

# Constraining galactic models through parallax and astrometry of microlensing events

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**Abstract.** Various models for the Galactic distribution of massive compact halo objects (MACHOs) have been proposed for the interpretation of microlensing toward the Large Magellanic Cloud (LMC). A direct way to fit the best model is by measuring the lens parameters, which can be obtained by measuring the Einstein crossing time and the parallax effect on the microlensing light curve and by astrometry of centroids of images. In this work, the theoretical distribution of these parameters is obtained for the various power-law Galactic dark halo models and MACHO mass functions (MF). For self-lensing as one of the models for the interpretation of LMC events, the maximum shift of image centroids and the parallax parameter are one order of magnitude smaller than for models with dark halos. This can be used as a test for self-lensing, although the shift of image centroids 0.02 mas for the LMC events is unlikely to be observed by the astrometric missions such as FAME, GAIA and SIM.

**Key words.** cosmology: dark matter – gravitational lensing – galaxy: halo – galaxy: structure

## 1. Introduction

One of the primary candidates for the dark matter in the Galaxy is the massive compact halo object (MACHOs) that can be in baryonic or even non-baryonic form. Paczyński (1986) proposed the gravitational microlensing method as an indirect means of detecting MACHOs. The microlensing effect is the magnification of a background star that occurs while a MACHO passes the line of sight of an observer to a background object. Due to the small separation between the images produced by the lensing, it is impossible to resolve them by ground-based telescopes and only the magnification and its duration can be measured by observing the light curve of the background star.

Many groups observed stars in the Large and Small Magellanic Clouds to measure the contribution of MACHOs to the dark halo mass and estimate the mass of MACHOs. Due to the degeneracy between lens parameters (i.e. mass of the lens, MACHO distance and the transverse velocity), it is impossible to obtain these parameters just by measuring the duration and magnification of events. The degeneracy problem also makes it impossible to find the distribution of MACHOs in the Galaxy although one can learn much from statistical methods by analyzing the duration distribution of events. Furthermore, the low number of events observed in the direction of the LMC causes considerable Poisson fluctuation; to properly constrain Galactic models we will need many more events to be detected, as will be carried out by the next generation of microlensing

surveys (Becker et al. 2004). One of the alternative ways to overcome the problem of statistical fluctuation is by improving the quality of light curves and measuring the other observable parameters that can break the degeneracy between the lens parameters.

Apart from the duration of microlensing events, there are two more observable parameters that can be measured by accurate observation of light curves. Gould (1992) showed that the effect of the rotation of the Earth around the Sun on the microlensing light curve is a slight deviation of the light curve from symmetry. The possibility of observing this (parallax) effect has been studied towards the Galactic bulge (Buchalter & Kamionkowski 1997) and the Large Magellanic Cloud (Gould 1998). An observational strategy to have sufficiently accurate microlensing light curves (with a significant parallax signature) has been proposed to distinguish two extreme models of the Milky Way and to test the hypothesis of self-lensing (Rahvar et al. 2003). However, using the parallax effect is not presently sufficient to distinguish between Galactic models with a small number of microlensing events.

The displacement of the image centroids due to microlensing can also be measured with high resolution astrometry. The typical angular separations of multiple images formed by microlensing in the direction of Magellanic Clouds is 1 milli arc-second (mas) (Paczynski 1996, 1998) and the image centroid is expected to move about 0.1 mas. The future astrometric

missions such as FAME<sup>1</sup>, SIM<sup>2</sup> and GAIA<sup>3</sup> can reach this high precision. Astrometric accuracies anticipated for those missions are 50 micro arcsecond ( $\mu\text{as}$ ) for FAME and 10  $\mu\text{as}$  (or possibly higher) for SIM and GAIA; this accuracy depends on the apparent magnitude of the stars. SIM can perform astrometry for stars as faint as 20th mag and it can measure the shift of image centroids of most of the LMC stars (Rajagopal et al. 2001). This positional shift of images can also be used to break one more degree of degeneracy. The advantage of astrometric microlensing is that the probability of observing a microlensing event is larger than that of photometric microlensing (Miralda-Escude 1996; Dominik & Sahu 2000; Honma 2001).

