A&A 447, L13–L16 (2006)
DOI: 10.1051/0004-6361:200500231


Astronomy & Astrophysics

Letter to the Editor

# High brightness temperatures and circular polarisation in extra-galactic radio sources

J. G. Kirk and O. Tsang

Max-Planck-Institut für Kernphysik, Saupfercheckweg 1, 69117 Heidelberg, Germany
e-mail: john.kirk@mpi-hd.mpg.de

Received 21 November 2005 / Accepted 23 December 2005


**ABSTRACT**

*Context.* Some rapidly variable extra-galactic radio sources show very high brightness temperatures $T_{\rm B} > 10^{12}$ K and high degrees of circular polarisation (∼1%). Standard synchrotron models that assume a power-law electron distribution cannot produce such high temperatures and have much lower degrees of intrinsic circular polarisation.

*Aims.* We examine the synchrotron and inverse Compton radiation from a monoenergetic electron distribution and discuss the constraints placed upon it by radio, optical and hard X-ray/gamma-ray observations.

*Methods.* The standard expressions of synchrotron theory are used. Observational constraints on the source parameters are found by formulating the results as functions of the source size, Doppler boosting factor, optical depth to synchrotron self-absorption and maximum frequency of synchrotron emission, together with a parameter governing the strength of the inverse Compton radiation.

*Results.* The model gives brightness temperatures $T_{\rm B} \sim 10^{13}$ to $10^{14}$ K for moderate ($\lesssim$10) Doppler boosting factors together with intrinsic degrees of circular polarisation at the percent level. It predicts a spectrum $I_\nu \propto \nu^{1/3}$ between the radio and the infra-red as well as emission in the MeV to GeV range. If the energy density in relativistic particles is comparable to or greater than the magnetic energy density, we show that electrons do not cool within the source, enabling the GHz emission to emerge without absorption and the potentially catastrophic energy losses by inverse Compton scattering to be avoided. Magnetically dominated sources can also fulfil these requirements at the cost of a slightly lower limit on the brightness temperature.

*Conclusions.* We suggest that sources such as PKS 1519−273, PKS 0405−385 and J 1819+3845 can be understood within this scenario without invoking high Doppler boosting factors, coherent emission mechanisms, or the dominance of proton synchrotron radiation.


**Key words.** galaxies: active – galaxies: high redshift – galaxies: jets

## 1. Introduction

The rapidly varying radio flux density observed in several extra-galactic sources implies a very high brightness temperature $T_{\rm B}$ at the source (Wagner & Witzel 1995). In cases such as PKS 1519−273 and PKS 0405−385, where the variability is identified as interstellar scintillation (Macquart et al. 2000; Rickett et al. 2002) realistic models of the scattering screen require $T_{\rm B} > 10^{13}$ K. The recently discovered diffractive scintillation in J 1819+3845 requires $T_{\rm B} > 2 \times 10^{14}$ K (Macquart & de Bruyn 2006). In other sources, the variability can be interpreted as intrinsic, in which case the implied temperature can be even higher.

Such high temperatures are difficult to understand within standard models of synchrotron emission, which assume a power-law distribution of electrons. The energy radiated by inverse Compton scattering in the Thomson regime rises dramatically when the intrinsic brightness temperature exceeds a certain threshold, roughly equal to $3 \times 10^{11}$ K in the case of a power-law electron distribution, although a flaring source might exceed this limit for a short time (Kellermann & Pauliny-Toth 1969; Slysh 1992; Melrose 2002). However, as first pointed out by Crusius-Waetzel (1991), the threshold temperature is higher for a monoenergetic electron distribution than for a power-law distribution, since, in the former case, photons of low frequency (compared to the characteristic synchrotron frequency) can escape without absorption by low energy electrons.

High brightness temperature sources frequently display circular polarisation at the 1% level or above (Macquart 2003). Because this is much higher than the value $m_{\rm e}c^2/k_{\rm B}T_{\rm B}$ conventionally estimated for the intrinsic emission of a power-law electron distribution, propagation effects are the favoured explanation (Jones & O'Dell 1977b; Wardle et al. 1998; Macquart & Melrose 2000; Beckert & Falcke 2002; Ruszkowski & Begelman 2002; Broderick & Blandford 2003; Wardle & Homan 2003). A monoenergetic electron model, on the other hand, predicts *intrinsic* circular polarisation at the 1% level, obviating the need for a conversion process. Additional predictions are a hard $I_\nu \propto \nu^{1/3}$ synchrotron spectrum extending from the radio at least up to the infra red, and an inverse Compton component in the MeV to GeV range.

