shows a steep drop at $\log t \simeq 9.5$ (with only one cluster in the last bin) and Fig. 2 shows that this corresponds to $M_i = 3 \times 10^4$ $M_\odot$. The predicted relation for twice higher or lower values of $M_{\rm max}$ agree worse with the observed relation. However, see discussion in Sect. 4. We have also calculated the expected age distribution in case there was no mass-radius relation for the clusters, i.e. for $\lambda = 0$. The downward slope of the resulting distribution (not shown here) is significantly less steep than the one for $\lambda = 0.1$ and does not fit the observed distribution.

The flattening of the predicted distribution at the low age end is due to the fact that clusters with an initial mass in the range of about 100 to 300 $M_\odot$ quickly reach 100 $M_\odot$ (see Fig. 2). The bump in the observed distribution around $\log(t) \simeq 8.5$ is due to a local starburst (see L05 and Piskunov et al. 2006). Notice the good agreement in the shapes of the predicted and observed distributions!

The vertical shift that is applied to the predicted curve to match the observed one gives a value of $C = 10^{-4.15}$ in Eq. (10), which corresponds to a SFR of 4×10^2 $M_\odot$ Myr^{-1} for bound clusters in the range of $10^2 < M_i/M_\odot < 3 \times 10^4$ within 600 pc from the sun.

4. Discussion and summary

We studied the dissolution of star clusters in the solar neighbourhood due to four effects: stellar evolution, tidal stripping, spiral arm shocks and encounters with GMCs. For this study we adopted the descriptions of GAPZ06 and GPZB06 for the dissolution of star clusters by spiral arms and encounters with GMCs. We found that the last effect plays a dominant role in the solar neighbourhood.

The cluster dissolution time due to spiral arms and GMCs depends on the density of the clusters, i.e. on M/r_h^3. This implies that the dissolution time is $t_\mathrm{dis} \sim M^\gamma$ with $\gamma = 1 - 3\lambda$ if the radius of a cluster depends on its mass as $r_\mathrm{h} \sim M^\lambda$. We adopted $\lambda \simeq 0.1$ as found by Larsen (2004) for clusters in spiral galaxies and so $\gamma \simeq 0.7$ for dissolution by both spiral arms and GMCs. This value is very similar to $\gamma = 0.62$ predicted for dissolution by the tidal field only (BM03) and empirically derived for cluster samples in four galaxies by Boutloukos & Lamers (2003). If there was no mass radius dependence for clusters in the solar neighbourhood, e.g. $\lambda = 0$, then the predicted age distribution would have a shallower slope than shown in Fig. 3, since more old (massive) clusters would have survived.

Our calculated dissolution times of clusters in the solar neighbourhood are about a factor five smaller than predicted by BM03 for clusters in the tidal field of our Galaxy, with stellar evolution, binaries and two-body relaxation taken into account. This is reminiscent of the short dissolution time of clusters in the central region of the interacting galaxy M51, where the empirical dissolution time is even ten times shorter than can be explained by stellar evolution and tidal fields (Gieles et al. 2005). GMCs severely limit the lifetime of clusters in that galaxy also (see the discussion in GPZB06).

The steep drop in the observed age distribution at $t \simeq 2$ Gyr can be explained by an upper mass limit for the initial cluster mass in the solar neighbourhood of about $3 \times 10^4\ M_\odot$. However, this value is uncertain because it depends crucially on the completeness of the used sample at ages above 1 Gyr. (The sample contains only six clusters older than 1 Gyr.) The mass versus age distribution of our adopted sample, shown in Fig. 8 of L05, suggests that the lower mass limit of the observed clusters increases steeply for clusters older than 1 Gyr. Since the predicted value of $\mathrm{d}N/\mathrm{d}t$ at any age depends on M_lim and M_max as given in Eq. (10), and M_lim increases when the lower mass limit increases, an increase in this limit implies an increase in M_max derived from the observed age distribution. Based on this argument and the small number of clusters older than about 1 Gyr in the observed sample, we conclude that the derived value of $M_\mathrm{max} = 3 \times 10^4\ M_\odot$ should be considered as a lower limit of the real maximum initial mass.

The vertical shift applied to the predicted age distributions to match the observed one indicates a star formation rate of $4 \times 10^2\ M_\odot\ \mathrm{Myr}^{-1}$ in bound clusters of $M_i > 100\ M_\odot$ within a distance of 600 pc, corresponding to a surface formation rate of $3.5 \times 10^{-10}\ M_\odot\ \mathrm{yr}^{-1}\ \mathrm{pc}^{-2}$. This is a factor 2 to 3 smaller than the SFR derived from the study of embedded stars by Lada & Lada (2003) because many of the stars are born in unbound clusters that dissolve within 10 Myr.

The very good agreement between the predicted and observed age distribution of clusters shows that dissolution of clusters in the solar neighbourhood is dominated by encounters with GMCs, as was already suggested by Oort (1958). In fact, the good agreement may be slightly fortuitous because we have underestimated the dissolution by two-body relaxation (see Sect. 1) and slightly overestimated the dissolution by encounters with GMCs, because we adopted the midplane density of GMCs whereas clusters may spend a faction of their lifetime above or under the galactic disk. Both effects are expected to be smaller than a factor two and may partially cancel out.

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