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Interferometry and asteroseismology: The radius of τ Cet[★]

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Abstract. We have determined from interferometry the radius of the nearby star τ Cet, using recent observations with the VINCI instrument on VLTI using the siderostats. The limb-darkened disk diameter is determined, with an unprecedented internal precision of 0.5%, to be $1.971 \pm 0.009_{\rm (int.)} \pm 0.05_{\rm (ext.)}$ mas, corresponding to a physical radius of $0.773 \pm 0.004_{\rm (int.)} \pm 0.02_{\rm (ext.)}$ $R_\odot$. With this determination τ Cet becomes a prime target for asteroseismic campaigns to determine its internal structure, and thereby test stellar evolution theory. We discuss implications for asteroseismology and present predictions for oscillation properties.

Key words. stars: fundamental parameters – stars: oscillations – stars: evolution – stars: individual: τ Cet

1. Introduction

The determination of the mass and age of stars is important all through astronomy, but those are parameters that, in most cases, can only be inferred from models. Stellar structure and evolution models have themselves a number of free, unconstrained parameters. And even the comparison between observations and models is hampered by the difficulties of translating magnitudes, colours and spectra into luminosity, effective temperature and metallicity. As a consequence, the estimates of masses and ages of stars suffer from low precision and accuracy, even in the case of "simple" solar-like, bright, nearby stars. It is therefore crucial to obtain independent, reliable observables with an accuracy better than 10% (Brown et al. 1994) as significant constraints that models and observational methods currently lack. One such observable is the stellar radius.

Stellar radii have been measured for giants and supergiants using interferometric techniques (cf. Nordgren et al. 1999, 2001; Mozurkewich et al. 2001; see also Richichi & Percheron 2002). But high-precision (<5%) measurements of radii of smaller stars, and in particular main-sequence stars, is only now becoming possible (e.g. Ségransan et al. 2003), with the advent of a new generation of interferometers like the Very Large Telescope Interferometer (VLTI).

This Letter presents the first interferometric measurement of the radius of the G8V star τ Ceti (HIP 8102) with the VINCI instrument on VLTI using the siderostats. The limb-darkened disk diameter is determined, with a precision of 0.5%, to be 1.971 ± 0.009 mas, corresponding to 0.773 ± 0.004 $R_\odot$. The accuracy on the measurement is limited by the (external) 1% uncertainty on the estimate of the radius of the available calibrator. This result for the radius of τ Cet implies an effective temperature that is more than 2σ larger than the effective temperature obtained from spectroscopy. Finally, we present theoretical models for τ Cet and predictions for its oscillation spectrum.

2. Observations and data reduction

τ Ceti (HIP 8102) is a G8V star (Hoffleit & Warren Jr. 1991) of low metallicity ([Fe/H] $= -0.50 \pm 0.03$, Soubiran et al. 1998), with $V = 2.496$ and $K = 1.68$ (Mermilliod et al. 1997), and a parallax of 274.17 ± 0.80 mas (ESA 1997). This target was chosen for being bright in the K band, for having a well-determined parallax and a large expected angular diameter ($\sim$2 mas), for being solar-like, and therefore a good potential target for asteroseismic campaigns (cf. Baglin 1991; Bedding et al. 1996; Pijpers 2003). This star has a faint optical neighbour, possibly a binary companion (Worley 1996), which has a V magnitude of 13.1 and is separated by more than $10''$ from τ Cet, and therefore does not influence the interferometric measurement.

τ Cet was observed with VINCI on the VLTI (Kervella et al. 2000), operating at 2.2 μm, using the siderostats in the B3-M0

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[★] Based on observations collected at the European Southern Observatory, Chile.