## 2. Model parameters

We consider an idealised model characterised by a single length scale $R$ and Doppler boosting factor $\mathcal{D} = \sqrt{1-\beta^2}/(1-\beta\cos\phi)$, where $c\beta$ is the source speed with respect to the rest frame of the host galaxy and $\phi$ is the angle between the velocity and the line of sight. A homogeneous, isotropic, monoenergetic distribution of electrons with Lorentz factor $\gamma$ and number density $n$, embedded in a uniform magnetic field $B$ is assumed in the rest frame of the source. This captures the relevant properties of a power-law electron distribution with a lower energy "cut-off", provided the synchrotron opacity at low frequency is dominated by electrons whose energy is close to that of the cut-off, as is the case if the distribution rises towards the cut-off sufficiently rapidly: $\mathrm{d}\ln n/\mathrm{d}\ln\gamma > -1/3$. Quasi-monoenergetic distributions of this type can account for the lack of Faraday depolarisation in parsec-scale emission regions (Wardle 1977; Jones & O'Dell 1977a) and have recently been discussed in connection with statistical trends in the observed distribution of superluminal velocities as a function of observing frequency and redshift (Gopal-Krishna et al. 2004). Monoenergetic models have been considered previously in connection with high brightness temperature sources by Crusius-Waetzel (1991), and, more recently, by Protheroe (2003), who, however, adopted the assumption of equipartition between particle and magnetic energy densities in the source and introduced an additional, potentially large, geometrical factor. Slysh (1992) also treated a monoenergetic model, but did not allow for the possibility of multiple Compton scatterings and further restricted his treatment to optically thick sources.

Neglecting, for a moment, the angle between the line of sight and the magnetic field, the source model contains five free parameters: $R$, $\mathcal{D}$, $\gamma$, $B$ and $n$. Of these, only two are directly constrained by observations. Firstly, an upper limit on the linear dimension $R$, is given by the maximum permitted angular size inferred from the presence of scintillation. Secondly, surveys of superluminal motion (Cohen et al. 2003) reveal apparent transverse velocities that are mostly less than $10c$, but extend up to $30c$. These data suggest that $\mathcal{D} \lesssim 10$ for most sources, although it may be up to 30 or 40 in a few cases. If the source variability arises intrinsically, rather than from scintillation, an upper limit to the quantity $R/\mathcal{D}^2$ is obtained.

Additional constraints can be found by eliminating $\gamma$, $B$ and $n$ in favour of three new parameters. The first of these is the optical depth to synchrotron self-absorption $\tau_\mathrm{s}$. Denoting the Thomson optical depth of the monoenergetic electrons by $\tau_\mathrm{T} = n\sigma_\mathrm{T}R$ (where $\sigma_\mathrm{T}$ is the Thomson cross-section) one has

$$\tau_\mathrm{s} = \frac{\sqrt{3}\,\tau_\mathrm{T} m_\mathrm{e} c^3 K_{5/3}(x)}{8\pi e^2 \nu_\mathrm{s}\gamma^3} \tag{1}$$

(e.g., Longair 1992). Here, $K_{5/3}(x)$ is a modified Bessel function,

$$x = \nu(1+z)/(\mathcal{D}\nu_\mathrm{s}) \tag{2}$$

$\nu$ is the observing frequency, $\nu_\mathrm{s} = 3eB\sin\theta\gamma^2/(4\pi m_\mathrm{e}c)$ is the characteristic synchrotron frequency, for an angle $\theta$ between the line of sight and the magnetic field $B$ (both measured in the frame in which the source is at rest) and $z$ is the redshift of the host galaxy. $\tau_\mathrm{s}$ is a convenient parameter because it controls the brightness temperature, which has a single maximum close to $\tau_\mathrm{s} = 1$, when the other parameters are fixed.