In this work we obtain the distributions of event duration and two parallax and astrometry parameters for various power law dark halo models with various MACHO mass functions (MF) and LMC models (so-called self-lensing). We show that for the self-lensing model, the parallax and the astrometric parameters are one order of magnitude smaller than for typical dark halo lensing.

In Sect. 2 we discuss the degeneracy problem in microlensing and show how astrometry accompanying the parallax measurements can break the degeneracy between the lens parameters. In Sect. 3 we introduce power law dark halo models in the Milky Way and obtain the theoretical distributions of event duration, parallax and astrometry parameters for various MACHO mass distributions. In Sect. 4 we discuss the hypothesis of self-lensing and the distribution of observable parameters in this model. The results are discussed in Sect. 5.

## 2. Non-standard microlensing and degeneracy breaking

The Einstein crossing time of the microlensing events, ( $t_c$ ), degenerately depends on the mass, distance and the transverse velocity of the lenses. To break this degeneracy, we can measure both the deviation of the light curve from the Paczyński form due to the parallax effect and the displacement of image centroid during the lensing. In this section we give a brief account of these effects.

### 2.1. Parallax effect on the microlensing light curve

As the Earth rotates around the Sun, its velocity component, projected on the deflector plane, is not negligible compared to the transverse speed of the deflector. The apparent velocity of the deflector with respect to the line of sight is a cycloid instead of a straight line and this effect causes the light curve of the microlensing event to deviate from that of a Heliocentric observer (Alcock et al. 1995). The deviation of the light curve from simple microlensing can be described by the parameter  $\delta u$  which is the projection of the Earth orbital radius ( $a_\oplus$ ) in the deflector plane in units of the Einstein radius and the projected transverse speed of the deflector ( $\tilde{v}$ ) on the Earth's orbit.

$\delta u$  and  $\tilde{v}$  are defined as follows:

$$\delta u = a_\oplus(1-x)/R_E, \quad (1)$$

$$\tilde{v} = \frac{R_E}{t_E(1-x)}, \quad (2)$$

where  $R_E$  is the Einstein radius and  $x$  is the ratio of the observer-lens distance to the observer-source distance. Equations (1) and (2) show that by measuring the signature of the parallax effect in the microlensing light curves, one can constrain the mass and the distance of the lenses. This effect can reduce one degree of degeneracy between the lens parameters.

### 2.2. Astrometric microlensing

The angular separations of the images due to the microlensing of the background stars at the LMC, lensed by the dark halo MACHOs, are of order of one mas. The image centroid sweeps out nearly one mas during the microlensing event. The ground-based telescopes, due to the seeing limit, or the HST, due to the diffraction limit, cannot achieve the resolution of mas shifts in image centroid, however observations based on interferometry in space may be able to do so. The shift of the image centroid during the lensing can be derived as follows:

$$\vec{\delta\theta} = \frac{1}{2+u^2} \vec{u}\theta_E, \quad (3)$$

where  $u$  is the separation between lens and the source in the lens plane normalized to the Einstein radius,  $\theta_E$  is the angular size of the Einstein radius ( $\theta_E = \frac{R_E}{D_{ol}}$ , where  $D_{ol}$  is the observer-lens distance). For  $u = \sqrt{2}$  the angular shift obtains its maximum value  $\delta\theta_{\max} = \frac{\theta_E}{\sqrt{2}}$ . Measuring  $\delta\theta_{\max}$  yields the angular Einstein radius and constrains the mass and distance of the lens from the observer.