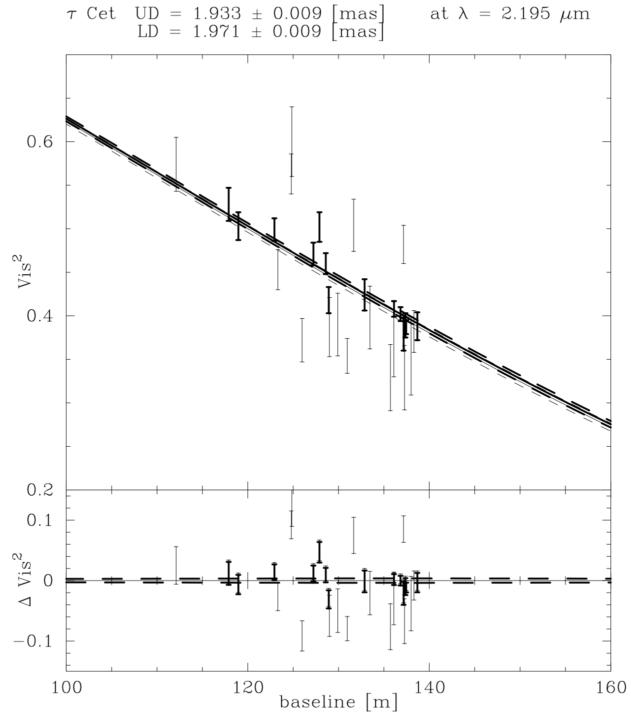

Fig. 1. Measured squared-visibility data points, with superimposed Uniform Disk visibility function for a UD diameter of 1.933 mas. Fits are shown for the full data set (thin lines) and for the clipped set (thick lines), with corresponding 1σ errors plotted as dashed lines (see text for explanation on the clipping). The bottom panel shows the residuals after subtracting the fit for the clipped set from the data.

configuration (corresponding to a baseline length of 140 m). The programme was run under the Service Mode, on a shared risk basis. The observations were carried out in the course of 12 nights between 19/10/2002 and 24/11/2002. The calibrator used was θ Cet (HIP 6537), which was observed immediately before or after the main target on every night except one, for which the data had to be discarded for lack of a suitable calibrator. The measurements for both the main target and the calibrator were obtained as described in Ballester et al. (2002) and Kervella et al. (2000), with the Optical Path Difference (OPD) scanned at a rate of 282 Hz, resulting in a total of 39 exposures (i.e. visibility points). The data were reduced using release vinci-2.0, of Sep. 2002 of the data-reduction pipeline, and using Morlet wavelets for fringe fitting as described by Ségransan et al. (2003), but with corrected error treatment taking into account the different statistical behaviour of random errors and (systematic) calibrator errors. The pipeline discarded 8 visibility points due to low quality, in most cases because of low signal-to-noise in the photometric channels. One further point, accepted by the pipeline when taking the default value for the threshold in signal-to-noise (S/N), was removed because of very low S/N in only one channel. The surviving 30 points at the end of the data reduction procedure are shown in Fig. 1 as a function of the projected baseline length.

To obtain the calibrated visibility (see Fig. 1), it was necessary to give as an input to the pipeline the value of the angular diameter of the calibrator. For θ Cet the uniform disk diameter has been estimated to be 2.69 ± 0.03 mas, using absolutely calibrated spectra over a wide spectral range (Cohen et al. 1999).

3. The radius of τ Cet

As described in e.g. Davis et al. (2000), the visibility V of a uniformly illuminated disk is given by:

$$V_{\rm UD} = 2J_1(x_{\rm UD})/x_{\rm UD}. \qquad (1)$$

J_1 is the Bessel function of first order, and $x_{\rm UD} = \pi b\theta_{\rm UD}/\lambda$, where b is the projected baseline length, λ is the effective wavelength of observation (2.195 μm, in the case of VINCI), and $\theta_{\rm UD}$ the uniform disk diameter, which is the parameter fitted for. The resulting angular diameter is 1.941 ± 0.011 mas for the full set of 30 measurements, and 1.933 ± 0.009 mas if only a sigma-clipped set of 14 data points with errors on the squared visibility <0.020 are used. This clipping criterion corresponds to excluding measurements with an error greater than 2.5 times that of the measurement(s) with the smallest error. The reason for doing this is that the distribution of measurement errors appears to have a strong tail, the adverse effect of which is insufficiently suppressed by standard error-weighted fitting. Fitting each visibility point individually and taking the mean of the UD diameters results in values within 1σ of the simultaneous fitting, which supports the high precision of the measurement. Moreover, Ségransan et al. (2003) have shown that dwarfs measured by VLTI and Palomar Testbed Interferometer follow theoretical mass-radius relations equally well, which implies that these instruments should have similar high accuracy.