The second new parameter, $\xi$, determines the inverse Compton luminosity. Synchrotron photons act as targets for inverse scattering by the same electron population that produced them, giving rise to a first generation of upscattered photons. These, in turn, act as targets off which the electrons produce a second generation, and so forth. We define $\xi$ as the ratio of the energy densities (or, equivalently, luminosities) in consecutive generations, assuming the scattering processes occur in the Thomson regime (e.g., Melrose 2002). This quantity is somewhat sensitive to the geometry and homogeneity of the source (Protheroe 2002), but, for a uniform, roughly spherical source that is optically thin to scattering, one can write

$$\xi = 4\gamma^2\tau_\mathrm{T}/3 \tag{3}$$

since the photon frequency is increased on average by the factor $4\gamma^2/3$ per scattering, and the probability of scattering is $\tau_\mathrm{T}$. The assumption of Thomson scattering leads to a divergent luminosity whenever $\xi > 1$, independent of the source of the initial (zeroth generation) photon targets. This phenomenon has acquired the name *Compton Catastrophe*. If the source is optically thin to most of the synchrotron photons, $\xi$ is equal to the ratio of the energy density in these photons to that in the magnetic field. This is, in fact, the most commonly used definition. However, using our definition (3) the condition for catastrophe remains $\xi > 1$ in both the optically thin and thick (to synchrotron absorption) cases as well as in the case where photons of the cosmic microwave background provide a more effective target than do the synchrotron photons. We show below that inverse Compton scattering indeed takes place at least initially in the Thomson regime. Therefore, in order to avoid inordinately large Compton losses, we require

$$\xi \lesssim 1 \tag{4}$$

which automatically ensures $\tau_\mathrm{T} \ll 1$.

The observed brightness temperature $T_\mathrm{B}$, related to the specific intensity of radiation $I_\nu$ in the direction of the source by $T_\mathrm{B} = c^2 I_\nu/(2\nu^2 k_\mathrm{B})$, follows straightforwardly from the solution of the equation of radiation transport:

$$\frac{k_\mathrm{B}T_\mathrm{B}}{m_\mathrm{e}c^2} = \frac{\mathcal{D}}{1+z}\left(\frac{\gamma F(x)}{2x^2 K_{5/3}(x)}\right)(1-\mathrm{e}^{-\tau_\mathrm{s}}) \tag{5}$$

where $F(x) = x\int_x^\infty \mathrm{d}t K_{5/3}(t)$ is the standard synchrotron function in the Airy integral approximation. Eliminating the parameters $\tau_\mathrm{T}$, $\nu_\mathrm{s}$ and $\gamma$ using Eqs. (1)–(3), gives

$$\frac{k_\mathrm{B}T_\mathrm{B}}{m_\mathrm{e}c^2} = \left(\frac{3^{3/2} m_\mathrm{e} c^3}{4^5\pi e^2\nu}\right)^{1/5}\left(\frac{\xi\mathcal{D}^6}{(1+z)^6}\right)^{1/5} \times\left(\frac{1-\mathrm{e}^{-\tau_\mathrm{s}}}{\tau_\mathrm{s}^{1/5}}\right)\left(\frac{F(x)}{x^{9/5}K_{5/3}^{4/5}(x)}\right). \tag{6}$$

The first term in parentheses on the right-hand side of this equation is independent of the source parameters. The second is constrained by observation to be $\lesssim$10. The third reaches a maximum of the order of unity at $\tau_s \sim 1$. The fourth, however, diverges for small $x$ as $x^{-2/15}$. Thus, even with $\xi < 1$ and $\mathcal{D} < 10$, it is possible to choose an $x$ for which this formula gives an arbitrarily high brightness temperature at any specified observing frequency. However, this model predicts a synchrotron spectrum that rises with frequency at least as fast as $I_\nu \propto \nu^{1/3}$ between the observing frequency $\nu$ and the frequency $\nu/x$. In principle, this can be constrained by observation. For PKS 1519−273 and PKS 0405−385, for example, the modest optical fluxes suggest that $\nu_{\rm max} \lesssim 10^{14}$ Hz (Heidt & Wagner 1996, and Wagner, priv. comm.), although it is not known whether these observations coincided with an episode of high brightness temperature radio emission in these variable sources. Nevertheless, for our third parameter, we adopt the frequency $\nu_{\rm max} = \nu/x$ above which the optically thin synchrotron radiation cuts off.