### 2.3. Degeneracy breaking

The Einstein crossing time,  $t_E$ , depends on three parameters: the mass,  $M$ , transverse velocity,  $v_t$ , and distance of lens from the observer,  $D_s$ , through

$$t_E = \frac{1}{v_t} \sqrt{\frac{4GMD_s}{c^2} x(1-x)}. \quad (4)$$

On the other hand the parallax and astrometry parameters ( $\delta u$  and  $\delta\theta_{\max}$ ) give two more equations depending on the mass and the distance of the lens. Using these three equations, we can obtain the lens parameters as following:

$$x = \frac{a_\oplus}{a_\oplus + \sqrt{2}D_s\delta u\delta\theta_{\max}}, \quad (5)$$

$$M = \frac{1}{2\sqrt{2}} \left( \frac{c^2 a_\oplus \delta\theta_{\max}}{G\delta u} \right), \quad (6)$$

$$v_t = \frac{\sqrt{2}}{t_E} \left( \frac{a_\oplus \delta\theta_{\max} D_s}{a_\oplus + \sqrt{2}D_s\delta u\delta\theta_{\max}} \right). \quad (7)$$

<sup>1</sup> <http://www.usno.navy.mil/FAME/>

<sup>2</sup> [http://planetquest.jpl.nasa.gov/SIM/sim\\_index.htm](http://planetquest.jpl.nasa.gov/SIM/sim_index.htm)

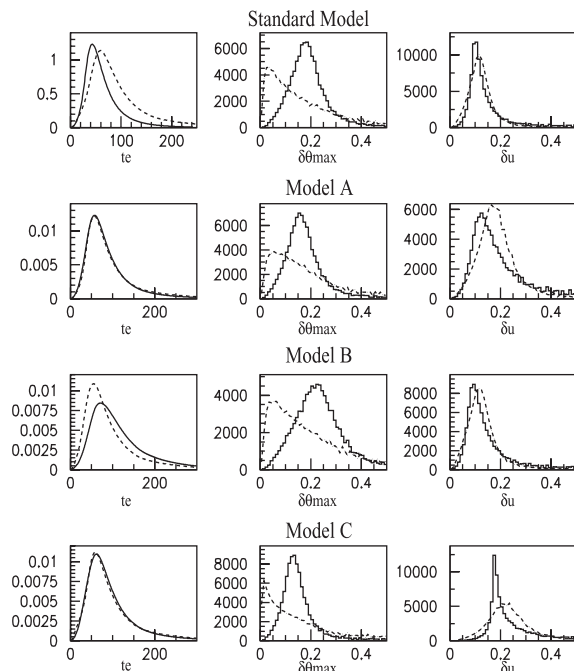
<sup>3</sup> <http://astro.estec.esa.nl/GAIA/>

### 3. Constraining galactic models via degeneracy breaking

In the last section, we showed that measuring the duration of events along with the astrometric and parallax parameters can break the degeneracy between the lens parameters. In spite of the large optical depth of astrometric microlensing compared to the photometric one (Honma 2001), we need to consider both astrometric and photometric microlensing to get the mass and the distance of the lenses. The mass and distance of lenses can be obtained by measuring the two parameters  $\delta u$  and  $\delta\theta_{\max}$  where their dependence on the distance of lenses from the observer is  $\sqrt{\frac{1-x}{x}}$ . This means that measuring the distance of lenses is more accurate the closer the lens is. The fraction of localized lenses among the overall microlensing events is expressed by the observation efficiency, which is a function of the accuracy of the apparatus and the observational strategy. The expected distributions of  $\delta u$  and  $\delta\theta_{\max}$  can be obtained by convolving the theoretical distributions with the observational efficiency of the experiment.