Since the actual intensity distribution over the visible disk of the star is not uniform but limb-darkened, it is necessary to perform the adequate correction in order to obtain the actual radius of the star which can then be compared to models. Fitting the intensity distribution over the disk with a limb-darkened intensity profile results in the following expression for the visibility (cf. Davis et al. 2000):

$$V_{\rm LD} = \frac{\int_0^1 I_\lambda(\mu) J_0\left[x_{\rm LD}(1-\mu^2)^{1/2}\right]\mu {\rm d}\mu}{\int_0^1 I_\lambda(\mu)\mu {\rm d}\mu} \qquad (2)$$

where J_0 is the Bessel function of zero order, $x_{\rm LD} = \pi b\theta_{\rm LD}/\lambda$, $\theta_{\rm LD}$ is the limb-darkened disk diameter, which is the parameter fitted for, and $I_\lambda(\mu)$ is the intensity profile as a function of $\mu = \cos\phi$ where ϕ is the angle between the line-of-sight to the observer and the normal to the stellar surface. We use the parameterised limb-darkening law of Claret (2000):

$$I_\lambda(\mu) = I(1)\left[1 - \sum_{n=1}^{4} a_n\left(1-\mu^{n/2}\right)\right]. \qquad (3)$$

where a_n are the limb-darkening coefficients, which depend on the spectral type of the target and the wavelength of observation. For a wavelength of 2.195 μm and for a G8V star the coefficients can be obtained by interpolation from the tables in Claret (2000), and have the values a_n = [0.5747, 0.1543, −0.4726, 0.2164]. Fitting the visibility points for τ Cet results in a limb-darkened disk diameter of 1.980 ± 0.011 mas for the full set and 1.971 ± 0.009 mas if the sigma-clipped set is used. We acknowledge that the limb-darkening coefficients depend on models of stellar atmospheres, but currently such coefficients are only available from 1D models, and we have no way to assess their behaviour for multi-dimensional stellar atmospheres models. No attempt was made to fit for the limb-darkening coefficients, rather than using the tabulated values of a_n, since the errors on the visibility points do not allow a reliable determination of more than just the diameter itself. Using the Hipparcos parallax for τ Cet of (274.17 ± 0.80) mas this angular diameter translates to a radius for τ Cet of (0.773 ± 0.004) $R_\odot$.

4. Discrepancies and uncertainties

This *measured* radius can now be compared with the *estimated* radius of 0.87 ± 0.04 $R_\odot$, which is obtained from the Stefan-Boltzmann relation between the luminosity, the radius and the effective temperature taking, for τ Cet, $T_{\rm eff} = 5264 \pm 100$ K (Soubiran et al. 1998) and $L = 0.52 \pm 0.03\ L_\odot$ (Pijpers 2003). Conversely, the interferometric radius of τ Cet can be converted into an effective temperature of (5525 ± 12) K with a luminosity of (0.500 ± 0.006) $L_\odot$, for a negligible visual extinction. (The conversion is performed by adjusting $T_{\rm eff}$ so that the measured radius is reproduced. In the procedure it is necessary to determine the bolometric correction by interpolation in tables of bolometric correction (Lejeune et al. 1998) which is a function of the $T_{\rm eff}$, $\log g$, and [Fe/H] of the star.) It is clear that the results obtained from interferometry and from spectroscopy are discrepant. What might explain such a discrepancy?

Our diameter measurement for τ Cet (0.5% error) is limited in accuracy by the uncertainty in the estimate of the diameter of the calibrator (1% error). This problem can be circumvented by using a point-source as a calibrator, which is difficult in general, and for this run none was available. In future observations this will be addressed. In any case, an uncertainty in the calibrator diameter of 0.03 mas results in an additional (external) uncertainty on the diameter of τ Cet of 0.05 mas which implies a 0.02 $R_\odot$ uncertainty on the radius of the target, or an uncertainty of 70 K in effective temperature. Despite this external source of uncertainty, the discrepancy between interferometric and spectroscopic values is still significant.