## 3. Brightness temperature and circular polarisation

Rewriting the brightness temperature Eq. (6) in terms of the three new parameters $\tau_s$, $\xi$ and $\nu_{\rm max}$, expressed in convenient units, we find that it is independent of the source size, and depends only weakly on $\xi$ and $\nu_{\rm max}$:

$$T_{\rm B} = 1.2\times 10^{14}\left(\frac{\mathcal{D}_{10}^6\xi}{(1+z)^6}\right)^{1/5}\left(\frac{1-{\rm e}^{-\tau_s}}{\tau_s^{1/5}}\right)\nu_{\rm max14}^{2/15}\nu_{\rm GHz}^{-1/3}\,{\rm K} \qquad (7)$$

where $\nu_{\rm max14} = \nu_{\rm max}/(10^{14}\,{\rm Hz})$, $\nu_{\rm GHz} = \nu/(1\,{\rm GHz})$, $\mathcal{D}_{10} = \mathcal{D}/10$, and the approximations $F(x) \approx 2.15x^{1/3}$ and $K_{5/3}(x) \approx 1.43x^{-5/3}$, valid for $x \ll 1$, are used. According to this equation, brightness temperatures of up to roughly $10^{13}$ K at GHz frequencies, as observed in PKS 1519−273 and PKS 0405−385, can be achieved with $\xi \lesssim 1$ and $\mathcal{D} = 1$. With $\mathcal{D} \approx 15$, the model permits $T_{\rm B} = 2\times 10^{14}$ K, as observed in J 1819+3845.

In addition to the high brightness temperature, a particularly interesting source property is the degree of intrinsic circular polarisation $r_c$. Assuming a pure electron-proton plasma that is optically thin to synchrotron self-absorption, this quantity is also independent of source size:

$$r_c = \frac{1}{3\gamma}\left(\frac{2}{x}\right)^{1/3}\cot\theta\,\Gamma(1/3)$$
$$= 0.019\times\left(\frac{(1+z)\tau_s}{\mathcal{D}_{10}\xi}\right)^{1/5}\nu_{\rm max14}^{1/5}\cot\theta \qquad (8)$$

(Melrose 1980). In the case of a power-law electron distribution, $r_c$ changes sign when the optically thick regime is entered (Jones & O'Dell 1977b). We are not aware of the corresponding calculation for a monoenergetic distriubtion, but, to order of magnitude, one can estimate the peak value using this expression, which is remarkably insensitive to all source parameters other than the magnetic field direction. Several extra-galactic sources of extremely high brightness temperature display circular polarisation at the percent level (Macquart 2003), in particular PKS 1519 −273 and PKS 0405 −385. In the absence of a low-energy cut off in the electron distribution, $r_c \sim 1/\gamma$, which is far too small to explain the observations. However, Eq. (8) shows that for a mono-energetic electron distribution, the intrinsic emission can be polarised at the % level, or above, depending on the geometry of the magnetic field configuration.

## 4. Discussion

The electron Lorentz factor implied by the above analysis is:

$$\gamma = 2.8\times 10^3\left(\frac{\xi\mathcal{D}_{10}\nu_{\rm max14}^{2/3}}{\tau_s(1+z)}\right)^{1/5}\nu_{\rm GHz}^{-1/3}. \qquad (9)$$

A key ingredient of this model is the absence of electrons of lower Lorentz factor, since these would absorb the GHz emission, leading to a reduction of the brightness temperature. Specifically, we require a quasi-monoenergetic distribution such that $d\ln n/d\ln\gamma > -1/3$ at Lorentz factors lower than that given by Eq. (9). Such a distribution is not a natural consequence of, for example, the first-order Fermi process at relativistic shocks (e.g., Kirk 2005). On the other hand, a relativistic thermal distribution, which rises at low energy as $\gamma^2$ is well-approximated by a monoenergetic distribution of energy roughly equal to the temperature. The addition of a power-law tail to higher energy would not change this conclusion.

The Lorentz factor implied by Eq. (9) is higher than the cut-off conventionally assumed when modelling radio sources (e.g., Gopal-Krishna et al. 2004). Nevertheless, scenarios exist which suggest such values. One example is an electron-proton jet with a bulk Lorentz factor $\Gamma \sim 10$ which is thermalised at a shock front. If the downstream electron and ion temperatures are equal, the distribution can be approximated as monoenergetic with an electron Lorentz factor of $\Gamma m_p/m_e$, where $m_p$ and $m_e$ are the proton and electron masses. Another possibility is that the electrons are accelerated at a current sheet in an electron-proton plasma in which the magnetic energy density is comparable to the rest-mass energy density (Kirk 2004). Each of these possibilities relies on the composition of the source plasma being electron-proton. Interestingly, so does the relatively high degree of intrinsic circular polarisation given by Eq. (8).