Here we obtain the theoretical distributions of  $t_E$ ,  $\delta u$  and  $\delta\theta_{\max}$  in various Galactic models for microlensing events in the direction of the LMC. The relevant components of the Galaxy, in the LMC microlensing events, are the Galactic disk, halo and the LMC itself. For the dark halo, we have considered a large set of axisymmetric models, so-called power-law models (Evans 1994). To model the density of the thin and thick disks of the Milky Way, we use a double exponential function (Binney & Tremaine 1987). Eight power-law halo models with a corresponding disk can be combined to build various Galactic models: these are the Standard Model (S), Medium halo (A), Large halo (B), Small halo (C), Elliptical (D), Maximal disk (E) and two thick disk models (F and G) where we follow the designations of Alcock et al. (1996).

One of the elements for generating  $t_E$ ,  $\delta u$  and  $\delta\theta_{\max}$  is the mass function (MF) of MACHOs. Usually, one uses a single MACHO mass (i.e. a Dirac-Delta function). Here we use two Dirac-Delta and spatially-varying MFs. Kerins & Evans (1998) and recently Rahvar (2005) have shown that a spatially-varying MF for the MACHOs not only can fit the distribution of microlensing event durations in the direction of LMC, but also can resolve the contradiction between the microlensing results and other astrophysical observations such as the unseen white dwarfs in the Galactic halo (Oppenheimer et al. 2001; Torres et al. 2002; Spagna et al. 2004). The Dirac-Delta MF is given by  $\delta(M - M_{\text{macho}})$ , where  $M_{\text{macho}}$  in the various Galactic models is given in Alcock et al. (1996, 2000). The spatially varying MF has the form  $MF(r) = \delta(M - M(r))$  where  $M(r)$  scales as  $M_U \left(\frac{M_L}{M_U}\right)^{r/R_{\text{halo}}}$ , where  $M_U$  and  $M_L$  are the upper and lower limits of mass of MACHOs,  $r$  is the distance from the center of Galaxy and  $R_{\text{halo}}$  is a halo scale parameter. The mean mass for the spatially varying MF in the standard dark halo model from the likelihood analysis is about the brown dwarf mass (Rahvar 2005). The physical meaning of this type of spatially-varying MF is that the inner part of dark halos contains heavier MACHOs than the outer part. The optimized parameters for the spatially-varying MF of MACHOs in



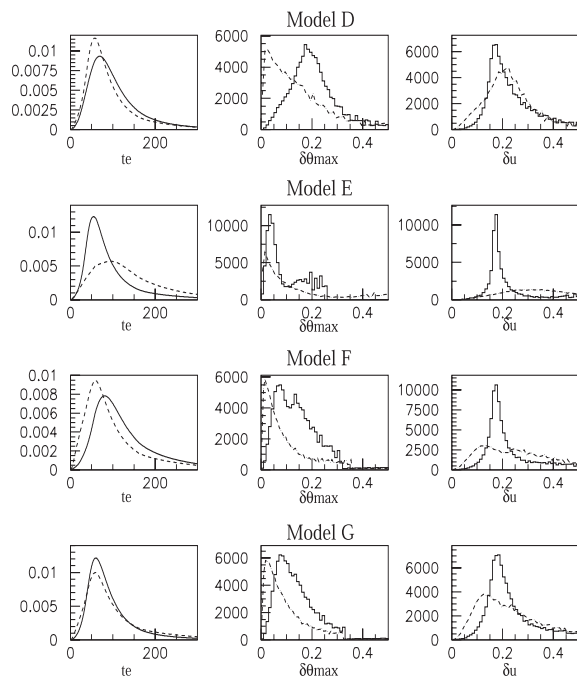
**Fig. 1.** Distributions of event duration,  $t_e$ , (in days, total number of events is normalized to one), parallax parameter,  $\delta u$ , and astrometry parameter,  $\delta\theta_{\max}$ , (in mas) for the Galactic halo models of S, A, B and C. The solid curves are the distributions derived using the Dirac-delta MF and the dashed lines for a spatially-varying MF.

power-law halo models to be compatible with the LMC microlensing candidates of MACHO experiment is given in Rahvar (2005). The MF of the Galactic disk also can be taken from the results of observations with the *Hubble Space Telescope* (Gould et al. 1997).