Another potential source for the discrepancy, and one which is harder to quantify, is models of stellar atmospheres. It must be emphasised that the actual observables for stars are magnitudes, colours and spectra. To compare those with the physical parameters describing a star (namely the effective temperature, luminosity, and metallicity) it is necessary to perform conversions which involve using models of stellar atmospheres to determine bolometric corrections, intrinsic colours, and the effective temperature, metallicity and surface gravity, which are themselves interrelated in the process. Such conversions are non-trivial and known to be plagued by calibration problems due to the simplicity of 1D stellar atmospheres models and to the difficulty of the spectroscopic observations to determine the metallicity (cf. Stetson et al. 2003). To resolve this calibration problem, it is necessary to measure independent observables that may provide tighter, external constraints. One such observable is a high-precision, high-accuracy measurement of the stellar radius by interferometry, and another observable is a set of oscillation frequencies of the star.

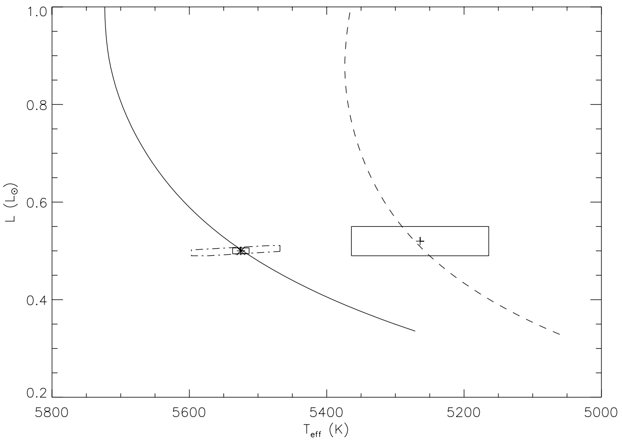

Fig. 2. Location in the Hertzsprung-Russel diagram of the points obtained from spectroscopy (+) and inferred from interferometry ($*$) with associated uncertainty boxes. The solid line corresponds to an evolution track for a 0.77 $M_\odot$ star with $X_i = 0.73$ and $\alpha = \alpha_\odot$, and the dashed line to the same mass and X_i but with $\alpha \simeq 0.6\,\alpha_\odot$. The dash-dotted region shows the influence of changing the calibrator diameter within its 1σ uncertainty.

5. Evolution tracks and oscillation frequencies

In order to determine the mass and age of τ Cet, evolutionary tracks were produced to obtain stellar structure models that reproduce $T_{\rm eff}$ and L, both in the case of the values derived from spectroscopy (5264 ± 100 K, 0.52 ± 0.03 $L_\odot$), and in the case of the values derived to reproduce the measured radius (5525 ± 12 K, 0.500 ± 0.006 $L_\odot$). The evolution tracks were computed using the evolution code of Christensen-Dalsgaard (1982), with the EFF equation of state (Eggleton et al. 1973), OPAL opacities (Iglesias & Rogers 1996), Bahcall & Pinsonneault (1995) nuclear cross sections, and the mixing-length formalism (MLT) for convection. This code has been calibrated in detail on the Sun (Christensen-Dalsgaard et al. 1996). The exploration of the parameter space defined by the uncertainties in $(T_{\rm eff}, L, Z)$ and by other parameters controlling the physics in the models is beyond the scope of this work. Here we only report on the model results for the expected mean values of the three parameters.

First, models were attempted taking an initial hydrogen abundance in mass $X_i = 0.70$, resulting in ages for the star larger than 15 Gyr: larger than the current inferred age of the Universe! Since τ Cet is a low-mass, low-metallicity star, X_i was accordingly increased to 0.73; a value somewhat larger than that taken for the standard solar model (cf. Christensen-Dalsgaard et al. 1996). That resulted in an acceptable age for the higher $T_{\rm eff}$ point, but still too large an age for the lower $T_{\rm eff}$ point. In order to produce acceptable models for the 5264 K point, it was necessary to lower the mixing-length parameter (α) to be ~60% of that which has been calibrated for the Sun. Figure 2 shows two tracks that reproduce $T_{\rm eff}$ and L for the two relevant points, both of which coincidentally correspond to a mass of 0.77 $M_\odot$. For both tracks the metallicity was taken to be [Fe/H] = −0.50 or equivalently, $Z = 0.0054$. The star has already evolved away from the main-sequence, and the best models correspond to an age of 10 Gyr in the case of the point from spectroscopy, and 9 Gyr in the case of the point from interferometry. In both cases the central hydrogen abundance has decreased to $\sim$30%, the core is not convective, and both models have an outer convective envelope with a size of $\sim 0.25\,R_*$.