Although it is conceivable that continuous re-acceleration prevents the accumulation of low energy electrons, both the current sheet and the shock scenario envisage a finite escape rate of particles from the acceleration or thermalisation region. Escaping particles subsequently cool by synchrotron and inverse Compton emission. Therefore, the model electron distribution is self-consistent only if these particles can be evacuated from the source in a time shorter than the cooling timescale. The ratio of the electron cooling time $t_{\rm cool}$ to the light-crossing time of the source can be written as:

$$ct_{\rm cool}/R = \eta/\xi \qquad (10)$$

where $\eta$ is the ratio of the energy density in relativistic electrons to that in the magnetic field. Writing $R = 0.01R_{-2}$ parsec, we find:

$$\eta = \frac{\gamma nmc^2}{(B^2/8\pi)}$$
$$= 2.9\left(\frac{\mathcal{D}_{10}^{13}\xi^8}{(1+z)^{13}\tau_s^3\nu_{\rm max14}^8}\right)^{1/5}\nu_{\rm GHz}^{-1}R_{-2}^{-1}\sin^2\theta. \qquad (11)$$

Clearly, very small sources tend to be particle dominated and, since $\xi < 1$, they satisfy the self-consistency requirement $ct_{\rm cool}/R > 1$. However, Eq. (11) shows that $\eta$ is also quite sensitive to $\mathcal{D}$, $\xi$ and $\nu_{\rm max}$, so that magnetically dominated sources are by no means ruled out, provided they have $\xi \lesssim \eta$. Since the brightness temperature is proportional to $\xi^{1/5}$ (Eq. (7)) it is slightly lower for magnetically dominated sources.

Although lacking a compelling physical justification, the assumption of equipartition, $\eta = 1$, can be used to define an "equipartition Doppler factor". This leads, in the standard model, to a relatively low limit on the brightness temperature $T_{\rm B,eq} \lesssim 3 \times 10^{10}$ K (Singal & Gopal-Krishna 1985; Readhead 1994), which has some observational support (Cohen et al. 2003). In the monoenergetic model, however, this assumption re-introduces a dependence on the source size, but does not substantially constrain the brightness temperature, as can be seen from Eqs. (7) and (11).

The parameter

$$\zeta = \frac{\gamma h \nu_{\rm s}}{m_{\rm e} c^2} = 2.3 \times 10^{-4} \left( \frac{(1+z)^4 \xi}{\tau_{\rm s} \mathcal{D}_{10}^4} \right)^{1/5} \nu_{\rm max14}^{17/15} \nu_{\rm GHz}^{-1/3} \quad (12)$$

which gives the ratio of the energy of a photon of the characteristic synchrotron frequency to the electron rest-mass, as seen in the rest frame of a relativistic electron is also independent of source size. For $\zeta \ll 1$, the first inverse Compton scattering takes place in the Thomson regime. In this case, it is consistent to require $\xi \lesssim 1$ in order to avoid an excessively large energy demand on the source i.e, in order to avoid the Compton Catastrophe. The first generation of inverse Compton photons has a frequency of approximately

$$\nu_1 \approx 4.3 \left( \frac{\mathcal{D}_{10}^2 \xi^2}{(1+z)^2 \tau_{\rm s}^2} \right)^{1/5} \nu_{\rm max14}^{19/15} \nu_{\rm GHz}^{-2/3} \,{\rm MeV} \quad (13)$$

and its flux can be estimated to be

$$\mathcal{F}_{\rm IC} \approx 4.5 \times 10^{-6} \left( \frac{(1+z)^3 \xi^3 \tau_{\rm s}^2}{\mathcal{D}_{10}^2} \right)^{1/5} \nu_{\rm max14}^{1/15} \nu_{\rm GHz}^{1/3} \mathcal{F}_{\rm GHz} \quad (14)$$

where $\mathcal{F}_{\rm GHz}$ is the flux observed in the radio at frequency $\nu_{\rm GHz}$ GHz. This estimate of the inverse Compton flux is generally above the detection threshold of instruments on the INTEGRAL satellite, as noted by Protheroe (2003). Subsequent generations of inverse Compton scattered photons are likely to fall into the Klein-Nishina regime, and the maximum photon energy achieved by multiple inverse Compton scattering is ultimately limited by the electron energy, as seen in the observer's frame, which takes the value:

$$\frac{\mathcal{D} \gamma m c^2}{1+z} = 14 \left( \frac{\mathcal{D}_{10}^6 \xi}{(1+z)^6 \tau_{\rm s}} \right)^{1/5} \nu_{\rm max14}^{2/15} \nu_{\rm GHz}^{-1/3} \,{\rm GeV}. \quad (15)$$