The distribution of event duration in the various Galactic models can be obtained by numerical simulation (Alcock et al. 1996). To obtain the distributions of  $\delta u$  and  $\delta\theta_{\max}$  during an observation time of  $T_{\text{obs}}$ , we perform a Monte-Carlo simulation, using the probability function of lenses along our line of sight. The rate of microlensing events due to the MACHOs between the distances  $x$  and  $x + dx$  from the observer is given by:

$$\frac{d\Gamma}{dx} = 4 \sqrt{\frac{GD_s}{M(x)c^2}} x(1-x)v_t \rho(x),$$

where  $M(x)$  can be substituted from the desired MF. The location of the selected lenses is used to calculate corresponding  $\delta u$  and  $\delta\theta_{\max}$ . Figures 1 and 2 show the results for the distributions of event duration, parallax parameter and maximum centroid shift in the power-law halo models. For the models with dominant halo such as S, A, B, C and D, the distribution of  $\delta u$  and  $\delta\theta_{\max}$  looks similar, while they are different to the disk dominant models (E, F and G). The distribution of  $\delta\theta_{\max}$  is sensitive to the type of MFs. For the spatially varying MF, the distribution of  $\delta\theta_{\max}$  shifts to significantly smaller values than single MACHO mass MFs.

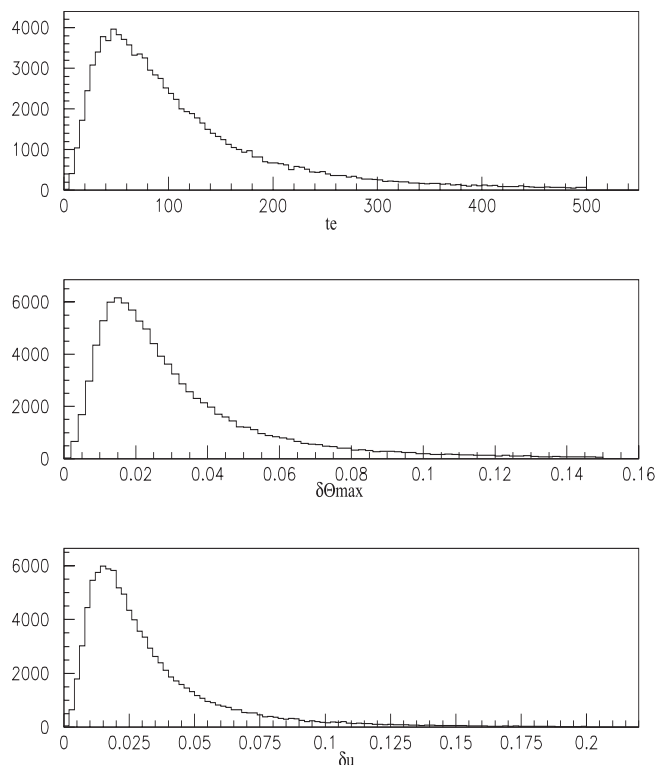


**Fig. 2.** Distributions of event duration,  $t_e$ , (in days, total number of events is normalized to one), parallax parameter,  $\delta u$ , and astrometry parameter,  $\delta\theta_{\max}$ , (in mas) for the Galactic halo models of D, E, F and G. The solid curves are the distributions derived using the Dirac-delta MF and the dashed lines for a spatially-varying MF.