Despite the fact that τ Cet is of lower metallicity and somewhat lower mass than the Sun, it is in fact rather similar to our own star: the central hydrogen content is roughly the same, neither has a convective core, and the convective envelope has roughly the same extent. It is therefore expected that both stars share one further property: like in the Sun, the convection in the envelope should stochastically excite resonant acoustic (sound) oscillations, making the star behave much like a musical instrument. (For a review on stellar oscillations the reader is referred to Brown & Gilliland 1994.) Being a roughly spherical "instrument", the eigenfunctions of the oscillations can be described by the product of a radial displacement function and a spherical harmonic, with radial order n (number of radial nodes of the eigenfunction) and angular degree l (number of node circles in the stellar hemisphere). The stellar oscillation spectrum is then characterised by the presence of frequencies that are approximately equidistant and that are modulated by a characteristic envelope. The separation between the frequencies of modes with the same degree l and consecutive radial order n, known as the *large separation* $\Delta\nu_{nl} = \nu_{n\,l} - \nu_{n\,l-1}$, is a complicated function of the stellar structure, but it in fact approximately scales with the mean stellar density as $\Delta\nu \propto \sqrt{\rho_*}$. The stellar mass and radius can therefore be determined robustly with only very weak dependence on evolution model parameters from the triplet $(T_{\rm eff}, L, \Delta\nu)$. The location and width of the envelope are currently poorly modelled; however, empirical evidence (H. Kjeldsen, private communication) suggests that the peak of the envelope is, like in the case of the Sun, located at approximately 0.6 times the acoustic cutoff frequency (which is the maximum frequency of waves that can be reflected at the stellar surface and contribute to a resonant mode), as originally proposed by Brown & Gilliland (1994). Taking the results from the evolution track fitting, we computed the oscillation frequencies for models at the appropriate ages using the pulsation code developed by Christensen-Dalsgaard & Berthomieu (1991), which has been extensively calibrated on the Sun. We predict the power in the oscillation frequencies of τ Cet to have a peak at $\sim 3570\,\mu$Hz in the case of the higher $T_{\rm eff}$ point, or at $\sim 2950\,\mu$Hz in the case of the lower $T_{\rm eff}$ point, with a large separation of $\sim 173\,\mu$Hz or $\sim 148\,\mu$Hz, respectively. By carrying out high accuracy, high resolution spectroscopy with an instrument like the ESO HARPS, it will be possible to obtain a time-series with frequency resolution better than $1\,\mu$Hz, which will allow the two possibilities to be easily distinguished.

6. Conclusions

The radius of τ Cet is determined to be $(0.773 \pm 0.004_{\rm (int.)} \pm 0.02_{\rm (ext.)})\,R_\odot$. This is somewhat smaller than previous estimates using the spectroscopically determined effective temperature, which points to limitations in 1D model atmospheres to produce accurate spectra. The luminosity determination (Pijpers 2003) together with the new radius determination points to a mass of $\sim 0.77\,M_\odot$ and an age $\sim$9 Gyr for τ Cet. We predict the oscillation frequencies to have a peak at $\sim 3570\,\mu$Hz if we adopt the interferometric radius, whereas the spectroscopic estimate would imply a value of $\sim 2950\,\mu$Hz, with a large separation of $\sim 173\,\mu$Hz or $\sim 148\,\mu$Hz, respectively. By measuring oscillation frequencies in τ Cet, these two possibilities can easily be distinguished, and thus be an independent check on the radius. Furthermore, with the interferometric constraint the frequencies are sensitive probes of interior structure, which can test the evolutionary state of τ Cet (age), as well as providing constraints on convection theory for metal-poor stars.

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