To summarise, synchrotron radiation from a monoenergetic electron distribution reproduces the extremely high brightness temperatures observed in variable extra-galactic radio sources, and explains the observed levels of circular polarisation. Therefore, it does not appear necessary to appeal to coherent mechanisms (Krishnan & Wiita 1990; Benford & Lesch 1998; Begelman et al. 2005) or to proton synchrotron radiation (Kardashev 2000) to understand these objects. Testable predictions of the theory are a hard radio to infra-red spectrum and gamma-ray emission in the MeV to GeV range.

## References


Beckert, T., & Falcke, H. 2002, A&A, 388, 1106
Begelman, M. C., Ergun, R. E., & Rees, M. J. 2005, ApJ, 625, 51
Benford, G., & Lesch, H. 1998, MNRAS, 301, 414
Broderick, A. E., & Blandford, R. D. 2003, Am. Astron. Soc. Meeting Abst., 203
Cohen, M. H., Russo, M. A., Homan, D. C., et al. 2003, in Radio Astronomy at the Fringe, ASP Conf. Ser., 300, 177
Crusius-Waetzel, A. R. 1991, A&A, 251, L5
Gopal-Krishna, Biermann, P. L., & Wiita, P. J. 2004, ApJ, 603, L9
Heidt, J., & Wagner, S. J. 1996, A&A, 305, 42
Jones, T. W., & O'Dell, S. L. 1977a, A&A, 61, 291
Jones, T. W., & O'Dell, S. L. 1977b, ApJ, 214, 522
Kardashev, N. S. 2000, Astron. Rep., 44, 719
Kellermann, K. I., & Pauliny-Toth, I. I. K. 1969, ApJ, 155, L71
Kirk, J. G. 2004, Phys. Rev. Lett., 92, 181101
Kirk, J. G. 2005, in X-Ray and Radio Connections, ed. L. O. Sjouwerman, & K. K. Dyer, Published electronically by NRAO, http://www.aoc.nrao.edu/events/xraydio, held 3–6 February 2004 in Santa Fe, New Mexico, USA, (E1.01) 6 pages
Krishnan, V., & Wiita, P. J. 1990, MNRAS, 246, 597
Longair, M. S. 1992, High energy astrophysics, Particles, photons and their detection (Cambridge: Cambridge University Press), 2nd edn., 1
Macquart, J.-P. 2003, New Astron. Rev., 47, 609
Macquart, J.-P., & de Bruyn, G. 2006, A&A, 446, 185
Macquart, J.-P., Kedziora-Chudczer, L., Rayner, D. P., & Jauncey, D. L. 2000, ApJ, 538, 623
Macquart, J.-P., & Melrose, D. B. 2000, ApJ, 545, 798
Melrose, D. B. 1980, Plasma astrohysics. Nonthermal processes in diffuse magnetized plasmas, Vol. 1: The emission, absorption and transfer of waves in plasmas (New York: Gordon and Breach)
Melrose, D. B. 2002, , 19, 34
Protheroe, R. J. 2002, Pub. Astron. Soc. Austr., 19, 486
Protheroe, R. J. 2003, MNRAS, 341, 230
Readhead, A. C. S. 1994, ApJ, 426, 51
Rickett, B. J., Kedziora-Chudczer, L., & Jauncey, D. L. 2002, ApJ, 581, 103
Ruszkowski, M., & Begelman, M. C. 2002, ApJ, 573, 485
Singal, K. A., & Gopal-Krishna 1985, MNRAS, 215, 383
Slysh, V. I. 1992, ApJ, 391, 453
Wagner, S. J., & Witzel, A. 1995, ARA&A, 33, 163
Wardle, J. F. C. 1977, Nature, 269, 563
Wardle, J. F. C., & Homan, D. C. 2003, Ap&SS, 288, 143
Wardle, J. F. C., Homan, D. C., Ojha, R., & Roberts, D. H. 1998, Nature, 395, 457