#### 4. Testing the hypothesis of LMC self-lensing

Due to the low optical depth of microlensing events in the direction of the Magellanic Clouds, the EROS and MACHO experiments have obtained only 3 and 13–17 microlensing candidates, respectively (Lassere et al. 2000; Alcock et al. 2000; Tisserand & Milsztajn 2005). These studies lead to an upper limit of about 3% (in EROS experiment) to 20% (in MACHO experiment) obtained for the contribution of MACHOs to the dark halo and a mean MACHO mass of about that of a white dwarf (Lassere et al. 2000; Alcock et al. 2000; Tisserand & Milsztajn 2005). However, the expected white dwarfs have not been seen by studies such as proper motion measurements (Oppenheimer et al. 2001; Torres et al. 2002; Spagna et al. 2004). This inconsistency can be resolved by (i) considering a spatially-varying MF for the MACHOs in the Galactic halo; or (ii) considering the LMC stars playing the role of lenses. If most MACHOs reside in the Galactic halo, the timescale analysis of LMC microlensing candidates implies that the typical mass of lenses is of the order  $0.5 M_{\odot}$  while for the SMC events, the mass of lenses is estimated to be 2 to  $3 M_{\odot}$  (Sahu 2003). This inconsistency can be solved by considering the lenses as belonging to the LMC.

The other motivation for the possibility of self-lensing was presented by Sahu (2003), where the number of localized binary lenses was compared with the overall microlensing candidates. If half of the MACHOs are considered to be in a binary system (similar to the stars), then we expect to observe half of the microlensing events via binary lenses. On the other hand only 20% of binary lenses are expected to show the caustic



**Fig. 3.** The distributions of duration of events  $t_e$  (in days), parallax parameter  $\delta u$  and the astrometry parameter  $\delta\theta_{\max}$  (in mas), for the self-lensing events. The typical values of  $\delta u$  and  $\delta\theta_{\max}$  for the self-lensing events comparing with Figs. 1 and 2 are one order of magnitude smaller than for the halo.

crossing feature, then we should see the caustic crossing for 10% of lenses. Two of the Magellanic Clouds' microlensing candidates had crossing a caustic feature and by breaking the degeneracy between the lens parameters, they were localized within the Magellanic Clouds. According to this rough estimation, about 20 microlensing events are expected via LMC self-lensing; this number is about the same number of events as has been observed.

Using the parallax effect and astrometric centroid shift, observations can test the hypothesis of self-lensing. We used a disk model for the LMC (Gyuk et al. 2000) and obtained the distributions of  $t_e$ ,  $\delta u$  and  $\delta\theta_{\max}$  for self-lensing. Comparing Fig. 3 with Figs. 1 and 2 shows that for self-lensing events  $\delta u$  and  $\delta\theta_{\max}$  are about one order of magnitude smaller than that of halo lensing. For the halo lensing the mean value of the maximum shift for the centroid of images is about 0.2 mas while for the self-lensing, it is about 0.02 mas. Comparing the resolution of the astrometric missions with the centroid shift for the halo and LMC lenses shows that FAME, SIM and GAIA have enough resolution to observe the halo lensing while for the self-lensing events the resolution is of the same order as the centroid shift and it is unlikely to be detectable.

#### 5. Conclusion

In this work we studied the degeneracy breaking between the lens parameters by measuring the duration of events and

deviation of light curves due to the parallax effect and the maximum displacement of image centroids during lensing.

We used various Galactic models and obtained the theoretical distributions of observable parameters. In order to have the observed distribution of parameters we need to convolve these distributions with details of the observing program such as the photometric and astrometric accuracy and the sampling rate. One of the applications of astrometry with parallax measurements is in testing the hypothesis of self-lensing. The maximum shift of the centroid of images and parallax parameter for halo lensing are one order of magnitude large than the self-lensing ones.

In practice, observations to measure the three parameters of microlensing can be done by a ground-based telescope accompanying a space-based interferometry telescope. The ground-based telescope signals the microlensing events and undertakes photometric measurements and the space-based telescope measures the displacement of image centroids. Rahvar et al. (2003) proposed a strategy for observation of microlensing events based on both alert and follow-up telescopes. They obtained the observational efficiency for the detection of the parallax effect and estimated the number of microlensing events required to distinguish between two extreme Galactic models and to test the self-lensing hypothesis. This type of observational strategy can be extended by adding a space-based astrometric telescope.

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